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Research Article

Buckling Analysis of Functionally Graded Beams Using the Finite Element Method

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Abstract

This study developed a finite element model according to higher-order shear deformation beam theory (HSDT) for the buckling analysis of functionally graded (FG) beams. Equilibrium equations of the FG beam are obtained from Lagrange's equations. The beam element to be discussed within the scope of the study has 5 nodes and 16 degrees of freedom (DOF). As a result of the buckling analysis, the critical buckling load of the beam was obtained for various boundary conditions, power-law index (*p*), and slenderness (*L/h*). When the critical buckling loads obtained as a result of the analysis were compared with the literature, it was seen that they were quite compatible.

Keywords: Finite element model, FG beam, Buckling analysis, Higher-order shear deformation beam theory.

Sonlu Elemanlar Yöntemi Kullanılarak Fonksiyonel Derecelendirilmiş Kirişlerin Burkulma Analizi

Öz

Bu çalışmada, fonksiyonel derecelendirilmiş (FG) kirişlerin burkulma analizi için yüksek mertebeden kayma deformasyonlu kiriş teorisine göre bir sonlu eleman modeli geliştirilmiştir. FG kirişin denge denklemleri Lagrange denklemlerinden elde edilmiştir. Çalışma kapsamında ele alınacak kiriş elemanı 5 düğüm noktasına ve 16 serbestlik derecesine sahiptir. Burkulma analizi sonucunda çeşitli sınır koşulları, kuvvet kuralı indeksi (*p*) ve narinlik (*L/h*) için kirişin kritik burkulma yükü elde edilmiştir. Analiz sonucunda elde edilen kritik burkulma yüklerinin literatür ile karşılaştırıldığında oldukça uyumlu oldukları görülmüştür.

Anahtar Kelimeler: Sonlu eleman modeli, FG kiriş, Burkulma analizi, Yüksek mertebeden kayma deformasyonlu kiriş teorisi.

1 Introduction

Functionally graded materials (FGM) have superior mechanical and thermal properties. Due to their superior properties are widely used in maritime, aviation, space, and engineering fields. Many studies have been performed in the literature using functionally graded materials, especially beam and plate-type elements, as structural elements in civil engineering, one of the engineering fields. When these materials are used as structural elements, it is very important to determine their behavior according to different loadings, ambient conditions, and material properties. When the literature on functionally graded (FG) beams is examined, it is seen that there is extensive literature. Studies that perform mechanical analysis of FG beams with the analytical and finite element methods can be summarized as follows.

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In his doctoral thesis, Turan [1] developed an analytical and finite element method (FEM) according to the first-order shear deformation beam theory (FSDT) to examine the mechanical behavior of FG beams. In addition, if we summarize the analytical studies carried out, Nguyen et al. [2] suggested a new method based on HSDT for the free vibration and buckling behaviors of FG sandwich and isotropic FG beams. With their proposed method, the transverse shear stresses of the FG beam were expressed with a new hyperbolic distribution. The motion equations of the beam were obtained with Lagrange's equations. Nguyen and Nguyen [3] developed a new method for free vibration, buckling, and static analysis of FG sandwich beams using HSDT. The equilibrium equations of the FG beam were obtained by Hamilton's principle and solved with the Navier-type solution method. The transverse shear stress of the FG beam is taken into account in their new solution theory. The axial displacement expression of the FG beam contains third-order and inverse trigonometric expressions. Nguyen et al. [4] proposed a new HSDT using different beam theories for the buckling and vibration of FG sandwich beams. With the theory they proposed, they analyzed the FG beam analytically with the Ritz method. Transverse and normal shear deformations of the FG beam were considered in the analysis. They obtained the equations of motion with Lagrange formulations. Turan [5] carried out bending analyses of two-directional FG beams using the FSDT, considering various boundary conditions according to the Navier method. Turan and Kahya [6] investigated the free vibration and buckling of FG sandwich beams with the Navier method under various boundary conditions. The displacement relations of the beam were obtained according to FSDT. They obtained the equations of motion according to Lagrange's principle. They defined trigonometric functions separately for different boundary conditions in the analytical solution. Liu et al. [7] recommended solving the buckling problem of FG sandwich beams with an analytical solution based on the boundary FEM. Avcar et al. [8] carried out the vibration of sigmoid FG sandwich beams, employing the HSDT. Keleshteri and Jelovica [9] performed static and dynamic analyses of the beams with the generalized differential quadrature method. This simple method has a low computational cost.

If we list the studies made using the FEM, which is widely used in the literature, Oyekoya et al. [10] carried out the buckling and free vibration analyses of the FG beam using the FEM according to different material distributions and various support conditions. The material property of the FG beam changes with the power law. The beam's axial and transverse displacement expressions were obtained according to the Euler-Bernoulli beam theory. Alshorbagy et al. [11] investigated the free vibration and buckling analysis of FG sandwich beams with a new method based on the FEM under various boundary conditions, using HSDT. They obtained the governing equations of the FG beam with Hamilton's principle. Vo et al. [12] proposed a new method based on the FEM for free vibration and buckling analysis of FG sandwich beams using HSDT. The governing equations of the beam were obtained with the help of Hamilton's principle. Vo et al. [13] developed a FEM based on HSDT for the vibration and buckling analysis of FG beams considering the effects of stress and shear deformation in the z-direction. Liu et al. [14] carried out the vibration analysis of FG isotropic beams and plates using the differential quadrature finite element method. Analyses were made under different boundary conditions using Voigt's mixing rule and the Mori-Tanaka model as FG material properties. Yarasca et al. [15] investigated static analyses of FG monolayer and sandwich beams with the FEM based on the quasi-3D hybrid theory with seven degrees of freedom. Kahya and Turan [16] developed a finite element based on FSDT for the free vibration and buckling of FG beams. Along with the thickness changes the material properties of the beam. The governing equations of the beam were obtained by the Lagrange formulation. Kahya and Turan [17] proposed a finite element model based on HSDT for the vibration and buckling of lamina composite and sandwich beams. Reddy et al. [18] developed a double mesh finite field and finite element model for the nonlinear behaviors of FG beams. The beam is analyzed separately according to various beam theories. Yaghoobi et al. [19] proposed a simple and efficient element according to the Timoshenko beam theory for the vibration and buckling analysis of FG beams. The assumption of constant shear strain partially reduces the number of unknowns and improves the efficiency of the new element. Koutoati et al. [20] analyzed multilayer composite and FG structures with a finite element approach. The material properties of the FG beam vary with the power-law distribution. This study investigated the vibration and static behaviors of the bi-directional FG beams. Belarbi et al. [21] investigated the static behaviors of FG sandwich curved beams with a novel refined shear deformation beam theory. The shear stress distribution along the thickness direction changes parabolic in this theory. Thus, a shear correction factor is not used.

This study aims to develop a finite element model based on high-order shear deformation beam theory for the buckling analysis of FG beams. When the literature is examined, it has been seen that the buckling behavior of FG beams has not been examined with the proposed finite element. The proposed finite element in which a third-order polynomial describes shear deformations can accurately predict critical buckling loads, especially in short beams. It also saves time and offers fast solutions when examining the buckling behavior of FG beams, as here. The proposed FG beam element has 5 nodes and 16 DOF. The material property of the beam changes along the beam thickness with the power-law distribution. The equilibrium equations are derived with Lagrange's equations. The buckling analysis of the FG beam with the proposed FEM was performed according to the various boundary conditions, power-law index, and slenderness. The results obtained were compared with the results of the literature, and it was seen that the results were in good agreement.

2 Material and Methods

As a functionally graded beam, an inhomogeneous isotropic rectangular beam with a length L of b x h dimensions, seen in Fig. 1, is used. The upper part of the beam is made of ceramic, and the lower part is made of metal. Pressure force P_0 was applied from the ends of the beam. Material behavior obeys Hooke's law. In addition, the material properties of the beam, according to power-law distribution, are constantly changing in the direction of thickness.

$$P(z) = P_m + (P_c - P_m)V_c(z) \tag{1}$$

where P_c ceramic P_m shows the material properties of the metal phase. The volumetric ratio function of the ceramic phase to be used is as follows.

$$V_c(z) = \left(0.5 + z/h\right)^p, \quad -h/2 \le z \le h/2 \tag{2}$$

where p is power-law index. The finite element considered in the finite element model is given in Fig. 2. The finite element has 5 nodes and 16 DOF. The displacement expressions u and w at any point of the beam are expressed as follows.

$$\begin{aligned} u(x,z,t) &= u^0(x,t) - z\phi^0(x,t) - z^2\beta_1(x,t) - z^3\beta_2(x,t), \\ w(x,z,t) &= w^0(x,t) \end{aligned} \tag{3}$$

where t is time, u^0 horizontal displacement, w^0 vertical displacement, ϕ^0 cross-sectional rotation, β_1 ve β_2 represents higher-order terms coming from the Taylor expansion. Strain and displacement expressions, respectively, are written as follows.

$$\begin{aligned} \varepsilon_{xx} &= u^0_{,x} - z\phi^0_{,x} - z^2\beta_{1,x} - z^3\beta_{2,x}, \\ \gamma_{xz} &= w^0_{,x} - \phi^0 - 2z\beta_1 - 3z^2\beta_2 \end{aligned} \tag{4}$$

$(\square)_{,x}$ shows the derivative with respect to x. σ_{xx} normal stress and τ_{xz} shear stress is expressed as follows.

$$\begin{aligned} \sigma_{xx} &= E(z)\,\varepsilon_{xx}, \\ \tau_{xz} &= G(z)\,\gamma_{xz} \end{aligned} \tag{5}$$

E modulus of elasticity, G represents the shear modulus and varies along the beam thickness. u^0, w^0, ϕ^0, β_1, and β_2 to solve the problem, it is defined as follows.

$$\begin{aligned} &u^0(x,t) = \sum_{j=1}^{3}\varphi_j(x)u_j(t), \quad w^0(x,t) = \sum_{j=1}^{4}\psi_j(x)\,w_j(t), \quad \phi^0(x,t) = \sum_{j=1}^{3}\theta_j(x)\,\phi_j(t), \\ &\beta_1(x,t) = \sum_{j=1}^{3}\varpi_{1j}(x)\,\beta_{1j}(t), \quad \beta_2(x,t) = \sum_{i=1}^{3}\varpi_{2j}(x)\;\beta_{2j}(t) \end{aligned} \tag{6}$$

where, $u_j(t)$, $w_j(t)$, $\phi_j(t)$, $\beta_{1j}(t)$ and $\beta_{2j}(t)$ generalized displacements, $\varphi_j(x)$, $\psi_j(x)$, $\theta_j(x)$, $\varpi_{1j}(x)$ and $\varpi_{2j}(x)$ shows the shape functions and are given as follows.

$$
\begin{aligned}
&\varphi_1=\theta_1=\varpi_{11}=\varpi_{21}=(1-x/L_e)(1-2x/L_e), \quad \varphi_2=\theta_2=\varpi_{12}=\varpi_{22}=4x/L_e(1-x/L_e),\\
&\varphi_3=\theta_3=\varpi_{13}=\varpi_{23}=-x/L_e(1-2x/L_e), \quad \psi_1=(1-x/L_e)\left(2-3x/L_e\right)(1-3x/L_e)/2,\\
&\psi_2=9x/L_e\left(2-3x/L_e\right)(1-x/L_e)/2, \quad \psi_3=-9x/L_e\left(1-x/L_e\right)(1-3x/L_e)/2,\\
&\psi_4=x/L_e\left(2-3x/L_e\right)(1-3x/L_e)/2
\end{aligned}
\tag{7}
$$

The equations of motion were obtained with Lagrange's formulation given as follows.

$$
\frac{d}{dt}\left(\frac{\partial \Pi}{\partial \dot{q}_i}\right)-\frac{\partial \Pi}{\partial q_i}=0 \tag{8}
$$

where $\dot{q}_i$ and q_i denote the independent variables and the Lagrangian formulation is expressed as follows.

$$
\Pi=T-U-V \tag{9}
$$

The strain energy of the FG beam is expressed as follows.

$$
\begin{aligned}
U&=\frac{1}{2}\int_0^{L_e}\int_A(\sigma_{xx}\,\varepsilon_{xx}+\tau_{xz}\,\gamma_{xz})dAdx\\
&=\frac{1}{2}\int_0^{L_e}\Big\{A_0\left(u_{,x}^0\right)^2-2A_1u_{,x}^0\phi_{,x}^0+A_2\left(\left(\phi_{,x}^0\right)^2-2u_{,x}^0\beta_{1,x}\right)+2A_3\left(\phi_{,x}^0\beta_{1,x}-u_{,x}^0\beta_{2,x}\right)\\
&\quad+A_4\left(\beta_{1,x}^2+\phi_{,x}^0\beta_{2,x}\right)+2A_5\beta_{1,x}\beta_{2,x}+A_6\beta_{2,x}^2+B_0\left(\left(\phi^0\right)^2-2\phi^0w_{,x}^0+\left(w_{,x}^0\right)^2\right)\\
&\quad+4B_1\beta_1\left(\phi^0-w_{,x}^0\right)+2B_2\left(2\beta_1^2+3\phi^0\beta_2-3\beta_2w_{,x}^0\right)+12B_3\beta_1\beta_2+9B_4\beta_2^2\Big\}dx
\end{aligned}
\tag{10}
$$

where the coefficients covering the material constants are given in Eq. 11.

$$
A_n=\int_A E\left(z\right)z^i dA\ \ (i=0,1,...,6), \quad B_m=\int_A G\left(z\right)z^k dA\ \ (k=0,1,...,4) \tag{11}
$$

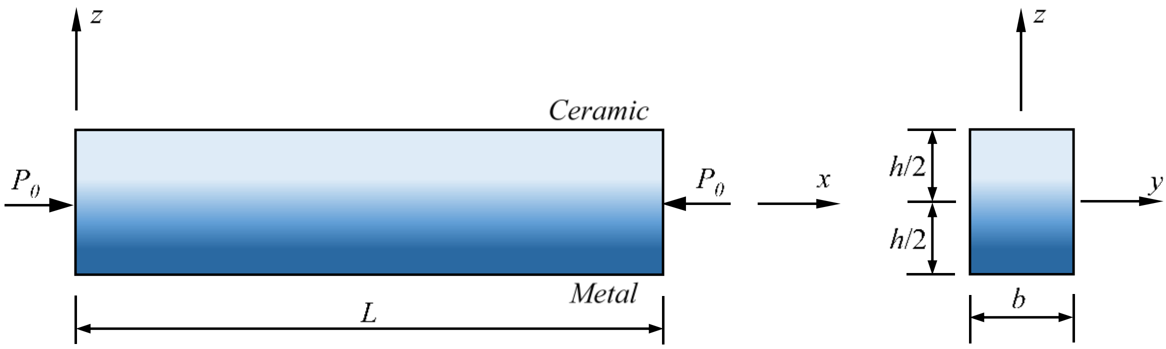

Figure 1. Dimensions and geometry of functionally graded beam

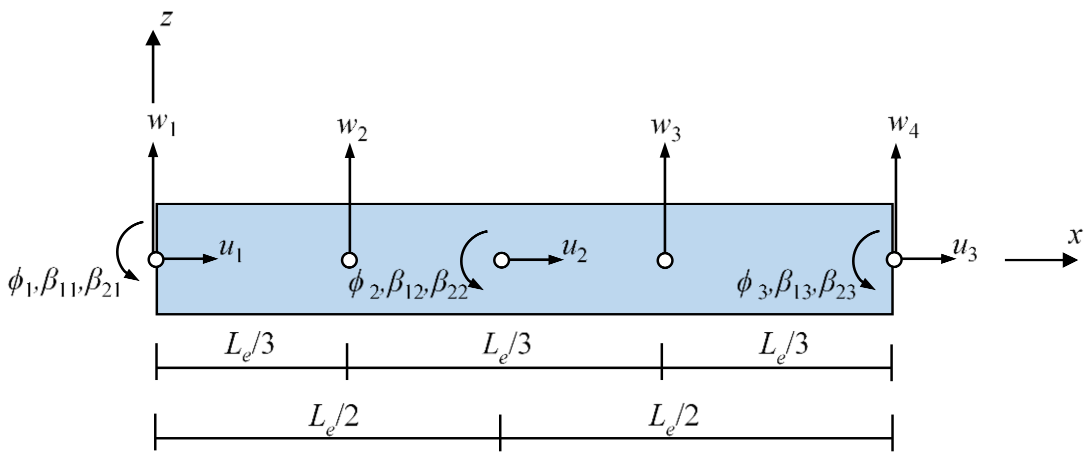

Figure 2. 16 degrees of freedom beam element with 5 nodes

The kinetic energy of the FG beam is expressed in Eq. 12.

$$\begin{aligned} T &= \frac{1}{2}\int_0^{L_e}\int_A \rho(z)(\dot{u}^2+\dot{w}^2)dAdx \\ &= \frac{1}{2}\int_0^{L_e}\Big\{I_0\left(\dot{u}^0\right)^2 - 2I_1\dot{u}^0\,\dot{\phi}^0 + I_2\left(\left(\dot{\phi}^0\right)^2 - 2\dot{u}^0\dot{\beta}_1\right) + 2I_3\left(\dot{\phi}^0\dot{\beta}_1 - \dot{u}^0\dot{\beta}_2\right) \\ &\quad + I_4\left(\dot{\beta}_1^2 + 2\dot{\phi}^0\dot{\beta}_2\right) + 2I_5\dot{\beta}_1\dot{\beta}_2 + I_6\dot{\beta}_2^2 + I_0\left(\dot{w}^0\right)^2\Big\}dx \end{aligned} \tag{12}$$

Explicit expressions of the coefficients depending on the material constants in the kinetic energy equation are given in Eq. 13.

$$I_n = \int_A \rho(z) z^n dA \;\; (n = 0,1,...,6) \tag{13}$$

The work done by the P_0 pressure force acting from the endpoints of the FG beam is as follows.

$$V = \frac{1}{2}\int_0^{L_e} P_0\left(w_{,x}^0\right)^2 dx \tag{14}$$

By substituting the displacement, stress, and work-energy equations described above in the Lagrangian equation, the equation of motion for a single layer higher-order beam element is obtained below.

$$\mathbf{m\ddot{u}} + (\mathbf{k_e} - P_0\mathbf{k_g})\mathbf{u} = \mathbf{f} \tag{15}$$

where $\mathbf{m}$ is the element mass matrix, $\mathbf{k_e}$ is the element stiffness matrix, $\mathbf{k_g}$ is the geometric stiffness matrix, $\mathbf{f}$ is the load vector. For a beam of length L, the equation of motion of the system is obtained by the finite element method as follows.

$$\mathbf{M}\ddot{\mathbf{X}} + (\mathbf{K} - P_0\mathbf{K_g})\mathbf{X} = \mathbf{F} \tag{16}$$

where **M** is the system mass matrix, **K** is the system stiffness matrix, $\mathbf{K_g}$ is the system geometric stiffness matrix and **F** is the system load vector. For the buckling analysis, if **M** = 0, **F** = 0, and $\mathbf{X} = \mathbf{U}e^{P_0 x}$ are taken in Eq.16, the buckling equation is obtained as follows.

$$(\mathbf{K} - P_0\mathbf{K_g})\mathbf{U} = \mathbf{0} \tag{17}$$

where $P_0 = (P_0)_{cr}$ is critical buckling load. Equation 17 is an eigenvalue problem. The P_0 values, which make the coefficient matrices of these systems of equations zero, are the critical buckling loads of the beam.

3 Results and Discussion

Numerical results of buckling analysis of FG beams with various boundary conditions are given with the help of the FORTRAN program in this section. As the different boundary conditions of the FG beam, clamped-clamped support (C-C), simply supported (S-S), and cantilever (C-F) beams are considered.

The material properties of the FG beam are given in Table 1. The critical buckling load of the beam is given below in the dimensionless form.

$$\bar{\lambda} = \frac{12L^2}{E_m b h^3}(P_0)_{cr} \tag{18}$$

Table 2 examines the variation of the dimensionless critical buckling load of the functionally graded beam with the number of elements. According to the table, it was seen that the number of 14 elements was sufficient.

Table 1. Properties of materials

Material	Properties		
	E (Pa)	ρ (kg/m^3)	v
Metal (Al)	70x10^9	2702	0.3
Ceramic (Al_2O_3)	380x10^9	3960	0.3

Table 2. Variation of dimensionless critical buckling loads of FG beams according to various boundary conditions with the number of elements (L/h=5, p =1)

Boundary	Number of Elements							
Conditions	4	8	10	12	14	16	18	20
C-C	79.8756	79.4617	79.4440	79.4377	79.4345	79.4334	79.4327	79.4322
S-S	24.5900	24.5792	24.5787	24.5786	24.5788	24.5785	24.5788	24.5785
C-F	6.5350	6.5349	6.5349	6.5334	6.5347	6.5350	6.5357	6.5347

Dimensionless critical buckling loads of the FG beam according to various boundary conditions and L/h are given in Tables 3 and 4. The results obtained were compared with the results of the literature, and it was seen that the results were in good agreement. It was observed that the $\bar{\lambda}$ decreased as the p value increased. As p = 0, the FG beam material is completely composed of ceramic material. The maximum $\bar{\lambda}$ is obtained when the beam is all-ceramic, as it has the largest elasticity modulus. As the value of p approaches infinity, the elasticity modulus decreases. When $p = \infty$ is reached, the FG beam consists entirely of metal material. (Looking at equation 2, p is seen as the power of the equation. Since this equation is a fractional expression, V_c becomes zero when its power goes to infinity. When we write the V_c in Equation 1, P_m, that is, metal remains.) Therefore, the smallest critical buckling load is obtained when the FG beam consists entirely of metal material. The maximum $\bar{\lambda}$ occurs at C-C. The minimum $\bar{\lambda}$ occurs at C-F. In addition, the $\bar{\lambda}$ increases with increasing slenderness L/h.

The solution methods of the comparison sources in Tables 3 and 4 are as follows, respectively.
(1) Finite element solution with the FSDT
(2) Analytical solution with the FSDT
(3) Analytical solution with the HSDT
(4) FEM with the HSDT
(5) FEM with the three-dimensional beam theory

Table 3. Dimensionless critical buckling loads ($\bar{\lambda}$) of FG beams (L/h = 5)

Beam	Theory	$p = 0$	$p = 0.5$	$p = 1$	$p = 2$	$p = 5$	$p = 10$	$p \to \infty$
C-C	Present	152.1482	102.2666	79.4345	60.6789	46.7314	40.9609	28.0274
	Turan [1]1)	151.9430	101.7439	79.3903	61.7449	49.5828	43.5014	27.9896
	Turan [1](2)	151.9319	101.7354	79.3842	61.7402	49.5791	43.4982	27.9875
	Nguyen et al. [2](3)	154.5610	103.7167	80.5940	61.7666	47.7174	41.7885	-
	Vo et al. [12](4)	154.5500	103.7490	80.6087	61.7925	47.7562	41.8042	-
	Vo et al. [13](5)	160.1070	107.6550	83.6958	64.1227	49.3856	43.1579	-
S-S	Present	48.5970	31.8657	24.5788	19.0487	15.6224	14.0464	8.9519
	Turan [1]1)	48.5907	31.8238	24.5815	19.1617	15.9417	14.3445	8.9510
	Turan [1](2)	48.5904	31.8231	24.5814	19.1616	15.9417	14.3445	8.9509
	Nguyen et al. [2](3)	48.8406	32.0013	24.6894	19.1577	15.7355	14.1448	-
	Vo et al. [12](4)	48.8401	32.0094	24.6911	19.1605	15.7400	14.1468	-
	Vo et al. [13](5)	49.5901	32.5867	25.2116	19.6124	16.0842	14.4116	-
C-F	Present	13.0587	8.4943	6.5347	5.0899	4.2688	3.8742	2.4057
	Turan [1]1)	13.0594	8.4899	6.5352	5.0981	4.2926	3.8970	2.4057
	Turan [1](2)	13.0594	8.4889	6.5352	5.0981	4.2925	3.8970	2.4057
	Nguyen et al. [2](3)	13.0771	8.5000	6.5427	5.0977	4.2772	3.8820	-
	Vo et al. [12](4)	13.0771	8.5020	6.5428	5.0979	4.2776	3.8821	-
	Vo et al. [13](5)	13.0993	8.5469	6.6067	5.1680	4.3290	3.9121	-

Table 4. Dimensionless critical buckling loads ($\bar{\lambda}$) of FG beams (L/h = 20)

Beam	Theory	$p = 0$	$p = 0.5$	$p = 1$	$p = 2$	$p = 5$	$p = 10$	$p \to \infty$
C-C	Present	208.9625	135.8767	104.5632	81.4435	68.3042	61.9945	43.0634
	Turan [1]1)	208.9707	135.8362	104.5734	81.5783	68.6873	62.3580	43.0792
	Turan [1](2)	208.9496	135.8224	104.5628	81.5700	68.6803	62.3517	43.0749
	Nguyen et al. [2](3)	209.2330	136.0490	104.7160	81.6035	68.4689	62.1282	-
	Vo et al. [12](4)	210.4890	137.3160	106.1200	82.9975	69.5392	62.8546	-
	Vo et al. [13](5)	208.9707	135.8362	104.5734	81.5783	68.6873	62.3580	43.0792
S-S	Present	53.2375	34.5368	26.5622	20.7171	17.4827	15.9097	10.9890
	Turan [1]1)	53.2369	34.5345	26.5622	20.7256	17.5075	15.9334	10.9899
	Turan [1](2)	53.2363	34.5342	26.5620	20.7253	17.5073	15.9332	10.9898
	Nguyen et al. [2](3)	53.2546	34.5488	26.5718	20.7275	17.4935	15.9185	-
	Vo et al. [12](4)	53.3075	34.7084	26.8174	21.0066	17.7048	16.0416	-
	Vo et al. [13](5)	53.2369	34.5345	26.5622	20.7256	17.5075	15.9334	10.9899
C-F	Present	13.3714	8.6714	6.6665	5.2029	4.3969	4.0040	2.7616
	Turan [1]1)	13.3730	8.67045	6.6673	5.2025	4.3984	4.0055	2.7616
	Turan [1](2)	13.3730	8.67046	6.6673	5.2026	4.3984	4.0055	2.7616
	Nguyen et al. [2](3)	13.3742	8.6714	6.6680	5.2027	4.3976	4.0046	-
	Vo et al. [12](4)	13.3896	8.7130	6.7307	5.2736	4.4512	4.0359	-
	Vo et al. [13](5)	13.3730	8.67045	6.6673	5.2025	4.3984	4.0055	2.7616

In Figures 3, 4, and 5, the $\bar{\lambda}$ of the FG beam at different p values according to L/h are compared under different boundary conditions. According to the comparison results, as the L/h value increases, the $\bar{\lambda}$ increases to a certain value and remains constant. According to the figures, the $\bar{\lambda}$ decreases as p increases.

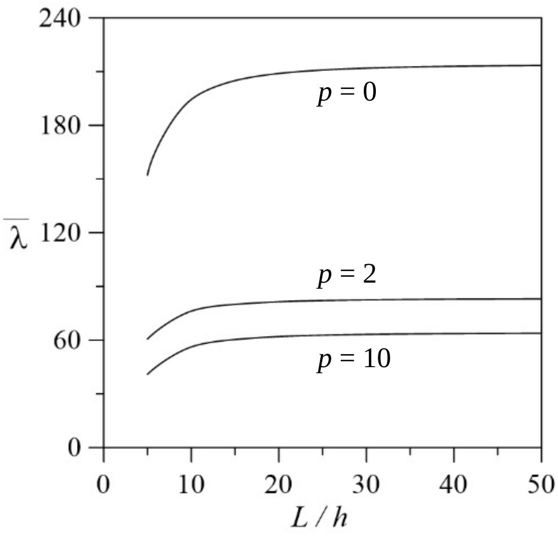

Figure 3. Variation of the $\bar{\lambda}$ of clamped-clamped FG beam with respect to L/h

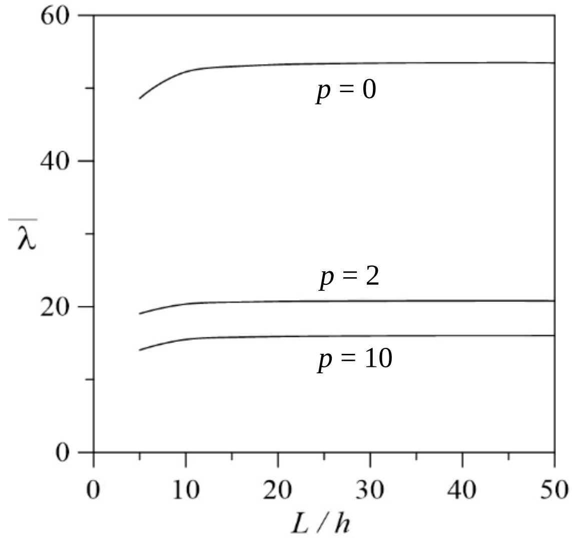

Figure 4. Variation of the $\overline{\lambda}$ of simply supported FG beam with respect to *L/h*

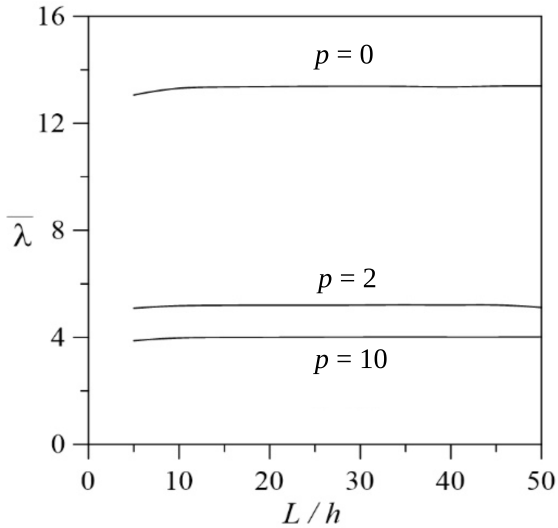

Figure 5. Variation of the $\overline{\lambda}$ of cantilever FG beam with respect to *L/h*

4 Conclusion

The buckling analysis of the FG beam was performed according to the finite element with 5 nodes and 16 degrees of freedom, using the FEM according to the HSDT. The results obtained were verified with the literature. The $\overline{\lambda}$ of FG beams under various boundary conditions have been accurately obtained with the proposed higher-order finite element model. In addition, the following conclusions were drawn from the study. The $\overline{\lambda}$ decrease as the *p*-value increase. The maximum $\overline{\lambda}$ occurs at C-C. The minimum $\overline{\lambda}$ occurs at C-F. The $\overline{\lambda}$ increases with increasing slenderness *L/h*.

Ethics in Publishing

There are no ethical issues regarding the publication of this study.

Author Contributions

The authors contributed equally.

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