Konuralp Journal of Mathematics
Volume 5 No. 1 pp. 92–98 (2017) ©KJM

DINI LIPSCHITZ FUNCTIONS FOR THE GENERALIZED FOURIER-DUNKL TRANSFORM IN THE SPACE $L^2_{\alpha,n}$

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Abstract. Using a generalized translation operator, we obtain an analog of Younis Theorem 5.2 in [5] for the generalized Fourier-Dunkl transform for functions satisfying the Fourier-Dunkl Dini Lipschitz condition in the space $L^2_{\alpha,n}$.

1. Introduction and Preliminaries

Younis Theorem 5.2 [5] characterized the set of functions in $L^2(\mathbb{R})$ satisfying the Cauchy Lipschitz condition by means of an asymptotic estimate growth of the norm of their Fourier transforms, namely we have

Theorem 1.1. *[5] Let $f \in L^2(\mathbb{R})$. Then the following are equivalents*

(a) $\|f(x+h) - f(x)\| = O\left(\frac{h^\delta}{(\log\frac{1}{h})^\gamma}\right)$, *as* $h \to 0, 0 < \delta < 1, \gamma \geq 0$

(b) $\displaystyle\int_{|\lambda|\geq r} |\widehat{f}(\lambda)|^2 d\lambda = O\left(\frac{r^{-2\delta}}{(\log r)^{2\gamma}}\right)$, *as* $r \to \infty$,

where $\widehat{f}$ stands for the Fourier transform of f.

In this paper, we consider a first-order singular differential-difference operator Λ on $\mathbb{R}$ which generalizes the Dunkl operator Λ_α. We prove an analog of Theorem 1.1 in the generalized Fourier-Dunkl transform associated to Λ in $L^2_{\alpha,n}$. For this purpose, we use a generalized translation operator.
In this section, we develop some results from harmonic analysis related to the differential-difference operator Λ. Further details can be found in [1] and [6]. In all what follows assume where $\alpha > -1/2$ and n a non-negative integer.
Consider the first-order singular differential-difference operator on $\mathbb{R}$

$$\Lambda f(x) = f'(x) + \left(\alpha + \frac{1}{2}\right)\frac{f(x) - f(-x)}{x} - 2n\frac{f(-x)}{x}.$$

Date: May 14, 2016 and, in revised form, August 23, 2016.
2000 *Mathematics Subject Classification.* 42B37.
Key words and phrases. Differential-difference operator, Generalized Fourier-Dunkl transform, Generalized translation operator.

For $n = 0$, we regain the differential-difference operator

$$\Lambda_\alpha f(x) = f'(x) + \left(\alpha + \frac{1}{2}\right) \frac{f(x) - f(-x)}{x},$$

which is referred to as the Dunkl operator of index $\alpha + 1/2$ associated with the reflection group $\mathbb{Z}_2$ on $\mathbb{R}$. Such operators have been introduced by Dunkl (see [3], [4]) in connection with a generalization of the classical theory of spherical harmonics. Let M be the map defined by

$$Mf(x) = x^{2n} f(x), \quad n = 0, 1, ...$$

Let $L^p_{\alpha,n}$, $1 \leq p < \infty$, be the class of measurable functions f on $\mathbb{R}$ for which

$$\|f\|_{p,\alpha,n} = \|M^{-1} f\|_{p,\alpha+2n} < \infty,$$

where

$$\|f\|_{p,\alpha} = \left(\int_{\mathbb{R}} |f(x)|^p |x|^{2\alpha+1} dx \right)^{1/p}.$$

If $p = 2$, then we have $L^2_{\alpha,n} = L^2(\mathbb{R}, |x|^{2\alpha+1})$.
The one-dimensional Dunkl kernel is defined by

$$e_\alpha(z) = j_\alpha(iz) + \frac{z}{2(\alpha+1)} j_{\alpha+1}(iz), z \in \mathbb{C}, \tag{1.1}$$

where

$$j_\alpha(z) = \Gamma(\alpha+1) \sum_{m=0}^{\infty} \frac{(-1)^m (z/2)^{2m}}{m! \Gamma(m+\alpha+1)}, z \in \mathbb{C}, \tag{1.2}$$

is the normalized spherical Bessel function of index α. It is well-known that the functions $e_\alpha(\lambda.)$, $\lambda \in \mathbb{C}$, are solutions of the differential-difference equation

$$\Lambda_\alpha u = \lambda u, u(0) = 1.$$

From (2) we see that

$$\lim_{z \to 0} \frac{j_\alpha(z) - 1}{z^2} \neq 0. \tag{1.3}$$

Lemma 1.1. *For $x \in \mathbb{R}$ the following inequalities are fulfilled*
i) $|j_\alpha(x)| \leq 1$,
ii) $|1 - j_\alpha(x)| \leq |x|$,
iii) $|1 - j_\alpha(x)| \geq c$ *with* $|x| \geq 1$, *where* $c > 0$ *is a certain constant which depends only on* α.

Proof. Similarly as the proof of Lemma 2.9 in [2]. □

For $\lambda \in \mathbb{C}$, and $x \in \mathbb{R}$, put

$$\varphi_\lambda(x) = x^{2n} e_{\alpha+2n}(i\lambda x),$$

where $e_{\alpha+2n}$ is the Dunkl kernel of index $\alpha + 2n$ given by (1).

Proposition 1.1. *i)* φ_λ *satisfies the differential equation*

$$\Lambda \varphi_\lambda = i\lambda \varphi_\lambda.$$

ii) For all $\lambda \in \mathbb{C}$, *and* $x \in \mathbb{R}$

$$|\varphi_\lambda(x)| \leq |x|^{2n} e^{|Im\lambda||x|}.$$

The generalized Fourier-Dunkl transform we call the integral transform

$$\mathcal{F}_\Lambda f(\lambda) = \int_{\mathbb{R}} f(x)\varphi_{-\lambda}(x)|x|^{2\alpha+1}dx, \lambda \in \mathbb{R}, f \in L^1_{\alpha,n}.$$

Let $f \in L^1_{\alpha,n}$ such that $\mathcal{F}_\Lambda(f) \in L^1_{\alpha+2n} = L^1(\mathbb{R}, |x|^{2\alpha+4n+1}dx)$. Then the inverse generalized Fourier-Dunkl transform is given by the formula

$$f(x) = \int_{\mathbb{R}} \mathcal{F}_\Lambda f(\lambda)\varphi_\lambda(x)d\mu_{\alpha+2n}(\lambda),$$

where

$$d\mu_{\alpha+2n}(\lambda) = a_{\alpha+2n}|\lambda|^{2\alpha+4n+1}d\lambda, \quad a_\alpha = \frac{1}{2^{2\alpha+2}(\Gamma(\alpha+1))^2}.$$

Proposition 1.2. *i) For every $f \in L^2_{\alpha,n}$,*

$$\mathcal{F}_\Lambda(\Lambda f)(\lambda) = i\lambda\mathcal{F}_\Lambda(f)(\lambda).$$

ii) For every $f \in L^1_{\alpha,n} \cap L^2_{\alpha,n}$ we have the Plancherel formula

$$\int_{\mathbb{R}} |f(x)|^2|x|^{2\alpha+1}dx = \int_{\mathbb{R}} |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda).$$

iii) The generalized Fourier-Dunkl transform $\mathcal{F}_\Lambda$ extends uniquely to an isometric isomorphism from $L^2_{\alpha,n}$ onto $L^2(\mathbb{R}, \mu_{\alpha+2n})$.

The generalized translation operators τ^x, $x \in \mathbb{R}$, tied to Λ are defined by

$$\begin{aligned} \tau^x f(y) &= \frac{(xy)^{2n}}{2}\int_{-1}^{1} \frac{f(\sqrt{x^2+y^2-2xyt})}{(x^2+y^2-2xyt)^n}\left(1+\frac{x-y}{\sqrt{x^2+y^2-2xyt}}\right)A(t)dt \\ &+ \frac{(xy)^{2n}}{2}\int_{-1}^{1} \frac{f(-\sqrt{x^2+y^2-2xyt})}{(x^2+y^2-2xyt)^n}\left(1-\frac{x-y}{\sqrt{x^2+y^2-2xyt}}\right)A(t)dt, \end{aligned}$$

where

$$A(t) = \frac{\Gamma(\alpha+2n+1)}{\sqrt{\pi}\Gamma(\alpha+2n+1/2)}(1+t)(1-t^2)^{\alpha+2n-1/2}.$$

Proposition 1.3. *Let $x \in \mathbb{R}$ and $f \in L^2_{\alpha,n}$. Then $\tau^x f \in L^2_{\alpha,n}$ and*

$$\|\tau^x f\|_{2,\alpha,n} \le 2x^{2n}\|f\|_{2,\alpha,n}.$$

Furthermore,

$$\mathcal{F}_\Lambda(\tau^x f)(\lambda) = x^{2n}e_{\alpha+2n}(i\lambda x)\mathcal{F}_\Lambda(f)(\lambda). \tag{1.4}$$

2. Fourier-Dunkl Dini Lipschitz condition

Definition 2.1. Let $f \in L^2_{\alpha,n}$, and let

$$\|\tau^h f(x) + \tau^{-h}f(x) - 2h^{2n}f(x)\|_{2,\alpha,n} \le C\frac{h^{\eta+2n}}{(\log\frac{1}{h})^\gamma}, \quad \gamma \ge 0, m = 0, 1, 2...$$

i.e.,

$$\|\tau^h f(x) + \tau^{-h}f(x) - 2h^{2n}f(x)\|_{2,\alpha,n} = O\left(\frac{h^{\eta+2n}}{(\log\frac{1}{h})^\gamma}\right),$$

for all x in $\mathbb{R}$ and for all sufficiently small h, C being a positive constant. Then we say that f satisfies a Fourier-Dunkl Dini Lipschitz of order η, or f belongs to $Lip(\eta, \gamma)$.

Definition 2.2. If however

$$\frac{\|\tau^h f(x)+\tau^{-h}f(x)-2h^{2n}f(x)\|_{2,\alpha,n}}{\frac{h^{\eta+2n}}{(\log\frac{1}{h})^{\gamma}}}\to 0, \quad as \quad h\to 0,\gamma\geq 0, m=0,1,2...$$

i.e.,

$$\|\tau^h f(x)+\tau^{-h}f(x)-2h^{2n}f(x)\|_{2,\alpha,n}=O\left(\frac{h^{\eta+2n}}{(\log\frac{1}{h})^{\gamma}}\right),$$

then f is said to be belong to the little Fourier-Dunkl Dini Lipschitz class $lip(\eta,\gamma)$.

Remark 2.1. It follows immediately from these definitions that

$$lip(\eta,\gamma)\subset Lip(\eta,\gamma).$$

Theorem 2.1. *Let $\eta>1$. If $f\in Lip(\eta,\gamma)$, then $f\in lip(1,\gamma)$.*

Proof. For $x\in\mathbb{R}$ and h small, $f\in Lip(\eta,\gamma)$ we have

$$\|\tau^h f(x)+\tau^{-h}f(x)-2h^{2n}f(x)\|_{2,\alpha,n}\leq C\frac{h^{\eta+2n}}{(\log\frac{1}{h})^{\gamma}}.$$

Then

$$(\log\frac{1}{h})^{\gamma}\|\tau^h f(x)+\tau^{-h}f(x)-2h^{2n}f(x)\|_{2,\alpha,n}\leq Ch^{\eta+2n}.$$

Therefore

$$\frac{(\log\frac{1}{h})^{\gamma}}{h^{1+2n}}\|\tau^h f(x)+\tau^{-h}f(x)-2h^{2n}f(x)\|_{2,\alpha,n}\leq Ch^{\eta-1},$$

which tends to zero with $h\to 0$. Thus

$$\frac{(\log\frac{1}{h})^{\gamma}}{h^{1+2n}}\|\tau^h f(x)+\tau^{-h}f(x)-2h^{2n}f(x)\|_{2,\alpha,n}\to 0, \quad h\to 0.$$

Then $f\in lip(1,\gamma)$. □

Theorem 2.2. *If $\eta<\nu$, then $Lip(\eta,0)\supset Lip(\nu,0)$ and $lip(\eta,0)\supset lip(\nu,0)$.*

Proof. We have $0\leq h\leq 1$ and $\eta<\nu$, then $h^{\nu}\leq h^{\eta}$. □

3. New results on Fourier-Dunkl Dini Lipschitz class

Theorem 3.1. *Let $\eta>2$. If f belong to the Fourier-Dunkl Dini Lipschitz class, i.e.,*

$$f\in Lip(\eta,\gamma), \quad \eta>2,\gamma\geq 0.$$

Then f is equal to the null function in $\mathbb{R}$.

Proof. Assume that $f\in Lip(\eta,\gamma)$. Then

$$\|\tau^h f(x)+\tau^{-h}f(x)-2h^{2n}f(x)\|_{2,\alpha,n}\leq C\frac{h^{\eta+2n}}{(\log\frac{1}{h})^{\gamma}}, \quad \gamma\geq 0.$$

From formulas (1.1), (1.2) and (1.4) we have the generalized Fourier-Dunkl transform of $\tau^h f(x)+\tau^{-h}f(x)-2h^{2n}f(x)$ is $2h^{2n}(j_{\alpha+2n}(\lambda h)-1)$.
By Plancherel equality, we obtain

$$4h^{4n}\int_{-\infty}^{+\infty}|j_{\alpha+2n}(\lambda h)-1|^2|\mathcal{F}_{\Lambda}f(\lambda)|^2d\mu_{\alpha+2n}(\lambda)\leq C^2\frac{h^{2\eta+4n}}{(\log\frac{1}{h})^{2\gamma}}.$$

Therefore

$$\int_{-\infty}^{+\infty} |j_{\alpha+2n}(\lambda h) - 1|^2 |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda) \leq \frac{C^2}{4} \frac{h^{2\eta}}{(\log \frac{1}{h})^{2\gamma}}.$$

Then

$$\frac{\displaystyle\int_{-\infty}^{+\infty} |j_{\alpha+2n}(\lambda h) - 1|^2 |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda)}{h^4} \leq \frac{C^2}{4} \frac{h^{2\eta-4}}{(\log \frac{1}{h})^{2\gamma}},$$

Since $\eta > 2$ we have

$$\lim_{h\to 0} \frac{h^{2\eta-4}}{(\log \frac{1}{h})^{2\gamma}} = 0.$$

Then

$$\lim_{h\to 0} \int_{-\infty}^{+\infty} \left(\frac{|1 - j_{\alpha+2n}(\lambda h)|}{\lambda^2 h^2} \right)^2 \lambda^4 |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda) = 0.$$

and also from the formula (1.3) and Fatou theorem, we obtain

$$\int_{-\infty}^{+\infty} \lambda^4 |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda) = 0.$$

Thus $\lambda^2 \mathcal{F}_\Lambda f(\lambda) = 0$ for all $\lambda \in \mathbb{R}$, then $f(x)$ is the null function. □

Analog of the Theorem 3.1, we obtain this theorem.

Theorem 3.2. *Let $f \in L^2_{\alpha,n}$. If f belong to $lip(2, 0)$. i.e.,*

$$\|\tau^h f(x) + \tau^{-h} f(x) - 2h^{2n} f(x)\|_{2,\alpha,n} = O(h^{2+2n}), \quad as \quad h \to 0.$$

Then f is equal to null function in $\mathbb{R}$

Now, we give another the main result of this paper analog of Theorem 1.1.

Theorem 3.3. *Let $f \in L^2_{\alpha,n}$. Then the following are equivalents*

(a) $f \in Lip(\eta, \gamma), \quad 0 < \eta < 1, \gamma \geq 0,$

(b) $\displaystyle\int_{|\lambda|\geq r} |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda) = O\left(\frac{r^{-2\delta}}{(\log r)^{2\gamma}}\right), \quad as \quad r \to \infty.$

Proof. $(a) \Rightarrow (b)$. Let $f \in Lip(\eta, \gamma)$. Then we have

$$\|\tau^h f(x) + \tau^{-h} f(x) - 2h^{2n} f(x)\|_{2,\alpha,n} = O\left(\frac{h^{\eta+2n}}{(\log \frac{1}{h})^\gamma}\right) \quad \text{as} \quad h \to 0.$$

From formulas (1.1), (1.2) and (1.4) we have the generalized Fourier-Dunkl transform of $\tau^h f(x) + \tau^{-h} f(x) - 2h^{2n} f(x)$ is $2h^{2n}(j_{\alpha+2n}(\lambda h) - 1)$.
By Plancherel equality, we obtain

$$\|\tau^h f(x)+\tau^{-h} f(x)-2h^{2n} f(x)\|^2_{2,\alpha,n} = 4h^{4n} \int_{-\infty}^{+\infty} |j_{\alpha+2n}(\lambda h)-1|^2 |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda).$$

If $|\lambda| \in [\frac{1}{h}, \frac{2}{h}]$, then $|\lambda h| \geq 1$ and (iii) of Lemma 1.1 implies that $1 \leq \frac{1}{c^2}|j_{\alpha+2n}(\lambda h) - 1|^2$. Then

$$\begin{aligned}
\int_{\frac{1}{h}\leq|\lambda|\leq\frac{2}{h}} |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda) &\leq \frac{1}{c^2}\int_{\frac{1}{h}\leq|\lambda|\leq\frac{2}{h}} |j_{\alpha+2n}(\lambda h) - 1|^2 |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda) \\
&\leq \frac{1}{c^2}\int_{-\infty}^{+\infty} |j_{\alpha+2n}(\lambda h) - 1|^2 |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda) \\
&\leq \frac{h^{-4n}}{4c^2}\|\tau^h f(x) + \tau^{-h} f(x) - 2h^{2n} f(x)\|^2_{2,\alpha,n} \\
&= O\left(\frac{h^{2\eta}}{(\log\frac{1}{h})^{2\gamma}}\right).
\end{aligned}$$

We obtain

$$\int_{r\leq|\lambda|\leq 2r} |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda) \leq C\frac{r^{-2\eta}}{(\log r)^{2\gamma}}, \quad r\to\infty.$$

where C is a positive constant. Now,

$$\begin{aligned}
\int_{|\lambda|\geq r} |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda) &= \sum_{i=0}^{\infty}\int_{2^i r\leq|\lambda|\leq 2^{i+1}r} |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda) \\
&\leq C\left(\frac{r^{-2\eta}}{(\log r)^{2\gamma}} + \frac{(2r)^{-2\eta}}{(\log 2r)^{2\gamma}} + \frac{(4r)^{-2\eta}}{(\log 4r)^{2\gamma}} + \cdots\right) \\
&\leq C\frac{r^{-2\eta}}{(\log r)^{2\gamma}}\left(1 + 2^{-2\eta} + (2^{-2\eta})^2 + (2^{-2\eta})^3 + \cdots\right) \\
&\leq K_\delta \frac{r^{-2\eta}}{(\log r)^{2\gamma}},
\end{aligned}$$

where $K_\delta = C(1-2^{-2\eta})^{-1}$ since $2^{-2\eta} < 1$.
Consequently

$$\int_{|\lambda|\geq r} |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda) = O\left(\frac{r^{-2\eta}}{(\log r)^{2\gamma}}\right), \quad as \quad r\to\infty.$$

$(b) \Rightarrow (a)$. Suppose now that

$$\int_{|\lambda|\geq r} |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda) = O\left(\frac{r^{-2\eta}}{(\log r)^{2\gamma}}\right), \quad as \quad r\to\infty.$$

and write

$$\|\tau^h f(x) + \tau^{-h} f(x) - 2h^{2n} f(x)\|^2_{2,\alpha,n} = 4h^{4n}(I_1 + I_2),$$

where

$$I_1 = \int_{|\lambda|<\frac{1}{h}} |j_{\alpha+2n}(\lambda h) - 1|^2 |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda),$$

and

$$I_2 = \int_{|\lambda|\geq\frac{1}{h}} |j_{\alpha+2n}(\lambda h) - 1|^2 |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda).$$

Firstly, we use the formulas $|j_{\alpha+2n}(\lambda h)| \leq 1$ and

$$I_2 \leq 4\int_{|\lambda|\geq\frac{1}{h}} |\mathcal{F}_\Lambda f(\lambda)|^2 d\mu_{\alpha+2n}(\lambda) = \left(\frac{h^{2\eta}}{(\log\frac{1}{h})^{2\gamma}}\right), \quad as \quad h\to 0.$$

Set

$$\phi(x)=\int_x^{+\infty}|\mathcal{F}_\Lambda f(\lambda)|^2d\mu_{\alpha+2n}(\lambda).$$

Integrating by parts we obtain

$$\begin{aligned}\int_0^x\lambda^2|\mathcal{F}_\Lambda f(\lambda)|^2d\mu_{\alpha+2n}(\lambda) &= \int_0^x-\lambda^2\phi'(\lambda)d\lambda=-x^2\phi(x)+2\int_0^x\lambda\phi(\lambda)d\lambda\\ &\leq C_1\int_0^x\lambda\lambda^{-2\eta}(\log\lambda)^{-2\gamma}d\lambda=O(x^{2-2\eta}(\log x)^{-2\gamma}),\end{aligned}$$

where C_1 is a positive constant.
We use the formula (ii) of Lemma 1.1

$$\begin{aligned}\int_{-\infty}^{+\infty}|j_{\alpha+2n}(\lambda h)-1|^2|\mathcal{F}_\Lambda f(\lambda)|^2d\mu_{\alpha+2n}(\lambda) &= O\left(h^2\int_{|\lambda|<\frac{1}{h}}\lambda^2|\mathcal{F}_\Lambda f(\lambda)|^2d\mu_{\alpha+2n}(\lambda)\right)\\ &+ \left(\frac{h^{2\eta}}{(\log\frac{1}{h})^{2\gamma}}\right)\\ &= O\left(h^2\frac{h^{2\eta-2}}{(\log\frac{1}{h})^{2\gamma}}\right)+O\left(\frac{h^{2\eta}}{(\log\frac{1}{h})^{2\gamma}}\right)\\ &= O\left(\frac{h^{2\eta}}{(\log\frac{1}{h})^{2\gamma}}\right),\end{aligned}$$

and this ends the proof. □

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