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Characterizations of Spacelike Curves according to Bishop Darboux Vector in Minkowski 3-Space E_1^3

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Abstract: In this paper, we obtained some characterizations of spacelike curves according to Bishop frame in Minkowski 3-space E_1^3 by using Laplacian operator and Levi-Civita connection. Furthermore we gave the general differential equations which characterize the spacelike curves according to the Bishop Darboux vector and the normal Bishop Darboux vector.

Keywords: Bishop frame, Darboux vector, Minkowski 3-Space, Laplacian operator.

1 Introduction

It is well-known that a curve of constant slope or general helix is defined by the property that the tangent of the curve makes a constant angle with a fixed straight line which is called the axis of the general helix. A necessary and sufficient condition for a curve to be a general helix is that the ratio of curvature to torsion be constant [12]. The study of these curves in E^3 has been given by many mathematicians. Moreover, Ilarslan studied the characterizations of helices in Minkowski 3-space E_1^3 and found differential equations according to Frenet vectors characterizing the helices in E_1^3 [17]. Then, Kocayigit obtained general differential equations which characterize the Frenet curves in Euclidean 3-space E^3 and Minkowski 3-space E_1^3 [13].

Analogue to the helix curve, Izumiya and Takeuchi have defined a new special curve called the slant helix in Euclidean 3-space E^3 by the property that the principal normal of a space curve α makes a constant angle with a fixed direction [21]. The spherical images of tangent indicatrix and binormal indicatrix of a slant helix have been studied by Kula and Yayl [18]. They obtained that the spherical images of a slant helix are spherical helices. Moreover, Kula et al. studied the relations between a general helix and a slant helix [19]. They have found some differential equations which characterize the slant helix. Position vectors of slant helices have been studied by Ali and Turgut [3]. Also, they have given the generalization of the concept of a slant helix in the Euclidean n-space E^n [4].

Furthermore, Chen and Ishikawa classified biharmonic curves, the curves for which $\Delta H = 0$ holds in semi-Euclidean space E_v^n where Δ is Laplacian operator and H is mean curvature vector field of a Frenet curve [10]. Later, Kocayigit and Hacisalihoglu studied biharmonic curves and 1-type curves i.e., the curves for which $\Delta H = \lambda H$ holds, where λ is constant, in Euclidean 3-space E^3 [14] and Minkowski 3-space E_1^3 [15]. They showed the relations between 1- type curves and circular helix and the relations between biharmonic curves and geodesics. Moreover, slant helices have been

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studied by Bukcu and Karacan according to Bishop Frame in Euclidean 3-space [5] and Minkowski space [6,7]. Characterizations of space curves according to Bishop Frame in Euclidean 3-space E^3 have been given in [15].

In this paper, we gave some characterizations of spacelike curves according to Bishop Frame in Minkowski 3-space E_1^3 by using Laplacian operator. We found the differential equations characterizing spacelike curves according to the Bishop Darboux vector and the normal Bishop Darboux vector.

2 Preliminaries

Let $\mathbb{R}^3 = \{(x_1,x_2,x_3) : x_1,x_2,x_3 \in \mathbb{R}\}$ be a 3-dimensional vector space and let $\overrightarrow{x} = (x_1,x_2,x_3)$ and $\overrightarrow{y} = (y_1,y_2,y_3)$ be two vectors in $\mathbb{R}^3$. The Lorentz scalar product of $\overrightarrow{x}$ and $\overrightarrow{y}$ is defined by

$$\langle \overrightarrow{x}, \overrightarrow{y} \rangle = -x_1y_1 + x_2y_2 + x_3y_3.$$

$E_1^3 = \left(\mathbb{R}^3, \langle \overrightarrow{x}, \overrightarrow{y} \rangle\right)$ is called 3-dimensional Lorentzian space, Minkowski 3-Space or 3-dimensional Semi-Euclidean space. The vector $\overrightarrow{x}$ in E_1^3 is called a spacelike vector, null vector or a timelike vector if $\langle \overrightarrow{x}, \overrightarrow{x} \rangle > 0$ or $\overrightarrow{x} = 0$, $\langle \overrightarrow{x}, \overrightarrow{x} \rangle = 0$ and $\overrightarrow{x} \neq 0$, $\langle \overrightarrow{x}, \overrightarrow{x} \rangle < 0$, respectively [10]. Similarly a curve α is called spacelike, timelike or null if $\left\langle \overrightarrow{\alpha'}, \overrightarrow{\alpha'} \right\rangle > 0$, $\left\langle \overrightarrow{\alpha'}, \overrightarrow{\alpha'} \right\rangle < 0$ or $\left\langle \overrightarrow{\alpha'}, \overrightarrow{\alpha'} \right\rangle = 0$, respectively. For $\overrightarrow{x} \in E_1^3$, the norm of the vector $\overrightarrow{x}$ defined by $\|\overrightarrow{x}\| = \sqrt{|\langle \overrightarrow{x}, \overrightarrow{x} \rangle|}$, and $\overrightarrow{x}$ is called a unit vector if $\|\overrightarrow{x}\| = 1$. For any vectors $\overrightarrow{x}, \overrightarrow{y} \in E_1^3$, Lorentzian cross product of $\overrightarrow{x}$ and $\overrightarrow{y}$ is defined by

$$\overrightarrow{x} \wedge \overrightarrow{y} = (x_2y_3 - x_3y_2, x_1y_3 - x_3y_1, x_1y_2 - x_2y_1).$$

Denoted the moving Frenet frame along a space curve α by $\left\{\overrightarrow{T}, \overrightarrow{N}, \overrightarrow{B}\right\}$ where $\overrightarrow{T}$, $\overrightarrow{N}$ and $\overrightarrow{B}$ are tangent, principal normal and binormal vector of α, respectively. If α is a spacelike curve, then this set of orthogonal unit vectors, known as the Frenet frame, has the following properties

$$\begin{aligned}
\overrightarrow{T'} &= \kappa \overrightarrow{N} \\
\overrightarrow{N'} &= -\varepsilon\kappa \overrightarrow{T} + \tau \overrightarrow{B} \\
\overrightarrow{B'} &= -\tau \overrightarrow{N}
\end{aligned}$$

where $\left\langle \overrightarrow{T}, \overrightarrow{T} \right\rangle = 1$, $\left\langle \overrightarrow{N}, \overrightarrow{N} \right\rangle = \varepsilon$ and $\left\langle \overrightarrow{B}, \overrightarrow{B} \right\rangle = -\varepsilon$.

The parallel transport frame is an allternative approach to defining a moving frame that is well-defined even when the curve has vanishing second derivative. We can parallel transport an orthonormal frame along a curve simply by paralel transporting each component of the frame [1].

Its mathematical properties derive from the observation that, while $\overrightarrow{T}(s)$ for a given curve model is unique, we may choose any convenient arbitrary basis $\left(\overrightarrow{N}_1(s), \overrightarrow{N}_2(s)\right)$ for the remainder of the frame, so long as it is in the normal plane perpendicular to $\overrightarrow{T}(s)$ at each point. If the derivatives of $\left(\overrightarrow{N_1}(s), \overrightarrow{N_2}(s)\right)$ depend only on $\overrightarrow{T}(s)$ and not each other, we can make $\overrightarrow{N_1}(s)$ and $\overrightarrow{N_2}(s)$ vary smoothly throughout the path regardless of the curvature [2,20].

Denote by $\left\{\overrightarrow{T}, \overrightarrow{N_1}, \overrightarrow{N_2}\right\}$ the moving Bishop frame along the spacelike curve $\alpha(s) : I \subset \mathbb{R} \to E_1^3$ in the Minkowski

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3-space E_1^3. For an arbitrary spacelike curve $\alpha(s)$ in the space E_1^3,the following Bishop formula are given by

$$\begin{bmatrix} \nabla_{\alpha'}\overrightarrow{T} \\ \nabla_{\alpha'}\overrightarrow{N_1} \\ \nabla_{\alpha'}\overrightarrow{N_2} \end{bmatrix} = \begin{bmatrix} 0 & k_1 & -k_2 \\ -\varepsilon k_1 & 0 & 0 \\ -\varepsilon k_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \overrightarrow{T} \\ \overrightarrow{N_1} \\ \overrightarrow{N_2} \end{bmatrix} \tag{1}$$

where $\left\langle \overrightarrow{T}, \overrightarrow{T} \right\rangle = 1$, $\left\langle \overrightarrow{N_1}, \overrightarrow{N_1} \right\rangle = \varepsilon$ and $\left\langle \overrightarrow{N_2}, \overrightarrow{N_2} \right\rangle = -\varepsilon$. The relations between κ, τ, θ and k_1, k_2 are given as follows.

$$\kappa(s) = \sqrt{\left|k_1^2 - k_2^2\right|},\ \theta(s) = \arg\tanh\left(\frac{k_2}{k_1}\right),\ k_1 \neq 0.$$

so that k_1 and k_2 effectively correspond to Cartesian coordinate system for the polar coordinates κ, θ with $\theta = \int \tau(s)\,ds$. The orientation of the parallel transport frame includes the arbitrary choice of integration constant θ_0, which disappears from τ due to the differentitation [7,9].

A regular spacelike curve $\alpha(s) : I \to E_1^3$ is called a slant helix if unit vector $\overrightarrow{N_1}(s)$ of α makes a constant angle θ with a fixed unit vector $\overrightarrow{U}$; that is, $\left\langle \overrightarrow{N_1}(s), \overrightarrow{U} \right\rangle = const.$, for all $s \in I$.

Let, $\alpha(s) : I \to E_1^3$ be a unit speed spacelike curve with nonzero naturel curvatures k_1, k_2. Then α is a slant helix if and only if $\frac{k_1}{k_2}$ is constant [5].

Let, ∇ denotes the Levi-Civita connection given by $\nabla_{\alpha'} = \frac{d}{ds}$ where s is the arclength parameter of the timelike curve α. The Laplacian operator of α is defined by

$$\Delta = -\nabla_{\alpha'}^2 = -\nabla_{\alpha'}\nabla_{\alpha'}$$

[15].

3 Characterizations of Spacelike Curves

In this section, we gave the characterizations of the spacelike curves according to Bishop frame in Minkowski 3-space E_1^3. Furthermore, we obtained the general differential equations which characterize the spacelike curves according to the Bishop Darboux vector $\overrightarrow{W}$ and the normal Bishop Darboux vector $\overrightarrow{W^{\perp}}$ in E_1^3.

Theorem 1. *Let $\alpha(s)$ be a unit speed spacelike curve in Minkowski 3-space E_1^3 with Bishop frame $\left\{\overrightarrow{T}, \overrightarrow{N_1}, \overrightarrow{N_2}\right\}$ and curvatures k_1, k_2. The Bishop Darboux vector $\overrightarrow{W}$ of the curve α is given by*

$$\overrightarrow{W} = \varepsilon k_2 \overrightarrow{N_1} - \varepsilon k_1 \overrightarrow{N_2} \tag{2}$$

Proof. The Bishop Darboux vector $\overrightarrow{W}$ of the curve α can be written as follow.

$$\overrightarrow{W} = a\overrightarrow{T} + b\overrightarrow{N_1} + c\overrightarrow{N_2} \tag{3}$$

Taking the cross product of (3) with $\overrightarrow{T}$, we get

$$\begin{aligned} \overrightarrow{T'} &= \overrightarrow{W} \wedge \overrightarrow{T} = \left(a\overrightarrow{T} + b\overrightarrow{N_1} + c\overrightarrow{N_2}\right) \wedge \overrightarrow{T} \\ &= -\varepsilon c\overrightarrow{N_1} - \varepsilon b\overrightarrow{N_2}. \end{aligned}$$

Then, $-\varepsilon c = k_1$ and $-\varepsilon b = -k_2$. Taking the cross product of (3) with $\overrightarrow{N_1}$, we get

$$\begin{aligned}\overrightarrow{N_1'} &= \overrightarrow{W} \wedge \overrightarrow{N_1} = \left(a\overrightarrow{T} + b\overrightarrow{N_1} + c\overrightarrow{N_2}\right) \wedge \overrightarrow{N_1} \\ &= c\overrightarrow{T} + \varepsilon a\overrightarrow{N_2}.\end{aligned}$$

Then, $a = 0$ and $c = -\varepsilon k_1$. Taking the cross product of (3) with $\overrightarrow{N_2}$, we obtain

$$\begin{aligned}\overrightarrow{N_2'} &= \overrightarrow{W} \wedge \overrightarrow{N_2} = \left(a\overrightarrow{T} + b\overrightarrow{N_1} + c\overrightarrow{N_2}\right) \wedge \overrightarrow{N_2} \\ &= -b\overrightarrow{T} + \varepsilon a\overrightarrow{N_1}.\end{aligned}$$

Then, $a = 0$ and $b = \varepsilon k_2$. Thus we can get the Bishop darboux vector $\overrightarrow{W}$ as follow.

$$\overrightarrow{W} = \varepsilon k_2\overrightarrow{N_1} - \varepsilon k_1\overrightarrow{N_2}.$$

Definition 1. *A regular spacelike curve α in E_1^3 said to have harmonic Darboux vector $\overrightarrow{W}$ if*

$$\Delta\overrightarrow{W} = 0.$$

Definition 2. *A regular spacelike curve α in E_1^3 said to have harmonic 1-type Darboux vector $\overrightarrow{W}$ if*

$$\Delta\overrightarrow{W} = \lambda\overrightarrow{W}, \quad \lambda \in \mathbb{R}. \tag{4}$$

Theorem 2. *Let $\alpha(s)$ be a unit speed spacelike curve in Minkowski 3-space E_1^3 with Bishop frame $\left\{\overrightarrow{T}, \overrightarrow{N_1}, \overrightarrow{N_2}\right\}$ and Bishop curvatures k_1, k_2. The differential equation characterizing α according to the Bishop Darboux vector $\overrightarrow{W}$ is given by*

$$\lambda_4\nabla^3_{\alpha'}\overrightarrow{W} + \lambda_3\nabla^2_{\alpha'}\overrightarrow{W} + \lambda_2\nabla_{\alpha'}\overrightarrow{W} + \lambda_1\overrightarrow{W} = 0 \tag{5}$$

where

$$\begin{aligned}\lambda_4 &= f^2 \\ \lambda_3 &= -f\left(f' - g\right) \\ \lambda_2 &= -\left[\left(f' - g\right)g + k_1\left(k_2''' + \varepsilon k_1 f\right)f + k_2\left(k_1''' + \varepsilon k_2 f\right)\right] \\ \lambda_1 &= -\left[\left(f' - g\right)\left(\frac{k_1'}{k_2'}\right)\left(k_2'\right)^2 + k_1'\left(k_2''' + \varepsilon k_1 f\right)f - k_2'\left(k_1''' + \varepsilon k_2 f\right)f\right]\end{aligned}$$

and

$$f = \left(\frac{k_1'}{k_2'}\right)k_2^2, \; g = k_1k_2'' - k_1''k_2.$$

Proof. Let $\alpha(s)$ be a unit speed spacelike curve in Minkowski 3-space E_1^3 with Bishop frame $\left\{\overrightarrow{T}, \overrightarrow{N_1}, \overrightarrow{N_2}\right\}$ and Bishop curvatures k_1, k_2. By differentiating $\overrightarrow{W}$ three times with respect to s, we obtain the followings.

$$\nabla_{\alpha'}\overrightarrow{W} = -k_2'\overrightarrow{N_1} + k_1'\overrightarrow{N_2} \tag{6}$$

$$\nabla^2_{\alpha'}\overrightarrow{W} = \left(k_1'k_2 - k_1k_2'\right)\overrightarrow{T} - k_2''\overrightarrow{N_1} + k_1''\overrightarrow{N_2} \tag{7}$$

$$\begin{aligned}\nabla^3_{\alpha'}\overrightarrow{W} = {} & \left[\left(k_1'k_2-k_1k_2'\right)'-k_1k_2''+k_1''k_2\right]\overrightarrow{T} \\ & +\left[-k_2''+k_1\left(k_1'k_2-k_1k_2'\right)\right]\overrightarrow{N_1} \\ & +\left[k_1'''-k_2\left(k_1'k_2-k_1k_2'\right)\right]\overrightarrow{N_2}\end{aligned} \tag{8}$$

From (2) and (6), we obtain

$$\overrightarrow{N_1}=\frac{k_1}{k_1'k_2-k_1k_2'}\nabla_{\alpha'}\overrightarrow{W}-\frac{k_1'}{k_1'k_2-k_1k_2'}\overrightarrow{W} \tag{9}$$

and

$$\overrightarrow{N_2}=\frac{-k_2}{\varepsilon\left(k_1'k_2-k_1k_2'\right)}\nabla_{\alpha'}\overrightarrow{W}+\frac{k_2'}{\varepsilon\left(k_1'k_2-k_1k_2'\right)}\overrightarrow{W}. \tag{10}$$

By substituting (9) and (10) in (7), we have

$$\overrightarrow{T}=\frac{1}{k_1'k_2-k_1k_2'}\nabla^2_{\alpha'}\overrightarrow{W}+\frac{k_1k_2''-k_1''k_2}{\left(k_1'k_2-k_1k_2'\right)^2}\nabla_{\alpha'}\overrightarrow{W}+\frac{k_1''k_2'-k_1'k_2''}{\left(k_1'k_2-k_1k_2'\right)^2}\overrightarrow{W}. \tag{11}$$

By substituting (9), (10) and (11) in (8), we obtain

$$\lambda_4\nabla^3_{\alpha'}\overrightarrow{W}+\lambda_3\nabla^2_{\alpha'}\overrightarrow{W}+\lambda_2\nabla_{\alpha'}\overrightarrow{W}+\lambda_1\overrightarrow{W}=0$$

where

$$\begin{aligned}\lambda_4 &= f^2\\ \lambda_3 &= -f\left(f'-g\right)\\ \lambda_2 &= -\left[\left(f'-g\right)g+k_1\left(k_2'''+\varepsilon k_1 f\right)f+k_2\left(k_1'''+\varepsilon k_2 f\right)\right]\\ \lambda_1 &= -\left[\left(f'-g\right)\left(\frac{k_1'}{k_2'}\right)\left(k_2'\right)^2+k_1'\left(k_2'''+\varepsilon k_1 f\right)f-k_2'\left(k_1'''+\varepsilon k_2 f\right)f\right]\end{aligned}$$

and

$$f=\left(\frac{k_1'}{k_2'}\right)k_2^2,\ \ g=k_1k_2''-k_1''k_2.$$

Corollary 1. *Let $\alpha(s)$ be a general helix in E_1^3 with Bishop frame $\left\{\overrightarrow{T},\overrightarrow{N_1},\overrightarrow{N_2}\right\}$ and Bishop curvatures k_1,k_2. The differential equation characterizing α according to the Bishop Darboux vector $\overrightarrow{W}$ is given by*

$$g\nabla_{\alpha'}\overrightarrow{W}+\left(\frac{k_1'}{k_2'}\right)'\left(k_2'\right)^2\overrightarrow{W}=0.$$

Theorem 3. *Let $\alpha(s)$ be a unit speed spacelike curve in Minkowski 3-space E_1^3 with Bishop frame $\left\{\overrightarrow{T},\overrightarrow{N_1},\overrightarrow{N_2}\right\}$ and Bishop curvatures k_1,k_2. The differential equation characterizing α according to the normal Bishop Darboux vector $\overrightarrow{W^{\perp}}$ is given by*

$$\lambda_3\nabla^2_{\alpha'}\overrightarrow{W^{\perp}}+\lambda_2\nabla_{\alpha'}\overrightarrow{W^{\perp}}+\lambda_1\overrightarrow{W^{\perp}}=0 \tag{12}$$

where

$$\lambda_3=f,\quad \lambda_2=g,\quad \lambda_1=\left(\frac{k_1'}{k_2'}\right)'\left(k_2'\right)^2$$

and

$$f=\left(\frac{k_1'}{k_2'}\right)k_2^2,\ \ g=k_1k_2''-k_1''k_2.$$

Proof. Let $\alpha(s)$ be a unit speed spacelike curve in Minkowski 3-space E_1^3 with Bishop frame $\left\{\vec{T},\vec{N_1},\vec{N_2}\right\}$ and Bishop curvatures k_1,k_2. By differentiating $\overrightarrow{W^{\perp}}$ two times with respect to s, we obtain the followings.

$$\overrightarrow{W^{\perp}}=\varepsilon k_2\vec{N_1}-\varepsilon k_1\vec{N_2} \tag{13}$$

$$\nabla_{\alpha'}\overrightarrow{W^{\perp}}=\varepsilon k_2'\vec{N_1}-\varepsilon k_1'\vec{N_2} \tag{14}$$

$$\nabla_{\alpha'}^2\overrightarrow{W^{\perp}}=\varepsilon k_2''\vec{N_1}-\varepsilon k_1''\vec{N_2} \tag{15}$$

From (13) and (14), we get

$$\vec{N_1}=\frac{-k_1}{\varepsilon\left(k_1'k_2-k_1k_2'\right)}\nabla_{\alpha'}\overrightarrow{W^{\perp}}+\frac{k_1'}{\varepsilon\left(k_1'k_2-k_1k_2'\right)}\overrightarrow{W^{\perp}} \tag{16}$$

and

$$\vec{N_2}=\frac{-k_2}{\varepsilon\left(k_1'k_2-k_1k_2'\right)}\nabla_{\alpha'}\overrightarrow{W^{\perp}}+\frac{k_2'}{\varepsilon\left(k_1'k_2-k_1k_2'\right)}\overrightarrow{W^{\perp}} \tag{17}$$

By substituting (16) and (17) in (15), we obtain

$$f\nabla_{\alpha'}^2\overrightarrow{W^{\perp}}+g\nabla_{\alpha'}\overrightarrow{W^{\perp}}+\left(\frac{k_1'}{k_2'}\right)'\left(k_2'\right)^2\overrightarrow{W^{\perp}}=0 \tag{18}$$

Corollary 2. *Let $\alpha(s)$ be a slant helix in E_1^3 with Bishop frame $\left\{\vec{T},\vec{N_1},\vec{N_2}\right\}$ and Bishop curvatures k_1,k_2. The differential equation characterizing α according to the normal Bishop Darboux vector $\overrightarrow{W^{\perp}}$ is given by*

$$g\nabla_{\alpha'}\overrightarrow{W^{\perp}}+\left(\frac{k_1'}{k_2'}\right)'\left(k_2'\right)^2\overrightarrow{W^{\perp}}=0.$$

Theorem 4. *Let $\alpha(s)$ be a unit speed spacelike curve in E_1^3 with Bishop frame $\left\{\vec{T},\vec{N_1},\vec{N_2}\right\}$. Then, α is of harmonic 1-type Darboux vector if and only if the curvatures k_1 and k_2 of the curve α satisfy the followings.*

$$-k_1''=\lambda k_1,\ \ k_1'k_2-k_1k_2'=0,\ \ k_2''=-\lambda k_2 \tag{19}$$

Proof. Let α be a unit speed spacelike curve and let Δ be the Laplacian associated with ∇. From (6) and (7) we can obtain following.

$$\Delta\vec{W}=-\left(k_1'k_2-k_1k_2'\right)\vec{T}-\varepsilon k_2''\vec{N_1}+\varepsilon k_1''\vec{N_2} \tag{20}$$

We assume that the spacelike curve α is of harmonic 1-type Darboux vector $\vec{W}$. Substituting (20) in (4), we get (19).

4 Conclusion

In this paper, by using Laplacian operator and Levi-Civita connection, some characterizations of spacelike curves according to Bishop frame in Minkowski 3-space are obtained. In additon the general differential equations which

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characterize the spacelike curves according to Bishop Darboux vector and normal Bishop Darboux vector are given. It is obtained some useful results. These characterizations are applied to other curves in different spaces.

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