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On Certain Properties of Bipolar Fuzzy Supra Preopen Sets

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Abstract
The goal of this paper is to investigate the certain properties of bipolar fuzzy supra preopen sets (BF supra preopen set). We introduce and study the concepts of bipolar fuzzy supra pre-interior (BF supra pre-interior) and bipolar fuzzy supra pre-closure (BF supra pre-closure) operators. Furthermore, we define new type of bipolar fuzzy supra continuous mapping based on the BF preopen sets.
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1. Introduction

Fuzzy set theory was introduced by Zadeh [1] in 1965 and it has been developed by many researchers. This theory is one of the most efficient decision methods providing the ability to deal with uncertainty. Several papers are published with various fuzzy set applications in the fields of pure and applied mathematics and also several kinds of fuzzy set extensions were defined such as intuitionistic fuzzy sets [2], interval-valued fuzzy sets [3], vague sets [4], etc. Bipolar-valued fuzzy set [5] is another an extension of fuzzy set and its membership degree range is different from the above sets. Membership degree range is [-1, 1] for bipolar valued fuzzy set. Lee [6] initiated an extension of fuzzy set named bipolar fuzzy set in 2000. In recent years a series of works was published related to bipolar fuzzy sets. Anitha et. al. [7] investigated the notion of bipolar valued fuzzy subgroup. Pazar Varol [8] defined bipolar fuzzy submodule and studied some certain properties. Azhagappan and Kamaraj [9] introduced the concept of bipolar fuzzy topology. Kim et. al. [10] introduced bipolar fuzzy points and examined the topological structures of bipolar fuzzy set, such as neighborhood, continuity, base, subbase.

Supra topology was defined by dropping a finite intersection condition of topological spaces by Mashhour et al. [11] in 1983. Although supra topological space is the weaker type of classical topological space, supra topology can be more convenient to solve some practical problems. The fuzzy supra topology were introduced by El Monsef and Ramadan [12]. They also investigated fuzzy supra continuous mappings and obtained some basic characterizations. Using the notion of intuitionistic fuzzy sets, Turanlı [13] defined intuitionistic fuzzy supra topology in 2003. Malkoç and Pazar Varol [14] discussed BF supra topological spaces as a generalization of supra topologies to bipolar fuzzy topologies.

The concept of preopen sets in topological spaces was defined by Mashhour et. al. [15] in 1982. Then, the concept of fuzzy preopen sets in fuzzy topological spaces was given by Singal et.al. [16] in 1991. Sayed [17] defined supra preopen set in 2010 and fuzzy version of this concept was studied by Srikirutika et. al. [18] as “fuzzy supra preopen sets” in 2018.

In this work, we introduce and investigate bipolar fuzzy supra preopen sets using bipolar fuzzy supra open sets. Then, we define new type of bipolar continuous mapping based on the bipolar fuzzy preopen sets.

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2. Preliminaries

We recall the following definitions and results which will be needed in this work.

Definition 2.1: [19] Let $U \neq \emptyset$ be universal set. $\mathcal{Q} = \{< v, \mu_{\mathcal{Q}}^{+}(v), \mu_{\mathcal{Q}}^{-}(v) >: v \in U\}$ denotes a bipolar fuzzy set (BF set) in U where $\mu_{\mathcal{Q}}^{+}: U \longrightarrow [0,1]$ and $\mu_{\mathcal{Q}}^{-}: U \longrightarrow [-1,0]$ are two mappings. Here, $\mu_{\mathcal{Q}}^{+}(v)$ denotes the positive memberships ranges over $[0,1]$ and $\mu_{\mathcal{Q}}^{-}(v)$ denotes the negative memberships ranges over $[-1,0]$.

We will use the notation $BPF(U)$ for the family of all bipolar fuzzy set in U.
For each bipolar fuzzy set $\boldsymbol{\mathcal{Q}}$ and $\boldsymbol{v} \in \boldsymbol{U}$, if $\mathbf{0} \leq \boldsymbol{\mu_{\mathcal{Q}}^{+}(v)} - \boldsymbol{\mu_{\mathcal{Q}}^{-}(v)} \leq \mathbf{1}$ then $\boldsymbol{\mathcal{Q}}$ is an intuitionistic fuzzy set [2]. $\boldsymbol{\mu_{\mathcal{Q}}^{+}(v)}$ (resp. $\boldsymbol{\mu_{\mathcal{Q}}^{-}(v)}$) denotes the membership degree (resp. non-membership degree) of $\boldsymbol{v} \in \boldsymbol{U}$.

Example 2.2: $\boldsymbol{\mathcal{Q}} = \{< \boldsymbol{a}, \mathbf{0.5}, -\mathbf{0.3} >, < \boldsymbol{b}, \mathbf{0.7}, -\mathbf{0.2} >, < \boldsymbol{c}, \mathbf{0.4}, -\mathbf{0.4} >\}$ is a BF set in $\boldsymbol{U} = \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$.

Definition 2.3: [9] 1. $\mathcal{Q} \in BPF(U)$ is called universal BF set if $\mu_{\mathcal{Q}}^{+}(v) = 1_{BP}^{+}(v) = 1$ and $\mu_{\mathcal{Q}}^{-}(v) = 1_{BP}^{-}(v) = -1$, for each $v \in U$ and we write $1_{BP} = (1_{BP}^{+}, 1_{BP}^{-})$.
2. $\mathcal{Q} \in BPF(U)$ is called empty BF set if $\mu_{\mathcal{Q}}^{+}(v) = 0_{BP}^{+}(v) = 0_{BP} = 0_{BP}^{-}(v) = \mu_{\mathcal{Q}}^{-}(v)$, for each $v \in U$ and we write $0_{BP} = (0_{BP}^{+}, 0_{BP}^{-})$.

Definition 2.4: [19] Let $\mathcal{Q}, \mathcal{R} \in BPF(U)$. Then;
1. $\mathcal{Q} \subseteq \mathcal{R} \iff \mu_{\mathcal{Q}}^{+}(v) \leq \mu_{\mathcal{R}}^{+}(v)$ and $\mu_{\mathcal{Q}}^{-}(v) \geq \mu_{\mathcal{R}}^{-}(v)$, $\forall\, v \in U$.
2. $\mathcal{Q} = \mathcal{R} \iff \mu_{\mathcal{Q}}^{+}(v) = \mu_{\mathcal{R}}^{+}(v)$ and $\mu_{\mathcal{Q}}^{-}(v) = \mu_{\mathcal{R}}^{-}(v)$, $\forall\, v \in U$.
3. The complement of $\mathcal{Q}$ is defined by, $\mathcal{Q}^{c} = \{< v, 1 - \mu_{\mathcal{Q}}^{+}(v), -1 - \mu_{\mathcal{Q}}^{-}(v) >: v \in U\}$.
4. $\mathcal{Q} \cap \mathcal{R} = \{< v, \mu_{\mathcal{Q}\cap\mathcal{R}}^{+}(v), \mu_{\mathcal{Q}\cap\mathcal{R}}^{-}(v) >: v \in U\}$, where $\mu_{\mathcal{Q}\cap\mathcal{R}}^{+}(v) = \min\{\mu_{\mathcal{Q}}^{+}(v), \mu_{\mathcal{R}}^{+}(v)\}$ and
$\mu_{\mathcal{Q}\cap Y}^{-}(v) = \max\{\mu_{\mathcal{Q}}^{-}(v), \mu_{\mathcal{R}}^{-}(v)\}$.
5. $\mathcal{Q} \cup Y = \{< v, \mu_{\mathcal{Q}\cup\mathcal{R}}^{+}(v), \mu_{\mathcal{Q}\cup\mathcal{R}}^{-}(v) >: v \in U\}$, where $\mu_{\mathcal{Q}\cup\mathcal{R}}^{+}(v) = \max\{\mu_{\mathcal{Q}}^{+}(v), \mu_{\mathcal{R}}^{+}(v)\}$ and
$\mu_{\mathcal{Q}\cup Y}^{-}(v) = \min\{\mu_{\mathcal{Q}}^{-}(v), \mu_{\mathcal{R}}^{-}(v)\}$.

Proposition 2.5: [10] Let $\mathcal{Q}, \mathcal{R}, \mathcal{Z} \in BPF(U)$. Then the following statements are satisfied:
1. $\mathcal{Q} \cup \mathcal{Q} = \mathcal{Q}$ and $\mathcal{Q} \cap \mathcal{Q} = \mathcal{Q}$.
2. $\mathcal{Q} \cup \mathcal{R} = \mathcal{Q} \cup \mathcal{R}$ and $\mathcal{Q} \cap \mathcal{R} = \mathcal{R} \cap \mathcal{Q}$.
3. $\mathcal{Q} \cup (\mathcal{R} \cup \mathcal{Z}) = (\mathcal{Q} \cup \mathcal{R}) \cup \mathcal{Z}$ and $\mathcal{Q} \cap (\mathcal{R} \cap \mathcal{Z}) = (\mathcal{Q} \cap \mathcal{R}) \cap \mathcal{Z}$.
4. $\mathcal{Q} \cup (\mathcal{R} \cap \mathcal{Z}) = (\mathcal{Q} \cup \mathcal{R}) \cap (\mathcal{Q} \cup \mathcal{Z})$ and $\mathcal{Q} \cap (\mathcal{R} \cup \mathcal{Z}) = (\mathcal{Q} \cap \mathcal{R}) \cup (\mathcal{Q} \cap \mathcal{Z})$.
5. $\mathcal{Q} \cup (\mathcal{Q} \cap \mathcal{R}) = \mathcal{Q}$ and $\mathcal{Q} \cap (\mathcal{Q} \cup \mathcal{R}) = \mathcal{Q}$.
6. $\mathcal{Q} \cap \mathcal{R} \subset \mathcal{Q}$ and $\mathcal{Q} \cap \mathcal{R} \subset \mathcal{R}$.
7. $\mathcal{Q} \subset \mathcal{Q} \cup \mathcal{R}$ and $Y \subset \mathcal{Q} \cup \mathcal{R}$.
8. $(\mathcal{Q}^{c})^{c} = \mathcal{Q}$.
9. $(\mathcal{Q} \cup \mathcal{R})^{c} = \mathcal{Q}^{c} \cap \mathcal{R}^{c}$ and $(\mathcal{Q} \cap \mathcal{R})^{c} = \mathcal{Q}^{c} \cup \mathcal{R}^{c}$.
10. If $\mathcal{Q} \subset \mathcal{R}$ and $\mathcal{R} \subset \mathcal{Z}$, then $\mathcal{Q} \subset \mathcal{Z}$.
11. If $\mathcal{Q} \subset \mathcal{R}$, then $\mathcal{Q} \cap \mathcal{Z} \subset \mathcal{R} \cap \mathcal{Z}$ and $\mathcal{Q} \cup \mathcal{Z} \subset \mathcal{R} \cup \mathcal{Z}$.

Definition 2.6: [19] Let $U \neq \emptyset$ and $(\mathcal{Q}_j)_{j\in J} \subset BPF(U)$.
1. The intersection of $(\mathcal{Q}_j)_{j\in J}$, represented by $\bigcap_{j\in J} \mathcal{Q}_j$, and defined as
$(\bigcap_{j\in J} \mathcal{Q}_j)(v) = \left(\bigwedge_{j\in J} \mu_{\mathcal{Q}_j}^{+}(v), \bigvee_{j\in J} \mu_{\mathcal{Q}_j}^{-}(v)\right)$, $\forall v \in U$.
2. The union of $(\mathcal{Q}_j)_{j\in J}$, represented by $\bigcup_{j\in J} \mathcal{Q}_j$ and defined as
$(\bigcup_{j\in J} \mathcal{Q}_j)(v) = \left(\bigvee_{j\in J} \mu_{\mathcal{Q}_j}^{+}(v), \bigwedge_{j\in J} \mu_{\mathcal{Q}_j}^{-}(v)\right)$, $\forall v \in U$.

Definition 2.7: [10] Let $\varphi: U_1 \longrightarrow U_2$ be a mapping and $\mathcal{Q} \in BPF(U_1)$, $\mathcal{R} \in BPF(U_2)$.
1. The image of $\mathcal{Q}$ under φ is represented by $\varphi(\mathcal{Q})(w) = \left(\mu_{\varphi(\mathcal{Q})}^{+}(w), \mu_{\varphi(\mathcal{Q})}^{-}(w)\right) = \left(\varphi(\mu_{\mathcal{Q}}^{+})(w), \varphi(\mu_{\mathcal{Q}}^{-})(w)\right)$, and it is a bipolar fuzzy set in U_2 defined as

$$\varphi(\mu_{\mathcal{Q}}^{+})(w) = \begin{cases} \bigvee \mu_{\mathcal{Q}}^{+}(v), & v \in \varphi^{-1}(w), \\ 0, & other \end{cases}$$

$$\varphi(\mu_{\mathcal{Q}}^{-})(w) = \begin{cases} \bigwedge \mu_{\mathcal{Q}}^{-}(v), & v \in \varphi^{-1}(w), \\ 0, & other \end{cases},$$

$\forall\, w \in U_2$.
2. The preimage of $\mathcal{R}$ under φ is represented by $\varphi^{-1}(\mathcal{R}) = \left(\varphi^{-1}(\mu_{\mathcal{R}}^{+}), \varphi^{-1}(\mu_{\mathcal{R}}^{-})\right)$, is a bipolar fuzzy set in U_1 defined by
$[\varphi^{-1}(\mu_{\mathcal{R}}^{+})](v) = \mu_{\mathcal{R}}^{+} \circ \varphi(v)$ and
$[\varphi^{-1}(\mu_{\mathcal{R}}^{-})](v) = \mu_{\mathcal{R}}^{-} \circ \varphi(v)$, $\forall\, v \in U_1$.

Corollary 2.8: [10] Let $\varphi: U_1 \longrightarrow U_2$ be a mapping and $\mathcal{Q}, \mathcal{Q}_1, \mathcal{Q}_2 \in BPF(U_1)$, $(\mathcal{Q}_j)_{j\in J} \subset BPF(U_1)$, $\mathcal{R}$, $\mathcal{R}_1, \mathcal{R}_2 \in BPF(U_2)$ and $(\mathcal{R}_j)_{j\in J} \subset BPF(U_2)$. Then the followings are satisfied;
1. If $\mathcal{Q}_1 \subset \mathcal{Q}_2$, then $\varphi(\mathcal{Q}_1) \subset \varphi(\mathcal{Q}_2)$,

2. $\varphi(\mathcal{Q}_1 \cup \mathcal{Q}_2) = \varphi(\mathcal{Q}_1) \cup \varphi(\mathcal{Q}_2)$, $\varphi(\bigcup_{j\in J}\mathcal{Q}_j) = \bigcup_{j\in J}\varphi(\mathcal{Q}_j)$,
3. $\varphi(\mathcal{Q}_1 \cap \mathcal{Q}_2) \subset \varphi(\mathcal{Q}_1) \cap \varphi(\mathcal{Q}_2)$, $\varphi(\bigcap_{j\in J}\mathcal{Q}_j) \subset \bigcap_{j\in J}\varphi(\mathcal{Q}_j)$,
4. If φ is 1-1, then $\varphi(\mathcal{Q}_1 \cap \mathcal{Q}_2) = \varphi(\mathcal{Q}_1) \cap \varphi(\mathcal{Q}_2)$, $\varphi(\bigcap_{j\in J}\mathcal{Q}_j) = \bigcap_{j\in J}\varphi(\mathcal{Q}_j)$,
5. If $\mathcal{R}_1 \subset \mathcal{R}_2$, then $\varphi^{-1}(\mathcal{R}_1) \subset \varphi^{-1}(\mathcal{R}_2)$,
6. $\varphi(\mathcal{Q}) = 0_{BP} \Leftrightarrow \mathcal{Q} = 0_{BP}$,
7. $\varphi^{-1}(\mathcal{R}_1 \cup \mathcal{R}_2) = \varphi^{-1}(\mathcal{R}_1) \cup \varphi^{-1}(\mathcal{R}_2)$, $\varphi^{-1}(\bigcup_{j\in J}\mathcal{R}_j) = \bigcup_{j\in J}\varphi^{-1}(\mathcal{R}_j)$,
8. $\varphi^{-1}(\mathcal{R}_1 \cap \mathcal{R}_2) = \varphi^{-1}(\mathcal{R}_1) \cap \varphi^{-1}(\mathcal{R}_2)$, $\varphi^{-1}(\bigcap_{j\in J}\mathcal{R}_j) = \bigcap_{j\in J}\varphi^{-1}(\mathcal{R}_j)$,
9. $\varphi^{-1}(\mathcal{R}) = 0_{BP} \Leftrightarrow \mathcal{R} \cap \varphi(1_{BP}) = 0_{BP}$,
10. $\mathcal{Q} \subset (\varphi^{-1}o\varphi)(\mathcal{Q})$, in particular $\mathcal{Q} = (\varphi^{-1}o\varphi)(\mathcal{Q})$ if φ is injective,
11. $(\varphi o\varphi^{-1})(\mathcal{R}) \subset \mathcal{R}$, in particular $\mathcal{R} = (\varphi o\varphi^{-1})(\mathcal{R})$ if φ is surjective,
12. $\varphi^{-1}(\mathcal{R}^c) = (\varphi^{-1}(\mathcal{R}))^c$.

Definition 2.9: [9] Let $U \neq \emptyset$ and $\tau \subset BPF(U)$. Then τ is called a BF topology on U if it satisfies the followings;
(BFT1) $0_{BP}, 1_{BP} \in \tau$.
(BFT2) $\mathcal{Q} \cap \mathcal{R} \in \tau$, for $\mathcal{Q}, \mathcal{R} \in \tau$.
(BFT3) $(\bigcup_{j\in J}\mathcal{Q}_j) \in \tau$ for every $(\mathcal{Q}_j)_{j\in J} \subset \tau$.
(U,τ) is named to be BF topological space and members of τ are called to be BF open sets. $\mathcal{Q} \in BPF(U)$ is called BF closed sets in (U,τ), if $\mathcal{Q}^c$ is BF open set.

Definition 2.10: [14] Let $U \neq \emptyset$ and $\tau \subset BPF(U)$. Then τ is called a BF supra topology on U if it satisfies the followings;
(BFST1) $0_{BP}, 1_{BP} \in \tau$.
(BFST2) $(\bigcup_{j\in J}\mathcal{Q}_j) \in \tau$ for every $(\mathcal{Q}_j)_{j\in J} \subset \tau$.
(U,τ) is named to be BF supra topological space and members of τ are called BF supra open sets. $\mathcal{Q} \in BPF(U)$ is called BF supra closed sets in (U,τ), if $\mathcal{Q}^c$ is BF supra open set. We denote the family of all bipolar fuzzy supra topologies on U as $BPFST(U)$.

Let τ^* be a BF topology and τ be a BF supra topology on U. Then, τ^* is named to be associated BF topology with the τ if and only if $\tau^* \subset \tau$.

Examples 2.11: [14]. Let $\tau \in BPFST(U)$. Then, the families $\tau^+ = \{\mu_{\mathcal{Q}}^+ \in I^U \mid \mathcal{Q} \in \tau\}$ and $\tau^- = \{-\mu_{\mathcal{Q}}^- \in I^U \mid \mathcal{Q} \in \tau\}$ are two fuzzy supra topologies in the sense of [12].

Theorem 2.12: [14] Let (U,τ) be a BF supra topological space and $\mathcal{K}$ be the family of all BF supra closed sets in U. Then the followings are true;
i. $0_{BP}, 1_{BP} \in \mathcal{K}$.
ii. $(\bigcap_{i\in J}\mathcal{Q}_i) \in \mathcal{K}$ for every $(\mathcal{Q}_i)_{j\in J} \subset \mathcal{K}$.

Definition 2.13: [14] Let $\tau \in BPFST(U)$ and $\mathcal{Q} \in BPF(U)$. Then;
i. Bipolar fuzzy supra interior of $\mathcal{Q}$, denoted by $int_\tau(\mathcal{Q})$, is defined as

$$int_\tau(\mathcal{Q}) = \bigcup\{O : O \subseteq \mathcal{Q} \text{ and } O \in \tau\}.$$

ii. Bipolar fuzzy supra closure of $\mathcal{Q}$, denoted by $cl_\tau(\mathcal{Q})$, is defined as

$$cl_\tau(\mathcal{Q}) = \bigcap\{K : \mathcal{Q} \subseteq K \text{ and } K^c \in \tau\}.$$

(1) The BF supra closure of $\mathcal{Q}$ is the smallest BF supra closed set containing $\mathcal{Q}$.
(2) The BF supra interior of $\mathcal{Q}$ is the largest BF supra open set contained in $\mathcal{Q}$.
(3) Let (U,τ) be an associated BF supra topological space with the BF topological space (U,τ^*) and $\mathcal{Q} \in BPF(U)$. Then $int_{\tau^*}(\mathcal{Q}) \subseteq int_\tau(\mathcal{Q}) \subseteq \mathcal{Q} \subseteq cl_\tau(\mathcal{Q}) \subseteq cl_{\tau^*}(\mathcal{Q})$.

Theorem 2.14: [14] Let $\mathcal{Q}, \mathcal{R} \in BPF(U)$ and $\tau \in BPFST(U)$. Following statements are true;
1. $\mathcal{Q}$ is a BF supra open (closed) set $\Leftrightarrow \mathcal{Q} = int_\tau(\mathcal{Q})$ $(\mathcal{Q} = cl_\tau(\mathcal{Q}))$
2. If $\mathcal{Q} \subseteq \mathcal{R}$, then $int_\tau(\mathcal{Q}) \subseteq int_\tau(\mathcal{R})$ and $cl_\tau(\mathcal{Q}) \subseteq cl_\tau(\mathcal{R})$.
3. $cl_\tau(\mathcal{Q}) \cup cl_\tau(\mathcal{R}) \subseteq cl_\tau(\mathcal{Q} \cup \mathcal{R})$.
4. $int_\tau(\mathcal{Q}) \cup int_\tau(\mathcal{R}) \subseteq int_\tau(\mathcal{Q} \cup \mathcal{R})$.
5. $int_\tau(\mathcal{Q} \cap \mathcal{R}) \subseteq int_\tau(\mathcal{Q}) \cap int_\tau(\mathcal{R})$.
6. $cl_\tau(\mathcal{Q} \cap \mathcal{R}) \subseteq cl_\tau(\mathcal{Q}) \cap cl_\tau(\mathcal{R})$.
7. $int_\tau(1_{BP} - \mathcal{Q}) = 1_{BP} - cl_\tau(\mathcal{Q})$.
8. $cl_\tau(1_{BP}) = 1_{BP} = int_\tau(1_{BP})$ and $cl_\tau(0_{BP}) = 0_{BP} = int_\tau(0_{BP})$.
9. $int_\tau(int_\tau(\mathcal{Q})) = int_\tau(\mathcal{Q})$, $cl_\tau(cl_\tau(\mathcal{Q})) = cl_\tau(\mathcal{Q})$.

Definition 2.15: [14] Let $\tau \in BPFST(U_1)$, $\sigma \in BPFST(U_2)$ and $\varphi: (U_1,\tau) \longrightarrow (U_2,\sigma)$ be a mapping. Then, φ is called a BF supra continuous mapping if $\varphi^{-1}(\mathcal{Q}) \in \tau$ for every $\mathcal{Q} \in \sigma$.

3. Bipolar Fuzzy Supra Preopen Sets

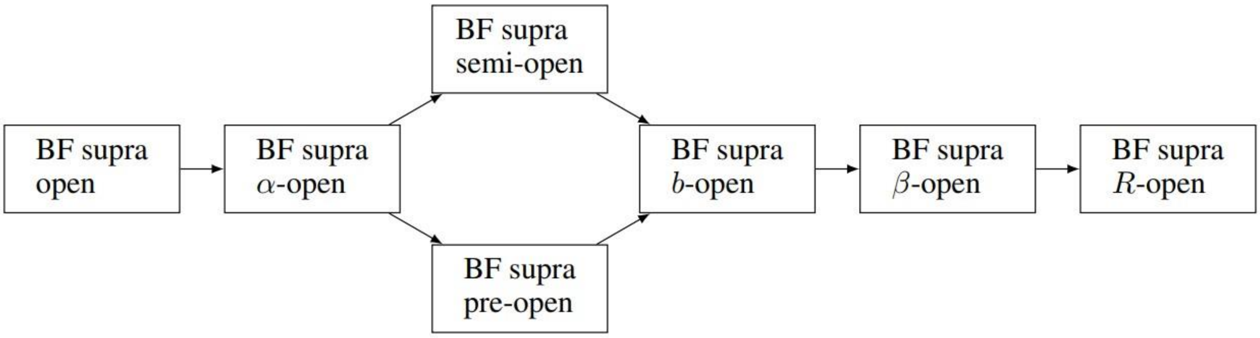
In this chapter, we initiate and study a new type of supra open sets named to be bipolar fuzzy supra preopen sets.

Definition 3.1: Let $\tau \in BPFST(U)$. A BF set $\mathcal{Q}$ is called
(i) a bipolar fuzzy supra preopen set (BF supra preopen set) if $\mathcal{Q} \subseteq int_{\boldsymbol{\tau}}(cl_{\boldsymbol{\tau}}(\mathcal{Q}))$.

(ii) a bipolar fuzzy supra preclosed set (BF supra preclosed set) if $cl_{\tau}(int_{\tau}(Q)) \subseteq Q$.
It is obvious that the complement of a bipolar fuzzy supra preopen set is a bipolar fuzzy supra preclosed set.

Definition 3.2: Let $\tau \in BPFST(U)$. A BF set Q is called
(i) a bipolar fuzzy supra α −open set if $Q \subseteq int_{\tau}(cl_{\tau}(int_{\tau}(Q)))$.
(ii) a bipolar fuzzy supra b −open set if $Q \subseteq int_{\tau}(cl_{\tau}(Q)) \cup cl_{\tau}(int_{\tau}(Q))$.
(iii) a bipolar fuzzy supra β −open set if $Q \subseteq cl_{\tau}(int_{\tau}(cl_{\tau}(Q)))$.
(iv) a bipolar fuzzy supra semi-open set if $Q \subseteq cl_{\tau}(int_{\tau}(Q))$.
(v) a bipolar fuzzy supra R −open set if $int_{\tau}(cl_{\tau}(Q)) \neq 0_{BP}$ or $Q = 0_{BP}$.

The following diagram gives the relations between the bipolar fuzzy open sets. In this work, we only focus on bipolar fuzzy supra pre-open set.



Theorem 3.3: Let $\tau \in BPFST(U)$. If Q is BF supra open set, then Q is BF supra preopen set.
Proof: Let $Q \in \tau$. Then $int_{\tau}(Q) = Q$. Since $Q \subseteq cl_{\tau}(Q)$ and properties of BF supra interior, $int_{\tau}(Q) \subseteq int_{\tau}(cl_{\tau}(Q))$.
Hence, $Q \subseteq int_{\tau}(cl_{\tau}(Q))$.

We show that the implication in above theorem is not reversible;

Example 3.4: Let $U = \{u_1, u_2\}$,
$Q = \{< u_1, 0.5, -0.3 >, < u_2, 0.3, -0.4 >\}$
$\mathcal{R} = \{< u_1, 0.3, -0.6 >, < u_2, 0.6, -0.5 >\}$
$\mathcal{Z} = \{< u_1, 0.5, -0.8 >, < u_2, 0.4, -0.5 >\}$
and $\tau = \{0_{BP}, 1_{BP}, Q, \mathcal{R}, Q \cup \mathcal{R}\}$. Then τ is a BF supra topology on U and $\mathcal{Z}$ is a BF supra preopen set (we obtain $int_{\tau}(cl_{\tau}(\mathcal{Z})) = 1_{BP}$, so $\mathcal{Z} \subseteq int_{\tau}(cl_{\tau}(\mathcal{Z}))$) but not BF supra open set.

Theorem 3.4: Let $\tau \in BPFST(U)$. If Q is BF supra closed set, then Q is BF supra preclosed set.
Proof: Let Q be a BF supra closed set in (U, τ). Then $Q = cl_{\tau}(Q)$. Since $int_{\tau}(Q) \subseteq Q$, we get $cl_{\tau}(int_{\tau}(Q)) \subseteq cl_{\tau}(Q) = Q$.

Theorem 3.5: Let Q be a BF set in BF supra topological space (U, τ), then Q is a BF supra preopen set if and only if Q^c is a BF supra preclosed set.

Proof: ($\Longrightarrow$) Let Q be a BF preopen set in U. Then we have $Q \subseteq int_{\tau}(cl_{\tau}(Q))$.
$\Longrightarrow \quad Q^c \supseteq \left(int_{\tau}(cl_{\tau}(X))\right)^c = cl_{\tau}(cl_{\tau}(Q)^c) = cl_{\tau}(int_{\tau}(Q^c))$.
$\Longrightarrow Q^c$ is a BF supra preclosed set.
($\Longleftarrow$) Let Q^c be a BF supra preclosed set. Hence, we have $cl_{\tau}(int_{\tau}(Q^c)) \subseteq Q^c$.
$\Longrightarrow (Q^c)^c \subseteq \left(cl_{\tau}(int_{\tau}(Q^c))\right)^c$
$\Longrightarrow Q \subseteq int_{\tau}(int_{\tau}(Q^c))^c = int_{\tau}(cl_{\tau}(Q^c)^c)$
$= int_{\tau}(cl_{\tau}(Q))$.
Hence, Q is a BF preopen set.

Theorem 3.6: Let $\tau \in BPFST(U)$.
(i) If $\{Q_i : i \in J\}$ is a collection of BF supra preopen sets, then $\bigcup_{i \in J} Q_i$ is a BF supra preopen set.
(ii) If $\{\mathcal{R}_i : i \in J\}$ is a collection of BF supra preclosed sets, then $\bigcap_{i \in J} \mathcal{R}_i$ is a BF supra preclosed set.
Proof: (i) Let $\{Q_i : i \in J\}$ be a collection of BF supra preopen sets.
Then for each $i \in J$, $Q_i \subseteq int_{\tau}(cl_{\tau}(Q_i))$.
$\Longrightarrow \quad \bigcup_{i \in J} Q_i \subseteq \bigcup_{i \in J} int_{\tau}(cl_{\tau}(Q_i)) \subseteq int_{\tau}(\bigcup_{i \in J} cl_{\tau}(Q_i)) \subseteq int_{\tau}(cl_{\tau}(\bigcup_{i \in J} Q_i))$.
(ii) By (i), we get $\bigcup_{i \in J} Q_i \subseteq int_{\tau}(cl_{\tau}(\bigcup_{i \in J} Q_i))$.
$\Longrightarrow \left(\bigcup_{i \in J} Q_i\right)^c \supseteq \left(int_{\tau}\left(cl_{\tau}(\bigcup_{i \in J} Q_i)\right)\right)^c$.

$\Longrightarrow \quad \bigcap_{i\in J}(\mathcal{Q}_i)^c \quad \supseteq cl_{\tau}\left(\left(cl_{\tau}(\bigcup_{i\in J}\mathcal{Q}_i)\right)^c\right) = cl_{\tau}\left(int_{\tau}(\bigcap_{i\in J}\mathcal{Q}_i^c)\right)$.

Definition 3.7: Let $\tau \in BPFST(U)$ and $\mathcal{Q} \in BPF(U)$.
(i) The bipolar fuzzy supra preinterior of a bipolar fuzzy set $\mathcal{Q}$ is defined by
$pint_{\tau}(\mathcal{Q}) = \bigcup\{ \mathcal{O} : \mathcal{O} \subseteq \mathcal{Q}$ and $\mathcal{O}$ is a bipolar fuzzy supra preopen set in $U\}$.
(ii) The bipolar fuzzy supra preclosure of a bipolar fuzzy set $\mathcal{Q}$ is defined by
$pcl_{\tau}(\mathcal{Q}) = \bigcap\{ \mathcal{K}: \mathcal{Q} \subseteq \mathcal{K}$ and $\mathcal{K}$ is a bipolar fuzzy supra preclosed set in $U\}$.

Remark 3.8: We see that $pint_{\tau}(\mathcal{Q})$ is a BF supra preopen set and $pcl_{\tau}(\mathcal{Q})$ is a BF supra preclosed set.

Theorem 3.9: Let $\mathcal{Q}$ be a BF set in (U,τ).
(i) $pint_{\tau}(\mathcal{Q}) \subseteq \mathcal{Q}$ and $pint_{\tau}(\mathcal{Q}) = \mathcal{Q}$ if and only if $\mathcal{Q}$ is a BF supra preopen set.
(ii) $\mathcal{Q} \subseteq pcl_{\tau}(\mathcal{Q})$ and $\mathcal{Q} = pcl_{\tau}(\mathcal{Q})$ if and only if $\mathcal{Q}$ is a BF supra preclosed set.
(iii) $(pint_{\tau}(\mathcal{Q}))^c = pcl_{\tau}(\mathcal{Q}^c)$.
(iv) $(pcl_{\tau}(\mathcal{Q}))^c = pint_{\tau}(\mathcal{Q}^c)$.
Proof: Straightforward.

Theorem 3.10: Let $\mathcal{Q}$ and $\mathcal{R}$ be two BF sets in (U,τ). If $\mathcal{Q} \subseteq \mathcal{R}$, then $pint_{\tau}(\mathcal{Q}) \subseteq pint_{\tau}(\mathcal{R})$ and $pcl_{\tau}(\mathcal{Q}) \subseteq pcl_{\tau}(\mathcal{R})$.
Proof: Since $pint_{\tau}(\mathcal{Q})$ is the largest BF supra preopen set contained in $\mathcal{Q}$, $pint_{\tau}(\mathcal{Q}) \subseteq \mathcal{Q}$. Then, $pint_{\tau}(\mathcal{Q}) \subseteq \mathcal{Q} \subseteq \mathcal{R}$. $pint_{\tau}(\mathcal{Q})$ is the BF supra preopen set contained in $\mathcal{R}$, but $pint_{\tau}(\mathcal{R})$ is the largest BF supra preopen set contained in $\mathcal{Q}$. So, $pint_{\tau}(\mathcal{Q}) \subseteq pint_{\tau}(\mathcal{R})$. For the other part, $\mathcal{Q} \subseteq \mathcal{R} \Leftrightarrow \mathcal{R}^c \subseteq \mathcal{Q}^c$. Then $pint_{\tau}(\mathcal{R}^c) \subseteq pint_{\tau}(\mathcal{Q}^c)$ and we obtain $(pcl_{\tau}(\mathcal{R}))^c \subseteq (pcl_{\tau}(\mathcal{Q}))^c$ and hence $(pcl_{\tau}(\mathcal{Q}))^c = pint_{\tau}(\mathcal{Q}^c)$.

Theorem 3.11: Let $\mathcal{Q}$ and $\mathcal{R}$ be two BF sets in (U,τ). Then:
(i) $pint_{\tau}(\mathcal{Q}) \cup pint_{\tau}(\mathcal{R}) \subseteq pint_{\tau}(\mathcal{Q} \cup \mathcal{R})$.
(ii) $pint_{\tau}(\mathcal{Q} \cap \mathcal{R}) \subseteq pint_{\tau}(\mathcal{Q}) \cap pint_{\tau}(\mathcal{R})$.
(iii) $pcl_{\tau}(\mathcal{Q} \cap \mathcal{R}) \subseteq pcl_{\tau}(\mathcal{Q}) \cap pcl_{\tau}(\mathcal{R})$.
(iv) $pcl_{\tau}(\mathcal{Q}) \cup pcl_{\tau}(\mathcal{R}) \subseteq pcl_{\tau}(\mathcal{Q} \cup \mathcal{R})$.
Proof: (i) We have $\mathcal{Q} \subseteq \mathcal{Q} \cup \mathcal{R}$ and $\mathcal{R} \subseteq \mathcal{Q} \cup \mathcal{R}$, then $pint_{\tau}(\mathcal{Q}) \subseteq pint_{\tau}(\mathcal{Q} \cup \mathcal{R})$ and
$pint_{\tau}(\mathcal{R}) \subseteq pint_{\tau}(\mathcal{Q} \cup \mathcal{R})$. Hence, $pint_{\tau}(\mathcal{Q}) \cup pint_{\tau}(\mathcal{R}) \subseteq pint_{\tau}(\mathcal{Q} \cup \mathcal{R})$.
The other statements are obtained by similar way.

4. Bipolar Fuzzy Supra Pre-continuous Mappings

Here, we define a new type of continuous mappings called a bipolar fuzzy supra pre-continuous mapping and also discussed their basic properties.

Definition 4.1: Let (U_1,τ) and (U_2,σ) be two BF supra topological spaces, τ^* and σ^* be two associated BF topologies with τ and σ, respectively. A mapping $\varphi: (U_1,\tau^*) \longrightarrow (U_2,\sigma^*)$ is called bipolar fuzzy supra pre-continuous mapping if $\varphi^{-1}(B) \subseteq U_1$ is BF supra preopen set for every $B \in \sigma^*$.

Theorem 4.2: Every BF supra continuous mapping is a BF supra pre-continuous mapping.
Proof: Let $\varphi: (U_1,\tau^*) \longrightarrow (U_2,\sigma^*)$ be a BF continuous mapping and $\mathcal{Q} \in \sigma^*$. Then $\varphi^{-1}(\mathcal{Q}) \in \tau^*$. Since τ is associated with τ^*, then $\tau^* \subseteq \tau$. Hence, $\varphi^{-1}(\mathcal{Q}) \in \tau$. By the Theorem 3.3, $\varphi^{-1}(\mathcal{Q})$ is a BF supra preopen set and φ is a BF supra pre-continuous mapping.

The implication in above theorem is not reversible;

Example 4.3: Let $U_1 = \{a,b\}$, $U_2 = \{u,v\}$ and
$\mathcal{Q} = \{< a, 0.5, -0.3 >, < b, 0.2, -0.4 >\}$
$\mathcal{R} = \{< a, 0.3, -0.6 >, < b, 0.4, -0.5 >\}$
$\mathcal{Z} = \{< u, 0.5, -0.3 >, < v, 0.4, -0.4 >\}$
$\mathcal{Y} = \{< u, 0.5, -0.6 >, < v, 0.4, -0.5 >\}$
Then $\tau = \{0_{BP}, 1_{BP}, \mathcal{Q}, \mathcal{R}, \mathcal{Q} \cup \mathcal{R}\}$ is a BF supra topology on U_1 and $\sigma = \{0_{BP}, 1_{BP}, \mathcal{Z}, \mathcal{Y}, \mathcal{Z} \cup \mathcal{Y}\}$ is a BF supra topology on U_2.
Define a mapping $\varphi: (U_1,\tau) \longrightarrow (U_2,\sigma)$ by $\varphi(a) = u$ and $\varphi(b) = v$. For $\mathcal{Z} \in \sigma$, we obtain $\varphi^{-1}(\mathcal{Z}) = \{< a, 0.5, -0.3 >, < b, 0.4, -0.4 >\} \notin \tau$ and $\varphi^{-1}(\mathcal{Z}) \subseteq int_{\tau}(cl_{\tau}(\varphi^{-1}(\mathcal{Z})))$, so $\varphi^{-1}(\mathcal{Z})$ is a BF supra preopen set. Therefore, φ is a BF supra pre-continuous mapping but not be a BF supra continuous mapping.

Theorem 4.4: Let $\tau \in BPFST(U_1)$ and $\sigma \in BPFST(U_2)$, τ^* and σ^* be two associated with BF topologies with τ and σ, respectively. Let $\varphi: (U_1,\tau^*) \longrightarrow (U_2,\sigma^*)$ be a function. Then the following statements are equivalent:
(1) φ is BF supra pre-continuous mapping.
(2) The inverse image of a closed set in U_2, is a BF supra pre-closed set in U_1.
(3) $pcl(\varphi^{-1}(\mathcal{R})) \subseteq \varphi^{-1}(cl(\mathcal{R}))$, $\forall\, \mathcal{R} \in BPF(U_2)$.

(4) $\varphi\big(pcl(\mathcal{Q})\big) \subseteq cl\big(\varphi(\mathcal{Q})\big)$, $\forall\, \mathcal{Q} \in BPF(U_1)$.
(5) $\varphi^{-1}\big(int(\mathcal{R})\big) \subseteq pint\big(\varphi^{-1}(\mathcal{R})\big)$, $\forall\, \mathcal{R} \in BPF(U_2)$.
Proof: (1 $\Longrightarrow$2) Let $\mathcal{R}$ be a BF supra closed in U_2. Then $\mathcal{R}^c$ is BF supra open set. By hypothesis $\varphi^{-1}(\mathcal{R}^c) = (\varphi^{-1}(\mathcal{R}))^c$ is BF supra preopen set in U_1. So, $\varphi^{-1}(\mathcal{R})$ is BF supra pre-closed set in U_1.
(2 $\Longrightarrow$3) Let $\mathcal{R} \in BPF(U_2)$. Since $cl(\mathcal{R})$ is BF supra closed set in U_2, $\varphi^{-1}(cl(\mathcal{R}))$ is BF supra pre-closed set in U_1. Then, $pcl\big(\varphi^{-1}(\mathcal{R})\big) \subseteq \varphi^{-1}\big(cl(\mathcal{R})\big)$.
(3 $\Longrightarrow$4) Let $\mathcal{Q} \in BPF(U_1)$. By hypothesis $\varphi^{-1}\big(cl(\varphi(\mathcal{Q}))\big) \supseteq pcl(\varphi^{-1}(\varphi(\mathcal{Q}))) \supseteq pcl(\mathcal{Q})$ and then $\varphi\big(pcl(\mathcal{Q})\big) \subseteq cl\big(\varphi(\mathcal{R})\big)$.
(4 $\Longrightarrow$5) Let $\mathcal{R} \in BPF(U_2)$. By hypothesis we get $\varphi\big(pcl((\varphi^{-1}(\mathcal{R}))^c\,)\big) \subseteq cl(\varphi((\varphi^{-1}(\mathcal{R}))^c))$ and $\varphi((pint(\varphi^{-1}(\mathcal{R}))^c) \subseteq cl(\mathcal{R}^c) = \big(int(\mathcal{R})\big)^c$.
Hence, we obtain $\big(pint\varphi^{-1}(\mathcal{R})\big)^c \subseteq \varphi^{-1}(\big(int(\mathcal{R})\big)^c)$ and then $\varphi^{-1}\big(int(\mathcal{R})\big) \subseteq pint\big(\varphi^{-1}(\mathcal{R})\big)$.
(5$\Longrightarrow$1) Let $\mathcal{R}$ be a BF supra open set in U_2 and $\varphi^{-1}\big(int(\mathcal{R})\big) \subseteq pint\big(\varphi^{-1}(\mathcal{R})\big)$. Then, $\varphi^{-1}(\mathcal{R}) \subseteq pint(\varphi^{-1}(\mathcal{R})$. Hence, $\varphi^{-1}(\mathcal{R}) \subseteq pint(\varphi^{-1}(\mathcal{R}))$, but $pint(\varphi^{-1}(\mathcal{R})) \subseteq \varphi^{-1}(\mathcal{R})$. Then, $\varphi^{-1}(\mathcal{R}) = pint(\varphi^{-1}(\mathcal{R}))$. Therefore, $\varphi^{-1}(\mathcal{R})$ is a BF supra preopen set in U_1.

Theorem 4.5: Let $\varphi: (U_1, \tau) \longrightarrow (U_2, \sigma)$ be a BF supra pre-continuous and $g: (U_2, \sigma) \longrightarrow (U_3, \delta)$ be a BF supra continuous. Then $g \circ \varphi$: $(U_1, \tau) \longrightarrow (U_3, \delta)$ is a BF supra pre-continuous mapping.
Proof: Straightforward.

Theorem 4.6: $\varphi: (U_1, \tau) \longrightarrow (U_2, \sigma)$ is a BF supra pre-continuous mapping if one of the followings holds:
(1) $\varphi^{-1}\big(pint(\mathcal{R})\big) \subseteq pint\big(\varphi^{-1}(\mathcal{R})\big)$, $\forall\, \mathcal{R} \in BPF(U_2)$.
(2) $cl(\varphi^{-1}(\mathcal{R})) \subseteq \varphi^{-1}(pcl\mathcal{R})$, $\forall\, \mathcal{R} \in BPF(U_2)$.
(3) $\varphi\big(cl(\mathcal{Q})\big) \subseteq pcl\big(\varphi(\mathcal{Q})\big)$, $\forall\, \mathcal{Q} \in BPF(U_1)$.
Proof: Let (1) be satisfied and $\mathcal{R}$ be a BF supra open set in U_2. Then, $\varphi^{-1}\big(pint(\mathcal{R})\big) \subseteq pint\big(\varphi^{-1}(\mathcal{R})\big)$. We have $\varphi^{-1}(\mathcal{R}) \subseteq int(\varphi^{-1}(\mathcal{R}))$. Hence $\varphi^{-1}(\mathcal{R})$ is a BF supra open set. We know that every BF supra open set is BF supra preopen set.
The others can be proven similarly.

4. Conclusion and Suggestions

We have established a new extension of bipolar fuzzy open sets named bipolar fuzzy supra preopen set. We hope that the finding in this manuscript will be helpful for the researchers concerned with kind of supra open sets.

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Contributions of the Authors

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