



Global Potential for $^{11}\text{Be}+^{64}\text{Zn}$ Elastic Scattering

Yusuf SERT^{1,2*}

¹Department of Physics, Faculty of Art & Sciences, Bozok University, 66100 Yozgat, Turkey

²Sorgun Vocational School, Bozok University, 66100 Yozgat, Turkey

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Abstract. In this analysis, the elastic scattering of $^{11}\text{Be}+^{64}\text{Zn}$ mechanism was investigated utilizing the theoretical phenomenological approximation potential within the optical model frame to determine a new and different global potential set that diagnosing the experimental results more coherently at near-Coulomb barrier energies. A set of global optical potentials have been produced to define the interaction of ^{11}Be . The reaction cross sections, volume integrals of the potentials, and error values have also been calculated.

Keywords: Optical model, elastic scattering, global potential

$^{11}\text{Be}+^{64}\text{Zn}$ Elastik Saçılması için Global Potansiyel

Özet. Bu analizde, $^{11}\text{Be}+^{64}\text{Zn}$ mekanizmasının elastik saçılması optik model çerçevesi içerisinde teorik fenomenolojik yaklaşım potansiyeli kullanılarak Coulomb bariyerine yakın enerjilerde deneysel sonuçları daha tutarlı bir şekilde teşhis edebilecek yeni ve farklı bir global potansiyel set belirlemek için incelenmiştir. Global optik potansiyel setleri ^{11}Be etkileşimlerini tanımlamak için üretilmiştir. Reaksiyon tesir kesitleri, potansiyellerin hacim integralleri ve hata değerleri de hesaplanmıştır.

Anahtar Kelimeler: Optik model, elastik saçılma, global potansiyel

1. INTRODUCTION

In the recent years, the analysis of the elastic scattering of halo nuclei at the energies near the Coulomb barrier have attracted a considerable interest, such importance comes from the strangeness of structure. The continuous developments in both experimental and theoretical methodologies make the study of reaction mechanism dynamics of nuclear interactions more consistent and more precise. In general halo nuclei are collected in two groups, which are either neutron halos or proton halos. The nuclei under the stability circle are considered as neutron halos while those above this circle are considered as proton halos. ^8B and ^{17}F are examples of proton halos, while ^6He (2-neutron halo) and ^{11}Be (1-neutron halo) are the best identified neutron halos. ^{11}Be nuclei are the most known single-neutron halo, its core is ^{10}Be and its binding energy, or 1 neutron breaking energy, is about 503 keV [1-3]. The $^{11}\text{Be}+^{12}\text{C}$ elastic scattering has been studied at $E/A=49.3$ MeV by using CDCC method based on the $^{10}\text{Be}+n+^{12}\text{C}$ three-body model, the dynamic polarization potential for reaction has been investigated as well and clearly observed the breakup effects [2]. In another Study, the elastic scattering of the $^{9,10,11}\text{Be}+^{64}\text{Zn}$ reaction was investigated [3], in this study the powerful efficiencies of the ^{11}Be halo nuclei on the elastic scattering has been estimated to be at energies near the Coulomb barrier (a center of mass energy of ≈ 24.5 MeV or $E_{\text{Lab}}=28.7$ MeV), it has also been observed an unusual attitude in specific interaction regions which is attributed to transfer or breakup. Furthermore a significant increase in total reaction

* Corresponding author. Email address: yusuf.sert@bozok.edu.tr

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cross section has been observed. As the continuation of the previous study, the $^{11}\text{Be}+^{64}\text{Zn}$ reaction has been investigated by using new theoretical calculations and, new experiments have been evaluated [4]. In these experiments the quasi elastic and transfer/breakup distributions of the $^{11}\text{Be}+^{64}\text{Zn}$ reaction have been observed, and the obtained results have been clarified within the optical model and compared with the CDCC model calculations. As a result of this study, authors have observed Igo-type ambiguity of the optical potentials for the $^{9,10}\text{Be}+^{64}\text{Zn}$ reaction [4]. In the following studies, researchers have generally concentrated on this reaction by applying different approaches [5-9]. Apart from these reported studies, there are many reaction studies using ^{11}Be nuclei. The ^{11}Be reactions at energies around the Coulomb barrier are important to understand and to compare the characteristics of halo nuclei.

Thus, in the theoretical investigations of these reactions a global potential set is necessary and very important. Therefore, the basic motivation of this article is to derive a global potential set for the system $^{11}\text{Be}+^{64}\text{Zn}$. New potential parameters in phenomenological potentials with the optical model calculations were investigated and derived a global potential set to define the elastic scattering of the ^{11}Be projectile nuclei from ^{64}Zn target nuclei at energies around Coulomb barrier.

2. OPTIC MODEL CALCULATIONS

The elastic scattering is the most common type of the reaction. In this process, the total kinetic energy of the system is conserved in the centre-of-mass frame. The angular distribution of elastic scattering for any reaction is investigated via different theoretical approaches. The optical model (OM) is accepted as one of the simplest and the most successful model of nuclear physics in the explanation of the elastic scattering. The OM defines the interaction of the projectile and the target nuclei in a complex potential. The potential generally consists of a Coulomb interaction term, a centrifugal interaction term and a nuclear interaction term. The real part in the nuclear potential is responsible for the scattering in the reaction while the imaginary part defines the lost flux [10, 11]. In this study, the OM is handled either using phenomenological (with 6 free parameter) or microscopical approximation (with 4 free parameter). In this section, the elastic scattering of ^{11}Be is examined as one-neutron halo nucleus on ^{64}Zn target by using the phenomenological approximation within the framework of the OM at the incident energies, E_{Lab} , of 28.7 and 29.8 MeV, respectively.

For calculation of the scattering system, the total effective potential in the OM consists of Nuclear, Coulomb and Centrifugal potential as,

$$V_{\text{total}}(r) = V_{\text{Coulomb}}(r) + V_{\text{Nuclear}}(r) + V_{\text{Centrifugal}}(r) \quad (1)$$

where the coulomb potential [10] due to a charge $Z_P e$ interacting with a charge $Z_T e$ distributed uniformly over a sphere of radius Rc is given by

$$\begin{aligned} V_{\text{Coulomb}}(r) &= \frac{1}{4\pi\epsilon_0} \frac{Z_P Z_T e^2}{r} \quad r \geq Rc \\ &= \frac{1}{4\pi\epsilon_0} \frac{Z_P Z_T e^2}{2Rc} \left(3 - \frac{r^2}{Rc^2} \right) \quad r < Rc \end{aligned} \quad (2)$$

where Rc is the radius of Coulomb interaction, received as $1.20(A_P^{1/3} + A_T^{1/3})$ fm within the optical model computations, Z_P and Z_T describes the charge of the projectile (incoming particle) P and target nuclei (fixed core or nuclei) T , respectively. Its Coulomb barrier has been calculated as about 20 MeV. The centrifugal potential is,

$$V_{centrifugal}(r) = \frac{\hbar^2 l(l+1)}{2\mu r^2} \quad (3)$$

where μ is the reduced mass of the interaction (^{11}Be - ^{64}Zn). The last term of the total potential, the complex nuclear potential $V_{Nuclear}(r)$ is defined as sum of the Woods-Saxon square shaped real and Woods-Saxon shaped imaginary potentials as,

$$V_N(r) = -\frac{V_0}{\left[1 + \exp\left(\frac{r-R_V}{a_V}\right)\right]^2} - i \frac{W_0}{\left[1 + \exp\left(\frac{r-R_V}{a_V}\right)\right]} \quad (4)$$

where V_0 , W_0 are the real and imaginary potential depths, respectively, and the nuclear radius are $R_i = r_i \left(A_P^{1/3} + A_T^{1/3}\right)$ ($i = v$ or w), where A_P and A_T are the masses of the projectile (incoming nuclei) and the target (fixed core-nuclei), respectively, and r_v and r_w are radius parameters of the real and imaginary parts of the nuclear potential, respectively. For the phenomenological model calculations, the real radius (r_v) and imaginary radius (r_w) parameters have been set to be 1.25 fm and similarly, the real diffusion (a_v) and imaginary diffusion (a_w) parameters have been set as 0.71 fm, in order to reduce uncertainty of free parameters. While the real and imaginary potential parameters are determined after r_v , r_w , a_v and a_w are set to convenient values, V_0 and W_0 are varied to fit the data. The depths of the real and imaginary potentials have been adjusted to obtain the best value for error analysis, which determines the quality of the fit between the experimental and theoretical results. If the parameters are released we could have a better agreement with experimental data. In the calculations, the code FRESKO, a general purpose reaction code [12], has been used to determine the parameters of OM to fit the experimental data.

After obtaining the best fit-good correlation for all the reactions, the depths of the real and imaginary parts have been identified, Eqs. (5) and (6) have been derived and obtained for the variation of the depths of the real and imaginary parts of the interaction potential in the nuclear section. From Eqs. (5) and (6) we can see that depths are depended on the incoming energy of the projectile (^{11}Be -as laboratory energy) with the charge number (Z) and the mass number (A) of the target (^{64}Zn). These equations are very important for similar projectiles since we can use them in nuclei with same number of nucleons.

$$V_0 = 0.461232 - 3.04636 \frac{Z_T}{A_T^3} + 11.18182 E_{LAB} (MeV) \quad (5)$$

$$W_0 = 79.3034 + 589.0902 \frac{Z_T}{A_T^{1/3}} - 146.091 E_{LAB} (MeV) \quad (6)$$

where E_{LAB} is the laboratory energy of ^{11}Be , and Z_T and A_T are the charge and mass numbers of the target nuclei (^{64}Zn).

3. RESULTS and DISCUSSION

The literatures, up to now, show that there is no global potential study on ^{11}Be nucleus. For this reason, this study has been done. Now let's continue our analyses by combining all the results with the literature knowledge. We would like to derive a new global potential, which does not fit the experimental data. Depending on the analyses motivation, we can conclude that our global phenomenological potential provides very good agreement with experimental data and, produces successfully the minimums and maximums of the elastic scattering angular distribution. Thus, the elastic scatterings of ^{11}Be from ^{64}Zn target nucleus was analyzed at energies (28.7 and 29.8 MeV) around Coulomb barrier by utilizing the

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derived new optical potential set obtained by equations (5) and (6) in the optical model frame. The experimental data are taken from the literature [3, 13]. The best fit parameters, reaction cross-sections and χ^2/N values have been obtained and, shown in Table 1.

Table 1. The OM parameters used in phenomenological model analysis of $^{11}\text{Be}+^{64}\text{Zn}$ reaction.

Energy (MeV)	V_0 (MeV)	r_v (fm)	a_v (fm)	W_0 (MeV)	r_w (fm)	a_w (fm)	σ mb	J_v MeV.fm ³	J_w MeV.fm ³	χ^2/N
28.7	298.5	1.25	0.71	210.1	1.25	0.71	1930	669.857	637.064	0.42691
29.8	310.8	1.25	0.71	150.1	1.25	0.71	1892	697.459	455.133	1.95929

There are 6 free parameters in the phenomenological model. To move away from ambiguities, one fixing in the parameters was done. r_v and r_w have been fixed at the same value of 1.25 fm and similarly, a_v and a_w at 0.71 fm. Other V_0 and W_0 potential depths have been determined according to χ^2/N values. The $^{9,10,11}\text{Be}+^{64}\text{Zn}$ elastic scattering mechanism have been examined within the optical model [3], the PTOLEMY program was used for the calculations. In that study for $^{9,10}\text{Be}$ nuclei, as a potential form the Wood Saxon volume has been utilized and the real and imaginary parameters have been fixed. Furthermore, to obtain a better harmony for ^{11}Be nucleus a surface term was inserted to the imaginary volume potential part. The best χ^2 obtained for a_{si} is around 3.5 fm. For this reaction the used parameters (^{11}Be) are $V=86.2$ MeV, $r_v=1.1$ fm, $a_v=0.7$ fm $V_i=43.4$ MeV, $r_i=1.2$ fm $a_i=0.7$ fm, $V_{si}=0.151$ MeV, $r_{si}=1.3$ fm, $a_{si}=3.5$ fm, $r_c=1.25$ fm, and the calculated cross-section has been found to be 2730 mb. Thus, it has been reported that a much larger cross section gives a clue for halo nucleus case. This comment has been made for 24.5MeV (center mass energy) [3]. Later, in Ref. [7] this reaction has been examined by using the Double Folding Model (both real and imaginary parts are Double Folding Model). In this model r_c (Coulomb radius) has been taken into account and the other parameters are taken as $N_v=1$ and $N_w=1$. It has been found that $\chi^2/N=48.6$ (very big according to our results) and $\sigma_R=1854$ mb for $E_{CM}\approx 24.5$ MeV. Considering the results of these literature, we can not find a theoretical analysis for $E_{lab}=29.8$ MeV, and can not find a global potential for the two systems. According to our results, there is a good correlation between the experimental and theoretical results. Table 1 shows that the best χ^2/N value has been derived for 28.7 MeV. The greatest error rate has been also derived for 29.8 MeV. Sometimes, the minimum χ^2/N does not necessarily mean a better visual result and, a subjective judgement (by eye) of the fit goodness may have more significance than χ^2/N [14]. Then, the visual fit must be noted by eye. Furthermore, the fit with the reaction cross-section data is simultaneously observed, which allow to derive new global potential forms. From the Eqs (5) and (6), the constants have been determined by using nonlinear equation solution method. Their elastic scattering angular distribution cross-sections have been given in Figs 1-2.

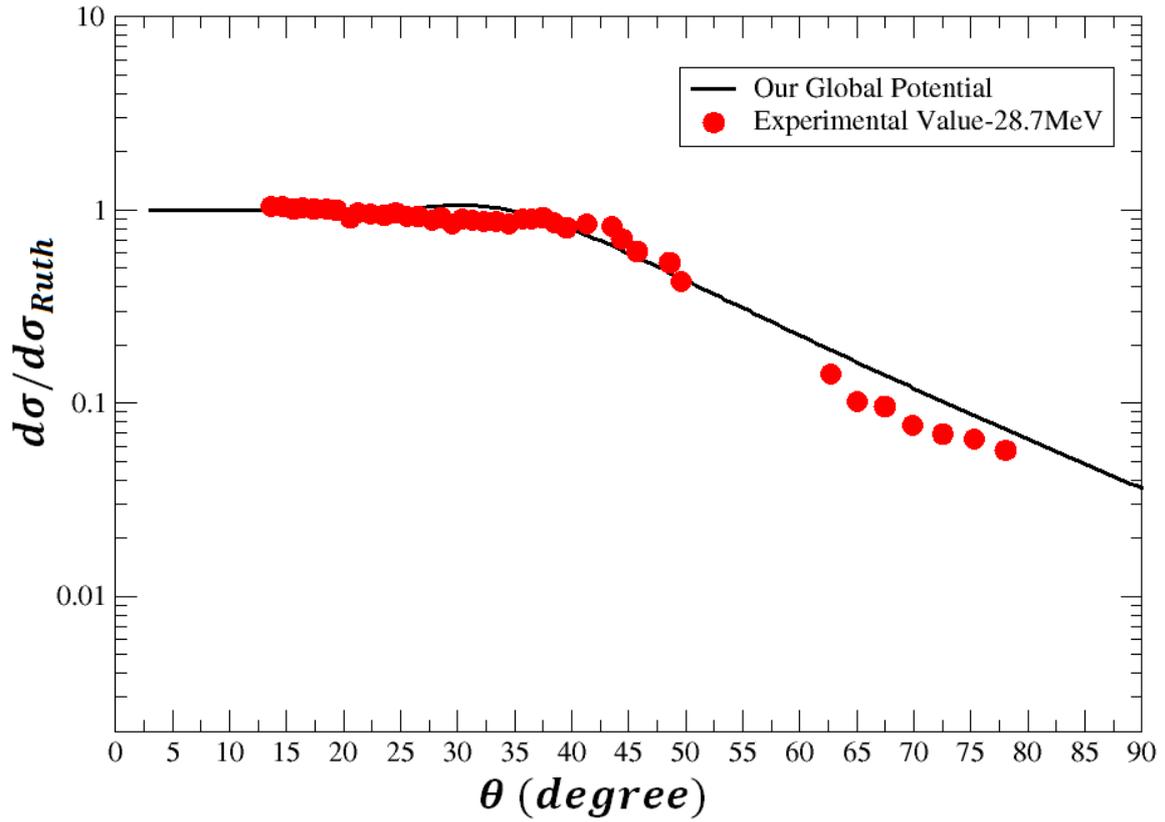


Figure 1. Angular distributions for the $^{11}\text{Be}+^{64}\text{Zn}$ elastic scattering at $E_{\text{Lab}}=28.7$ MeV for our global potential set.

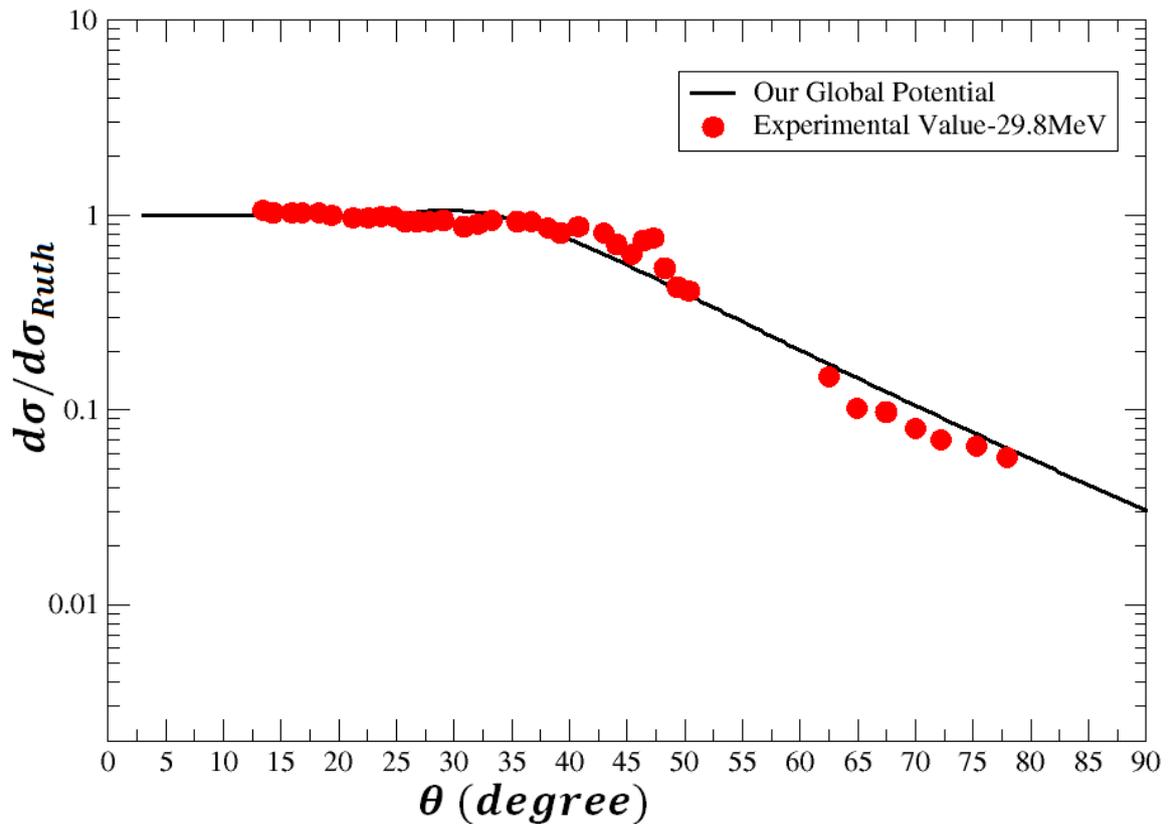


Figure 2. Angular distributions for the $^{11}\text{Be}+^{64}\text{Zn}$ elastic scattering at $E_{\text{Lab}}=29.8$ MeV for our global potential set.

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Figs. 1 and 2 show that our potential is quite acceptable. Perhaps different approaches could be applied between certain angles, so that the potential could slightly be revised. These potential sets can be used for similar nuclei and, also can be applied to define scatterings of halo or exotic type nuclei, which is very significant to their interaction mechanism and scattering. Furthermore, It was realized that the reaction cross-sections decrease with increasing energy.

Finally, let's look at the calculated volume integrals for $^{11}\text{Be}+^{64}\text{Zn}$ elastic scattering. These volume integral values have been shown in Table 1. Examining the relation between the real and imaginary potentials, will be the most logical way to investigate relationship between the real and imaginary volume integral values. Since the determined volume integrals are directly connected to dispersion relation, it is not quite right to comment on this relation because for the $^{11}\text{Be}+^{64}\text{Zn}$ elastic scattering we have very little data (only two data). The volume integrals can be calculated by using the following formulas;

$$J_V = \frac{4\pi}{A_P A_T} \int V(r, E) r^2 dr \quad (7)$$

$$J_W = \frac{4\pi}{A_P A_T} \int W(r, E) r^2 dr \quad (8)$$

where A_P is the mass number of projectile, and A_T is the mass number of target nucleus. These volume integrals were calculated in Ref. [3] as $J_V = 193 \text{ MeV}\cdot\text{fm}^3$ and $J_W = 129 \text{ MeV}\cdot\text{fm}^3$ for $E_{C.M.} = 24.5 \text{ MeV}$. We have calculated as $J_V = 669.857 \text{ MeV}\cdot\text{fm}^3$ and $J_W = 637.064 \text{ MeV}\cdot\text{fm}^3$ for 28.7 MeV and $J_V = 697.459 \text{ MeV}\cdot\text{fm}^3$ and $J_W = 455.133 \text{ MeV}\cdot\text{fm}^3$ for 29.8 MeV .

4. CONCLUSION

In this study, the elastic scattering of the $^{11}\text{Be}+^{64}\text{Zn}$ system has been analyzed by using phenomenological model potentials within the framework of the optical model at two different energies (28.7 and 29.8 MeV) close to the Coulomb barrier. In the calculations we have used the new parameters to explain the reactions. The main motivation of the study is to derive a new global potential for the $^{11}\text{Be}+^{64}\text{Zn}$ elastic scattering. We have concluded that our derived potentials are close to the experimental values. So we suggest that this obtained potential sets can be used for similar nuclei and halo nuclei with 11 nucleons. Additionally, the real and imaginary volume integrals have been and compared with the literature values.

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