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Research Paper

The Effect of α on The Behavior of Entropy Functions

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Abstract— Entropy is a measure used to measure uncertainty and disorder. The entropy measure contributes to the solution of many problems. There are different definitions of entropy used. The α value used in different ways offers new perspectives, new solutions and methods. Being able to choose the appropriate α value brings it closer to the solution. In this study, the effect of α value, which is used in some entropy definitions, on entropy criteria was examined. It was tried to understand in which direction it behaved and its effect on the criterion. With different variations and different α values, it was shown in which direction the entropy values took. It has been determined how it behaves through the $xlogx$ and $logx$ functions. Entropic behaviors were shown with graphs.

Keywords : *Entropy, Uncertainty, Alpha.*

1. Introduction

Over the more than a century of entropy's history, many attempts have been made to interpret and understand entropy. The oldest and most common interpretation of entropy is in terms of any of the related concepts such as 'disorder', ‘uncertainty’, 'complexity', 'spread of energy', 'randomness', 'chaos'. High entropy value means high disorder. First, Boltzmann implied in his writings that entropy changes are related to increased disorder (Boltzmann, 1964). Bridgman is believed to be the first to explain the relationship between disorder and entropy (Bridgman, 1950). The relationship of entropy to disorder is uncertain, qualitative, and highly subjective. It is based on observations that when viewed at the molecular level, it can be perceived as a disorder occurring in the system. This actually applies to many processes, but not all. Shannon never defined entropy as an expression of disorder in his definitions regarding his work on information theory. But he may have defined it as uncertainty. In conclusion, we can say that the increase in disorder can sometimes be qualitatively associated with the increase in entropy. On the other hand, “information” can always be associated with entropy and therefore means more than disorder (Ben-Naim, 2008).

Graphs give us a lot of information about the nature of the function. It is necessary to know what the maximum and minimum point of a function is, or in which direction this function is bent. It is tried to be understood that the trend of the function is in the direction of decreasing or increasing. The direction tells us whether the function is concave upwards or downwards. Shannon entropy is a concave function (Savare & Toscani, 2014). The concavity is clear when the uncertainty is maximum, i.e. in cases of equal probability. $xlogx$ refers to the concave functions to us.

Entropy is an extensive property as it changes with the size of the system. Probability values used in calculating entropy can tell us a lot. The maximum and minimum entropy of the system can be a guide when evaluating the probability distribution. It is important to be able to choose the appropriate uncertainty measure for different systems. In this study, it was tried to understand how different measures behave.

α is a value used in various entropy calculations. Each value of α gives a possible entropy measure. The α value, which is used in different ways in entropy, offers new perspectives, new solutions and methods. The value of α used in entropy measurements may vary according to the measurement system. This value can be used in different ways in entropic functions to see how this value behaves. Choosing the appropriate entropic function and α value for the problem brings it closer to the solution. Therefore, in this study, the effect of α in different entropy functions was tried to be understood. By giving values between 0 and 1.2 and between 0 and 30, the behavior of α was examined. It has been tried to understand how the α value changes the entropy value in these intervals in different uses, and which use will contribute better to the solution of the problems. Which α value are reasonable? It is necessary to pay attention to which α value or entropy function with α will be better.

2. Entropy

There is a need to measure the state-to-state disorder of systems. The definition of entropy was first introduced by Rudolf Clausius in 1865 for use in thermodynamics (Ribeiro et al., 2021). Afterwards, Ludwig Boltzman and Gibbs interpreted entropy statistically (Tsallis, 1988). Shannon proposed entropy by extending the theories of Nyquist and Hartley to measure uncertainty in information transmission (Shannon, 1948). Many entropy definitions have been made based on these studies, and the use of entropy has increased over time.

If there is a single state occurrence in the entropy function, the entropy value will be 0. So, there is no uncertainty. There is no other expected situation. If the probability of occurrence of all states is equal, the entropy value will be maximum. The effect of equalizing state probabilities increases entropy value. The entropy value changes when the states affecting the system change. If these situations increase uncertainty, the entropy value will be high.

Generalized any entropy expression can be defined using the generalized logarithm definition as in the Equation 1, instead of the standard logarithm. where γ and β are two parameters and x is the argument of the function.

$$\ln(x) = \frac{x^{\gamma} - x^{-\beta}}{\gamma + \beta} \tag{1}$$

Under this definition of logarithm, the most general form of generalized entropy is given as in the Equation 2 (Coraddu et al., 2006). W is the number of states in the phase space. According to the values of γ and β, various entropy definitions can be made.

$$S(t) = \langle\ \sum_{i=1}^{W} p_i(t)\widetilde{\ln}(\frac{1}{p_i(t)})\ \rangle = \langle\ \sum_{i=1}^{W} \frac{p_i^{1-\gamma}(t) - p_i^{1+\beta}(t)}{\gamma + \beta}\ \rangle \tag{2}$$

For example, when $\gamma = 1 - q$ and $\beta = 0$ for the Tsallis logarithm, the following entropy is obtained.

$$\widetilde{ln}(x) = ln_{\mathrm{q}}(x) = \frac{x^{1-q} - 1}{1-q} \tag{3}$$

$$S_q = \langle\ \frac{\sum_{i=1}^{W} p_i^q(t) - 1}{1-q}\ \rangle \tag{4}$$

If q>0, S_q is concave and S_q is convex when $q < 0$ (Uslu, 2009). If we write α instead of the q value, the Tsallis entropy will be as follows.

$$Tsallis\ entropy = S_{Tsallis}(X) = \frac{1}{1-\alpha}(\sum_{i=1}^{W} {x_i}^{\alpha} - 1) \tag{5}$$

When $\alpha = 2$ in Tsallis entropy, the entropy alternative Gini index equation is obtained, which is used in machine learning when placing the attributes in the decision trees.

$$Gini\ index = 1 - \sum_{i=1}^{W} {x_i}^2 \tag{6}$$

We get different results depending on the values we assign to α. Renyi(Rényi, 1961) and Karcı(Karci, 2016)(Tuğal & Karcı, 2019) entropy also use the α value. Renyi is defined as $\alpha \geq 0$ and $\alpha \neq 1$ in entropy. As α approaches zero, Renyi entropy more and more evenly weighs all events with nonzero probability. In the limit $\alpha \to 0$, it is Shannon entropy. As α approaches infinity, the Rényi entropy is increasingly determined by the events with the highest probability (wikipedia.org, 2022).

$$Renyi\ Entropy = S_{Renyi}(X) = \frac{1}{1-\alpha} log_2(\sum_{i=1}^{W} {x_i}^{\alpha}) \tag{7}$$

$$Karcı\ Entropy = S_{Karcı}(X) = \sum_{i=1}^{n} (x_i)^{\alpha} * log_2(x_i) \tag{8}$$

When the value of α approaches 1, Shannon entropy, which is a measure of uncertainty in communication, is obtained.

$$Shannon\ Entropy = S_{Shannon}(X) = \sum_{i=1}^{n} x_i * log_2(x_i) \tag{9}$$

We see that different entropies are formed according to the values of α. So, I wanted to examine α with different values.

3. Results and Discussion

The behavior of Shannon entropy and logarithmic function can be seen in Figure 1. According to these basic behaviors, a comparison can be made according to the values that α will take. The aim of this study is to see how

α changes the entropy value in different settlements. So, will its behavior be $xlogx$ or $logx$. Randomly generated 50 x-state probability values were used. The values of x range from 0 to 1. The $xlogx$ entropy was calculated for each x value. It was observed how the value changed as the probability increased according to the location of α.

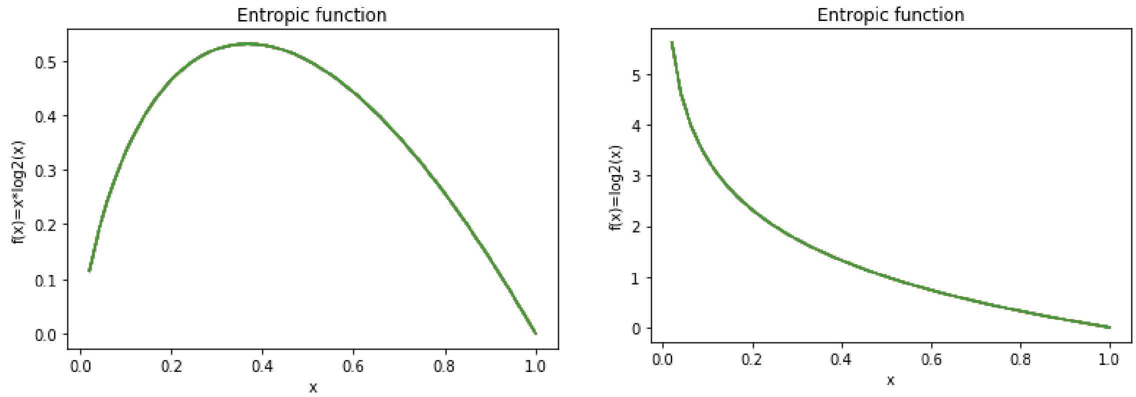

Figure 1. Entropic function

$$f(x_i) = -x_i * log_2(x_i) \quad (10)$$

$$Shannon\ Entropy = \sum_{i=1}^{n} f(x_i) = -\sum_{i=1}^{n} x_i * log_2(x_i) \quad (11)$$

In Figure 1, the behavior of the Shannon entropy function can be seen with serial x values between 0 and 1. The difference between $xlogx$ and $logx$ can be seen. In fact, the α value and the behavior between these two states can be adjusted. The $logx$ function takes a decreasing value as x increases. $xlogx$ increases up to a certain value, then decreases.

Accordingly, in this study, it was examined how the behavior of the function changes according to the use of α and the values it receives.

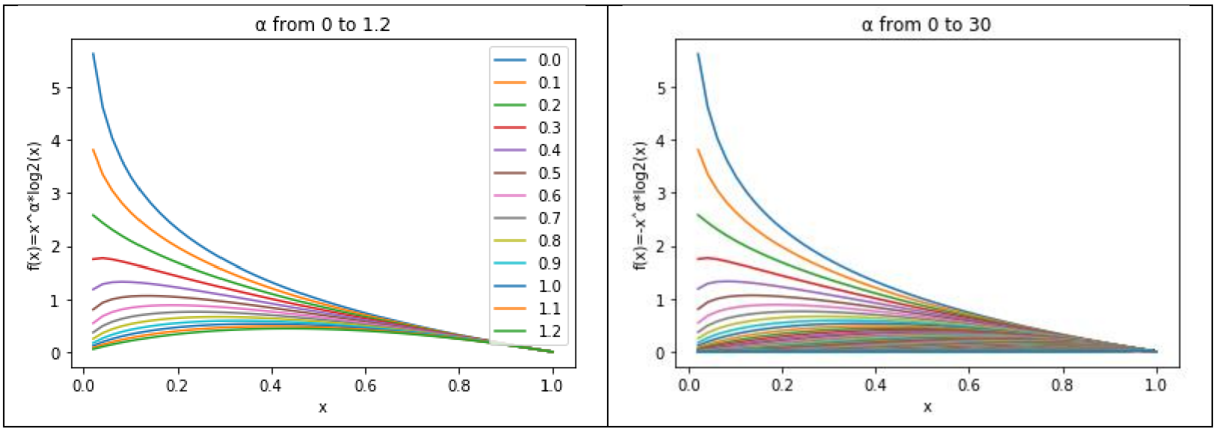

Figure 2. $f(x) = x^{\alpha} * log_2(x)$ function values

According to the equation $f(x) = x^{\alpha} * log_2(x)$, there is a change from $logx$ to $xlogx$ as the α value increases. As seen in Figure 2.

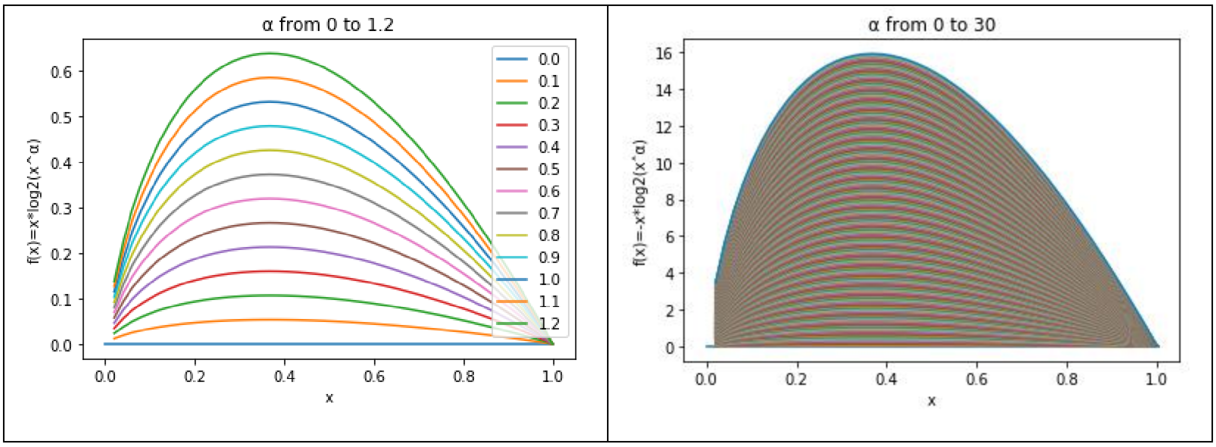

Figure 3. $f(x) = x * log_2(x^{\alpha})$ function values

With the Equation $f(x) = x * log_2(x^{\alpha})$, a similar behavior to $xlogx$ is observed when the value of α starts from 0 and increases as seen in in Figure 3. Here, when values greater than 1.2 are given to α, it is seen that its entropic behavior does not change, and the entropy value increases.

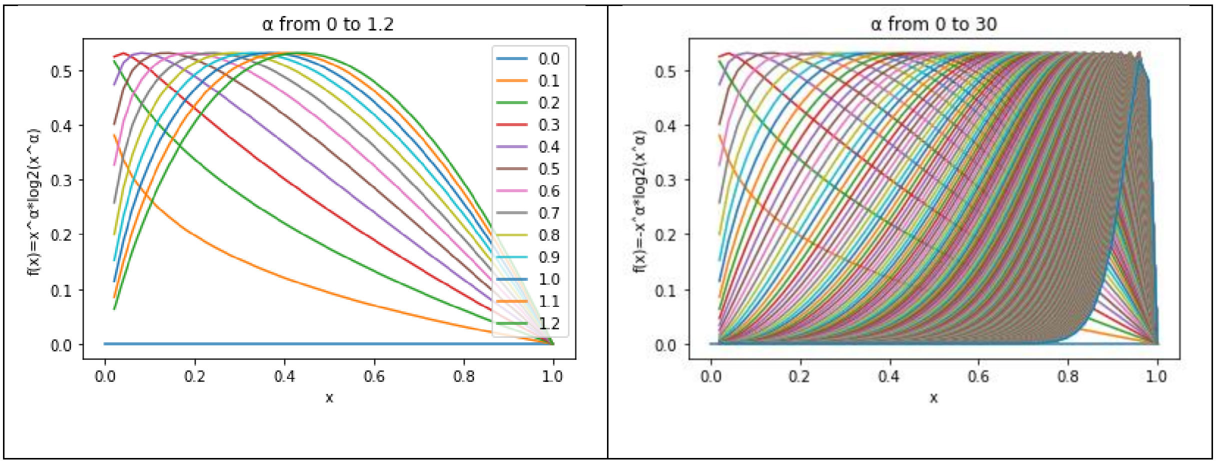

Figure 4. $f(x) = x^{\alpha} * log_2(x^{\alpha})$ function values

Equation $f(x) = x^{\alpha} * log_2(x^{\alpha})$, as the value of α increased, there was a slow change from $logx$ to $xlogx$. It is expected to behave similarly to Shannon entropy with smaller values.

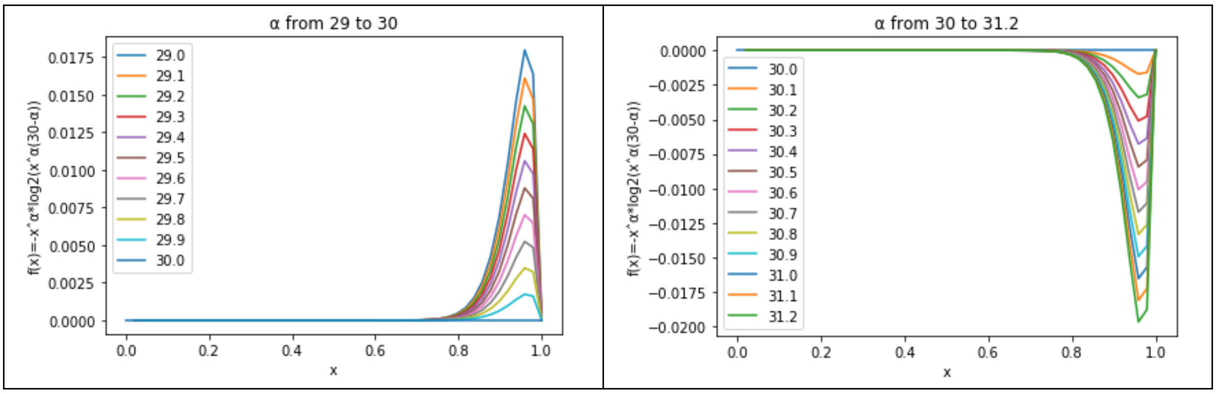

Figure 5. $f(x) = x^{\alpha} * log_2(x^{(30-\alpha)})$ function values

When Equation $f(x) = x^{\alpha} * log_2(x^{(30-\alpha)})$ selected, the behavior of the function can change according to the value of α in the entropy presented using a certain threshold value. As can be seen, for the assigned threshold value as 30, it takes positive or negative values depending on whether the α is greater or less than this value. According to the Equation, as the value of α increases, there is a change from $logx$ to $xlogx$. Negative α values defined in the log section make the entropy value negative.

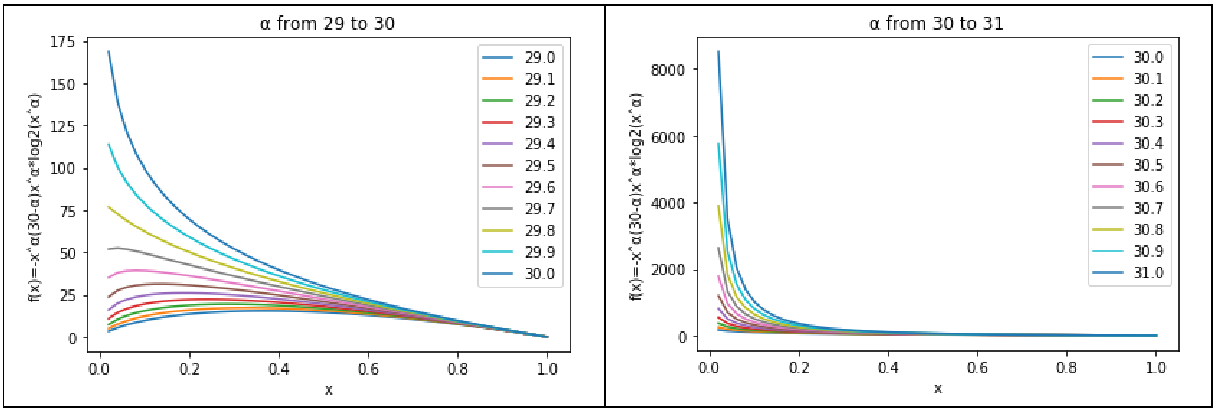

Figure 6. $f(x) = x^{(30-\alpha)} * log_2(x^\alpha)$ function values

According to the Equation $f(x) = x^{(30-\alpha)} * log_2(x^\alpha)$, the α range that will not exceed the defined threshold values such as 30 should be selected. As the α value above increases, there is a change from $xlogx$ to $logx$. When the α value is chosen small, it shows entropic behavior after certain values of x. We have seen that large entropy values are returned in negative α values defined outside the log part.

4. Conclusion

The entropy function gives us information about the direction of the changes that occur without external influence. In this work, the effect of α used in different entropy definitions was tried to be understood. It has been seen how the transition between $logx$ and $xlogx$ is achieved with α. In fact, the values between 0 and 1.2 given to α show the result about the direction of the behavior. With a value up to 30, it was shown that the behavior did not change. One of these entropic functions can be used to measure the uncertainty according to the system and need, considering the direction of behavior that can be seen in the graphs.

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