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RESEARCH ARTICLE

A HYBRID MODIFIED SUBGRADIENT ALGORITHM THAT SELF-DETERMINES THE PROPER PARAMETER VALUES

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ABSTRACT

A successful solution algorithm for non-convex optimization problems is the Modified Subgradient Algorithm (MSGA), which solves dual problems based on the sharp augmented lagrangian function. However, its performance highly depends on its parameter values, and determining the appropriate parameter values is difficult as they can be completely different for each problem. In this study, a new hybrid solution approach that a tabu search algorithm to find the appropriate MSGA parameter values and the MSGA algorithm run together is proposed. Although it seems like a contradiction to use an algorithm that also has its parameters to determine the most appropriate parameter values of an algorithm, this contradiction is eliminated by fixing the parameter values of the tabu search algorithm. The proposed algorithm does not need appropriate values of any algorithm parameter. It can find appropriate parameter values for each problem itself starting with the same fixed initial values. To show the success of the developed algorithm, especially on 0-1 quadratic problems, it is compared with the classical MSGA algorithm by using the quadratic knapsack test instances taken in the literature. According to the obtained solutions, the superiority of the hybrid algorithm has been demonstrated.

Keywords: *Modified Subgradient Algorithm (MSGA), Tabu Search Algorithm, Quadratic Knapsack Problem.*

1. INTRODUCTION

Lagrangian relaxation and subgradient algorithms have been widely applied to integer or mixed integer programming problems. However, classic Lagrangian techniques often result in a duality gap and generally cannot determine the optimum value of the primal integer optimization problems, such as the quadratic 0-1 problems that are non-convex [1].

Recently, a considerable amount of literature has been published on different augmented Lagrangian duality methods that can eliminate the duality gap in most non-convex problems and obtain good solutions. The MSGA is developed by Gasimov [2], and then, a general version of the algorithm including generalized augmented Lagrangian dual problems is presented by Gasimov and Rubinov [3]. They applied this algorithm to a different kind of optimization problem. Gasimov and Ustun [4] demonstrated the performance of the MSGA for non-convex 0-1 quadratic assignment problems. Gasimov and Ustun [5] focussed on a generalized version of the algorithm to handle sharp augmented Lagrangian dual problems. In another study, Burachik et.al. [6] developed an inexact version of the algorithm that could obtain solutions to the problems with less computational time. Sipahioglu and Saraç [7] examined the algorithm's performance for QKP with an inequality constraint. To solve a general portfolio optimization problem, Ustun and Kasimbeyli [8] applied the feasible value-based modified subgradient (F-MSG) algorithm, which is a generalized version of the MSGA. Ulutas and Sarac [9], handled a multi-period facility layout problem where the sum of material handling and re-layout costs are minimized. They proposed the MSGA for this problem and determined its parameters of MSGA by using the design of the experiment. In another study, Ozcelik and Sarac [10] addressed the cell formation problem with alternative part routes to minimize the weighted sum of the voids and the exceptional elements. They proposed a hybrid genetic algorithm based on MSGA. Takan and Kasimbeyli [11] developed a new hybrid subgradient algorithm for solving the capacitated vehicle routing problem. In another recent study conducted by Bulbul and Kasimbeyli [12], a new version of the aircraft maintenance routing problem is addressed. The authors proposed a hybrid solution approach for this problem, which hybridized the F-MSGA and the ant colony optimization metaheuristic. As can be seen from these studies, MSGA is a successful solution method that is widely used in solving discrete problems with linear or quadratic objective functions. However, two difficulties can be encountered when using this algorithm. The first is that solving the dual problem can be very difficult. So, the studies on the MSGA in the literature in which the hybrid solution approach is suggested have generally focused on the solution of the dual problem and used metaheuristic algorithms to solve the dual problem. However, another important issue affecting the performance of the MSGA is the determination of appropriate parameter values. In the literature, only one study [9] has been accessed to determine the parameters of the MSGA by using the experimental design method.

The design of the experiment is usually used for determining the parameter sets of the algorithms to improve the solutions in the literature. However, it has two important disadvantages: (1) the success of the parameter set depends on the problem type so for each test instance the parameter sets must be redetermined by resolving it for each parameter combination in the experiment plan (2) in the design of experiment, parameter values are the factor levels so only a few predefined values are available for every parameter and combinations of these levels can be examined. In fact, the optimum values of the parameters may not be one of these levels.

In this study, a new hybrid solution approach that a tabu search algorithm (TSA) to find the appropriate MSGA parameter values and the MSGA run together is proposed. Although at first glance, it seems like a contradiction to use an algorithm with parameters to determine the most appropriate parameter values of an algorithm, this is eliminated by fixing the parameter values of TSA. Different from the design of the experiment, the developed algorithm has two significant

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advantages: (1) The TSA searches the appropriate parameter values for each instance automatically, and (2) The parameter values are not selected among only a few predefined numbers, the algorithm searches the appropriate values in a very large set. So, the optimum values can be obtained. To show the performance of the hybrid algorithm, it is compared with the classical MSGA by using the quadratic knapsack problem instances taken from the literature.

The remainder of this study is organized as follows. In the following section, the hybrid solution approach is introduced. TSA and the proposed hybrid algorithm are explained, respectively. Quadratic knapsack problems which are used for testing the hybrid algorithm's performance and test results are given in the third section. In the last section, conclusions are presented.

2. HYBRID SOLUTION APPROACH

In this section, the proposed solution approach combining the MSGA and a TSA, to find the appropriate parameter values of the MSGA is explained in detail. Before the steps of the algorithm, MSGA and the TSA are explained in the following sub-sections.

2.1. Modified Subgradient Algorithm

The MSGA can be applied to non-convex problems with the equality constraint. For non-convex problems, using the classical Lagrangian method can result in a non-zero duality gap. However, this problem can be eliminated by using the sharp-augmented Lagrangean function. It is proven that when the objective and constraint functions are all Lipschitz then the sharp augmented Lagrangean guarantees the zero-duality gap [2]. The algorithm has many other remarkable features. In the primal problem, it is not required convexity or differentiability conditions and the penalty parameter. This means, when the parameters are selected accurately, the optimal value can be found with the MSGA. So, the success of the MSGA depends on the selection of parameter values. The MSGA is briefly as follows:

$$\min P = \min_{x \in S} f(x) \tag{1}$$

$$subject\ to\ g(x) = 0 \tag{2}$$

P represents the primary problem. (f:$X \rightarrow R$, g: $X \rightarrow R^n$) S is a compact subset of X. The sharp-augmented Lagrangean function (L) associated with P is given in Eq. (3) (L: $S \times R^n \times R_+ \rightarrow R$):

$$L(x,u,c) = f(x) + c\|g(x)\| - \langle g(x), u \rangle \tag{3}$$

Where, $\|g(x)\|$ is the Euclidean-Norm of $g(x)$. $\langle g(x), u \rangle$ is the Euclidean Inner-Product on R^n. c and u are the dual variables. $H(u,c)$ is the dual function. (P^*) dual the problem of (P).

$$H(u,c) = \min_{x \in S} L(x,u,c), \quad for\ u \in R^n, and\ c \in R_+ \tag{4}$$

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$$(P^*) \quad \max_{(u,c)\in R^n \times R_+} H(u,c) \tag{5}$$

The steps of the MSGA are given below:

Step 0. Select $(u_1, c_1) \in R^n \times R_+$ and k= 1.

Step 1. Solve the sub-problem with Lagrange multipliers (u_k, c_k).

$$(P)$$
$$\underset{x_k \in S}{Minimize}\; f(x_k) + c_k\|g(x_k)\| - \langle u_k, g(x_k)\rangle \tag{6}$$

$$subject\ to\ f(x_k) + c_k\|g(x_k)\| - \langle u_k, g(x_k)\rangle \leq \overline{H} \tag{7}$$

If $\|g(x_{k)}\| \neq 0$ for the obtained solution x_k, x_k is not a feasible solution for (P), go to *Step 2*, if $\|g(x_{k)}\| = 0$, x_k is a feasible solution for (P) and (u_k, c_k) is a solution to (P^*) STOP.

Step 2. Calculate (u_{k+1}, c_{k+1}) values by using the formulas in Eq. (8) and go to *Step 1.*

$$u_{k+1} = u_k - \alpha s_k g(x_k),\;\; c_{k+1} = c_k + (1+\alpha)s_k\|g(x_k)\| \tag{8}$$

Here, s_k denote positive scalar step size and it can be calculated by the formula in Eq. (9).

$$s_k = \frac{\delta\alpha\left(\overline{H} - L(x_k, u_k, c_k)\right)}{(\alpha^2 + (1+\alpha)^2)\|g(x_k)\|^2} \tag{9}$$

Where, $\overline{H}$ is an upper bound value in the (P^*) and δ is a step size parameter. α > 0 and 0 < δ < 2.

Parameters of step size formulation (9) are $\overline{H}$, α and δ. Selecting the values for these parameters is important for the success of the MSGA. In the following two sections, it is explained how the values of these parameters are determined.

2.2. Tabu Search Algorithm

The TSA is a metaheuristic originally developed by Glover, which has been successfully applied to a variety of combinatorial optimization problems [13]. In this algorithm, to escape from local optimal, it also accepts unimproved solutions when not all neighborhoods are improved. It is briefly a single-solution-based metaheuristic that is used to explore a search space beyond local optimization, guided by a local search algorithm. The feature that distinguishes it from other algorithms is that the successive moves are limited by keeping them in a list to prevent the algorithm from repeatedly visiting the same solutions. The list in which successive moves are kept is called the *Tabu List (TL)*. *TL* operates with short-term memory and this memory is updated in each iteration. It is very critical for the performance of the TSA. Another critical feature is the *aspiration criterion*. If a move leads to a better solution than the current solution, that is, if it is a good move, but it is available in the *TL*, this

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move is removed from the *TL* and the move is allowed thanks to the aspiration criterion. For this reason, diversity is ensured in the algorithm by allowing previously the tabu moves.

In TSA, another important term is tabu list size. It determines how long a move will remain in the tabu list. Accordingly, when the number of tabu moves reaches the tabu list size, the oldest element in the tabu list is removed from the list. Thus, the tabu list has an innovative memory structure.
The steps of the basic TSA are given below:

Step 1. Determine the TSA parameter values (Tabu list size (s), number of iterations(t), the structure of the neighborhood etc.)

Step 2. Generate a feasible initial solution (x). Define this solution as the best-obtained solution and an empty tabu list ($x^*=x$, $TL=\{\}$).

Step 3. Generate a feasible new solution ($x^{'}$) by applying a move that except the tabu move in the *TL* or ensure the aspiration criterion in the *TL*

Step 4. Update the *TL* and the current solution (x= $x^{'}$, $TL=\{move_{xt}\}$)
Step 5. If the tabu list size is equal to s, delete the oldest element from the tabu list.

Step 6. If the current solution is better than the best-obtained solution, update the best-obtained solution ($x^*=x$).

Step 7. If the termination criterion is met, stop. If not, update the t ($t=t+1$) and return to *Step 3.*

The parameters of the TSA are described and their fixed values are given below.

Neighbour generation scheme: There are 3 parameters we try to find their proper values. By adding a certain amount (Δ_1, Δ_2, Δ_3) to the current parameter values ($\overline{H}$, α, δ) or by reducing a certain amount from the current parameter values, 6 neighbors ($\overline{H}$-Δ_1, $\overline{H}$+Δ_1, α-Δ_2, α+Δ_2, δ- Δ_3, δ+Δ_3) are obtained in each iteration.

Tabu list size: The tabu list size is fixed to six.
Termination criterion: This parameter decides how many iterations are allowed. It is fixed to 30.

2.3. Hybrid MSGA
In this section, the steps of the developed algorithm are given.

Step 0. Determine the maximum iterations number (*IMAX).* And select the maximum number of updates of the dual variables allowed (*KMAX).* Set the initial parameter values of MSGA as $\overline{H}$ = 0, α = 5, δ = 1 and set the certain amounts (adding to the parameters or reducing from the parameters) as (Δ_1 = 500, Δ_2 = 1, Δ_3 = 0.2).

Step 1. Generate the 6 neighbors ($\overline{H}$-Δ_1, $\overline{H}$+Δ_1, α-Δ_2, α+Δ_2, δ- Δ_3, δ+Δ_3) of the current parameter values. If δ- $\Delta_3 \leq 0$ or $\delta+\Delta_3 \geq 2$ add this neighbor to the tabu list. If all generated neighbors are in the tabu list go to *Step* 4. Otherwise, compute the objective function for each neighbor, which is not in the tabu list, by using the MSGA. To solve the problem, repeat *Step* 1.0-*Step*1.2 for all neighbors.

Step 1.0. Select a vector (u_1, c_1) as zero vector. Let k = 1.

Step 1.1. Solve the sub-problem with Lagrange multipliers (u_k, c_k).

$$\underset{x \in S}{Minimize}\ f(x) + c_k\|g(x)\| - \langle u_k, g(x)\rangle \tag{10}$$

$$subject\ to\ f(x) + c_k\|g(x)\| - \langle u_k, g(x)\rangle \leq \overline{H} \tag{11}$$

In this problem, x_k is the global solution. If $\|g(x_{k)}\| = 0$, or k ≥ *KMAX* then *go to step 2*. x_k is a solution to (*P*) and (u_k, c_k) is a solution to (*P**). If $\|g(x_k)\| \neq 0$, go to *Step* 1.2.
Step 1.2. Calculate the dual variables u_{k+1} and c_{k+1} by using formulation (3) and calculate step size by using formulation (8). $k = k + 1$ and repeat *Step* 1.1.

Step 2. If $\|g(x_k)\| = 0$ for any neighbor, *f*(*S**) and *S** are the objective function value and the relevant parameter values of the best neighbor and go to *Step* 3. If $\|g(x_k)\| \neq 0$, go to *Step* 4.

Step 3. Set the best neighbor (*S**) as move and memorize it (*S*: =*S**) if *f* (*S**)< *f*(*S*) and update the *TL* and go to *Step* 5.

Step 4. If *k* ≥ *KMAX* for all neighbors then *KMAX* = *KMAX* + 10. Divide all the Δ_i's by 2 and then delete all the moves from the *TL*. Go to *Step*5.

Step 5. If the number of iterations is equal to IMAX, stop and the best solution is *S**. If not equal, increase the number of iterations by 1 and go to *Step* 1.

3. COMPUTATIONAL RESULTS

Quadratic knapsack test problems, their optimal solutions are known, have been used to demonstrate the effectiveness of the hybrid algorithm. First, all test problems are solved with MSGA using the same initial parameter values for $\overline{H}$, α and δ. Then, the test problems have been also solved with the hybrid algorithm using the same initial parameter values. Thus, it is aimed to show how much the solution performance can be increased by intelligently determining the $\overline{H}$, α and δ parameters of the MSGA with the hybrid algorithm. Before discussing the computational results, the quadratic knapsack problem to be used in testing the algorithm is explained in the following sub-section.

3.1. Quadratic Knapsack Problem

The knapsack problem (KP) is one of the well-known combinatorial optimization problems. It consists of selecting from a finite set of given objects in such a way that a linear function of selected objects is maximized while a set of knapsack constraints are satisfied.

The Quadratic Knapsack Problem (QKP) is given as follows:

(QKP)

$$\max z = \sum_{i=1}^{n} p_i x_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} p_{ij} x_i x_j \tag{12}$$

$$subject\ to\ \sum_{i=1}^{n} w_i x_i \leq a \tag{13}$$

$$x_i, x_j \in \{0,1\}, i, j = 1, \dots, n \tag{14}$$

Objective (12) is maximizing the total profit. If object *i* is chosen to knapsack, its profit (p_i), and if object *j* is also selected with it, additional profit (p_{ij}) is also earned. Total profit contains both kinds of profits. Constraint (13) guarantees that the total weights of the selected objects do not exceed the knapsack capacity. Constraint (14) is the sign constraint of decision variables.

In the first step of the hybrid solution approach, QKP should be transformed into a continuous form. The continuous nonlinear formulation of the problem with equality constraints (CNP) is given in below:

(*CNP*)

$$\min z = -\sum_{i=1}^{n} p_i x_i - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} p_{ij} x_i x_j \tag{15}$$

$$subject\ to\ \sum_{i=1}^{n} w_i x_i + slack - a = 0 \tag{16}$$

$$\sum_{i=1}^{n} (x_i - x_i^2) = 0 \tag{17}$$

$$0 \leq x_i \leq 1\ \ i = 1, \dots, n\ ,\ \ slack \geq 0 \tag{18}$$

The maximization objective function was transformed to the minimization which is given in equation (15). Capacity constraint (16) was converted to equality constraint with the help of a slack variable. To transform binary variables into a continuous form, a specific constraint (17) which was developed by Li [14] is used. Constraint (18), is the sign constraint of decision variables.

3.2. Test Results

In this section, ten QKP examples in the literature are used to compare the proposed hybrid algorithm with the classical MSGA. All instances have been solved using MSGA parameters and GAMS solver Minos on the HP6000 workstation. Obtained solutions and times are reported and compared. The *n* and *d* values are used to define the structure of the test problems. (*n:* The number of available objects, *d:* density or percentage of non-zero *pij*) The test instances have 100 objects with a density of 0.25.

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The optimal solutions for these test instances have been known as all instances have been solved to optimal by Billionnet and Soutif [15] and the related website (http://cedric.cnam.fr/~soutif/QKP/) reports their optimum values.

The test problems have been solved with the parameter values given in Table 1.

Table 1. Test results.

Parameter	*Value*
s	6
$IMAX$	500
$KMAX$	30
$(\overline{H}, \alpha, \delta)$	(0, 5, 1)
$(\Delta_1, \Delta_2, \Delta_3)$	(500, 1, 0.2)

We solve all the test problems by using both MSGA and hybrid algorithm. To show the performance of the hybrid algorithm, we utilized percentage gap $(G\%)$ which is indicate a gap between the optimal solution and solution of the MSGA/hybrid algorithm. The formulation of the gap is given in (19) and the results of test problems are given in Table 2.

$$Gap\ (G\%) = \frac{100\times(value - opt.value)}{opt.value} \tag{19}$$

The optimum values of each instance are given in the first column. The second, third and fourth columns show the obtained objective value, gap (G%) and solution time in seconds using the MSGA, respectively. Similarly, last three columns of the table represent obtained results by the hybrid algorithm. To solve the augmented Lagrangean function, Minos solver of the GAMS is used for both algorithms. According to the table, if appropriate values of MSGA parameters are not selected and if they are taken as $\Delta_1 = 500$, $\Delta_2 = 1$, and $\Delta_3 = 0.2$ for all test problems, MSGA has poor performance for solving QKP. Feasible solutions have been obtained for only three instances of ten by using the MSGA.

Table 2. Test results.

Instances	*MSGA*			*Hybrid Algorithm*		
Opt. Value	*Value*	*G%*	*Time*	*Value*	*G%*	*Time*
18558	-	-	-	**18558**	0.00	53.63
56525	56452	0.13	1.17	56452	0.13	0.82
3752	-	-	-	3717	0.93	32.31
50382	-	-	-	**50382**	0.00	43.93
61494	-	-	-	60955	0.88	3.00
36360	-	-	-	36137	0.61	2.73

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14657	-	-	-	**<u>14657</u>**	0.00	9.00
20452	**<u>20452</u>**	0.00	0.56	**<u>20452</u>**	0.00	4.86
35438	35139	0.84	0.67	35325	0.32	0.47
24930	-	-	-	24748	0.73	2.37
				mean	0.36	56.84

As you can be seen from Table 2, the hybrid algorithm has high performance for solving QKP, even if it used the same initial parameter set ($\Delta_1 = 500$, $\Delta_2 = 1$, $\Delta_3 = 0.2$) with MSGA. It is a remarkable result because finding appropriate values of MSGA may not be easy for some test instances. Although the solution time of the hybrid algorithm is longer than MSGA, since the algorithm does not need a pre-successful parameter set and can determine the appropriate values of these parameters itself, this solution time can be considered reasonable.

4. CONCLUSION

Although the MSGA algorithm is successful in solving 0-1 quadratic programming problems, its performance largely depends on the correct determination of its parameter values. However, it is not easy to predict the values of MSGA parameters. Moreover, the appropriate values for each different problem can be very different. Therefore, in this study, a new hybrid solution approach that a TSA to find the appropriate MSGA parameter values and the MSGA algorithm run together. The proposed algorithm does not need appropriate parameter values of the MSGA or TSA. The proposed hybrid algorithm starts with the same fixed parameter values for all problems and the appropriate values of the parameters are determined dynamically by the algorithm. Computational results show great success in solving QKPs. Most of the problems were solved optimally. Even for problems that were not solved optimally, the solution was only 1% away from the optimal solution. In addition, a wide class of non-convex problems can be solved by using this proposed algorithm. In a further study, the performance of the hybrid algorithm may be demonstrated on the different types of non-convex problems.

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