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Pre-service Mathematics Teachers' Conceptual Knowledge Related To Basic Concepts And Operations

Zeki Aksu[1]
Artvin Çoruh University

Senem Kalaç[2]
Atatürk University

Merve Özkaya[3]
Atatürk University

Alper Cihan Konyalıoğlu[4]
Atatürk University

Abstract

The aim of this study is to discuss the conceptual dimensions of pre-service mathematics teachers' approaches to the basic concepts and operations of mathematics. 25 pre-service mathematics teachers in the first semester of the 2022-2023 academic year, who were educated at a state university, participated in the research. Four open-ended questions about the basic concepts of mathematics were asked to pre-service mathematics teachers and their written opinions were received. Content and descriptive analysis were applied to the responses. As a result of the analyzes, pre-service teachers did not approach these questions in the context of conceptual knowledge, but generally gave the answers in the context of procedural knowledge. Pre-service mathematics teachers used reflective thinking, which they used to compare the procedural knowledge processes they gained in the formal education process with the previous solutions, in these question solutions as well.

Key Words

Conceptual knowledge • Procedural knowledge • Pre-service teacher

[1]**Correspondance to:** Artvin Çoruh University, Artvin, Turkey. E-mail: zekiaksu25@artvin.edu.tr **ORCID:** 0000-0001-6839-6847
[2]Atatürk University, Erzurum, Turkey. E-mail: senemkalac@gmail.com **ORCID:** 0000-0002-2365-941X
[3]Atatürk University, Erzurum, Turkey. E-mail: mdurkaya@atauni.edu.tr **ORCID:** 0000-0002-0436-4931
[4]Atatürk University, Erzurum, Turkey. E-mail ackonyali@atauni.edu.tr **ORCID:** 0000-0002-6009-4251

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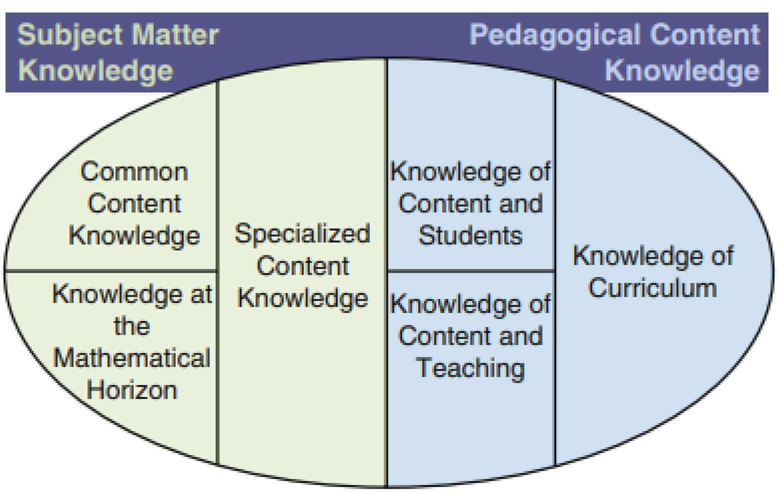
One of the researchers who formed the basis of research on teacher and teacher knowledge is Shulman. Shulman (1986), teacher knowledge; analyzed by establishing a theoretical framework on the categories of content knowledge (this refers to the amount and organization of knowledge per se in the mind of the teacher), pedagogical content knowledge (a type of content knowledge that is more related to the teachability of the subject), and curriculum knowledge (contains the sequence of materials and topics to be used in teaching). The effect of content knowledge on the formation of mathematics teaching knowledge of teachers or pre-service teachers is undoubtedly very important. Various studies have also revealed that there is a close relationship between content knowledge and pedagogical content knowledge (Boz, 2004; Capraro et al ., 2005; Even , 1993; Türnüklü , 2005).

Shulman (1987) defined content knowledge as a type of knowledge that includes teachers' concepts, operations, proofs, and problem-solving skills related to the subject they will teach. It is related to the basic concepts and content of the field (mathematics, biology, chemistry, etc.) to be taught by the teacher. The teacher should create appropriate learning environments that allow students to understand the part of the content in the curriculum. They should know and use the teaching methods and techniques used in their field. Shulman mentions two basic structures while describing content knowledge. The first of these structures is a set of ways to determine the truth or falsity, validity or invalidity of (mathematics) concepts and facts in the field, and the second is the different ways of producing and structuring knowledge in the field.

Ball et al. (2008), as a result of their study on what mathematics teachers should know, revealed a classification as in Figure 1 under the title of *"Content Knowledge for Teaching Mathematics"*.

Figure 1

Ball et al. (2008) model of Content Knowledge for Teaching Mathematics (CKTM)



Common Content Knowledge; It is kept related to the knowledge of mathematics that not only teachers but also others can have. Specifically, it includes teachers or pre-service teachers to be able to do the activities they will present to their students and to use mathematical terms for related concepts correctly.

Horizontal content knowledge; on the other hand, includes teachers having a general knowledge of the curriculum by knowing how the mathematics topics in the curriculum are related to each other.

Specialized content knowledge; is the mathematical knowledge and skills required for teaching used by mathematics teachers and pre-service teachers. What is expected from teachers and pre-service teachers is to know mathematical concepts, explanations and ideas in a conceptual way and to be able to transfer them to their students with appropriate representations and methods.

Both the *subject content knowledge* expressed by Shulman and the *customized content knowledge revealed by* Ball et al. are examined, mathematics teachers are expected to have in-depth procedural and conceptual content knowledge about mathematics subjects. According to Ball (1990), the concept and process knowledge of mathematics teachers should be correct; They also need to understand the principles underlying this information. Chick et al. (2006) stated the expectations from mathematics teachers regarding content knowledge in the pedagogical context as follows:

Profound Understanding of Fundamental Mathematics; Exhibits deep and thorough conceptual understanding of identified aspects of mathematics.

Deconstructing Content to Key Components; Identifies critical mathematical components within a concept that are fundamental for understanding and applying that concept

Mathematical Structure and Connections; Makes connections between concepts and topics, including interdependence of concepts

Procedural Knowledge; Displays skills for solving mathematical problems (conceptual understanding need not be evident).

Methods of Solution; Demonstrates a method for solving a mathematical problem.

Purpose of the Study

It is undoubtedly very important that mathematics teachers and pre-service teachers should have a deep conceptual knowledge of the subjects they will teach. It is undoubtedly very important that mathematics teachers and pre-service teachers should have a deep conceptual knowledge of the subjects they will teach. Skemp (1971), who investigated mathematics knowledge for the first time in terms of learning psychology , conceptual knowledge; "as the ability to know what to do and understand why", also procedural knowledge; He defined it as "the ability to use the rules without understanding the reasons". In other words, while in procedural knowledge there is only the state of knowing how to use a concept or process without knowing the reason for it, in conceptual knowledge the state of comprehension comes to the fore (Baki 1997). Permanent and functional learning in mathematics can only be possible by balancing procedural and conceptual knowledge (Baki, 1998).

The aim of this study is to evaluate the content knowledge of pre-service mathematics teachers about natural numbers in line with the explanations given in the introduction.

Method

In this section, the design of the research, the research group, the data collection tool, data analysis of the study are mentioned.

Research Design

This study, it was tried to determine the approaches of pre-service teachers to the basic concepts and operations of mathematics. For this purpose, the case study method was adopted from qualitative research approaches. The case study method focuses on a specific situation, describes this situation and enables the reader to better understand the existing situation (Merriam, 2013). This method seeks to answer the question of how while explaining the existing situation. But apart from that, it provides a deeper understanding of a phenomenon. Data are described and classified using themes (Creswell, 2020; Yıldırm & Şimşek, 2013). Similarly, in this study, the pre-service teachers' phenomenon about basic mathematical concepts and operations were described and described in depth.

Research Group

The research group of the study consists of 25 pre-service teachers attending a mathematics teacher education programme in a university in Turkey. Qualitative research generally involves a small sample recruited using purposive sampling (Patton, 2014). It was placed the names of the pre-service teachers within the ethical rules. Their names were not given, coded as PT1, PT2…, PT25.

Research Instruments and Processes

Data were collected with a concept test consisting of four questions. While preparing the concept test, the students were asked to interpret whether the statements in the question were always correct so that they could think from multiple perspectives. The questions that were included in the data collection tool are listed in the table below.

Table 1

Questions in the data collection tool

Question numbers	Questions
1	Does $\frac{a}{b}$ always make sense? Please comment.
2	Does $a - b$ always make sense? Please comment.
3	Is it correct to replace the parentheses in the $(x - y) + z = x - (y - z)$ equation? Please comment.
4	"Addition, subtraction, multiplication, and division are operations, and these are called four operations." Is the statement always true? Please comment.

Data Analysis

When the selected questions are examined, these four questions determine what kind of approach the pre-service teachers put forward regarding elementary concepts and operations at the basic level. The questions are at the basic level, as it is aimed to reveal the understanding of the concepts rather than revealing the knowledge levels of the pre-service teachers. It was applied to 10 primary school pre-service mathematics teachers in order to control the concept test questions prepared by one of the researchers and to provide preliminary information about the research. As a result of the pilot study, questions involving the search for significance in natural numbers were used throughout the test. In the study, it was concluded that the final version of the data collection tool, which was examined by two mathematics educators who are experts in their fields, is suitable for the research problem.

The researchers applied the concept test to the pre-service teachers during a class hour. It was said that it was left to them how to interpret the questions in the data collection tool and the data collection tool was presented to the pre-service teachers. They were asked to evaluate and interpret each question.

First of all, it was determined how many of the pre-service teachers who participated in the research answered the questions. Twenty-five pre-service teachers answered the first and second questions each, and twenty pre-service teachers answered each of the third and fourth questions. The answers of the pre-service teachers who did not answer, that is, left the question blank, were not taken into consideration. The obtained data were subjected to content and descriptive analysis. Categorization was made according to the common features in the answers given and direct quotations from the answers were given.

Results

In this section, content and descriptive analysis of the answers to the data collection tool consisting of four questions were made. The answers given by the pre-service teachers to the questions were examined and the codes were created. Interpretations were made by giving examples selected from the answers.

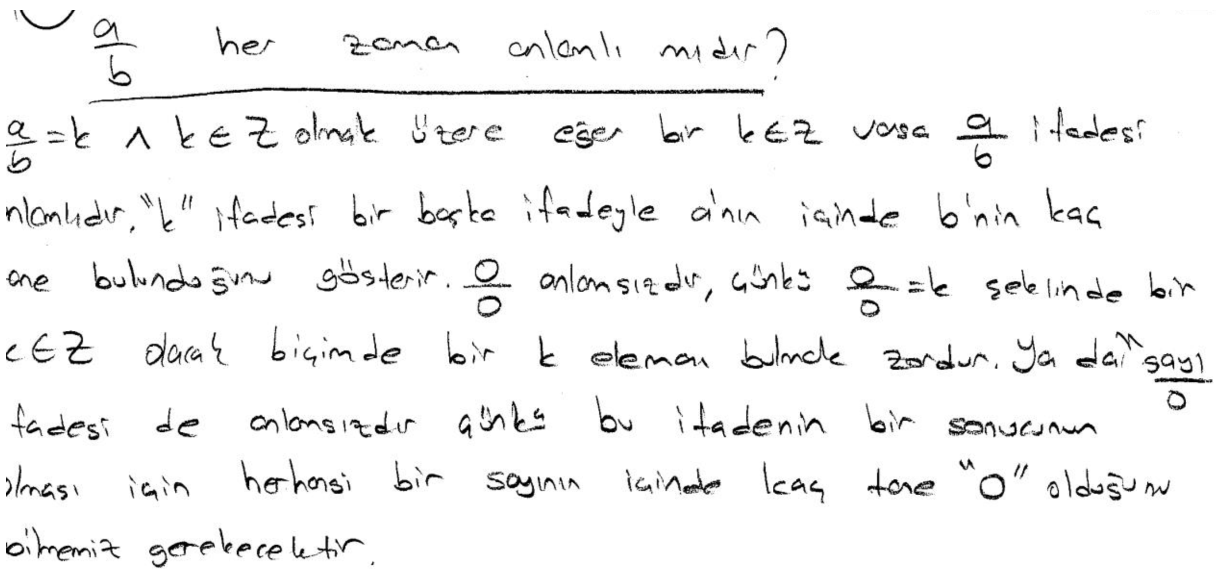
"Question 1 *:* Does $\frac{a}{b}$ always make sense? Please comment.*"* The findings of the answers to the question are presented in Table 2.

Table 2

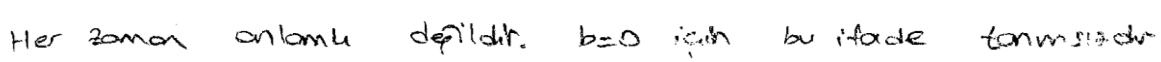
Responses to Question 1

Responses	Respondents	Percent
if b=0 the expression is undefined so it doesn't always make sense.	12	48
If b=0, it is undefined. The set we work with is important	9	36
Depends on the set being studied	3	12
a/b=k and makes sense if k elements are Z.	1	4
Total	25	100

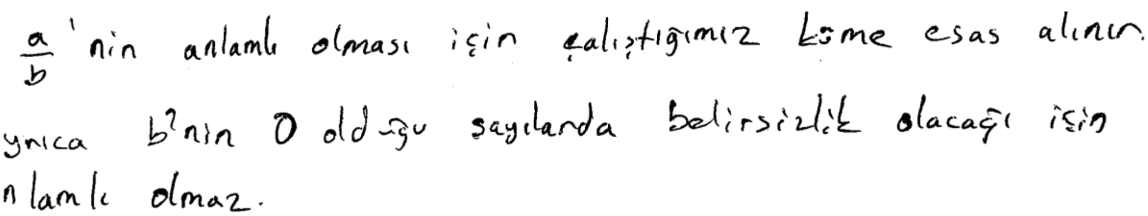
When Table 2 is examined, it is seen that pre-service teachers mostly focus on the situation where the denominator is not zero. While 12 people argued that the expression b should be different from 0, 9 people also emphasized the importance of the cluster being studied. 84% of the respondents stated that b should be nonzero. Examples of answers are given below.



It said: Makes sense if a/b=k and k elements are Z. The expression k shows how many b are in the moment. 0/0 is meaningless. Because it is difficult to find a k element such that 0/0=k. Again number/0 is undefined.



It said: It doesn't always make sense. For b=0 this expression is undefined.



It said: In order for it to be meaningful, it is based on the cluster we are working with. When b=0, it is not significant because there will be uncertainty.

a ve b reel sayılar olmak üzere $b \neq 0$ ifade tanımlı olup anlamlı dur. Farz edelim eşylar kümesinde çalışıyoruz a için masa olsun b için için sandelye olsun diyelim

$\frac{a}{b} = \frac{\text{masa}}{\text{sandalye}}$ anlamlı bir ifade olmayacaktır.

It said: Significant if a and b are real numbers and b is nonzero. Suppose we are working on a set of items. If a is a table and b is a chair, a/b= table/chair is not significant.

Hayır her zaman anlamlı olmayabilir, rasyonel sayılar tanımından payda sıfırdan farklı olacaktır. bu ifadede paydanın sıfır olma ihtimalini düşündüğümüzde tanımsız bir ifade olacağını görüyoruz Bu nedenle her zaman anlamlı olmayabilir.

It said: It may not always make sense. If the denominator is zero, it is undefined.

Although the respondents emphasized the importance of the studied cluster, they also linked the condition of being defined to different variables. Although it is one of the common ideas that the denominator should not be zero, it is also written that the expression can be made defined with limit approaches.

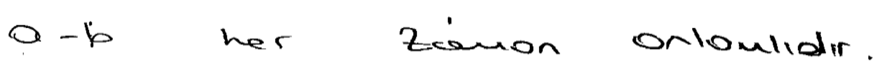
"Question 2: Does $a - b$ always make sense? Please comment." The findings of the students' answers to the question are presented in Table 3.

Table 3

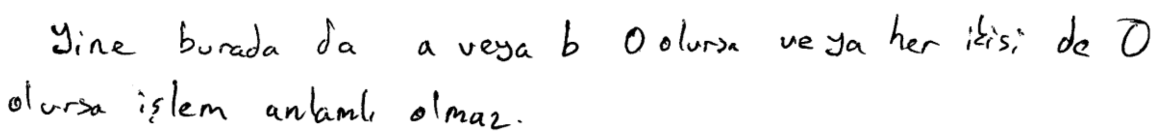
Responses to Question 2

Responses	Respondents	Percent
Always makes sense	10	40
∞-∞ becomes meaningless. That's why it doesn't always make sense.	5	20
Depends on the cluster being studied	4	16
Not meaningful if the result is zero	2	8
The expression is meaningful only if a is positive and b is negative (a+b).	1	4
It doesn't make sense mathematically. It is physically meaningful.	1	4
In physical life it is sometimes meaningless.	1	4
0-0 makes no sense	1	4
Total	25	100

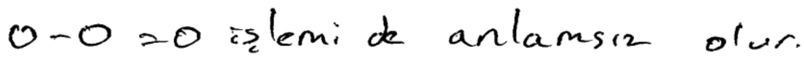
As seen in Table 3, 10 of the respondents stated that the expression was significant. 5 people argued that the expression is meaningless in the ∞-∞ ambiguity. 4 people stated the importance of the studied cluster. Examples of answers are given below.



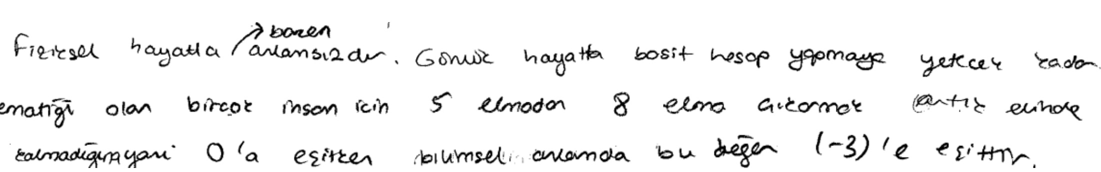
It said: Always ıt makes sense



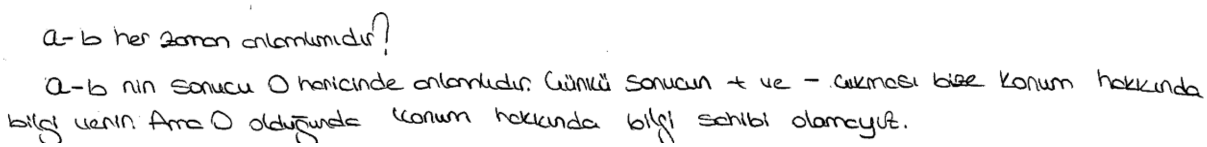
It said: Doesn't make sense if a or b are 0 or both are zero.



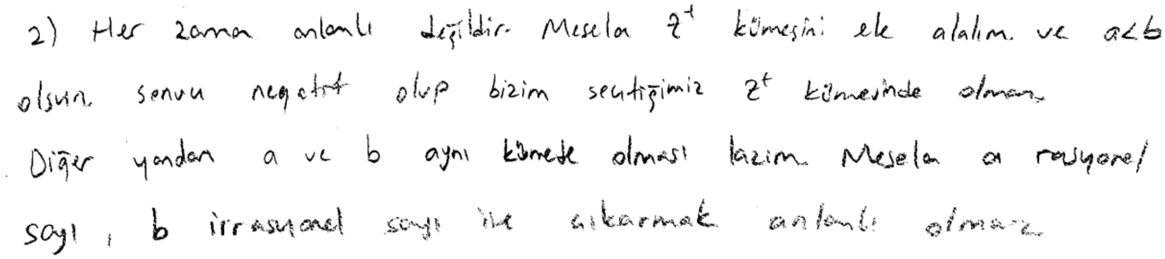
It said: It is meaningless in 0-0=0 operation.



It said: In physical life it is sometimes meaningless.



It said: It is significant except that the result is 0.



It said: It's not always meaningful. Let's take the set of positive integers for example. If a<b, the result will be negative and not in the set we selected. a and b must be in the same set.

When the answers were examined, it was seen that those who defended the correctness of the statement could not prove their answers mathematically. The cases ∞-∞ and 0-0 create uncertainty and therefore the expressions will not be meaningful are some of the answers given.

"Question 3: Is it correct to replace the parentheses in the $(x-y)+z=x-(y-z)$ equation? Please comment." The answers to the question are given in Table 4 below.

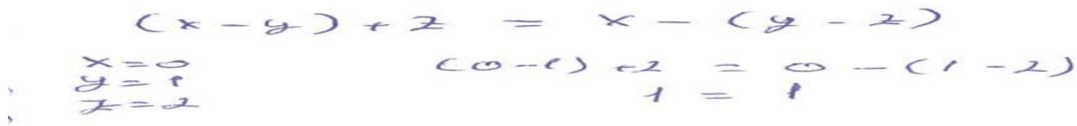
Table 4

Responses to Question 3

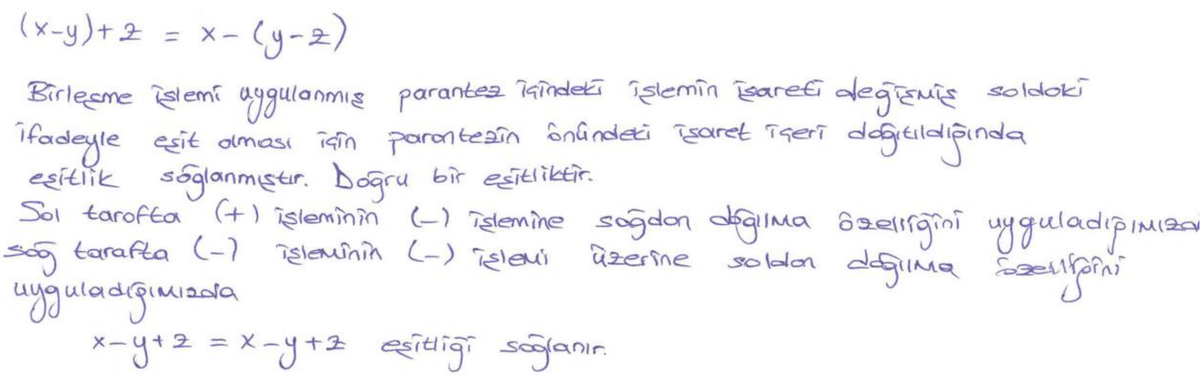
Responses	Respondents	Percent
Correct	7	28
Dispersion and merging properties are used and correctly said	10	40
The accuracy of the expression is shown by giving numerical values.	1	4
Depends on the cluster being studied	2	8
No responses	5	20
Total	25	100

When Table 4 is examined, 90% of those who answered the question stated that the statement was correct. While those who emphasized the importance of the studied cluster remained at 5%, 55% of those who defended the correctness of the expression tried to prove the correctness of the expression by distributing the parenthesis or giving numerical values. Examples of answers are given below.

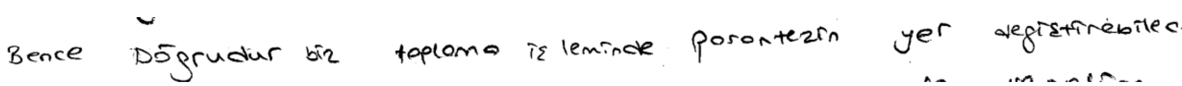
It said: It may change location due to addition process. It relates to the given cluster.



It said: The accuracy of the expression is shown by giving numerical values.



It said: Dispersion and merging properties are used and correctly said



It said: I think it's true, the parentheses can be swapped.

In the answers, it was tried to show the correctness of the expression by using the distribution and associativite properties in general. However, there were also those who tried to prove the correctness of the expression by giving it a numerical value. There have been some who have argued that it is correct to replace the parentheses without resorting to any proof method. Those who emphasized the importance of the studied cluster did not write in which clusters and conditions the operation was valid.

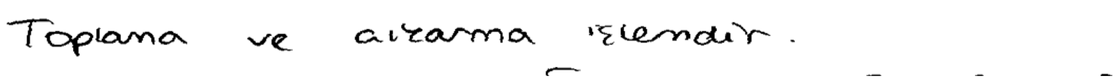
“Question 4: "Addition, subtraction, multiplication, and division are operations, and these are called four operations." Is the statement always true? Please comment.” The answers to the question are given in Table 5 below.

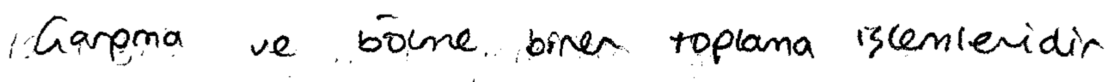
Table 5

Responses to Question 4

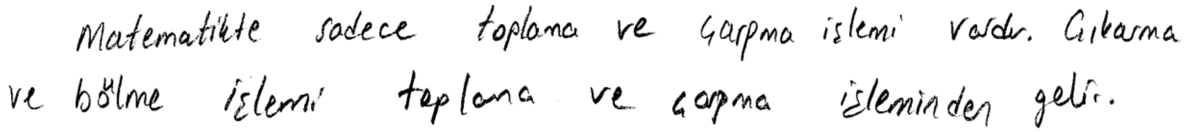
Responses	Respondents	Percent
Statement is correct	15	60
Only addition and multiplication are operations, the others are inverse operations.	4	16
Addition and subtraction are operations. Multiplication and division are addition operations.	1	4
No responses	5	20
Total	25	100

When the Table 5 is examined, those who stated that the statement is correct constitute 75% of the respondents. Although 5 people stated that addition is an operation, 4 of them added multiplication and 1 added subtraction. Below are examples of answers to the question.


Toplama ve çıkarma işlemdir.


Çarpma ve bölme, birer toplama işlemleridir.

It said: Addition and subtraction are operations. Multiplication and division are addition operations.


Matematikte sadece toplama ve çarpma işlemi vardır. Çıkarma ve bölme işlemi toplama ve çarpma işleminden gelir.

It said: In mathematics, there are only addition and multiplication operations. Subtraction and division operations come from them.


Evet bunlar işlemdir ve dört işlem denir.

It said: Yes these are operations and called four operations.

None of the respondents explained the feature of being a transaction. Those who claimed that the statements were true did not give reasons. While there are some who state that subtraction and division are not operations other than addition and multiplication, there are also some who state that only addition and subtraction are operations.

Discussion, Conclusion & Suggestions

Considering the general structure of mathematics, it can be said that mathematics is roughly based on the concepts of sets and functions. Here, the set shows the place and its properties, and the function shows how to act on this work place (Konyalıoğlu, 2008). In this context, these concepts are the concepts that should be taken into consideration in mathematical question and problem solutions. At the same time, it can be accepted as a separator of operational and conceptual knowledge that cannot be fully differentiated.

According to the literature, mathematics lessons are beginning to emphasize procedural learning rather than conceptual learning (Baki, 1998), and mathematics courses are taught with a significant emphasis on conceptual learning and operations are remembered rather than conceptually understood (Aksu et al., 2018).

In this study, the conceptual learning levels of pre-service mathematics teachers about some basic concepts and operations were examined by taking written opinions; showed that the explanations of the pre-service teachers were generally at the operational level, while the conceptual learning remained at the superficial level. This is consistent with some studies that found that there is procedural learning rather than conceptual learning in mathematics education (Aksu et al., 2018; Baki, 1998; Hiebert, 2013; Schoenfeld , 1985).

The findings roughly show that; pre-service teachers did not approach these questions in the context of conceptual knowledge, and they generally gave answers in the context of procedural knowledge. They tried to answer operationally without mentioning whether a relation satisfies the necessary conditions for it to be processed . Procedural knowledge and conceptual knowledge are not independent of each other. Conceptual knowledge and procedural knowledge interact with each other cyclically and with the problem according to the situation of the problem, without specifying priority in the solution of the problem (Rittle-Johnson et al., 2001).

Conceptual knowledge includes the skills to symbolize mathematical concepts, to establish relationships between concepts and between concepts and symbols, and to perform the necessary operations (Hogg & Vaughan, 2014). The currunt study, pre-service teachers were insufficient in answering the questions by paying attention to important issues such as being defined or meaningful, and the characteristics of the studied cluster, which they should deal with in the context of conceptual knowledge. In fact, this study shows that the basis of procedural knowledge is conceptual knowledge oriented and this is ignored by the pre-service mathematics teachers.

This situation can be a constant indication of both the prior knowledge and the way of thinking of the pre-service mathematics teachers. It is noted that the pre-service mathematics teachers used the process prototype they gained in the formal education process and the reflective thinking, which they compared to the previous solutions, in these question solutions.

Ethic

This study was conducted in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki declaration and its later amendments.

Author Contributions

All stages of the study were organized and conducted by the authors.

Conflict of Interest

In the research, the authors declare no conflict of interest.

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