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Research Article

Maximum entropy production and constructal law: Variable conductance and branched flow

Phillip Ralston JOHNSON*

Thales College, Wake Forest, North Carolina, USA

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ABSTRACT

Within the scope of modern thermodynamics, several new principles have been advanced. Yet a coherent picture has not yet fully emerged beyond classical thermodynamics, in part because of the disparate nature of these principles. This work analyzes two such principles, that of maximum entropy production and also the constructal law. These ideas are compared in two examples scenarios, atmospheric convection and riverbed generation. It is shown that both ideas utilized a flow network with variable conductivity, particularly that of a branched system. Comparison is made to show underlying shared principles.

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1. INTRODUCTION

Classical thermodynamics is the study of the driver of motion. This name was derived etymologically from the Greek, "the power of heat", by those interacting with Carnot's work, "On the Motive Power of Heat". Thermodynamics provides powerful insights into equilibrium states and stability status of a system. Modern thermodynamics aspires to unite the powers of motion (δυναμις) with the path-dependent motion (κινησις) of a system. Yet there is a lack of consensus on a unified theory with several varying (or even contradictory) ideas in the literature. Two of the more prominent, Maximum Entropy Production (MEP) and Constructal Law (CL), are here briefly reviewed and later compared directly with an attempt at unifying these systems.

1.1. Non-equilibrium thermodynamics

Drawing from various phenomenological laws, with forces X (gradients of temperature, pressure, etc.) and fluxes J (heat, mass flow, etc.), Onsager (Onsager, 1931a) proposed a generalized relation describing entropy generation:

$$\sigma=\Sigma_j \Sigma_i J_i X_j \quad (1)$$

Near equilibrium, fluxes can be assumed to be linear:

$$J_i=L_{ij} X_j \quad (2)$$

With conductance (L) and resistance ($R_{ij}=L_{ij}^{-1}$) assumed to be constant. In practice, the linear assumption is often made even for non-equilibrium systems. At equilibrium, the force is depleted and the flux has ceased, leading also to a cessation of entropy production:

$$X=0 \rightarrow J=0 \rightarrow \sigma=0 \quad (3)$$

Prigogine pioneered a theorem that near equilibrium the entropy generation is at a minimum given certain constraints (De Groot & Mazur, 1984; Glansdorff & Prigogine, 1971; Prigogine, 1955). These constraints are critical to understanding the theorem and fundamentally change the underlying concept if misunderstood. The

***Corresponding author.**
*E-mail address: phillip.johnson@thalescollege.org

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approach divides the force/flux pairs into two classes: 1) non-zero fluxes whose conjugate forces are invariant, 2) variable forces for which the conjugate flux is zero. When the first condition is met (Eq. 4), then the second condition is true if and only if the entropy production is at an extremal point (Eq. 5). Further, since σ is positive definite, this extremal point describes a minimum. The reader is referred to the references for further details.

$$dX_i=0,\ J_i>0,\ \forall_i=1,2,\ldots,k \quad (4)$$

$$\frac{\partial\sigma}{\partial X_i}=0 \leftrightarrow J_i=0,\ \forall_i=k+1,k+2,\ldots,n \quad (5)$$

An example of this behavior would be an electrical circuit placed in a temperature gradient with no other forces. We will arbitrarily consider the temperature gradient to be invariant according to Eq. 4. Then Eq. 5 states that the entropy production will not change with respect to voltage when there is no current and vice-versa. This can occur by the following scenarios: a) there is no current for a state with no voltage and 2) there could be a non-zero voltage but perhaps an amount smaller than required to pass by an idealized diode. By Eq. 5, lower voltage does not increase flux and so entropy production remains nil. If, however, the voltage is increased sufficiently to net a non-zero flux, the entropy production will increase, which indicates σ to previously have been a minimum. It is noted then, that Prigogine's principles concern steady-state processes with invariant forces or nil fluxes. Such conditions are not shared for systems with an evolutionary nature, which as will be shown below are reflective of the Constructal Law or systems undergoing MEP.

1.2. Maximum entropy production

When Onsager formulated Eq. 1 above, he included analysis of a "dissipation function" which is maximized at steady state (Onsager, 1931a, 1931b). From its beginning, studies in entropy production have been led by a desire to find an extremal operating principle. There are several to note in a general review; but for this context, the genealogy begins with Paltridge (Paltridge, 1978), who developed simple climate models. These models analyzed the radiation in and out of the earth system and invoked MEP for the intermediate atmospheric heat transfer from tropics to poles. The principle of MEP states that among the available steady-states, a system will chose the one with the highest entropy production. By invoking this principle, Paltridge was able to accurately predict zonal temperatures and heat transfer coefficients. Kleidon continued this work, applying such an analysis to atmospheric systems (Kleidon, 2009; Kleidon et al., 2010) and river basins (Kleidon et al., 2013) among other examples which are presented in more detail in sections 2.1.1 and 2.2.2. below. Other work includes that of Dewar (Dewar, 2003), who built upon theories from Jaynes to provide statistical-based arguments beyond the scope of this work. Further applications of MEP have been seen in electrical circuit analysis (Bruers et al., 2007; Županović et al., 2004) and other fluid networks (Niven, 2010). Here, we note the advances made on MEP principles and point to the review by Martyushev and Selenez (Martyushev & Seleznev, 2006).

1.3. Constructal law

While attempting to optimize heat transfer for a volumetrically generated heat source (Bejan, 1997), Bejan discovered what he named the 'Constructal Law' (CL) from the Latin *construere*, "to construct" (Bejan, 1996, 1997). The law states that: "for a system to persist (to live) in time, it evolves in such as a way to provide greater access to flow". Bejan has written several summaries on the various applications of CL (Bejan, 2016). At its core, CL is concerned with the shape and configuration of a flow network, its evolution, and the consequences for flow. CL addresses the question of how flow can occur from a single point to a volume in an optimal manner (Bejan & Lorente, 2004). This optimal distribution is characterized by svelteness, a non-dimensional shape parameter (function of volume and length) which determines the optimal number, length, and size of branches in a network.

Seminal papers address the way energy flows through thermal-fluid systems, particularly how a fluid flow network reduces flow resistance/friction (maintaining a constant flow rate and thereby reducing the pressure gradient) (Bejan & Lorente, 2004). There are various spatial aspects to this minimization, which include: 1) perimeter smoothing, or the demonstration that for a polygon cross-section, the friction of a laminar, fully-developed (Poiseuille flow), as the number of sides increase the friction coefficient decreases, 2) noting that branching systems reduce friction and that 2a) bifurcations are more effective than trifurcations and 2b) there is an optimal amount of branching levels. There are further abstractions to note that 3) branching systems effectively fill space through a point-to-volume interaction (noted previously under various fractal studies). Nature uses tree-like structures (bifurcating networks) to facilitate flow because systems with branched configurations have lower overall flow resistances (higher conductivities) than series configurations. Tree configurations are used to demonstrate CL in various transport phenomena such as heat transfer optimization and fluid flow in piping networks; in the latter case, the Hess-Murray law is predicted and generalized as a confirmation case for the validity of CL.

Many other examples have been listed: particle coalescence, crack-formation, dendric growth (due to heat loss), convective cells (as in Bénard flow). Additionally, a model was developed for animals minimizing heat loss through their vascular systems which a) delivers flow through a branched network and b) transfers heat in a counterflow heat exchanger. Predictions include breathing rate and the Hess-Murray law. Further ideas are explored ranging from animal mobility to socioeconomic relationships. For this work, focus is restricted to flows in 'inanimate' systems which are driven by thermal-fluid sources. Notwithstanding the consequences of interpretation when removing arguments about flow access for bipedalism or wing-flapping, focusing on convective or heat flow will bring clarity in analysis of these natural processes.

The final aspect of constructal theory is that systems operate on an Engine and Brake model, whereby "engines" minimize entropy generation and provide energy flow to "brakes" which maximize entropy production (Bejan & Lorente, 2010). Examples of engines and brakes include river systems where the flow network is the engine (minimize frictional head loss) and the ejection of water into the sink is the brake, which purportedly maximize dissipation as the pressure head is lost.

2. CASE COMPARISONS

We now consider these thermodynamic ideas side-by-side for two case studies. References and high-level details are noted. Comparison of the models then follows in the subsequent discussion section.

2.1. Atmospheric heat transfer

2.1.1. Kleidon

Kleidon analyzed atmospheric convection rates and the effect of radiation input or output on these rates by using a two-box model (Kleidon, 2009; Kleidon, 2010) shown in Figure 1. The sun radiates heat into the atmosphere, with a larger quantity transferred to the tropics compared with the poles. This excess heat is then transported from tropics to poles through atmospheric convection, where it is radiated outward into space. Model variations include radiation emission from the tropics as well as additional tropic-to-pole mechanisms such as convection. The key hypothesis to this model is that the tropic-to-pole convection mechanism varies in order to produce a state of MEP in the atmosphere. It has been argued that doing so yields an accurate prediction of the known convection coefficient without resorting to additional sub-models of atmospheric convection or turbulence – which purportedly shows both the accuracy and benefit of utilizing the MEP hypothesis.

Note that a model variation exists with only one pathway for heat flow (an equivalent series configuration): radiation into tropics, convection to poles, radiation out of poles. While perhaps physically unrealistic for the earth, this model yields the minimum entropy production (mEP) state rather than the MEP state. Further details can be found in the references for the relevant equations and plots showing EP principles.

2.1.2. Reis, Bejan

Reis and Bejan had reached similar conclusions earlier (Reis & Bejan, 2006), that heat transfer is maximized from tropics to poles. In doing so, the authors did not indicate that conductivity is maximized or that MEP applies here (or anywhere else). Their analysis uses sub-models with more detail than Kleidon's; yet high-level predictions such global temperature difference are the similar to Kleidon's and consistent with observations. There are additional matches with zone latitudes and convective cell size. Local entropy production is plotted with a maximum or minimum depending on the zone – although the overall value is such that minimum and maximum quantities are less than 1% of each other.

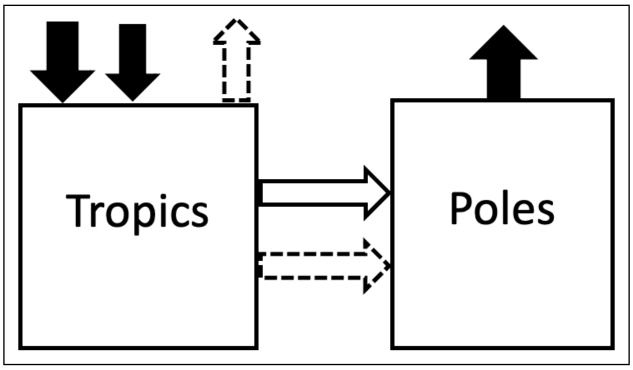

Figure 1. Kleidon's box model framework of the atmosphere. Arrows indicate heat fluxes: solid and filled arrows for constant fluxes used in all models, solid and unfilled arrows for variable fluxes used in all models, dashed and unfilled arrows for variable fluxes used only for specific model variations.

2.2. River network evolution

2.2.1. Reis (and earlier Bejan work)

Bejan and Errera addressed network configurations for porous flow by considering a combination of high and low permeability materials for drainage systems (Bejan & Errera, 1997). They discovered a single optimization principle whereby flow resistance is minimized when the high-permeability material is distributed in a tree network. The highest permeability material is that which creates a channel and Darcy flow becomes Poiseuille flow.

This early model was intentionally simple to exhibit the concept. Later work (Errera & Bejan, 1998) would add physically realistic specifics regarding branch angles and initial distributions of high permeability material. These later models better explain the geometric partitioning between seepage and channeling regions. Next, the development of a channel was considered from a given collection area of porous material with a cross-section ejection channel. Constructal Law analysis seeks to lower the peak pressure on the collection area, positing that flow resistance monotonically decreases in time by modifying the channel. This occurs due to erosion whereby "portions of the system can be dislodged and ejected through the sink." This process has an energetic cost which effects the shape of the channel network.

Reis applied CL to river networks (Reis, 2006) by taking max conductivity to find the non-dimensional ratios relating branch lengths and cross-sectional areas. These were in agreement with known relationships for branching networks including Horton's laws for the number of and length of consecutive streams and Hack's law relating the length and area of a given branch. To give visualization of a real branched-network system, the main pathways of the Nile River Delta are shown in Figure 2.

2.2.2. *Kleidon's river model*

In Kleidon, et al. (Kleidon et al., 2013), a model was developed to explore the development of river networks. It was built upon three sub-models: 1) precipitation and water flowing downhill (towards sea level) and removing sediment due to drag, 2) the creation of channel networks (branched) and their optimization (steeping sides, ideal amount and length of branches) which leads to enhanced sediment removal and 3) subterranean forces to restore the geopotential when the sediment weight has been removed. The strength of the model is the ability to a) recover the flow regime equations for various modes of porous transport, b) predict various flow regimes where sediment transport is proportional to different power relationships with geopotential gradients, and c) the demonstration of optimal dimensionless quantities which characterize the sediment drag or removal. The models were then used to describe the evolutionary development of a riverbed from a generic slope. Next, a discussion was made concerning the interactions of these various systems and how they produce feedback effects (see Gaia hypothesis in section 3.5 below). Finally, these observations were abstracted to a higher level to demonstrate that nature operates on certain extrema principles, many of which have similar names but are functionally the same: maximum power or minimum dissipation.

3. DISCUSSION

3.1. Common features

What is the relationship of these two theories? Are they two opposing camps within the landscape of thermodynamic principles? Both claim to subsume the other (see section 3.2 below). It will be shown that both MEP and CL are operating on the same framework and using the same logic but flipping the premise and conclusions.

To demonstrate this, we reduce the models used respectively by Kleidon and Bejan to their equivalent electrical networks using the circuit analogy. The power of circuit analogies can by demonstrated by observing Figure 3 with the common feature to both models becoming apparent: variable conductivity/resistivity. A second shared feature from their works is branched flow networks. It could be further noted that adding branches to a network inherently increases the network conductivity (given certain conditions are met regarding cross-sectional area, etc.), making this second point an extension of the first. However, the novelty of increasing conductivity by adding a branch compared to other mechanics, e.g. increasing cross-sectional area in a particular channel, is sufficient to merit its own designation.

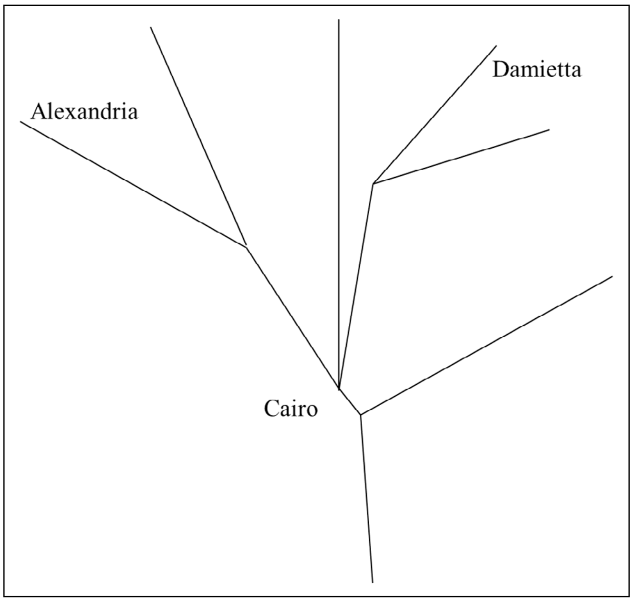

Figure 2. Mapping of main branches of the Nile River Delta to illustrate bifurcating flow network as used in Constructal Law analysis. Pathways have been straightened.

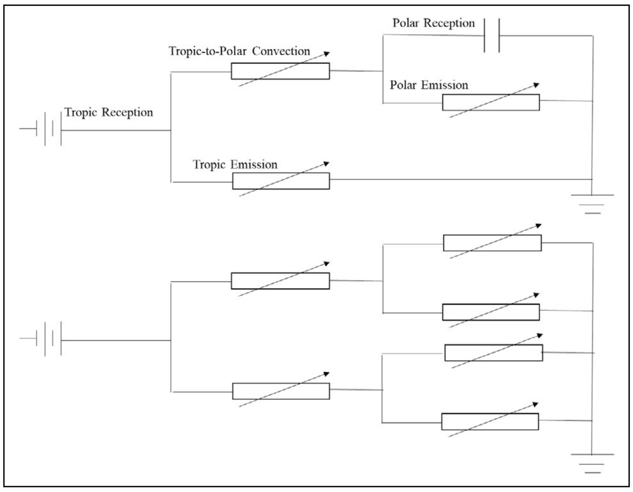

Figure 3. Circuit-Equivalent flow networks for Kleidon's atmospheric model (above) and Bejan's piping network (below) as seen for Hess-Murray type systems.

For Kleidon, the variable "conductivity" is found in the convection coefficient and affected by the driving force: radiation or channel size post-erosion. The latter is achieved by a maximization of conductance, both 1) in time and 2) due to force increases. Over time, the fluctuations in overland water flow allow for the optimal (branched) structure to be created. With force, as increased waterflow leads to more sediment ejection (more conductive channels). Similarly with Bejan, the type of conductivity changes depend on the system being analyzed: erosion for riverbeds, perimeter smoothing of pipes, self-lubrication for oil flows, etc. Further comparisons are made for the flow friction and the shear stress between laminar and turbulent flow with the latter becoming a dominant mechanism for transport under certain conditions. Finally, it is noted that as flow systems, these two models share a third commonality in that they are 'open' with energy flowing through them.

3.2. Priority and unity of models

After these similarities, the two idea-systems bifurcate. Kleidon has commented that his analysis of river network formation extends Bejan's work by quantifying the enhanced sediment flux and "does not need to rely on an ill-defined concept of 'access'" (Kleidon et al., 2013). For example, several applications listed for CL variously pose alternative characterizations such as minimum flow resistance, maximum flow rate, maximum rate of gradient dissipation (Bejan, 2016). Bejan has criticized MEP in general and Kleidon in particular (Bejan, 2010) on several points such that MEP is a consequence of CL and that MEP is one of many extremum principles (MEP, mEP, maximum power, etc.) which are "mutually contradictory" and yet simultaneously both corollaries of CL.

An expedient way to mitigate any ambiguity is to reduce these principles to mathematical formulation. Reis, a frequent co-author with Bejan has interpreted "greater access to flow" to mean maximum transport coefficients (Reis, 2014). Although Bejan has never explicitly stated such a definition to this author's knowledge, he has cited it elsewhere (Bejan, 2018, 2020; Bejan & Tsatsaronis, 2021). While the flow access of constructal theory has numerous implications, for systems pertaining to transport theory, we find it helpful to interpret CL as the process of maximizing conductance (MC); the specifics are particular to the flow system, but generally CL predicts that "surviving" systems increase conductivity in time.

There would appear to be a correlation between MC and MEP. In Kleidon's system, MEP applied to branched flow networks indicates MC when the entire system is considered (see σ_{tot} in Fig. 3c of Ref. (Kleidon, 2010)). For Bejan, MC applied to branched networks results in MEP – but only in certain conditions such as when the driving force is fixed. Alternatively, Bejan has argued for the presence of CL in fixed flux systems such as rivers with constant flow rates due to steady atmospheric precipitation or flow networks designed to reduce pressure drop while maintain a constant flowrate (Bejan, 2016). In these cases, the driving pressure is dissipated at the fastest possible rate, leading to a reduction in entropy production and apparently towards mEP. An operational scheme might then be proposed similar to Eqs. 6-7 below:

$$dX = 0 \rightarrow \left(\frac{\partial k}{\partial t} > 0 \leftrightarrow \frac{\partial \sigma}{\partial t} > 0 \right) \tag{6}$$

$$dJ = 0 \rightarrow \left(\frac{\partial k}{\partial t} > 0 \leftrightarrow \frac{\partial \sigma}{\partial t} < 0 \right) \tag{7}$$

This result is similar to that found by Reis (Reis, 2014) who argued that MC leads to MEP for fixed forces (Eqs. 8-9) but results in mEP for fixed fluxes (see Eqs. 10-11) by using Eqs. 1-2 above and substituting based upon the invariant quantity.

$$d\sigma = d \left(\Sigma_j X_i \cdot \left(\frac{X_i}{R_j} \right) \right) = 0 \tag{8}$$

$$d^2\sigma > 0 \; for \; d \left(\frac{1}{R} \right) > 0 \tag{9}$$

$$d\sigma = d \left(\Sigma_i J_i R_i \cdot J_i \right) = 0 \tag{10}$$

$$d^2\sigma < 0 \; for \; d \left(\frac{1}{R} \right) > 0 \tag{11}$$

Can it be said that the CL is the more general system and MEP a mere subsystem of thought? It might appear so in the above context. Yet this result is built upon an assumption that the system under analysis can operate indefinitely under a mode of either fixed transport force or fixed flux. However, the second law of thermodynamics forbids such transport regimes in isolated systems; they are only possible for subsystems within an isolated system. A consequence of this finding is that for isolated systems with finite driving forces, any increase in flow conductivity corresponds to an increase in entropy production. An example of a unified theory could be observed for a river network (with the inclusion of the source of the flow and the sink so that the system is thermodynamically isolated). It is clear that riverbeds widen with time due to erosion, thereby increasing the conductivity or "flow access". Consequently, the increased conductivity increases the flow speeds, augmenting the entropy production as expected from MEP. We note that these ideas balance for evolutionary flow systems rather than for a system at steady state.

3.3. Engines and brakes

However, these analyses tend to describe systems in their later/final/optimized state and must propose alternative processes which achieve such a state. Related to the issues with juxtaposed principles about flow resistance is that Bejan has proposed that the engine / brake model should be used instead of Kleidon's minimum expenditure and maximum power. It has also been claimed that MEP implies maximum irreversibility, which makes it unfit to explain any entities that grow, especially biological ones. Finally, Bejan has addressed two points which link to the question of teleology. He states that optimal principles are not true and that CL is not an optimal principle and that the principle we are searching for (constructal law) is not an end but about evolution. CL is about evolution to easier access or less reversibility or a symbiosis of less-reversibility-engine with more-reversibility-brake (moving toward a goal is not teleological but evolutionary).

These criticisms originate in a linguistic confusion about energy flow optimization. Minimum flow resistance in this context means minimum pressure/mass-flow resistance and maximum temperature/heat flow resistance. By taking "dissipation" instead of "resistance", it becomes clear that these viewpoints are not only consistent but can be the same principle: minimizing heat flow is equivalent to the maximization pressure flow. Failing to distinguish pressure-based and temperature-based energy and calling them both 'flow' is the source of this false contradiction. The engine/brake model builds upon this wording about dissipation. "Engines" maximize pressure-flow by minimizing temperature-flow out. It delivers the pressure to a "brake" where pressure is converted to temperature through dissipation which is maximally dissipated. Engines and brakes both maximize flow; they are merely designed to maximize different types of flows: pressure-flow or heat. Thus both 'engines' and 'brakes' operate under principles of MEP, for to maximize the flux for a given force is to max EP (Eqs. 1, 8). Naturally then, MEP is not max irreversibility. To say so is to restrict EP to heat dissipation only. From Onsager's equations, pressure-driven flow also contributes to EP. Thus, MEP can maximize pressure-driven flow exactly by minimizing temperature-driven dissipation and still effectively minimize "irreversibility" if this term means heat generation, viscous dissipation, etc.

Drawing further upon the engine/brake discussion, we now provide brief mathematical analysis of a flow system with heat dissipation. Often, transport systems are only treated for single modes of transport. Here, we account for multiple paths of energy dispersal and recall the argument above that pressure dissipation and temperature dissipation are both forms of entropy generation.

$$\sigma = \Sigma_i X_i \Sigma_k J_{ik} = X_P \left(J_{P,P} + J_{P,T}\right) + X_T \left(J_{T,T} + J_{T,P}\right) \tag{12}$$

$$\sigma = X_P \left(L_{P,P} X_P + L_{P,T} X_T\right) + X_T \left(L_{T,T} X_T + L_{T,P} X_P\right) \tag{13}$$

$$\sigma = L_{P,P} X_P^2 + \left(L_{P,T} + L_{T,P}\right) X_T X_P + L_{T,T} X_T^2 \tag{14}$$

Here, the pressure gradient is dissipated by the pressure-driven mass flow (P,P) and also by any existing temperature gradients (P,T) which might arise due to viscous dissipation or some other mechanism. Bejan's description of engines to brakes is implicitly utilizing the idea that systems such as rivers or other devices which perform pressure-work will transition temporally from an environment where X_P is dominant to a regime where X_T is dominant. In both cases, σ is maximized – what changes is the transport mode responsible for the maximizing. What Bejan calls "engines" appear to be systems where X_P is primarily responsible for σ while "brakes" are systems where X_T is the main driver of σ.

3.4. Other Model Features

It must be noted in concluding that there are additional hypotheses to these idea-systems which are not evaluated here. MEP as promulgated by Kleidon has focused on a particular subsystem (tropic-to-poles) rather than the entire system (the atmosphere). Further, the invocation of the Gaia hypothesis is related to and is an extension of MEP; but it is an addition to, and not a necessity of, MEP. There are two ways in which these ideas interact with Prigogine's. Firstly, feedback plays significant roles both in increasing the channel conductivity (due to sediment removal) and in restoring the geopotential force. Note how Prigogine talked about the autocatalytic nature of dissipative systems. Secondly, the apparent randomness in channel creation and smoothing over time (see conductance change with time above) is akin to Prigogine's idea of 'order from chaos' where system perturbations allow access to new structures with higher levels of dissipation.

On the other side, Constructal Theory tells us that when systems maintain existence, they operate with increased conductivity/"flow access". Yet the theory does not predict if this will occur, whether the system under scrutiny will increase conductivity and "survive", only that these two features are correlated. Indeed, the evolutionary nature of Constructal Theory has an inherent tension with principles for systems at steady-state: such systems are stagnant, not growing, and therefore will not ultimately survive. Such tensions can be resolved by appealing to the larger isolated system with a depleting force. This framework demonstrates that steady-state is only a subsystem feature. A steady-state subsystem within an isolated system will yield a depleting flux if the conductivity is not increased. Consequently, the entropy production will diminish and the system will eventually cease to experience flow. This provides a complete link between entropy production principles and Constructal Theory: just as maximizing entropy production is equivalent to maximizing flow conductivity, so stagnant flow conductivity in an isolated system is equivalent to minimizing entropy production which ultimately ends in equilibrium – a form of system death where flow is no longer channeled.

4. CONCLUSIONS

An important mark has been achieved by uniting key aspects of constructal law and maximum entropy production. It has been shown that advocates of both ideas operate with systems whose flow networks increase conductivity, often by generating new branches in the network. One method applies a hypothesis of increased transport conductivity which can lead to increased entropy production as a result. Another method assumes that entropy production will increase and finds an increased conductivity as a result. It has been implied that the proportional relationship of these two quantities makes either approach valid. Finally, a unity of entropy production from pressure-driven flow and from heat is shown, collapsing the engine/brake dichotomy into a single phenomenon.

DATA AVAILABILITY STATEMENT

The published publication includes all graphics and data collected or developed during the study.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

FINANCIAL DISCLOSURE

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