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RESEARCH ARTICLE

EFFECTS OF GRAVITY'S RAINBOW ON A RELATIVISTIC SPIN-1 OSCILLATOR

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ABSTRACT

We consider a relativistic spin-1 particle with non-minimal coupling in the context of gravity's rainbow in the three dimensional background spacetime spanned by static cosmic string. In this context, we acquire an exact solution of the associated spin-1 equation in the modified three dimensional static cosmic string-spanned background spacetime. This relativistic wave equation includes a reducible spinor and this allows us to acquire a non-perturbative expression including the modification functions in the energy domain. In the low energy limit, our results agree well with current literature and provide a basis to discuss the fundamental features of the relativistic spin-1 oscillator. Afterwards, we try to discuss the effects of gravity rainbow functions on the considered spin-1 oscillator in three different scenarios for the modification functions.

Keywords: *Gravity rainbow, Spin-1 oscillator, Planck energy, Doubly special relativity, Cosmic string, Topological defect*

1. INTRODUCTION

A useful way to determine the effects of a curved space or a non-trivial topology on the relativistic dynamics of quantum mechanical systems (QMSs) is to solve the corresponding forms of the covariant wave equations such as Duffin Kemmer Petiau (DKP), Vector boson (VB) , the generalized Klein Gordon (KG), Dirac and fully-covariant many-body equations. It was showed the relativistic wave equations describing the dynamics of spinning particles, such as spin-1/2, spin-1, spin-2 etc., can be derived from the canonical quantization of the action for spinning classical particles [1]. There is a unifying principle to derive the well-known relativistic wave equations. That is, these equations can be reproduced as different quantum states of the same classical system [1]. The well-known DKP equation's spin-1 sector in three dimensions corresponds to the spin-1 equation, which was obtained as an excited state of Zitterbewegung [2–5]. This spin-1 equation includes a reducible spinor and yields 3×3 dimensional matrix equation in any 2+1 dimensional spacetime [6, 7]. In the current literature, we see that relativistic dynamics of the spin-0 bosons are studied through solving the spin-0 sector of the DKP equation [8] and the KG equation [9]. Due to the relative complexity, there is not much research based on the dynamics of spin-1 bosons in curved spaces [4–7,10]. The announced results have shown that the covariant VB equation can provide a strong basis to analyse the dynamics of relativistic spin-1 particles in any 2+1 dimensional spacetime [4–7, 10]. Furthermore, this equation was also used to analyse the quantum gravity effects on the Hawking temperature of three dimensional black holes [11, 12]. It is thought that studies focus on the relativistic quantum systems's (QSs) dynamics in curved spaces can serve to establish a complete theory combining the main areas of physics such as relativity (c), quantum theory ($\hbar$) and gravity (G) since we do not yet have a complete theory that merges these areas. A useful way to determine the influence of a curved space on the physical systems is to use the exactly soluble systems such as the relativistic quantum oscillators [6, 13–16] and Hydrogen or Positronium like low-energy bound state systems [17–19]. After the Dirac oscillator (DO) [20], describing the interaction of a changing (linearly) electric field with an anomalous magnetic moment, had been introduced, the KG oscillator [21], DKP oscillator [22] and VB oscillator [23, 24] were introduced through establishing an analogy to the DO. The relativistic oscillators describe real physical systems [25, 26] and have several applications [27–35] in many areas of modern physics. Furthermore, the relativistic oscillators are the most preferred systems to determine the effects of topological defects on the associated systems and one of the most famous topological defects is the cosmic string. The cosmic strings [36] were introduced first by one-body solution of Einstein field equations in three dimensions [37, 38] and the results naturally extended to 3+1 dimensions [39] where there is a dynamical symmetry [15,19]. These objects are stable one-dimensional topological defects that are thought to have formed at the early stages of the universe [36] and they may cause several interesting phenomena in the universe [19,36]. Moreover, it is also known that the spatial part of the line element representing the static cosmic string-spanned spacetime describes the topological defect appearing in materials backgrounds [15]. For this reason, exact results acquired for relativistic QMSs in topological defect-induced background spacetimes have great importance in modern physics and such investigations have attained great attention by many research groups. Moreover, we are aware of relationships between the topological characteristics of the space and local physical laws, such that the local intrinsic geometry of the space is insufficient to exactly describe the physics of any system. Therefore, it is crucial to look at how, for instance, a nontrivial topology affects a QS. Also, in addition to the use of QMSs to detect gravitational waves [40], the effect of gravitational fields produced by cosmic strings on QSs has long drawn significant interest [41-43]. In what follows, we will deal with a relativistic spin-1 oscillator in the three dimensional background geometry induced by static cosmic string (see also [44]) and will try to obtain non-perturbative results including the effects of gravity's rainbow functions on such a spin-1 oscillator.

On the other hand, some semi-classical approaches are thought as very helpful to investigate the affects of gravity on the dynamics of physical systems due to the fact that we have not a well-established or a fully-fledged theory to determine the influence of gravity on the QSs. One of these interesting semi-classical approaches is the context of the gravity's rainbow scenario called sometimes also as doubly general relativity [45–47]. This interesting semi-classical approach has been applied to determine the effects of gravity on the relativistic QMSs [48, 49]. Briefly, in this context, as a result of a nonlinear Lorentz transformation in momentum space, the metric describing the background spacetime becomes energy-dependent and accordingly the relativistic dispersion relation is modified [48, 49]. In this scenario, the test fields feel a different geometry for each different frequency and also Planck length (or energy) and speed of light remain observer-independent [50–52]. Hence, at very high energy, we may investigate the influence of gravity on the QSs by using the spacetime metrics

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modified according to the modified relativistic dispersion relation [53, 54]. This is also suggested according to observational consequences [46, 47, 50, 54]. The gravity's rainbow approach was used to investigate black hole thermodynamics [55], casimir effect [56], spin-0 field in the Schwarzschild metric [57], relativistic spin-1/2 oscillator in a topological defect-generated background spacetime [48] relativistic spin-0 oscillator in a global monopole spacetime [49], wormhole geometry [58,59] and the massive scalar field with Casimir effect [60]. In addition to these works, a massless spin-1 field was studied in the cosmic string background spacetime in the context of gravity's rainbow [61]. In this contribution, we will use the gravity's rainbow approach to determine the effects of gravity on a relativistic spin-1 oscillator through obtaining an analytical solution of the associated covariant VB equation in the modified 2+1 dimensional spacetime generated by a static point source. Then, we will compare our results with the literature in the low energy limit (LEL) where gravity's rainbow corrections vanish.

We organized this contribution as follows: in the second section we give the mathematical procedure, in the third section we derive a 3× 3 matrix equation for the considered test field and determine the corresponding solution function. Accordingly, we arrive at a nonperturbative energy expression including the gravity rainbow corrections in general form. Then, we analyse the results in both LEL and very high energy limit in three different scenarios for the modification functions. In sec. (4), we give a summary and discuss the findings in details.

2. MATHEMATICAL PROCEDURE

Here, we introduce the relativistic spin-1 equation for a general spin-1 field and derive its' associated form to analyse the dynamics of a VB particle in the point source-generated background spacetime (2+1 dimensional cosmic string spacetime) in the context of gravity's rainbow. In any three dimensional curved space, the spin-1 equation can be written as follows [1, 4–6]

$$\left\{\frac{1}{2}[\gamma^{\mu}\otimes I_2+I_2\otimes\gamma^{\mu}]\nabla_{\mu}+i\mathcal{M}I_4\right\}\Phi=0, \tag{1}$$

$$\nabla_{\mu}=\partial_{\mu}-\Gamma_{\mu}\otimes I_2-I_2\otimes\Gamma_{\mu},\ \mu=0,1,2.$$

In this equation, γ^{μ} stand for the space-free Dirac matrices (DMs), Γ_{μ} represent spinorial affine connections for a Dirac field, I_2 and I_4 are the 2×2 and 4×4 unit matrices and Φ is the relativistic spin-1 field with mass of M and we will use the units $\hbar=c=1$. Now, we will try to obtain a matrix equation describing the dynamics of the system under scrutiny. To do this, we should obtain the associated operators such as the space-dependent DMs and the spinorial connections in the Eq. (1). These operators can be determined through the following relation [19]:

$$\Gamma_{\xi}=\frac{1}{4}\,g_{\mu\alpha}\left[e^{(a)}_{\nu,\xi}e^{\alpha}_{(a)}-\Gamma^{\alpha}_{\nu\xi}\right]\gamma^{\mu}\gamma^{\nu}\ ,\quad \xi,\mu,\nu,\alpha,a=0,1,2. \tag{2}$$

Here, the $\Gamma^{\alpha}_{\nu\xi}$ are the Christoffel symbols that can be determined by [19],

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$$\Gamma^{\alpha}_{\nu\xi} = \frac{1}{2} g^{\alpha\epsilon}\{\partial_{\nu}\, g_{\xi\alpha} + \partial_{\xi}\, g_{\epsilon\nu} - \partial_{\epsilon}\, g_{\nu\xi}\}, \tag{3}$$

and $g_{\mu\alpha}$ is the metric tensor in covariant form. Here, $g^{\alpha\epsilon}$ is inverse of the covariant metric tensor. It should also be noted that the Greek indices represent the curved spacetime's coordinates. To determine the spinorial affine connections, we need to construct the space dependent DMs, γ^{μ}. These matrices can be constructed through the relation: $\gamma^{\mu} = e^{\mu}_{(a)}\ \gamma^{(a)}$ where $e^{\mu}_{(a)}$ are the tetrads (inverse) and $\gamma^{(a)}$ indicate the flat DMs [19]. It is known that the flat DMs can be expressed in terms of the Pauli matrices in any 2+1 dimensional spacetime background. The tetrad fields, $e^{(a)}_{\mu}$, are obtained through the relation: $g_{\mu\tau} = e^{(a)}_{\mu}\, e^{(b)}_{\tau} \eta_{(a)(b)}$, where $\eta_{(a)(b)}$ represents the Minkowski metric tensor (flat). It should be noted that the tetrad choices are not unique. That is, other choices can be preferred as long as the orthonormality, orthogonality and Clifford-Dirac algebra requirements are satisfied [13, 15]. Here, we should underline that the Latin indices indicate the coordinates of the flat Minkowski spacetime while the Greek indices represent the curved background's coordinates. It is useful to mention that there is no consensus about the shape of the modification functions within the framework of the gravity rainbow. Hence, for each case the relativistic dispersion relation is modified differently [48, 49]. For the used three cases [48, 49], the momentum operators in the relativistic wave equations are altered and accordingly the total energy (at very high energies) is changed. Although, there are different suggestions on how to choose the modification functions [48, 49], for each choice the corresponding dispersion relation must result in the usual relativistic dispersion relation in the LEL. This means that the obtained results for QMSs in the context of gravity rainbow must give the usual results obtained in the usual relativistic framework. Now, we can write the modified relativistic dispersion relation and then give the static cosmic string-generated 2+1 dimensional background geometry in the context of the gravity's rainbow scenario. The rainbow gravity originates from deformations of the Lorentz symmetry. This deformation can be represented, formally, through the following relativistic dispersion relation [48, 49]

$$\mathcal{E}^2\mathcal{G}_0(\mathcal{X})^2 - \mathcal{P}^2\mathcal{G}_1(\mathcal{X})^2 = \mathcal{M}^2,$$

where, $\mathcal{E}$ is energy of the VB in question and $\mathcal{P}$ is momentum of this particle possessing mass of $\mathcal{M}$. The modification functions $\mathcal{G}_0(\mathcal{X})$ and $\mathcal{G}_1(\mathcal{X})$ are the rainbow functions in which the argument $\mathcal{X} = \mathcal{E} / \mathcal{E}_p$ and $\mathcal{E}_p \sim 1.22 \times 10^{19}$ GeV is the Planck energy. Here, it is very important that the gravity rainbow functions are reduced into 1 in the LEL where $\mathcal{E} / \mathcal{E}_p \to 0$. In this limit, it is clear that one can recover the usual relativistic dispersion relation. In this scenario, the 2+1 dimensional static cosmic string spacetime known also as point source-generated spacetime can be written, with negative signature $(+, -, -)$ as follows [48]

$$dS^2 = \mathcal{G}_0(\mathcal{X})^{-2}dt^2 - \mathcal{G}_1(\mathcal{X})^{-2}[d\rho^2 + \eta^2\rho^2 d\phi^2] \tag{4}$$

This metric describes the usual polar space when $\eta = 1$ in the LEL, $\mathcal{E} \to 0$ for which $\mathcal{G}_k(\mathcal{X}) \to 1$, (k = 0, 1.). In the presence of the topological defect ($0 < \eta < 1$), the background cannot be flat globally even though it is flat locally. Thus, the topology and accordingly symmetry of the background geometry is altered by the string tension [19]. For more details about the 2+1 dimensional static

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cosmic string (point source)-induced spacetime can be found in the Refs. [19, 37, 38]. Now, we can easily write the covariant ($g_{\mu\alpha}$) and contravariant ($g^{\mu\alpha}$) metric tensors

$$g_{\mu\alpha} = diag(\mathcal{G}_0(\mathcal{X})^{-2}, -\mathcal{G}_1(\mathcal{X})^{-2}, -\mathcal{G}_1(\mathcal{X})^{-2}\eta^2\rho^2)$$

$$g^{\mu\alpha} = diag(\mathcal{G}_0(\mathcal{X})^{2}, -\mathcal{G}_1(\mathcal{X})^{2}, -\mathcal{G}_1(\mathcal{X})^{2}\eta^{-2}\rho^{-2})$$

Through these expressions and Eq. (3), one can evaluate the components (non-vanishing) of the Christoffel symbols as follows, by using the relation given in [19],

$$\Gamma^{\rho}_{\phi\phi} = -\eta^2\rho, \quad \Gamma^{\phi}_{\rho\phi} = \frac{1}{\rho}. \tag{5}$$

It is also known that we can chose the space-free DMs by means of Pauli matrices and such a choice is not unique. So, different choices are acceptable provided that the signature in the Eq. (4) are satisfied. Here we choice the flat DMs as the following: $\gamma^0 = \sigma^3$, $\gamma^1 = i\sigma^1$, and $\gamma^2 = i\sigma^2$ [19]. To construct the space-dependent DMs we need to the tetrad fields. Through the mentioned procedure before, one can obtain the tetrad fields and can arrive at the following results

$$e^{(a)}{}_{\mu} = diag(\mathcal{G}_0(\mathcal{X})^{-1}, -\mathcal{G}_1(\mathcal{X})^{-1}, -\mathcal{G}_1(\mathcal{X})^{-1}\eta\rho)$$

$$e^{(\mu)}{}_{a} = diag(\mathcal{G}_0(\mathcal{X}), -\mathcal{G}_1(\mathcal{X}), -\mathcal{G}_1(\mathcal{X})\eta^{-1}\rho^{-1}). \tag{6}$$

Now, by using these expressions, we can construct the generalized DMs

$$\gamma^t = \mathcal{G}_0(\mathcal{X})\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \quad \gamma^\rho = i\mathcal{G}_1(\mathcal{X})\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad \gamma^\phi = \frac{\mathcal{G}_1(\mathcal{X})}{\eta\rho}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}. \tag{7}$$

Furthermore, by using the obtained results it can be found that the spinorial affine connections have only one non-vanishing component. This one is obtained as follows

$$\Gamma_\phi = \frac{i}{2}\eta\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \tag{8}$$

In this manuscript, we interest in a spin-1 oscillator in the 2+1 dimensional spacetime generated by static cosmic string in the context of gravity's rainbow. To derive the corresponding matrix equation for the system in question, we need to introduce the oscillator coupling. The spin-1 oscillator [6, 10] is introduced by adding a non-minimal interaction term in such a way that $\partial_\rho \to \partial_\rho + \mathcal{M}\omega_0(\sigma_3 \otimes \sigma_3)\rho$ where ω_0 is oscillator frequency and $\sigma_3 \otimes \sigma_3 = \mathrm{diag}\,(1, -1, -1, 1)$. For the considered system, the symmetric spinor Φ can be factorised as $\Phi = e^{-iEt}e^{is\phi}\,(\vartheta_1(\rho), \vartheta_2(\rho), \vartheta_3(\rho), \vartheta_4(\rho))^{\mathbf{T}}$, in which **T** means transpose of the matrix, according to the line element given in the Eq. (4).

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3. EFFECTS OF GRAVITY'S RAINBOW FUNCTIONS

Here, we obtain a matrix equation consisting of coupled equations and then try to solve this equation. By inserting the obtained results in the Eq. (7) and Eq. (8) into the Eq. (1), one obtains a 4 × 4 dimensional matrix equation in reducible form (see also [62]). This equation can be rewritten by defining a new dimensionless independent variable, $z = \kappa\rho^2$, as follows

$$\begin{pmatrix} \widehat{\mathcal{D}}^* & \widetilde{\mathcal{M}} & -\frac{\tilde{\mathcal{S}}}{\sqrt{z/\kappa}} \\ \tilde{\mathcal{E}} & -\frac{\tilde{\mathcal{S}}}{\sqrt{z/\kappa}} & -\widetilde{\mathcal{M}} \\ -\mathcal{M} & -\widehat{\mathcal{D}}^\dagger & \tilde{\mathcal{E}} \end{pmatrix} \begin{pmatrix} \vartheta_+(z) \\ \vartheta_0(z) \\ \vartheta_-(z) \end{pmatrix} = 0 \tag{9}$$

where

$$\widehat{\mathcal{D}}^* = 2\kappa\sqrt{\frac{z}{\kappa}}\left(\partial_z + \frac{1}{2} + \frac{1}{2z}\right), \quad \widehat{\mathcal{D}}^\dagger = 2\kappa\sqrt{\frac{z}{\kappa}}\left(\partial_z - \frac{1}{2}\right)$$

$$\widetilde{\mathcal{M}} = \frac{\mathcal{M}}{\mathcal{G}_1(\mathcal{X})}\ , \ \tilde{\mathcal{E}} = \mathcal{E}\frac{\mathcal{G}_0(\mathcal{X})}{\mathcal{G}_1(\mathcal{X})}\ , \quad \tilde{\mathcal{S}} = \frac{s}{\eta},$$

$\vartheta_\pm(z) = \vartheta_1(z) \pm \vartheta_2(z)$ and $\vartheta_0(z) = 2\vartheta_2(z)$ since $\vartheta_2(z) = \vartheta_3(z)$. Under these definitions, one of these equations becomes algebraic and this allows us to derive a non-perturbative wave equation. That is, by solving this set of equations for the $\vartheta_0(z)$ one derives a second order differential equation and moreover the resulting equation can be reduced into the following familiar form

$$\left(\frac{d^2}{dz^2} - \frac{1}{4} + \frac{q}{z} + \frac{\frac{1}{4} - p^2}{z^2}\right)\vartheta(z) = 0, \tag{10}$$

by considering an ansatz function, $\vartheta_0(z) = \vartheta(z)/\sqrt{z}$. By this way we arrive at the Whittaker equation and its regular solution [26] can be expressed as $\vartheta(z) = C\ W_{q,p}(z)$ where

$$q = -\frac{1}{2} - \frac{(\widetilde{\mathcal{M}}^2 - \tilde{\mathcal{E}}^2)}{4\kappa} + \frac{2\tilde{\mathcal{E}}\tilde{\mathcal{S}}}{4\widetilde{\mathcal{M}}}\ , \quad p = \frac{\tilde{\mathcal{S}}}{2}.$$

and C is a normalization constant. This solution function can also be expressed by means of confluent hypergeometric function, ${}_1F_1$ and it diverges when $z \to \infty$. Thus, we need to impose the following condition: $\frac{1}{2} + p - q = -n$, in which $n = 0,1,2..$ is the overtone number, to acquire polynomial solution [26]. At that rate, the solution function becomes well-behaved and this termination leads a non-perturbative expression in energy domain

$$\mathcal{E}_{n,s}(\mathcal{X}) = -\frac{s\omega_0\mathcal{G}_1(\mathcal{X})^2}{n\mathcal{G}_0(\mathcal{X})} \pm \frac{\mathcal{M}}{\mathcal{G}_0(\mathcal{X})}\sqrt{1 + \frac{4\omega_0\mathcal{G}_1(\mathcal{X})^2}{\mathcal{M}}\mathcal{N} + \frac{s^2\omega_0{}^2\mathcal{G}_1(\mathcal{X})^4}{\eta^2\mathcal{M}^2}}, \tag{11}$$

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$\mathcal{N} = (n + 1 + \frac{s}{2\eta})$.

This energy spectrum becomes as follows

$$\mathcal{E}_{n,s} = -s\omega_0 \pm \mathcal{M}\sqrt{1 + \frac{4\omega_0}{\mathcal{M}}(n + 1 + \frac{s}{2}) + \frac{s^2{\omega_0}^2}{\mathcal{M}^2}}, \tag{12}$$

when η=1 in the LEL where $\lim_{\mathcal{X}\to 0}\mathcal{G}_k(\mathcal{X}),\ k = 0,1$. It can be important that the result given by Eq. (12) gives exact result for a composite structure holding together by DO coupling, when s = 0 [26], and it can be also seen that it agrees well with the result obtained for one dimensional DO [26]. In a three dimensional spacetime background there are two spatial degrees of freedom. Accordingly, any spectra obtained for QSs must include two quantum numbers. This is why our results give the previously obtained results for one dimensional systems when we ignore the spin contributions. Now, we can discuss the energy spectrum in the Eq. (11). At first look, we see that the oscillator frequency couples with the spin and this coupling is altered by the string tension ($\propto \eta$) since there exists $\frac{s\omega_0}{\eta}$ terms in the spectrum even in the LEL. Also, we see that relativistic energy ($\mathcal{E}_{n,s}$) becomes $\mathcal{E}\sim \pm\mathcal{M}$ when $\omega \to 0$ and $\mathcal{X} \to 0$. In the LEL, dependence of the relativistic energy levels on the oscillator frequency can be seen in the Figure (1). This also shows that the VB oscillator does not stop oscillating even in the ground state and positive-negative energy states never mix. Now, we can discuss the results in three different scenarios for the gravity rainbow functions. Let we start the first case where [48, 49]

$$\mathcal{G}_0(\mathcal{X}) = \frac{e^{\mathcal{X}}-1}{\mathcal{X}},\ \ \mathcal{G}_1(\mathcal{X}) = 1. \tag{13}$$

This choice was proposed to describe gamma-ray burst phenomena in the universe [63–65]. In this case, the energy expression in the Eq. (11) can be written as the following

$$\mathcal{X} = -\frac{s\omega_0\mathcal{X}}{\eta\mathcal{E}_p(e^{\mathcal{X}}-1)} \pm \frac{\mathcal{M}\mathcal{X}}{\mathcal{E}_p(e^{\mathcal{X}}-1)}\sqrt{1 + \frac{4\omega_0}{\mathcal{M}}\mathcal{N} + \frac{s^2{\omega_0}^2}{\eta^2\mathcal{M}^2}}, \tag{14}$$

and by solving this expression for $\mathcal{X}$ one can acquire the following expressions

$$\mathcal{X}_{1,2} = ln\left[\pm 1 \mp \frac{s\omega_0}{\eta\mathcal{E}_p} \pm \frac{\mathcal{M}}{\mathcal{E}_p}\sqrt{1 + \frac{4\omega_0}{\mathcal{M}}\mathcal{N} + \frac{s^2{\omega_0}^2}{\eta^2\mathcal{M}^2}}\right]. \tag{15}$$

Here, it is clear that the energy of the considered system is altered by one of the gravity rainbow functions and this alterations on the relativistic energy levels can be seen in the Figure (2) and Figure (3). In this parts, we see that the magnitude of the relativistic energy levels is increased by the gravity rainbow effect according to the first scenario.

Also, we can choose the modification functions according to proposal of both non-commutative geometry and loop quantum gravity [56, 63]. In this case the modification functions were considered as $\mathcal{G}_0(\mathcal{X}) = 1$, $\mathcal{G}_1(\mathcal{X}) = \sqrt{1-\gamma\mathcal{X}^2}$ for which the energy expression can be written as the following form

$$\mathcal{X} = -\frac{\mathrm{s}}{\eta}\Omega(1-\gamma\mathcal{X}^2) \pm \mathrm{m}\sqrt{1+\frac{4\Omega}{\mathrm{m}}(1-\gamma\mathcal{X}^2)\mathcal{N}+\frac{\mathrm{s}^2\Omega^2}{\eta^2\mathrm{m}^2}(1-\gamma\mathcal{X}^2)^2}\,, \tag{16}$$

$\Omega = \frac{\omega_0}{\mathcal{E}_p}$, $m = \frac{\mathcal{M}}{\mathcal{E}_p}$.

Here, it seems not possible to obtain an expression in closed form for $\mathcal{X}$ except for $\mathrm{s} = 0$. However, we can discuss the effects of gravity's rainbow functions on the corresponding wave function(s). In this second scenario, the altered wave function(s) can be seen in the Figure (4). In this figure, we can also see the low energy (”usual”) case for $\gamma = 0$. The others include the effects of one of the gravity rainbow functions, $\mathcal{G}_1(\mathcal{X})$, since $\mathcal{G}_0(\mathcal{X}) = 1$. The Figure (4) shows that amplitude of the normalized wave function(s) increases as the energy of the VB oscillator increases. In the third scenario, stemming from the need for constant velocity of light, proposed for solve the horizon problem puzzle, the modification functions are $\mathcal{G}_0(\mathcal{X}) = \frac{1}{1-\gamma\mathcal{X}}$ and $\mathcal{G}_1(\mathcal{X}) = \frac{1}{1-\gamma\mathcal{X}}$. For this, the energy expression can be expressed as follows

$$\mathcal{X} = -\frac{\mathrm{s}\Omega}{\eta(1-\gamma\mathcal{X})} \pm \mathrm{m}(1-\gamma\mathcal{X})\sqrt{1+\frac{4\Omega}{\mathrm{m}(1-\gamma\mathcal{X})^2}+\frac{\mathrm{s}^2\Omega^2}{\eta^2\mathrm{m}^2(1-\gamma\mathcal{X})^2}}\,. \tag{17}$$

For this latest case, one can arrive that it is not possible to obtain an expression in closed-form for $\mathcal{X}$. Hence, we cannot see clearly what the dependence of the energy levels on the modification functions. However, we have plotted the effects of modification functions on the wave function(s) in the Figure (5). In this figure, we see that amplitude of the wave functions decreases in the presence of gravity's rainbow effect. However, space-dependence of the normalized wave functions seems to be same whether $\gamma = 0.1$ or $\gamma = 0.9$.

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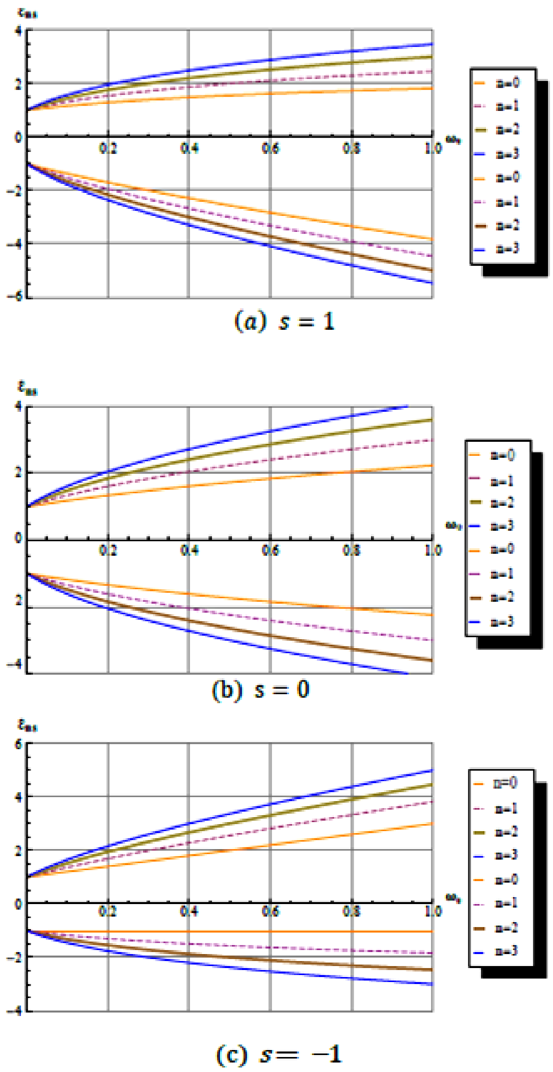
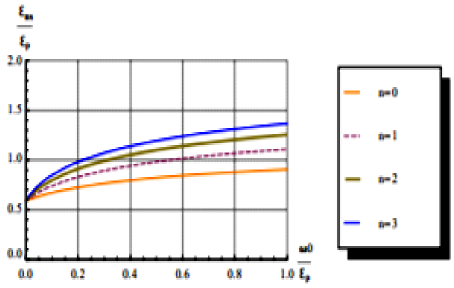
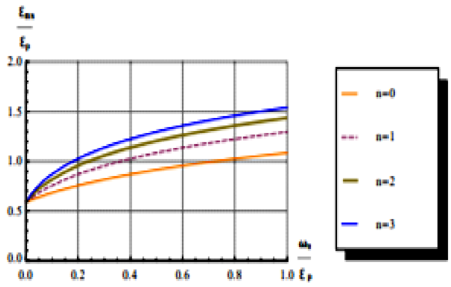


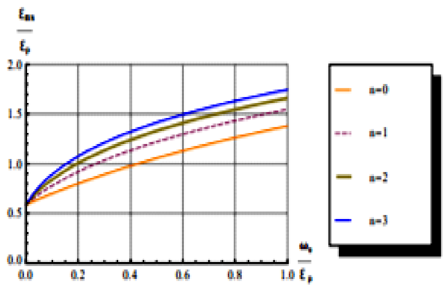
Figure 1. Behaviour of the relativistic energy levels with respect to the oscillator frequency in the LEL. Here, we get that $\mathcal{M} = 1,\ \eta = 1$.



(a) $s = 1$.



(b) $s = 0$.



(c) $s = -1$.

Figure 2. Alterations on the energy levels in high energy limit. Here, we get than $\frac{\mathcal{M}}{\varepsilon_p} = 0.8,\ \eta = 0.9$.

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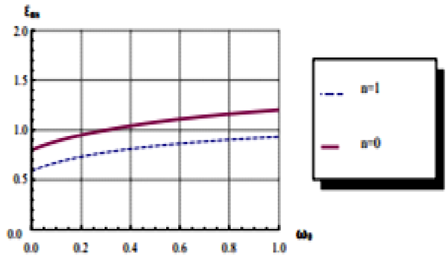
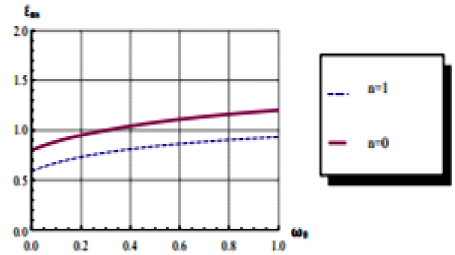
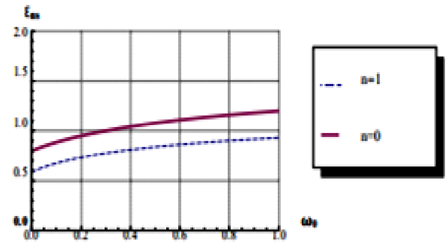


(a) $s=1$.



(a) $s=0$.



(c) $s=-1$.

Figure 3. Effects of gravity's rainbow on the possible spin states for $\mathcal{M} = 0.8$, $\eta = 1$. Here, the dashed lines show the usual case and the others show the energy levels altered by the gravity rainbow effect if $\mathcal{G}_0(\mathcal{X}) = \frac{e^{\mathcal{X}}-1}{\mathcal{X}}$, $\mathcal{G}_1(\mathcal{X}) = 1$.

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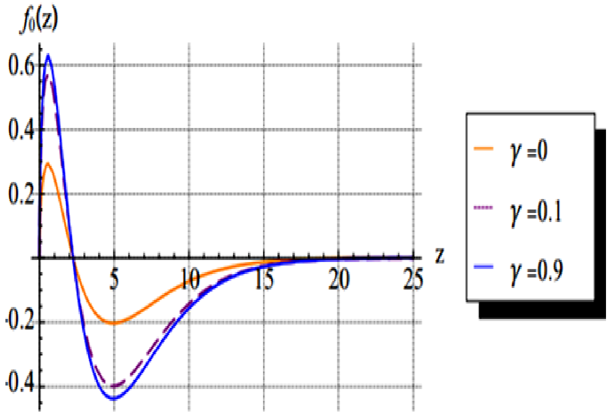

Figure 4. Space dependence of the wave functions (normalized) for different values of the parameter γ in the second scenario where $\mathcal{G}_0(\mathcal{X}) = 1$, $\mathcal{G}_1(\mathcal{X}) = \sqrt{1-\gamma\mathcal{X}^2}$. Here, we get that $\mathcal{M} = 0.6,\ n = 1,\ \eta = 0.8,\ \ \omega = 0.6,\ s = 1,\ \mathcal{E}_p = 1$.

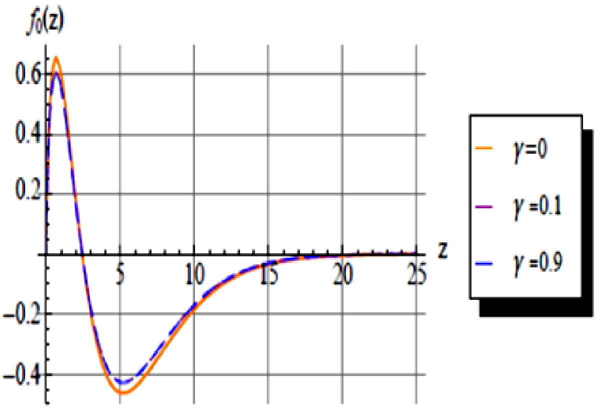

Figure 5. The normalized wave functions for the cases given in third case, according to the changing values of the parameter γ. Here, we get that $\mathcal{M} = 1, n = 1,\ \eta = 0.8,\ \ \omega = 0.6,\ s = 1,\ \mathcal{E}_p = 1$.

4. SUMMARY AND RESULTS

Here, we have interested in a relativistic VB oscillator in the context of gravity's rainbow in the 2+1 dimensional cosmic string-induced background geometry. According to the modified dispersion relation, by solving the fully covariant spin-1 equation in this background we have arrived at a non-perturbative energy expression including the modification functions. This expression is given in the

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Eq. (11) and it can be reduced into the results in the Eq. (12) in the low energy limit (LEL) where relativistic energy ($\mathcal{E}$) of the particle is very small according to the Planck energy ($\mathcal{E}_{\mathrm{p}}$) by assuming the background geometry is globally flat ($\eta = 1$). In the $\mathcal{E}/\mathcal{E}_{\mathrm{p}} \to 0$ case, our results show that spin of the relativistic VB couples with the oscillator frequency and this influences the symmetry of the energy levels for particle-antiparticle states even when $\eta = 1$. For this usual case, behaviour of the energy levels according to the oscillator frequency (ω_0) is plotted in the Figure (1). This also shows that the VB oscillator does not stop oscillating even in the ground state and positive-negative energy states never mix. In this figure, one can see that magnitude of the energy levels increases for increasing values of the ω_0 and it can be seen also that $\mathcal{E} \sim \pm \mathcal{M}$, where $\mathcal{M}$ is the rest mass of the VB particle, if $\omega_0 \to 0$. Here, we have observed that the effects of string tension ($\propto \eta$) on spin states ($s = 0, \pm 1$) of the spin-1 oscillator cannot be same due to the $\frac{s\omega_0}{\eta}$ terms in the energy spectrum. In the LEL, the resulting energy spectrum gives exact result obtained for a composite structure composed by a fermion-antifermion system holding together by DO interaction [26] if $s = 0$ when $\eta = 1$ and agrees well with the current literature (see also [10, 26]). That is, our results provide a strong basis for the effects of gravity's rainbow on the relativistic spin-1 oscillator. Here, it is very important to note that it may not be possible to show what the effect of the modification functions on the energy levels of the system in each shape of the modification functions even though the Eq. (11) is an algebraic equation. This is because the parameter $\mathcal{X}$ depends explicitly on the energy of the system in question. According to the first scenario where the modification functions are $\mathcal{G}_0(\mathcal{X}) = \frac{e^{\mathcal{X}}-1}{\mathcal{X}}$, $\mathcal{G}_1(\mathcal{X}) = 1$ and it is clear that the energy levels are altered by only one of these functions. For this, we have observed that the magnitude of the energy levels increases as oscillator frequency increases (see Figure 3). In the second scenario where $\mathcal{G}_0(\mathcal{X}) = 1$, $\mathcal{G}_1(\mathcal{X}) = \sqrt{1-\gamma\mathcal{X}^2}$ we gave an expression including the effect of modification function in energy domain (see Eq. 16). In this case, we could not find any way to show what the dependence of the energy levels on the modification functions without loss of generality since the energy expression in the Eq. (16) do not allow to obtain an expression (in closed-form) that gives clearly the altered particle-antiparticle energy states. However, we showed the effects of the modification functions on the normalized wave functions with respect to the second scenario. In the Figure (4), one can see that magnitude of the normalized wave functions increases if the relativistic energy of the VB oscillator closes to Planck energy. In the latest case, the third case, we have encountered a similar technical problem. That is, we could not find explicit expressions showing the dependence of the particle-antiparticle energy levels on the modification functions even though we have arrived at a non-perturbative expression for the energy if $\mathcal{G}_0(\mathcal{X}) = \frac{1}{1-\gamma\mathcal{X}}$ and $\mathcal{G}_1(\mathcal{X}) = \frac{1}{1-\gamma\mathcal{X}}$. In this case, we have observed that amplitude of the normalized wave functions decreases for big values of the parameter and also we have seen that the amplitude seems not dramatically change whether the energy of the spin-one oscillator closes to Planck energy or very small with respect to it.

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