Süleyman Demirel Üniversitesi
Fen Bilimleri Enstitüsü Dergisi
Cilt 27, Sayı 3, 366-373, 2023

Süleyman Demirel University
Journal of Natural and Applied Sciences
Volume 27, Issue 3, 366-373, 2023

DOI: 10.19113/sdufenbed.1196523

Q- rung orthopair probabilistic hesitant fuzzy hybrid aggregating operators in multi-criteria decision making problems

Şerif ÖZLÜ[*1]

[1]Gaziantep University, Nizip Vocational HighSchool, Food Processing, 27300, Gaziantep, Türkiye

(Alınış / Received: 30.10.2022, Kabul /Accepted: 31.10.2023, Online Yayınlanma / Published Online: 25.12.2023)

Keywords
Q- Rung orthopair hesitant fuzzy sets,
q-ROPHHWAG,
q-ROPHHOWAG
Decision Making

Abstract: With the increase of complex information in applications of decision making problems, the use of probabilistic hesitant fuzzy set structure has expanded. Therefore, this paper aims to present two new operators namely q-rung orthopair probabilistic hesitant fuzzy hybrid weighted arithmetic and geometric (q-ROPHHWAG) operator and q-rung orthopair probabilistic hesitant fuzzy hybrid ordered weighted arithmetic and geometric (q-ROPHHOWAG) operator for q>0. The presented operators are better than existing operators in many respects as adding a new parameter, having more flexible structure and presenting comparative analysis in its own. Moreover, we mention from some properties of the proposed operators. In addition to, we give an algorithm and example to indicate effective, reality and flexible of presented method and operators. Then, we solve an example over Pythagorean probabilistic hesitant fuzzy sets with our operators and the results are agreement and the offered operators have superior effect than other operators.

Multi-kriterli karar verme problemleri içindeki Q- katsayılı ortopair olasılıksal karmaşık Bulanık Ağırlaştırılmış Hibrid operatörler

Anahtar Kelimeler
Q Rank Orthopair kararsız bulanık küme,
q-ROPHHWAG,
q-ROPHHOWAG
Karar verme

Öz: Karar verme problemlerinin uygulamalarında karmaşık bilgilerin artması ile olasılıklı tereddütlü bulanık küme yapısının kullanımı genişlemiştir. Bu nedenle, bu makale, q>0 için q-katsayılı ortopair olasılıksal kararsız bulanık hibrit ağırlıklı aritmetik ve geometrik (q-ROPHHWAG) operatörü ve q-katsayılı ortopair olasılıklı tereddütlü bulanık hibrit sıralı ağırlıklı aritmetik ve geometrik (q-ROPHHOWAG) operatörü olmak üzere iki yeni operatör sunmayı amaçlamaktadır. Sunulan operatörler, yeni bir parametre eklenmesi, daha esnek bir yapıya sahip olması ve kendi içinde karşılaştırmalı analizler sunması bakımından birçok açıdan mevcut operatörlerden daha iyidir. Ayrıca önerilen operatörlerin bazı özelliklerinden de bahsettik. Ek olarak, sunulan yöntem ve operatörlerin etkili, gerçek ve esnek olduğunu belirtmek için bir algoritma ve örnek veriyoruz. Daha sonra operatörlerimizle Pisagor olasılıklı tereddütlü bulanık kümeler üzerinden bir örnek çözüyoruz ve sonuçlar diğer operatörlere göre uyumlu ve daha büyük bir etkiye sahiptir.

1. Introduction

Probabilistic hesitant fuzzy set (PHFS) is an effective construction adding probability value to HFS and helping to carry more information as an extension of HFS. Therefore, it can completely explain the fuzzy of decision-making information, which has attracted more and several researchers' attention. The basic operation of PHFSs was defined in [1] and some aggregation operators were penned by Zheng and coauthors [2]. Moreover, Zhai et al. [3] obtained measures of probabilistic interval-valued intuitionistic hesitant fuzzy sets and the application in reducing excessive medical examinations, Batool and others [4] realized Pythagorean probabilistic hesitant fuzzy aggregation operators in MCDM. Furthermore, Batool et al. worked decision making mechanism based EDAS method by utilizing Pythagorean probabilistic hesitant fuzzy sets, Ren and coauthors [5, 6, 7] introduced to calculation and aggregation of Q-rung orthopair probabilistic hesitant fuzzy information and gave q-rung orthopair probabilistic hesitant fuzzy power Muirhead mean

*Corresponding author: serif.ozlu@hotmail.com

operator, respectively. Moreover, some works can be ordered [8,9,10,11,12,13].

Despite the above studies, the desire to model more information, integration in the world, the desire for a quick solution has prevented the resolution of fuzzy information but generalized structures are the most basic structures to reach this solution. In this paper, we obtain two new operators called q-ROPHHWAG and q-ROPHHOWAG combining two different aggregating operator namely q-ROPHWA and q-ROPHWG. These operators include a new parameter as different unlike the base operators. Generalized concepts have more advantages as follow;

1. Combining two operators in the same formula will prevent separate calculations and will enable us to obtain fast solutions in the future;
2. The use of two different variables is important for the decision makers in terms of the precision of the results.
3. Self-comparison analysis is very necessary in this age of noisy information.

Then, we give some properties q-ROPHHWAG and q-ROPHHOWAG, and define an algorithm. We give an investment example through algorithm. The end of paper, a comparative analysis is revealed with Batool's aggregation operators and superior results are obtained.

The remaining of paper is organized as follow; in section 2, some basic definitions are given as HFS, PHFS so on, in section 3, we give definitions of q-ROPHWA and q-ROPHWG; define q-ROPHHWAG and q-ROPHHOWAG, in section 4, an algorithm and an application are presented, in section 5, comparative analysis is made with Batool' method.

2. Material and Method

Q- Rung Orthopair Probabilistic Hesitant Fuzzy Sets Based On Hybrid Aggregating Operators

The concept of q- Rung Orthopair probabilistic hesitant fuzzy sets was defined by Ren and others [31] in 2021. In this section, this concept is applied for Hybrid aggregating Operators.

Definition 2.1 [6] Let $X = \{x_1, x_2, \ldots, x_n\}$ be a reference set. A q- Rung Orthopair probabilistic hesitant fuzzy sets $\hbar$ is defined as follows:

$$\hbar = \{\langle(x, \mu_\hbar(x), \nu_\hbar(x))\rangle : x \in X\},$$

for $\mu_\hbar = \{\mathfrak{I}_1(\kappa_1), \mathfrak{I}_2(\kappa_2), \ldots, \mathfrak{I}_n(\kappa_n)\}$ and $\nu_\hbar = \{\mathfrak{R}_1(\breve{\kappa}_1), \mathfrak{R}_2(\breve{\kappa}_2), \ldots, \mathfrak{R}_m(\breve{\kappa}_m)\}$ in here, $\mathfrak{I}_n$ and $\mathfrak{R}_m$ indicate possible membership values and possible non-membership values, respectively and also, $0 < \mathfrak{I} < 1$, $0 < \mathfrak{R} < 1$ and $0 < \mathfrak{I}^q + \mathfrak{R}^q < 1$. In addition to, $0 < \kappa_n < 1$ and $0 < \breve{\kappa}_m < 1$ where $\sum_{n=1}^{|\mu_\hbar|} 1\kappa_n \le 1$ $\sum_{m=1}^{|\nu_\hbar|} 1\breve{\kappa}_m \le 1$ for $|\mu_\hbar|$ and $|\nu_\hbar|$ are number of elements of membership and non-membership values.

Definition 2.2 [6] Let $\hbar = \langle \mu_\hbar(x), \nu_\hbar(x)\rangle$, $\hbar_1 = \langle \mu_{\hbar_1}(x), \nu_{\hbar_1}(x)\rangle$ and $\hbar_2 = \langle \mu_{\hbar_2}(x), \nu_{\hbar_2}(x)\rangle$ be three q-ROPHFEs. Then,

1. $\hbar_1 \oplus \hbar_2 = \langle \cup_{\mathfrak{I}_{1n}\in\mu_{\hbar_1},\mathfrak{I}_{2n}\in\mu_{\hbar_2}} \{[(\mathfrak{I}_{1n}^q + \mathfrak{I}_{2n}^q + \mathfrak{I}_{1n}\mathfrak{I}_{2m})^{\frac{1}{q}}](\kappa_{1n}\kappa_{2m}/ \sum_{n=1}^{|\mu_{\hbar_1}|} \kappa_{1n} \sum_{m=1}^{|\mu_{\hbar_2}|} \kappa_{2m}), \cup_{\mathfrak{R}_{1\iota}\in\nu_{\hbar_1},\mathfrak{R}_{2\ell}\in\nu_{\hbar_2}} [\mathfrak{R}_{1\iota}\mathfrak{R}_{2\ell}](\breve{\kappa}_{1\iota}\breve{\kappa}_{2\ell}/ \sum_{\iota=1}^{|\mu_{\hbar_1}|} \breve{\kappa}_{1\iota} \sum_{\ell=1}^{|\mu_{\hbar_2}|} \breve{\kappa}_{2\ell})\}\rangle$
2. $\hbar_1 \otimes \hbar_2 = \langle \cup_{\mathfrak{I}_{1n}\in\mu_{\hbar_1},\mathfrak{I}_{2n}\in\mu_{\hbar_2}} \{[\mathfrak{I}_{1m}\mathfrak{I}_{2n}](\kappa_{1m}\kappa_{2n}/ \sum_{m=1}^{|\mu_{\hbar_1}|} \kappa_{1m} \sum_{n=1}^{|\mu_{\hbar_2}|} \kappa_{2n}), \cup_{\mathfrak{R}_{1\iota}\in\nu_{\hbar_1},\mathfrak{R}_{2\ell}\in\nu_{\hbar_2}} [(\mathfrak{R}_{1\iota}^q + \mathfrak{R}_{2\ell}^q + \mathfrak{R}_{1\iota}\mathfrak{R}_{2\ell})^{\frac{1}{q}}](\breve{\kappa}_{1\iota}\breve{\kappa}_{2\ell}/ \sum_{\iota=1}^{|\mu_{\hbar_1}|} \breve{\kappa}_{1\iota} \sum_{\ell=1}^{|\mu_{\hbar_2}|} \breve{\kappa}_{2\ell})\}\rangle$,
3. $\hbar^\lambda = \langle \cup_{\mathfrak{I}_n\in\mu_\hbar} \{[(1-(1-\mathfrak{I}_n^q)^\lambda)^{\frac{1}{q}}(\kappa_n)]\}, \cup_{\mathfrak{R}_m\in\nu_\hbar} \mathfrak{R}_m^\lambda(\breve{\kappa}_m)\rangle$,
4. $\lambda\hbar = \langle \cup_{\mathfrak{I}_n\in\mu_\hbar} \mathfrak{I}_n^\lambda(\kappa_n) \cup_{\mathfrak{R}_m\in\nu_\hbar} \{[(1-(1-\mathfrak{R}_m^q)^\lambda)^{\frac{1}{q}}(\breve{\kappa}_m)]\}\rangle$.

Definition 2.3 [6] Let $\hbar_j = \langle \mu_{\hbar_j}(x), \nu_{\hbar_j}(x)\rangle$ be collection of q-ROPHFSs for $(j = 1,2,\ldots,\pi)$ and in here, $\mu_{\hbar_j} = \{\mathfrak{I}_{j_n}(\kappa) : n = 1,2,\ldots,|\mu_{\hbar_j}|\}$ and $\nu_{\hbar_j} = \{\mathfrak{R}_{j_n}(\kappa) : n = 1,2,\ldots,|\nu_{\hbar_j}|\}$ define q-rung orthopair probabilistic hesitant fuzzy weighted average (q-ROPHWA) operator for $w_j \in [0,1]$ and $\sum_{j=1}^{\pi} w_j = 1$ as follow;

1. $q-ROPHWA: \Phi^n \to \Phi$ is a mapping called as q-rung orthopair probabilistic hesitant fuzzy weighted average (q-ROPHWA) operator for $q > 0$ and $w_j \in [0,1]$ and $\sum_{j=1}^{\pi} w_j = 1$ is defined as below;
2.

$$q-ROPHWA(\hbar_1, \hbar_2, \ldots, \hbar_\pi) = w_1\hbar_1 \oplus w_2\hbar_2 \oplus \ldots \oplus w_\pi\hbar_\pi$$
$$= \langle \bigcup_{\mathfrak{I}_{1n}\in\mu_{\hbar_1},\mathfrak{I}_{2n}\in\mu_{\hbar_2},\ldots,\mathfrak{I}_{\pi n}\in\mu_{\hbar_\pi}} \left\{\left((1-\prod_{j=1}^{\pi}(1-\mathfrak{I}_{j_n}^q)^{w_j})^{\frac{1}{q}}\right)\right\} \left(\frac{\prod_{j=1}^{\pi}\kappa_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_{\hbar_j}|}\kappa_{j_n}}\right),$$
$$\bigcup_{\mathfrak{R}_{1n}\in\mu_{\hbar_1},\mathfrak{R}_{2n}\in\mu_{\hbar_2},\ldots,\mathfrak{R}_{\pi n}\in\mu_{\hbar_\pi}} \left\{\left(\prod_{j=1}^{\pi}(\mathfrak{R}_{j_n})^{w_j}\right)\right\} \left(\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}/\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_{\hbar_j}|}\breve{\kappa}_{j_n}\right)\rangle$$

Example 2.4 *Let accept two q-ROPHFEs that* $\hbar_1 = \langle\{0.3(0.5)\},\{0.5(0.5),0.2(0.2)\}\rangle$ *and* $\hbar_2 = \langle\{0.2(0.4),0.3(0.1)\},\{0.4(0.3)\}\rangle$, *and also* $w = (0.6,0.4)$ *for* $q = 2$. *In this statement, if we calculate to q-ROPHWA;*

$$q-ROPHWA(\hbar_1,\hbar_2) = \langle\{(1-((1-0.3^2)^{0.6}\times(1-0.2^2)^{0.4})^{\frac{1}{2}}((0.5\times 0.4)/((0.5)\times(0.4+0.1))),(1-((1-0.3^2)^{0.6}\times(1-0.3^2)^{0.4})^{\frac{1}{2}}((0.5\times 0.1)/((0.5)\times(0.4+0.1)))\},\{((0.5)^{0.6}\times(0.4)^{0.4}((0.5\times 0.3)/((0.5+0.2)\times(0.3))),((0.2)^{0.6}\times(0.4)^{0.4}((0.2\times 0.3)/((0.5+0.2)\times(0.3)))\}\rangle = \langle\{0.2651(0.8),0.1924(0.2)\},\{0.4573(0.7142),0.2639(0.2857)\}\rangle$$

Definition 2.5 [6] Let $\hbar_j=\langle\mu_{\hbar_j}(x),\nu_{\hbar_j}(x)\rangle$ be collection of q-ROPHFSs for $(j=1,2,\ldots,\pi)$ and in here, $\mu_{\hbar_j}=\{\Im_{j_n}(\kappa): n=1,2,\ldots,|\mu_{\hbar_j}|\}$ and $\nu_{\hbar_j}=\{\Re_{j_n}(\kappa): n=1,2,\ldots,|\nu_{\hbar_j}|\}$ define q-rung orthopair probabilistic hesitant fuzzy weighted geometric (q-ROPHWG) operator for $w_j\in[0,1]$ and $\sum_{j=1}^{\pi} w_j=1$ as follow;

1. $q-ROPHWG:\Phi^n\to\Phi$ is a mapping called as q-rung orthopair probabilistic hesitant fuzzy weighted geometric (q-ROPHWG) operator for $q>0$ and $w_j\in[0,1]$ and $\sum_{j=1}^{\pi} w_j=1$ is defined as below;

$$q-ROPHWG(\hbar_1,\hbar_2,\ldots,\hbar_\pi) = \hbar_1^{w_1}\otimes\hbar_2^{w_1}\otimes\ldots\otimes\hbar_\pi^{w_\pi} = \langle\bigcup_{\Im_{1n}\in\mu_{\hbar_1},\Im_{2n}\in\mu_{\hbar_2},\ldots,\Im_{\pi n}\in\mu_{\hbar_\pi}}\left\{\left(\prod_{j=1}^{\pi}(\Im_{j_n})^{w_j}\right)\right\}\left(\frac{\prod_{j=1}^{\pi}\kappa_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_{\hbar_j}|}\kappa_{j_n}}\right),$$
$$\bigcup_{\Re_{1n}\in\mu_{\hbar_1},\Re_{2n}\in\mu_{\hbar_2},\ldots,\Re_{\pi n}\in\mu_{\hbar_\pi}}\left\{\left((1-\prod_{j=1}^{\pi}(1-\Re_{j_n}^{q})^{w_j})^{\frac{1}{q}}\right)\right\}\left(\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}/\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_{\hbar_j}|}\breve{\kappa}_{j_n}\right)\rangle$$

Example 2.6 *Let accept two q-ROPHFEs that* $\hbar_1=\langle\{0.3(0.5)\},\{0.5(0.5),0.2(0.2)\}\rangle$ *and* $\hbar_2=\langle\{0.2(0.4),0.3(0.1)\},\{0.4(0.3)\}\rangle$ *, and also* $w=(0.6,0.4)$ *for* $q=2$*. In this statement, if we calculate to q-ROPHWG;*

$$q-ROPHWG(\hbar_1,\hbar_2) = \langle\{((0.3)^{0.6}\times(0.2)^{0.4}((0.5\times 0.4)/((0.5)\times(0.4+0.1))),((0.3)^{0.6}\times(0.3)^{0.4}((0.5\times 0.1)/((0.5)\times(0.4+0.1)))\},\{(1-((1-0.5^2)^{0.6}\times(1-0.4^2)^{0.4})^{\frac{1}{2}}((0.5\times 0.3)/((0.5+0.2)\times(0.3))),(1-((1-0.2^2)^{0.6}\times(1-0.3^2)^{0.4})^{\frac{1}{2}}((0.2\times 0.3)/((0.5+0.2)\times(0.3)))\}\rangle = \langle\{0.255(0.8),0.3(0.2)\},\{0.4639(0.7142),0.2998(0.2857)\}\rangle$$

Definition 2.7 Let $\hbar_j=\langle\mu_{\hbar_j}(x),\nu_{\hbar_j}(x)\rangle$ be q-ROPHFE and in here, $\mu_{\hbar_j}=\{\Im_{j_n}(\breve{\kappa}): n=1,2,\ldots,|\mu_{\hbar_j}|\}$ and $\nu_{\hbar_j}=\{\Re_{j_m}(\kappa): n=1,2,\ldots,|\nu_{\hbar_j}|\}$. In this statement score function of $h_{\Re}$ is defined as follow;

$$S(\hbar_j(x)) = \left(\frac{1}{|\mu_{\hbar_j}|}\sum_{\Im_{j_n}\in\mu_{\hbar_j}}(\Im_{j_n}\kappa_n)\right)^q - \left(\frac{1}{|\nu_{\hbar_j}|}\sum_{\Re_{j_n}\in\nu_{\hbar_j}}(\Re_{j_n}\breve{\kappa}_n)\right)^q$$

If score values are same, the following accuracy function is used;

$$A(\hbar_j(x)) = \left(\frac{1}{|\mu_{\hbar_j}|}\sum_{\Im_{j_n}\in\mu_{\hbar_j}}(\Im_{j_n}\kappa_n)\right)^q + \left(\frac{1}{|\nu_{\hbar_j}|}\sum_{\Re_{j_n}\in\nu_{\hbar_j}}(\Re_{j_n}\breve{\kappa}_n)\right)^q$$

Q- Rung Orthopair Probabilistic Hesitant Fuzzy Sets Based On Hybrid Operators

In this section, we define two new operators by combining geometric and averaging operator based on probabilistic hesitant fuzzy sets.

Definition 2.8 Let $\hbar_j=\langle\mu_{\hbar_j}(x),\nu_{\hbar_j}(x)\rangle$ be collection of q-ROPHFSs for $(j=1,2,\ldots,\pi)$ and in here, $\mu_{\hbar_j}=\{\Im_{j_n}(\kappa): n=1,2,\ldots,|\mu_{\hbar_j}|\}$ *and* $\nu_{\hbar_j}=\{\Re_{j_n}(\kappa): n=1,2,\ldots,|\nu_{\hbar_j}|\}$ *for* $\lambda\in[0,1]$.

1. $q-ROPHHWAG:\Phi^n\to\Phi$ is a mapping called as q-rung orthopair probabilistic hesitant fuzzy hybrid weighted arithmetic and geometric (q-ROPHHWAG) operator for $q>0$ and $w_j\in[0,1]$ and $\sum_{j=1}^{\pi} w_j=1$ is defined as below;
2.

$$\left(\sum_{j=1}^{n} w_j\hbar_j\right)^{\lambda}\left(\sum_{j=1}^{n}\hbar_j^{w_j}\right)^{1-\lambda} = \langle\bigcup_{\Im_{1n}\in\mu_{\hbar_1},\Im_{2n}\in\mu_{\hbar_2},\ldots,\Im_{\pi n}\in\mu_{\hbar_\pi}}\left\{\left((1-\prod_{j=1}^{\pi}(1-\Im_{j_n}^{q})^{w_j})^{\frac{\lambda}{q}}\right)\left((\prod_{j=1}^{\pi}(\Im_{j_n})^{w_j})^{(1-\lambda)}\right)\right\}$$
$$\left(\frac{\prod_{j=1}^{\pi}\kappa_{j_n}\prod_{j=1}^{\pi}\kappa_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_{\hbar}|}\kappa_{j_n}\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_{\hbar}|}\kappa_{j_n}}\right)\bigcup_{\Re_{1n}\in\mu_{\hbar_1},\Re_{2n}\in\mu_{\hbar_2},\ldots,\Re_{\pi n}\in\mu_{\hbar_\pi}}\left\{(1-a\left((\prod_{j=1}^{\pi}(1-\Re_{j_n}^{q})^{w_j})^{(1-\lambda)}\right))^{\frac{1}{q}}\right\}$$
$$a=\left((1-(\prod_{j=1}^{\pi}(\Re_{j_n})^{w_j})^{q})^{\lambda}\right)$$
$$\left(\frac{\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_{\hbar}|}\breve{\kappa}_{j_n}\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_{\hbar}|}\breve{\kappa}_{j_n}}\right)\rangle$$

3. $q-ROPHHOWAG: \Phi^n \rightarrow \Phi$ is a mapping called as q-rung orthopair probabilistic hesitant fuzzy hybrid ordered weighted arithmetic and geometric (q-ROPHOWAG) operator for $q>0$ and $w_j \in [0,1]$ and $\sum_{j=1}^{\pi} w_j = 1$ is defined as below;

$$(\sum_{j=1}^{n} w_j \hbar_{\sigma(j)})^{\lambda}(\sum_{j=1}^{n} \hbar_{\sigma(j)}^{w_j})^{1-\lambda}$$
$$= \langle \bigcup_{\Im_{\sigma(1)n}\in\mu_{\hbar_1},\Im_{\sigma(2)n}\in\mu_{\hbar_2},\ldots,\Im_{\sigma(\pi)n}\in\mu_{\hbar_\pi}}$$
$$\left\{a\left((\prod_{j=1}^{\pi}(\Im_{\sigma(j)_n})^{w_j})^{(1-\lambda)}\right)\right\}$$
$$a=\left((1-\prod_{j=1}^{\pi}(1-\Im_{\sigma(j)_n}^{q})^{w_j})^{\frac{\lambda}{q}}\right)$$
$$\left(\frac{\prod_{j=1}^{\pi}\kappa_{j_n}\prod_{j=1}^{\pi}\kappa_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_\hbar|}\kappa_{j_n}\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_\hbar|}\kappa_{j_n}}\right)$$
$$\bigcup_{\Re_{\sigma(1)n}\in\mu_{\hbar_1},\Re_{\sigma(2)n}\in\mu_{\hbar_2},\ldots,\Re_{\sigma(\pi)n}\in\mu_{\hbar_\pi}}$$
$$\left\{(1-a\left((\prod_{j=1}^{\pi}(1-\Re_{\sigma(j)_n}^{q})^{w_j})^{(1-\lambda)}\right))^{\frac{1}{q}}\right\}$$
$$a=\left((1-(\prod_{j=1}^{\pi}(\Re_{\sigma(j)_n})^{w_j})^{q})^{\lambda}\right)$$
$$\left(\frac{\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}}\right)\rangle$$

where $\sigma(1),\sigma(2),\ldots,\sigma(j)$ is a permutation of $j=1,2,\ldots,\pi$ and also $\Re_{\sigma(j-1)} \geq \Re_{(j)}$ and $\Im_{\sigma(j-1)} \geq \Im_{(j)}$.

Theorem 2.9 *Let $\hbar_j = \langle \mu_{\hbar_j}(x), \nu_{\hbar_j}(x)\rangle$ be collection of q-ROPHFSs for $(j=1,2,\ldots,\pi)$ and $\lambda\in[0,1]$ where $w_j\in[0,1]$ and $\sum_{j=1}^{\pi} w_j = 1$;*

$$= (\sum_{j=1}^{n} w_j \hbar_j)^{\lambda}(\sum_{j=1}^{n} \hbar_j^{w_j})^{1-\lambda}$$
$$= \langle \bigcup_{\Im_{1n}\in\mu_{\hbar_1},\Im_{2n}\in\mu_{\hbar_2},\ldots,\Im_{\pi n}\in\mu_{\hbar_\pi}}$$
$$\left\{\left((1-\prod_{j=1}^{\pi}(1-\Im_{j_n}^{q})^{w_j})^{\frac{\lambda}{q}}\right)\left((\prod_{j=1}^{\pi}(\Im_{j_n})^{w_j})^{(1-\lambda)}\right)\right\}$$
$$\left(\frac{\prod_{j=1}^{\pi}\kappa_{j_n}\prod_{j=1}^{\pi}\kappa_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_\hbar|}\kappa_{j_n}\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_\hbar|}\kappa_{j_n}}\right)$$
$$\bigcup_{\Re_{1n}\in\mu_{\hbar_1},\Re_{2n}\in\mu_{\hbar_2},\ldots,\Re_{\pi n}\in\mu_{\hbar_\pi}}$$
$$\left\{(1-a\left((\prod_{j=1}^{\pi}(1-\Re_{j_n}^{q})^{w_j})^{(1-\lambda)}\right))^{\frac{1}{q}}\right\}$$
$$a=\left((1-(\prod_{j=1}^{\pi}(\Re_{j_n})^{w_j})^{q})^{\lambda}\right)$$
$$\left(\frac{\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}}\right)\rangle$$

Proof. Firstly, we can write by utilizing operational rules for q-ROPHFSs;

$$= (\sum_{j=1}^{n} w_j \hbar_j)^{\lambda}(\sum_{j=1}^{n} \hbar_j^{w_j})^{1-\lambda}$$
$$= \langle \bigcup_{\Im_{1n}\in\mu_{\hbar_1},\Im_{2n}\in\mu_{\hbar_2},\ldots,\Im_{\pi n}\in\mu_{\hbar_\pi}}$$
$$\left\{\left((1-\prod_{j=1}^{\pi}(1-\Im_{j_n}^{q})^{w_j})^{\frac{1}{q}}\right)\right\}\left(\frac{\prod_{j=1}^{\pi}\kappa_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_\hbar|}\kappa_{j_n}}\right),$$
$$\bigcup_{\Re_{1n}\in\mu_{\hbar_1},\Re_{2n}\in\mu_{\hbar_2},\ldots,\Re_{\pi n}\in\mu_{\hbar_\pi}}$$
$$\{(\prod_{j=1}^{\pi}(\Re_{j_n})^{w_j})\}\left(\frac{\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}}\right)\rangle^{\lambda}$$
$$\langle \bigcup_{\Im_{1n}\in\mu_{\hbar_1},\Im_{2n}\in\mu_{\hbar_2},\ldots,\Im_{\pi n}\in\mu_{\hbar_\pi}}$$
$$\left\{\left(\prod_{j=1}^{\pi}(\Im_{j_n})^{w_j}\right)\right\}\left(\frac{\prod_{j=1}^{\pi}\kappa_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_\hbar|}\kappa_{j_n}}\right),$$
$$\bigcup_{\Re_{1n}\in\mu_{\hbar_1},\Re_{2n}\in\mu_{\hbar_2},\ldots,\Re_{\pi n}\in\mu_{\hbar_\pi}}$$
$$\{a\}\left(\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}/\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}\right)\rangle^{1-\lambda}$$
$$a=\left((1-\prod_{j=1}^{\pi}(1-\Re_{j_n}^{q})^{w_j})^{\frac{1}{q}}\right)$$

from here

$$= \langle \bigcup_{\Im_{1n}\in\mu_{\hbar_1},\Im_{2n}\in\mu_{\hbar_2},\ldots,\Im_{\pi n}\in\mu_{\hbar_\pi}}$$
$$\left\{\left((1-\prod_{j=1}^{\pi}(1-\Im_{j_n}^{q})^{w_j})^{\frac{\lambda}{q}}\right)\right\}\left(\frac{\prod_{j=1}^{\pi}\kappa_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_\hbar|}\kappa_{j_n}}\right),$$
$$\bigcup_{\Re_{1n}\in\mu_{\hbar_1},\Re_{2n}\in\mu_{\hbar_2},\ldots,\Re_{\pi n}\in\mu_{\hbar_\pi}}$$
$$\left\{\left((1-(1-(\prod_{j=1}^{\pi}(\Re_{j_n})^{w_j})^{q})^{\lambda})^{\frac{1}{q}}\right)\right\}\left(\frac{\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}}\right)\rangle$$
$$\langle \bigcup_{\Im_{1n}\in\mu_{\hbar_1},\Im_{2n}\in\mu_{\hbar_2},\ldots,\Im_{\pi n}\in\mu_{\hbar_\pi}}$$
$$\left\{\left((\prod_{j=1}^{\pi}(\Im_{j_n})^{w_j})^{(1-\lambda)}\right)\right\}\left(\frac{\prod_{j=1}^{\pi}\kappa_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_\hbar|}\kappa_{j_n}}\right),$$
$$\bigcup_{\Re_{1n}\in\mu_{\hbar_1},\Re_{2n}\in\mu_{\hbar_2},\ldots,\Re_{\pi n}\in\mu_{\hbar_\pi}}$$
$$\left\{\left((1-(\prod_{j=1}^{\pi}(1-\Re_{j_n}^{q})^{w_j})^{(1-\lambda)})^{\frac{1}{q}}\right)\right\} b\rangle.$$
$$b=\left(\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}/\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}\right)$$

Then,

$$= \langle \bigcup_{\Im_{1n}\in\mu_{\hbar_1},\Im_{2n}\in\mu_{\hbar_2},\ldots,\Im_{\pi n}\in\mu_{\hbar_\pi}}$$
$$\left\{\left((1-\prod_{j=1}^{\pi}(1-\Im_{j_n}^{q})^{w_j})^{\frac{\lambda}{q}}\right)\left((\prod_{j=1}^{\pi}(\Im_{j_n})^{w_j})^{(1-\lambda)}\right)\right\}$$
$$\left(\frac{\prod_{j=1}^{\pi}\kappa_{j_n}\prod_{j=1}^{\pi}\kappa_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_\hbar|}\kappa_{j_n}\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_\hbar|}\kappa_{j_n}}\right)$$
$$\bigcup_{\Re_{1n}\in\mu_{\hbar_1},\Re_{2n}\in\mu_{\hbar_2},\ldots,\Re_{\pi n}\in\mu_{\hbar_\pi}}$$

$$(\left\{\left((1-(1-(\prod_{j=1}^{\pi}(\Re_{j_n})^{w_j})^q)^{\lambda})\right)\right\}$$

$$\left(\frac{\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}}\right)+\left\{\left((1-(\prod_{j=1}^{\pi}(1-\Re_{j_n}^q)^{w_j})^{(1-\lambda)})\right)\right\}$$

$$\left(\frac{\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}}\right)-\left\{\left((1-(1-(\prod_{j=1}^{\pi}(\Re_{j_n})^{w_j})^q)^{\lambda})\right)\right\}$$

$$\left(\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}/\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}\right)$$

$$\times\left\{\left((1-(\prod_{j=1}^{\pi}(1-\Re_{j_n}^q)^{w_j})^{(1-\lambda)})\right)\right\}b)^{\frac{1}{q}}\rangle$$

$$b=\left(\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}/\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}\right)$$

and thus

$$=\langle\bigcup_{\Im_{1_n}\in\mu_{\hbar_1},\Im_{2_n}\in\mu_{\hbar_2},\ldots,\Im_{\pi_n}\in\mu_{\hbar_\pi}}$$

$$\left\{\left((1-\prod_{j=1}^{\pi}(1-\Im_{j_n}^q)^{w_j})^{\frac{\lambda}{q}}\right)\left((\prod_{j=1}^{\pi}(\Im_{j_n})^{w_j})^{(1-\lambda)}\right)\right\}$$

$$\left(\frac{\prod_{j=1}^{\pi}\kappa_{j_n}\prod_{j=1}^{\pi}\kappa_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_\hbar|}\kappa_{j_n}\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_\hbar|}\kappa_{j_n}}\right)$$

$$\bigcup_{\Re_{1_n}\in\mu_{\hbar_1},\Re_{2_n}\in\mu_{\hbar_2},\ldots,\Re_{\pi_n}\in\mu_{\hbar_\pi}}$$

$$(\left\{\left((1-(1-(\prod_{j=1}^{\pi}(\Re_{j_n})^{w_j})^q)^{\lambda})\right)\right\}$$

$$\left(\frac{\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}}\right)+$$

$$\left\{\left((1-(\prod_{j=1}^{\pi}(1-\Re_{j_n}^q)^{w_j})^{(1-\lambda)})\right)\right\}$$

$$\left(\frac{\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}}\right)-$$

$$\left\{\left((1-(1-(\prod_{j=1}^{\pi}(\Re_{j_n})^{w_j})^q)^{\lambda})\right)\right\}$$

$$\left(\frac{\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}}\right)\times\left\{\left((1-(\prod_{j=1}^{\pi}(1-\Re_{j_n}^q)^{w_j})^{(1-\lambda)})\right)\right\}$$

$$\left(\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}/\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}\right))^{\frac{1}{q}}\rangle$$

and if the basic operations are made;

$$=\langle\bigcup_{\Im_{1_n}\in\mu_{\hbar_1},\Im_{2_n}\in\mu_{\hbar_2},\ldots,\Im_{\pi_n}\in\mu_{\hbar_\pi}}$$

$$\left\{\left((1-\prod_{j=1}^{\pi}(1-\Im_{j_n}^q)^{w_j})^{\frac{\lambda}{q}}\right)\left((\prod_{j=1}^{\pi}(\Im_{j_n})^{w_j})^{(1-\lambda)}\right)\right\}$$

$$\left(\frac{\prod_{j=1}^{\pi}\kappa_{j_n}\prod_{j=1}^{\pi}\kappa_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_\hbar|}\kappa_{j_n}\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_\hbar|}\kappa_{j_n}}\right)$$

$$\bigcup_{\Re_{1_n}\in\mu_{\hbar_1},\Re_{2_n}\in\mu_{\hbar_2},\ldots,\Re_{\pi_n}\in\mu_{\hbar_\pi}}$$

$$\left\{(1-\left((1-(\prod_{j=1}^{\pi}(\Re_{j_n})^{w_j})^q)^{\lambda}\right)b)^{\frac{1}{q}}\right\}$$

$$b=\left((\prod_{j=1}^{\pi}(1-\Re_{j_n}^q)^{w_j})^{(1-\lambda)}\right)$$

$$\left(\frac{\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}\prod_{j=1}^{\pi}\breve{\kappa}_{j_n}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{j_n}}\right)\rangle$$

Now, we discuss some special cases of q-ROPHHWAG as following;

- If $\lambda=1$, q-ROPHHWAG is reduced to (q-ROPHWA).
- If $\lambda=0$, q-ROPHHWAG is reduced to (q-ROPHWG).
- If $\lambda=0.5$, q-ROPHHWAG is reduced to (q-ROPHWA) and (q-ROPHWG).

Theorem 3.2 *Let* $\hbar_j=\langle\mu_{\hbar_j}(x),\nu_{\hbar_j}(x)\rangle$ *and* $\hbar_j^*=\langle\mu_{\hbar_j}^*(x),\nu_{\hbar_j}^*(x)\rangle$ *be collection of q-ROPHFSs for* $(j=1,2,\ldots,\pi)$ *. Thus, q-ROPHHWAG provides following properties;*

1. (Idempotency) Let be $\hbar_j=\hbar$ for $(j=1,2,\ldots,\pi)$. Thus, $q-ROPHHWAG(\hbar_1,\hbar_2,\ldots\hbar_\pi)=\hbar$.
2. (Boundedness) Let be $\hbar_j^+$ and $\hbar_j^-$ maximum and minimum elements for $j=1,2,\ldots,\pi$. Thus, $\hbar_j^-\leq q-ROPHHWAG(\hbar_1,\hbar_2,\ldots,\hbar_\pi)\leq\hbar_j^+$.
3.(Monotonicity)Let be $\mu_{\hbar_j}\geq\mu_{\hbar_j}^*$ and $\nu_{\hbar_j}\leq\nu_{\hbar_j}^*$. In this statement,
$q-ROPHHWAG(\hbar_1,\hbar_2,\ldots,\hbar_\pi)\geq$
$q-ROPHHWAG(\hbar_1^*,\hbar_2^*,\ldots,\hbar_\pi^*)$.

It is open that q-ROPHHWAG carries above all of the properties owing to q-ROPHWA and q-ROPHWG. As similar, q-ROPHHOWAG carries to above all of the properties.

Example 3.1 *Let accept two q-ROPHFEs that* $\hbar_1=\langle\{0.3(0.5)\},\{0.5(0.5),0.2(0.2)\}\rangle$ *and* $\hbar_2=\langle\{0.2(0.4),0.3(0.1)\},\{0.4(0.3)\}\rangle$ *, and also* $w=(0.6,0.4)$ *for* $q=2$ *and* $\lambda=0.3$*. In this statement, if we calculate to q-ROPHHWAG;*

$$q-ROPHHWAG(\hbar_1,\hbar_2)=$$
$$\langle\{(1-(1-0.3^2)^{0.6}\times(1-0.2^2)^{0.4})^{\frac{0.3}{2}}\times(0.3^{0.6}\times0.2^{0.4})^{(1-0.3)}(\frac{0.5^2\times0.4^2}{(0.5)^2\times(0.4+0.1)^2}),(1-(1-0.3^2)^{0.6}\times(1-0.3^2)^{0.4})^{\frac{0.3}{2}}\times(0.3^{0.6}\times0.3^{0.4})^{(1-0.3)}(\frac{0.5^2\times0.1^2}{(0.5)^2\times(0.4+0.1)^2})\},\{(1-(1-(0.5^{0.6}\times0.4^{0.4})^2)^{0.3}\times((1-0.5^2)^{0.6}\times(1-0.4^2)^{0.4})^{(1-0.3)})^{\frac{1}{2}}(\frac{0.5^2\times0.3^2}{(0.5+0.2)^2\times(0.3)^2}),(1-(1-(0.2^{0.6}\times0.4^{0.4})^2)^{0.3}\times((1-0.2^2)^{0.6}\times(1-0.4^2)^{0.4})^{(1-0.3)})^{\frac{1}{2}}(\frac{0.2^2\times0.3^2}{(0.5+0.2)^2\times(0.3)^2})\}\rangle$$
$$=\langle\{0.258(0.64),0.3(0.04)\},\{0.4619(0.862),0.2896(0.1379)\}\rangle$$

For q-ROPHHOWAG, firstly we calculate to score values for $\hbar_1$ and $\hbar_2$ as follow; $S(\hbar_1) = -0.0616$ and $S(\hbar_2) = -0.0023$. Thus, $\hbar_1 < \hbar_2$ and

$$\langle\{(1-(1-0.3^2)^{0.4}\times(1-0.2^2)^{0.6})^{\frac{0.3}{2}}\times(0.3^{0.4}\times 0.2^{0.6})^{(1-0.3)}\left(\frac{0.5^2\times 0.4^2}{(0.5)^2\times(0.4+0.1)^2}\right),(1-(1-0.3^2)^{0.4}\times(1-0.3^2)^{0.6})^{\frac{0.3}{2}}\times(0.3^{0.4}\times 0.3^{0.6})^{(1-0.3)}\left(\frac{0.5^2\times 0.1^2}{(0.5)^2\times(0.5+0.1)^2}\right)\},\{(1-(1-(0.5^{0.4}\times 0.4^{0.6})^2)^{0.3}\times((1-0.5^2)^{0.4}\times(1-0.4^2)^{0.6})^{1-0.3})^{\frac{1}{2}}\left(\frac{0.5^2\times 0.3^2}{(0.5+0.2)^2\times(0.3)^2}\right),(1-(1-(0.2^{0.4}\times 0.4^{0.6})^2)^{0.3}\times((1-0.2^2)^{0.4}\times(1-0.4^2)^{0.6})^{(1-0.3)})^{\frac{1}{2}}\left(\frac{0.2^2\times 0.3^2}{(0.5+0.2)^2\times(0.3)^2}\right)\}\rangle$$

$$= \langle\{0.2382(0.64),0.3(0.04)\},\{0.442(0.862),0.3276(0.1379)\}\rangle$$

3. Results

3.1. An Application Of Multi-Attribute Decision-Making Method Under Q-Rophhwag

In this section, we apply the presented q-ROPHHWAG into an algorithm and test over a MCDM problem with n alternatives and m criteria to indicate effective of averaging operators over NDHPFS. Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ be a set of criterions and let $w_j = (w_1, w_2, \ldots, w_n)$ be a weight vector of criterions where $w_j > 0, j = 1,2,\ldots,n$ and $\sum_{j=1}^{n} w_j = 1$. Then, the following steps have been defined for algorithm.

1. Consist of Decision making matrix as $(\phi_{ij})_{m\times n}$ for $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$,

$$[\phi_{ij}]_{m\times n} = \begin{pmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1n} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \delta_{m1} & \delta_{m2} & \cdots & \delta_{mn} \end{pmatrix}$$

2. Determine q- Rung Orthopair probabilistic hesitant fuzzy elements by utilizing $\phi_i = q - ROPHHWAG(\phi_{i1}, \phi_{i2}, \ldots, \phi_{in})$ or $\phi_i = q - ROPHHOWAG(\phi_{i1}, \phi_{i2}, \ldots, \phi_{in})$ operator for $i = 1,2,\ldots,m$,
3. Calculate score values of q- Rung Orthopair probabilistic hesitant fuzzy elements,
4. Determine alternatives rankings in descending order.

3.2. Numerical example

A company that wants to invest is doing some work to identify different alternatives and as a result of these researches, it determines four criteria and five alternatives as following; A_1; A hybrid car production company, A_2; A flue filter company, A_3; A recycle production company, A_4; A aircraft manufacturing factory responding to forest fire company; if criterions, C_1; minimum cost, maximum profitability, C_2; the proximity to raw material, C_3; least harm to the environment; C_4; experience and for criterions determined weights by decision makers as follow; $w = (0.3,0.2,0.2,0.3)$.

Decision makers construct decision making matrix as follow in Table 1.

Obtain aggregated values by using q-ROPHHWAG based on data in Table 2 for $q = 2$ and $\lambda = 0.3$.

$$\langle \bigcup_{\Im_{1n}\in\mu_{\hbar_1},\Im_{2n}\in\mu_{\hbar_2},\ldots,\Im_{\pi n}\in\mu_{\hbar_\pi}} \left\{\left((1-\prod_{j=1}^{\pi}(1-\Im_{jn}^{q})^{w_j})^{\frac{\lambda}{q}}\right)b\right\}$$

$$b = \left((\prod_{j=1}^{\pi}(\Im_{jn})^{w_j})^{(1-\lambda)}\right)\left(\frac{\prod_{j=1}^{\pi}\kappa_{jn}\prod_{j=1}^{\pi}\kappa_{jn}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_\hbar|}\kappa_{jn}\prod_{j=1}^{\pi}\sum_{n=1}^{|\mu_\hbar|}\kappa_{jn}}\right)$$

$$\bigcup_{\Re_{1n}\in\mu_{\hbar_1},\Re_{2n}\in\mu_{\hbar_2},\ldots,\Re_{\pi n}\in\mu_{\hbar_\pi}} \left\{(1-\left((1-(\prod_{j=1}^{\pi}(\Re_{jn})^{w_j})^{q})^{\lambda}\right)b)^{\frac{1}{q}}\right\}$$

$$b = \left((\prod_{j=1}^{\pi}(1-\Re_{jn}^{q})^{w_j})^{(1-\lambda)}\right)\left(\frac{\prod_{j=1}^{\pi}\breve{\kappa}_{jn}\prod_{j=1}^{\pi}\breve{\kappa}_{jn}}{\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{jn}\prod_{j=1}^{\pi}\sum_{n=1}^{|\nu_\hbar|}\breve{\kappa}_{jn}}\right)\rangle$$

and results are as follow;

ϕ_1
= {{0.3642(0.0044),0.4558(0.0044),0.2682(0.0177), 0.3982(0.04),0.3573(0.0177), 0.4963(0.01),0.3051(0.04),0.3959(0.04)}, {0.4735(0.0123),0.3868(0.0030),0.4598(0.0493), 0.5503(0.0123), 0.3703(0.0123), 0.4798(0.0044), 0.5503(0.0493),0.4798(0.0123)}}

ϕ_2
= {{0.3628(0.1111),0.3900(0.0493),0.4386(0.0044) , 0.3275(0.0011),0.4665(0.0019), 0.3525(0.0123), 0.3969(0.0011), 0.4222(0.0004)}, {0.4691(0.0004),0.4509(0.000069), 0.5123(0.0270), 0.5224(0.0051), 0.4926(0.0010), 0.5050(0.0001), 0.5224(0.3319), 0.5050(0.0132)}}

ϕ_3
= {{0.3497(0.0024),0.5091(0.1560),0.3645(0.0024) , 0.3014(0.00008103),0.5283(0.1560), 0.4492(0,0024), 0.3176(0.0000813), 0.4688(0.0024)}, {0,3274(0.0097),0.3730(0.0390), 0.3435(0.0152), 0.4114(0.0024), 0.3874(0.0609), 0.4484(0.0270), 0.4114(0.0038), 0.4484(0.0152)}}

ϕ_4
$= \{\{0.3416(0.0062), 0.4080(0.0062), 0.3606(0.0393), 0.3770(0.0251), 0.4294(0.0393), 0.4465(0.0040), 0.3965(0.0251), 0.4688(0.0251)\}, \{0.5872(0.0066), 0.4083(0.0066), 0.6009(0.0416), 0.5656(0.0037), 0.4259(0.0416), 0.3756(0.0066), 0.5656(0.0234), 0.37568(0.0234)\}\}$

ϕ_5
$= \{\{0.3976(0.0051), 0.4592(0.0318), 0.4555(0.0051), 0.3976(0.0051), 0.5193(0.0318), 0.4592(0.0318), 0.4555(0.0051), 0.5193(0.0318)\}, \{0.3589(0.0123), 0.4769(0.0123), 0.4155(0.0123), 0.4182(0.0192), 0.5221(0.0123), 0.5228(0.0192), 0.4182(0.0192), 0.5228(0.0192)\}\}$

Table 1. Evaluations of alternatives made by decision makers

	C_1	C_2
A_1	{{0.3(0.1)}, {0.7(0.4), 0.5(0.2)}}	{{0.3(0.2), 0.5(0.3)}, {0.4(0.1)}}
A_2	{{0.5(0.2)}, {0.3(0.5), 0.2(0.1)}}	{{0.2(0.4), 0.1(0.2)}, {0.8(0.3)}}
A_3	{{0.4(0.3)}, {0.5(0.2), 0.6(0.4)}}	{{0.7(0.8), 0.4(0.1)}, {0.3(0.9)}}
A_4	{{0.5(0.2)}, {0.8(0.2), 0.3(0.2)}}	{{0.4(0.5), 0.6(0.4)}, {0.5(0.9)}}
A_5	{{0.5(0.6)}, {0.4(0.5), 0.7(0.5)}}	{{0.4(0.5), 0.4(0.5)}, {0.5(0.8)}}
	C_3	C_4
A_1	{{0.8(0.2), 0.3(0.4)}, {0.3(0.1), 0.2(0.2)}}	{{0.2(0.1), 0.5(0.1)}, {0.3(0.5), 0.6(0.5)}}
A_2	{{0.3(0.5), 0.7(0.1)}, {0.2(0.1), 0.5(0.8)}}	{{0.4(0.3), 0.5(0.2)}, {0.4(0.2), 0.6(0.7)}}
A_3	{{0.5(0.4), 0.6(0.4)}, {0.2(0.4), 0.3(0.5)}}	{{0.1(0.1), 0.5(0.8)}, {0.2(0.6), 0.5(0.3)}}
A_4	{{0.3(0.2), 0.4(0.5)}, {0.3(0.2), 0.4(0.5)}}	{{0.2(0.5), 0.4(0.5)}, {0.5(0.4), 0.4(0.3)}}
A_5	{{0.4(0.5), 0.7(0.5)}, {0.2(0.1), 0.5(0.1)}}	{{0.3(0.2), 0.5(0.5)}, {0.3(0.4), 0.5(0.5)}}

Table 2. Score Values under q-ROPHHWAG

q values	Ranking Alternatives
$q = 2$	$A_3 > A_5 > A_4 > A_1 > A_2$
$q = 3$	$A_3 > A_5 > A_4 > A_1 > A_2$
$q = 5$	$A_3 > A_5 > A_4 > A_1 > A_2$
$q = 8$	$A_3 > A_5 > A_1 > A_4 > A_2$
$q = 10$	$A_3 > A_5 > A_1 > A_4 > A_2$
$q = 15$	$A_3 > A_5 > A_1 > A_4 > A_2$
$q = 25$	$A_3 > A_5 > A_1 > A_4 > A_2$
$q = 40$	$A_3 > A_5 > A_1 > A_4 > A_2$
$q = 75$	$A_3 > A_5 > A_1 > A_4 > A_2$

Table 3. Score Values under q-ROPHHWAG

λ values	Ranking Alternatives
$\lambda = 0.1$	$A_3 > A_5 > A_4 > A_1 > A_2$
$\lambda = 0.2$	$A_3 > A_5 > A_4 > A_1 > A_2$
$\lambda = 0.3$	$A_3 > A_5 > A_4 > A_1 > A_2$
$\lambda = 0.4$	$A_3 > A_5 > A_4 > A_1 > A_2$
$\lambda = 0.5$	$A_3 > A_5 > A_4 > A_1 > A_2$
$\lambda = 0.6$	$A_3 > A_5 > A_4 > A_1 > A_2$
$\lambda = 0.7$	$A_3 > A_5 > A_4 > A_1 > A_2$
$\lambda = 0.8$	$A_3 > A_5 > A_4 > A_1 > A_2$
$\lambda = 0.9$	$A_3 > A_5 > A_4 > A_1 > A_2$

If the score values are surveyed according to orderings of Alternatives, alternative A_3 is the most desirable for all of alternatives and the most undesirable alternative is A_2 for different q values. Now let's keep the q values constant and change the λ values.

As seen from score values; the best alternative is same for all λ and $q = 2$ values. The proposed operator is reality, objective and effective.

5. Comparative and discussion

In this section, the proposed operator under probabilistic hesitant fuzzy (PHFS) environment is compared with some operators which defined over q-ROPHFS. If we solve with our method to example over coronavirus disease of Batool [4], the results are as following;

Table 4. Comparative Analyzes according to Score Values under q-ROPHHWAG for different pairs of (λ,q)

Methods	Ranking Alternatives
The proposed method	$A_2 > A_4 > A_1 > A_3$
PyPHFWA[4]	$A_2 > A_4 > A_1 > A_3$
PyPHFWG[4]	$A_2 > A_4 > A_1 > A_3$
PyPHFOWA[4]	$A_2 > A_4 > A_1 > A_3$
PyPHFOWG[4]	$A_2 > A_4 > A_1 > A_3$
PyPHFHWA[4]	$A_2 > A_4 > A_1 > A_3$
PyPHFHWG[4]	$A_2 > A_4 > A_1 > A_3$

6. Discussion and Conclusion

Q- rung orthopair fuzzy sets revealed as generalization of pythagorean fuzzy sets and intuitionistic fuzzy sets is very important that it presents a comparative analysis within itself, contains multiple structures within own of it, and changes according to the desire, request and need of the decision makers. Probabilistic hesitant fuzzy sets (PHFS) propose to evaluate for an each element in cluster probabilistic concept for experts. Q- rung orthopair probabilistic hesitant fuzzy sets (q-ROPHs) are to presented by combining these both structures. Aggregation operators based on q-ROPHs called q-ROPHWA and q-ROPHWG are significant mathematical tool to aggregate presented information. In this paper, we define $q-ROPHHWAG$ and $q-ROPHHOWAG$ because of some drawbacks q-ROPHWA and q-ROPHWG. Then, some properties of both operators are given. The presented operators are superior according to existing operators as q-ROPHWA and q-ROPHWG to overcome with fuzzy and ambiguous information.

1. Combining two operators in the same formula will prevent separate calculations and will enable us to obtain fast solutions in the future;
2. The use of two different variables is important for the decision makers in terms of the precision of the results.
3. Self-comparison analysis is very necessary in this age of noisy information.

Furthermore, we established an algorithm and example to indicate effective our operators and gave comparative analysis, and results are almost agreement.

In the future, we will use our work to solve other real life MCDM problems by using different aggregation operators, TOPSIS, VIKOR, ELECTRE family and PROMETHEE based on interval probabilistic hesitant fuzzy sets, dual probabilistic hesitant fuzzy sets and pythagorean dual probabilistic fuzzy sets.

Declaration and Ethical Code

In this study, we undertake that all the rules required to be followed within the scope of the "Higher Education Institutions Scientific Research and Publication Ethics Directive" are complied with, and that none of the actions stated under the heading "Actions Against Scientific Research and Publication Ethics" are not carried out.

References

[1] Xu Z.S., Zhou W. 1986. Consensus building with a group of decision makers under the hesitant probabilistic fuzzy environment. Fuzzy Optim. Decis. Mak. 16(4), 481-503, 2017.Atanassov, K. T., Intuitionistic fuzzy sets. Fuzzy Sets Syst. 20(1), 87-96.

[2] Zhang, S., Xu ZS, He, Y. 2017. Operations and integrations of probabilistic hesitant fuzzy information in decision making" Inf Fusion 38, 1-11.

[3] Zhai ,Y., Xu, Z., Liao, H. 2017. Measures of probabilistic interval-valued intuitionistic hesitant fuzzy sets and the application in reducing excessive medical examinations. IEEE Transactions on Fuzzy Systems, 26(3), 1651-1670.

[4] Batool,, B., Abdullah, S., Ashraf, S., Ahmad, M. 2021. Pythagorean probabilistic hesitant fuzzy aggregation operators and their application in decision-making. Kybernetes 6, 688-694.

[5] Batool, B., Abosuliman, SS, Abdullah, S., Ashraf, S. 2021. EDAS method for decision support modeling under the Pythagorean probabilistic hesitant fuzzy aggregation information. Journal of Ambient Intelligence and Humanized Computing, 16(5) 1-14.

[6] Ren, Y., Yuan, X., Zhao, X., Yu, B. 2021. Calculation and aggregation of Q-rung orthopair probabilistic hesitant fuzzy information. IEEE 5th Advanced Information Technology, Electronic and Automation Control Conference (IAEAC) 5, 2146-2150.

[7] Ren ,Y., Yuan, X., Lin, R. 2021. A novel MADM algorithm for landfill site selection based on q-rung orthopair probabilistic hesitant fuzzy power Muirhead mean operatör. Plos one, 16(10), 258-275.

[8] Ashraf, S., Kousar, M., Hameed, M. S. 2023. Early infectious diseases identification based on complex probabilistic hesitant fuzzy N-soft information. *Soft Computing*, 1-26.

[9] Attaullah, Ashraf, S., Rehman, N., & Khan, A. 2023. q-Rung Orthopair Probabilistic Hesitant Fuzzy Rough Aggregation Information and Their Application in Decision Making. International Journal of Fuzzy Systems, 25(5), 2067-2080.

[10] Wan, B., Zhang, J., Garg, H., & Huang, W. (2023). Q-rung orthopair hesitant fuzzy preference relations and its group decision-making application. Complex & Intelligent Systems, 1-22.

[11] Qahtan, S., Alsattar, H. A., Zaidan, A. A., Deveci, M., Pamucar, D., Delen, D., & Pedrycz, W. 2023. Evaluation of agriculture-food 4.0 supply chain approaches using Fermatean probabilistic hesitant-fuzzy sets based decision making model. Applied Soft Computing, 138, 110170.

[12] Liao, N., Wei, G., & Chen, X. 2022. TODIM method based on cumulative prospect theory for multiple attributes group decision making under probabilistic hesitant fuzzy setting. International Journal of Fuzzy Systems, 1-18.