e-ISSN: 2458-8156

Journal Homepage: http://dergipark.gov.tr/ejens

EJENS

European Journal of Engineering and Natural Sciences

DergiPark AKADEMİK

CODEN: EJENS

First Order Integral Sliding Mode Control Of The Magnetically Levitated 4-Pole Type Hybrid Electromagnet

H.Fatih Ertugrul[1]*, Kadir Erkan[2]

[1]*TUBITAK, Energy Institute, 41470, Gebze/Kocaeli, Turkey.*

[2]*Yildiz Technical University, Department of Mechatronic Engineering, 34220, Esenler/İstanbul, Turkey.*

**Corresponding Author email: hasan.ertugrul@tubitak.gov.tr*

Publication Info

Paper received:
29 May 2016

Revised received:
15 October 2016

Accepted:
01 March 2017

Abstract

In this study, 4-pole type yoke hybrid electromagnet is modeled with respect to motion dynamics of the system. The hybrid electromagnet inherently has a non-linear characteristic and from the point view of controllability, it is unstable. This paper concerns the design of robust controller using first order integral sliding mode control method. Thus, the system becomes stable and robust against parametric uncertainties, nonlinearity, unmodeled uncertainties and external disturbance. Magnetic levitation system includes sensors that only measure the air gap. In order to estimate other states of the system, the full order disturbance observer is designed and integrated into the control loop. The estimated disturbance value is factored by the appropriate conversion gain and added to the input signal of the plant. The efficiency of control algorithm will be given in the paper by computer simulations.

Key words

Magnetic levitation, Integral Sliding mode control, Disturbance observer

1. INTRODUCTION

The non-contant magnetic levitation systems can operate without mechanical problem such as vibration, noise, abrasion, friction and so on. They also meet high accuracy and precision specifications. Because of these advantages, they are used in passenger transport vehicles, vibration isolation systems, biomedical devices, wind turbine, space studies and clean rooms as a key technology [1, 2].

The U and E-shaped electromagnets are often used in magnetic levitation systems, but they have only one degree of freedom control.4-Pole type hybrid electromagnet, which proposed by Koseki et al, has 3 degree of freedom. Each pole includes a coil (to control the field intensity) and a laminate permanent magnet leading to hybrid structure as shown in Fig.1.

The hybrid electromagnet inherently has a non-linear characteristic and from the point view of controllability it is unstable. In order to run such a system, it is required to actively control the hybrid electromagnet in multi axes. Several approaches have been proposed in the literature to control 4-pole type hybrid electromagnet [1-3]. In this paper, a sliding mode based control algorithm is proposed.

Conventional sliding mode control method does not guarantee robustness throughout the entire system. The control system response is sensitive against uncertainty during the reaching phase. After sliding mode occurs, the system response remains insensitive to variations of system parameters and external disturbance. By adding the integral component to sliding mode control, which is named integral sliding mode control (ISMC),the system response is became robust in both the reaching and sliding phase. In addition, ISMC can be used to eliminate the control chattering, which is the high-frequency vibrations (oscillations) of the control signal [4].

The performance of feedback control algorithm is associated with the measurement of the state variables of the system model. However, in practice, all the state variables are not measurable or the measurements of the state variables can be costly because of the high price of the sensors [5]. In order to obtain all state variables, the observer can be designed.

This paper is organized as follows. First, the mathematical model of the system is briefly derived. Then, integral sliding mode control and disturbance observer are introduced and designed using pole placement method. Finally simulation results verify the effectiveness of controller.

2. 4-POLE TYPE HYBRID ELECTROMAGNET

4-pole type hybrid electromagnet has three degrees of freedom of movement (along Z plus rotation around X and Y).

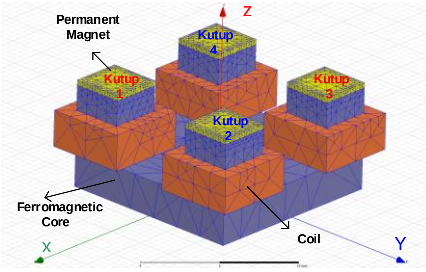

Figure 1. Basic structure of 4-pole hybrid electromagnet

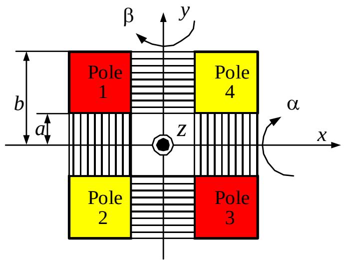

Figure 2. 4-pole hybrid electromagnet movement axis.

Independently control of each pole air gap is one of the way that can be followed to active control of the system. However, the implementation of this approach is difficult for controlling inclination axis motions and for compensating the unbalanced load. For this reason, system dynamics are developed independently using coordinate transformation. 4-Pole winding currents (i_1, i_2, i_3, i_4) are transformed virtual axis currents to provide control of each axis separately. Three virtual winding currents (i_z, i_α, i_β) are employed to control motion of vertical direction z, and inclinations α,β respectively.The relationships between virtual currents of the each degree of freedom and actual winding currents are represented by (Eq.1-2).

$$
\begin{aligned}
i_z &= \frac{1}{4}(i_1 + i_2 + i_3 + i_4) \\
i_\alpha &= \frac{1}{4}(-i_1 + i_2 + i_3 - i_4) \\
i_\beta &= \frac{1}{4}(-i_1 - i_2 + i_3 + i_4)
\end{aligned}
\qquad
\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} * \begin{bmatrix} i_z \\ i_\alpha \\ i_\beta \end{bmatrix} \qquad (1)
$$

In considering figure-2, the axial displacements are subjected to the following conversion.

$$
\begin{aligned}
z &= \frac{1}{4}(z_1 + z_2 + z_3 + z_4) \\
\alpha &= \frac{1}{2b}\left(\frac{z_1 + z_4}{2} - \frac{z_2 + z_3}{2}\right) \\
\beta &= \frac{1}{2b}\left(\frac{z_1 + z_2}{2} - \frac{z_3 + z_4}{2}\right)
\end{aligned}
\qquad
\begin{bmatrix} z \\ \alpha \\ \beta \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1/b & -1/b & -1/b & 1/b \\ 1/b & 1/b & -1/b & -1/b \end{bmatrix} * \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} \qquad (2)
$$

Controller output signals are transformed to pole coil signals using H transformation matrix which is obtained from (Eq.1). Similarly, pole displacements are transformed to axial displacements using T transformation matrix which is obtained from (Eq.2). This axial transformation is shown in Fig. 3.

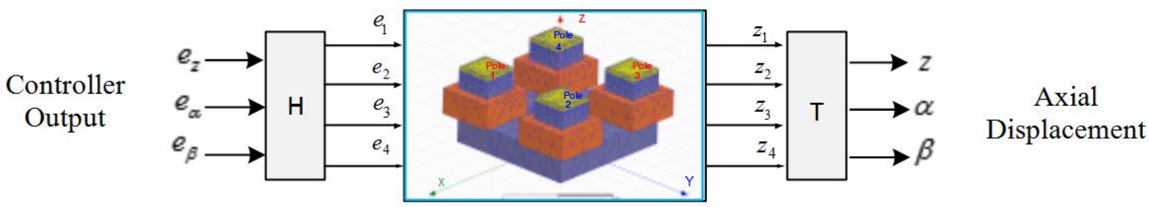

Figure 3. Axial transformation schematic.

The linearized mechanical system dynamics is given in below equation for z-axis motion [1-3].

$$Z(s) = \frac{K_B}{ms^2 - K_A} I_Z(s) - \frac{1}{ms^2 - K_A} F_d(s) \tag{3}$$

In the above equation, electrical input is the current form. In general, the voltage source is used to energize the coil of the magnetic levitation system.

$$I_Z(s) = \frac{1}{L_Z s - R_Z}\left[E_Z(s) - \frac{K_A L_Z}{K_B} sZ(s)\right] \tag{4}$$

The linearized system block diagram is shown in Figure 4(a) for z-axis motion. Changes are only encountered in the relevant parameters for inclination (α and β-axis) motion model (Figure 4(b)).

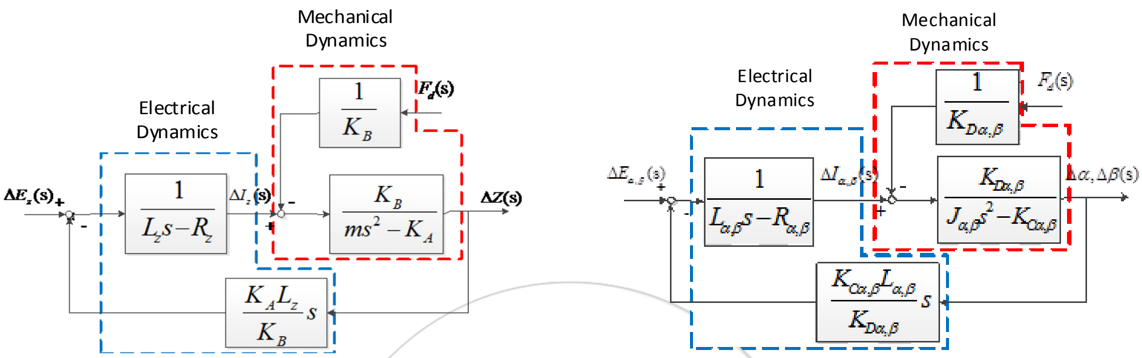

Figure 4. (a) System block diagram for z-axis motion. (b) System block diagram for α, β axis motions.

The state-space representation of the system is given in below.

$$\dot{x}(t) = Ax(t) + Bu(t) + EF_d(t) \tag{5}$$

$$\frac{d}{dt}\begin{bmatrix}\Delta z(t)\\ \Delta\dot{z}(t)\\ \Delta i_z(t)\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ \frac{K_A}{m} & 0 & \frac{K_B}{m}\\ 0 & -\frac{K_A}{K_B} & -\frac{R_z}{L_z}\end{bmatrix}\begin{bmatrix}\Delta z(t)\\ \Delta\dot{z}(t)\\ \Delta i_z(t)\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{1}{L_z}\end{bmatrix}\Delta e_z(t) + \begin{bmatrix}0\\ -\frac{1}{m}\\ 0\end{bmatrix}F_d(t)$$

3. DISTURBANCE OBSERVER-BASED INTEGRAL SLIDING MODE CONTROL

3.1. Integral Sliding Mode Controller Design

Sliding mode control signal is separated two components to achieve asymptotic output tracking; one is linear component u_l, and the other is nonlinear component u_{nl}.

$$u = u_l + u_{nl} \tag{6}$$

u_l component of the control signal drive the sliding variable (σ) to zero in finite time. The sliding starts after the sliding variable reaches zero at time t_r. After that time point, $\sigma = \dot{\sigma} = 0$ is valid for all time [6].

The sliding surface can be defined as follows:

$$\sigma = Sx = \begin{bmatrix}S_1 & S_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = 0 \qquad x_2 = \frac{-S_1}{S_2}x_1 \tag{7}$$

The sliding function is defined:

$$S = \begin{bmatrix}S_1\\ S_2\end{bmatrix} \quad where \;\; k = S_2^{-1}S_1 \qquad S = \begin{bmatrix}S_2 k\\ S_2\end{bmatrix} = S_2\begin{bmatrix}k\\ 1\end{bmatrix} \tag{8}$$

The sliding function (S) is obtained with selecting (S_2) parameter and calculating$k$. The integral of the position error is added to eliminate the error at the steady-state. The extended steady-space equation decomposed as follows

$$\frac{d}{dt}\begin{bmatrix} \int 0-\Delta z(t)dt \\ \Delta z(t) \\ \Delta \dot{z}(t) \\ \Delta i_z(t) \end{bmatrix} = \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{K_A}{m} & 0 & \frac{K_B}{m} \\ \hline 0 & 0 & -\frac{K_A}{K_B} & -\frac{R_z}{L_z} \end{array}\right] \left[\begin{array}{c} \int 0-\Delta z(t)dt \\ \Delta z(t) \\ \Delta \dot{z}(t) \\ \hline \Delta i_z(t) \end{array}\right] + \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline \frac{1}{L_z} \end{array}\right] \Delta e_z(t) \tag{9}$$

The (Eq.10) can be obtained from the (Eq.8) and (Eq.9):

$$\begin{aligned} \dot{x}_1 &= A_{11}x_1 + A_{12}x_2 = (A_{11} - A_{12}S_2^{-1}S_1)x_1 \\ &= (A_{11} - A_{12}k)x_1 \ \ where\ k = S_2^{-1}S_1 \end{aligned} \tag{10}$$

Since (A_{11}, A_{12}) is controllable, pole placement method is used to select the gain k[7,8]. Kessler canonical form (KCF) approach is used to obtain the gain$k$. KCF is an effective method to find the coefficients of characteristics polynomial of the SISO system. The basic idea behind this approach is to determine proper and stable characteristic polynomial using stability index and equivalent time constant [10].The equivalent time constant specifies the output response speed while stability index determines robustness, stability and output response of the system against parameter changes.

The characteristic equation of the closed-loop control system is given as:

$$P\ (s) = a_3s^3 + a_2s^2 + a_1s + a_0 \tag{11}$$

Stability index (γ) and equivalent time constant (τ) can be described as follows:

$$\gamma_i = \frac{{a_i}^2}{a_{i+1}a_{i-1}} \qquad (\ i = 1,2,3) \qquad\qquad \tau = \frac{a_1}{a_0} \tag{12}$$

In 1960s, Kessler has proposed that values of the γi should be two. In 1980s, Manabe proposed small modification of making γ1=2.5 instead of 2 to obtain no overshoot condition.It is practically acceptable to take the equivalent time constant smaller than 0.1[s] in the magnetic levitation based system[1,2].

$$\frac{P_3(s)}{a_0} = \frac{a_3}{a_0}s^3 + \frac{a_2}{a_0}s^2 + \frac{a_1}{a_0}s + \frac{a_0}{a_0} = b_3s^3 + b_2s^2 + b_1s + b_0$$

$$b_0 = 1 \qquad\qquad b_1 = \frac{a_1}{a_0} = \tau$$

$$\frac{b_1^2}{b_2b_0} = \frac{\tau^2}{b_2} = \gamma_1 \qquad\qquad b_2 = \frac{\tau^2}{\gamma_1}$$

$$\frac{b_2^2}{b_3b_1} = \gamma_2 \qquad\qquad b_3 = \frac{b_2^2}{b_1\gamma_2} \tag{13}$$

The characteristic equation is solved to obtain desired poles of the system. In this study, pole placement is performed with the following MATLAB command.

$$k = ac\ker(A_{11}, A_{12}, [\mathrm{p1} \quad \mathrm{p2} \quad \mathrm{p3}]) \tag{14}$$

Thus, the sliding function S is obtained from (Eq.14) u_{nl} and selecting S_2.

The linear component of control signal u_l can be calculated by:

$$
\begin{aligned}
&\dot{\sigma} = S\dot{x} = S(Ax + Bu_l) = 0 \\
&u_l = -(SB)^{-1} SAx
\end{aligned}
\tag{15}
$$

The nonlinear component of control signal is selected as follows:

$$u_{nl} = -(SB)^{-1} \rho \,\mathrm{sgn}(\sigma) \quad where\ \rho \succ 0 \tag{16}$$

To show stability of system, a positive definite Lyapunov function is selected as [9]:

$$V = \frac{1}{2}\sigma^2 \tag{17}$$

The time derivative of Lyapunov function is negative definite:

$$
\begin{aligned}
&\dot{V} = \sigma\dot{\sigma} = \sigma S(Ax + Bu) \\
&= \sigma SAx + \sigma SB[-(SB)^{-1} SAx - (SB)^{-1} \rho \,\mathrm{sgn}(\sigma)] \\
&= -\rho\sigma \,\mathrm{sgn}(\sigma) < 0 \qquad\qquad where\ \sigma \neq 0
\end{aligned}
\tag{18}
$$

Hence, the system becomes asymptotically stable.

In nonlinear component of control signal u_{nl} contain signum function which cause chattering problem because of the discontinuity. In practical case, the sigmoid function is used instead of signum function to eliminate chattering problem [6].

$$\mathrm{sgn}(\sigma) \approx \frac{\sigma}{|\sigma| + \varepsilon} \tag{19}$$

where ε is a small positive scalar. u_{nl} is commonly selected by:

$$u_{nl} = -\rho \,\mathrm{sgn}(\sigma) \approx -\rho \frac{\sigma}{|\sigma| + \varepsilon} \tag{20}$$

where ρ is a design parameter.

3.2. Disturbance Observer Based Design

Pole assignment is a basic design method for linear state feedback control system. In this method, it is assumed that all state variables are available for feedback. However, some state variables are not measurable directly or refrain from using of sensor because of noise occurring during the measurement. In order to estimate all state variables, the observer can be designed.

Disturbance force is added as a variable to obtain expanded system model (Eq.21). The disturbance force is generally step input in magnetic levitation system, so a zero line is added the system model. The expanded state equation is completely observable, thus observer can be designed. Figure 5 shows the block diagram of disturbance observer.

$$\frac{d}{dt}\begin{bmatrix} \Delta z(t) \\ \overset{\bullet}{\Delta z}(t) \\ \Delta i_z(t) \\ F_d \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{K_A}{m} & 0 & \frac{K_B}{m} & -\frac{1}{m} \\ 0 & -\frac{K_A}{K_B} & -\frac{R_z}{L_z} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta z(t) \\ \overset{\bullet}{\Delta z}(t) \\ \Delta i_z(t) \\ F_d \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_z} \\ 0 \end{bmatrix} \Delta e_z(t) \tag{21}$$

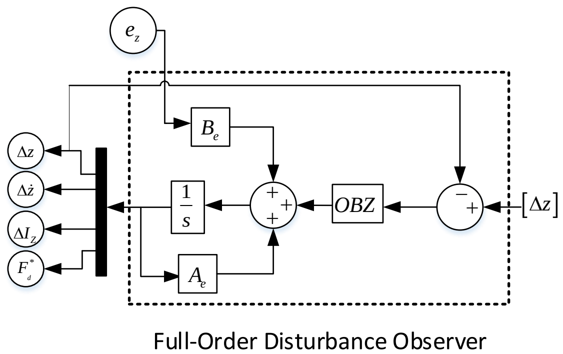

Figure 5. Full-Order Disturbance Observer Block Diagram.

Due to the separation principles, observer and controller can be designed independently of each other. The pole placement method is used to design full order disturbance observer similar to the controller design. Desired poles of the controllers and observers are decided by using Kessler's canonical form. However, the observer poles must be three to eight times faster than the controller poles to make sure the observation error converges to zero quickly [5].

Disturbance compensation gain is given as follows [2].

$$K_{Fd} = \frac{\Delta e_z(\infty)}{F_d} = \frac{R_z}{K_B} \tag{22}$$

Disturbance compensation gain is used to convert estimated force (N) into the control voltage (V). Figure 6 shows the Simulink model of the disturbance observer based integral sliding mode controller.

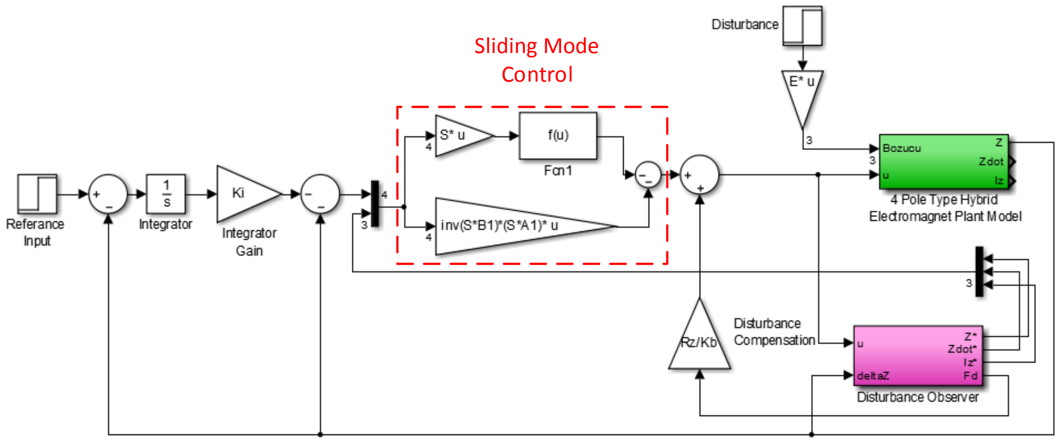

Figure 6. Simulink Model of the Control System

4. SIMULATION RESULTS AND EVALUATION

The effectiveness of integral sliding mode controller and disturbance observer, as discussed in the previous section, are shown using MATLAB environment. The simulation parameters for the 4-pole hybrid electromagnetic levitation system are shown in Table 1.

Table 1. System Parameters

Size / Unit	Value	Size/ Unit	Value	Size / Unit	Value
m [kg]	10	z_0 [mm]	4.3	α_0, β_0[rad]	0.0
$J_{\alpha,\beta}$[kg.m^2]	0.3	i_{z0} [A]	0.0	$i_{\alpha 0}$, $i_{\beta 0}$ [A]	0.0
k [N^2/A^2]	6.84*10^{-6}	K_A[N/m]	20991	K_C[Nm/rad]	106.43
I_m [A]	13.44	K_B[N/m]	14.87	K_D [Nm/A]	3.13
$R_{z,\alpha,\beta}$ [Ω]	1.50	$L_{z,\alpha,\beta}$[H]	0.016	E_{pm} [AT]	2689

Figure 7-8 show z-axes position and control signal waveform when a reference input is applied at 0.5 sec. of simulation time and 1.5 kg mass loads to the 4 pole type hybrid electromagnet at 2 sec. In Figure 7, the system can track step reference input with zero steady-state error. The disturbance compensator does not have much effect on the system response. This result indicates that proposed controller is insensitive to disturbance input. In Figure 9-10, the results show that the sliding mode control approach is achieved not only for vertical axis but also for inclinations. The chattering occurs particularly as shown on the control signals when a reference input and disturbance are applied. ρ parameter in (Eq.20) is used to adjust chattering effect. The higher value of ρ causes the high frequency chattering. The actual and observed disturbance values are shown in Figure 11.

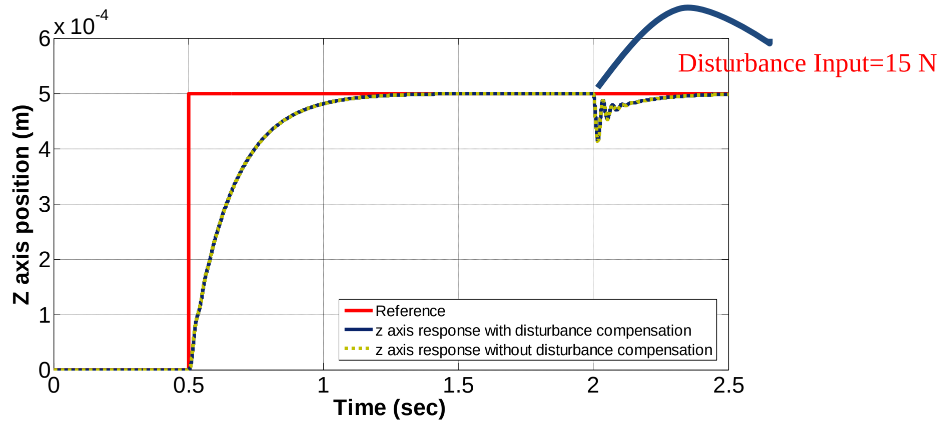

Figure 7. z-axis response for the step reference input and disturbance.

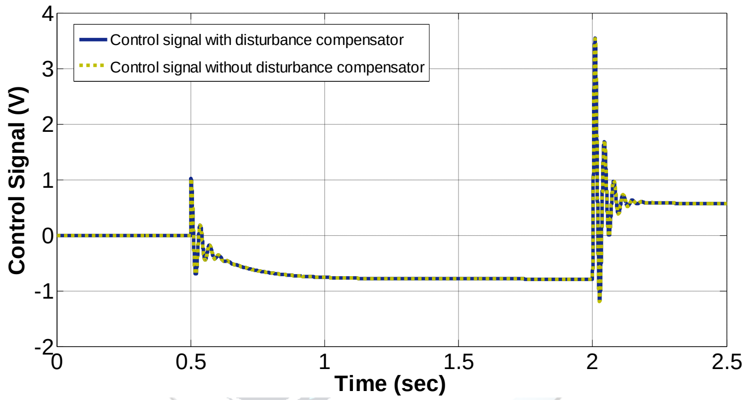

Figure 8. Control signal for z axes response.

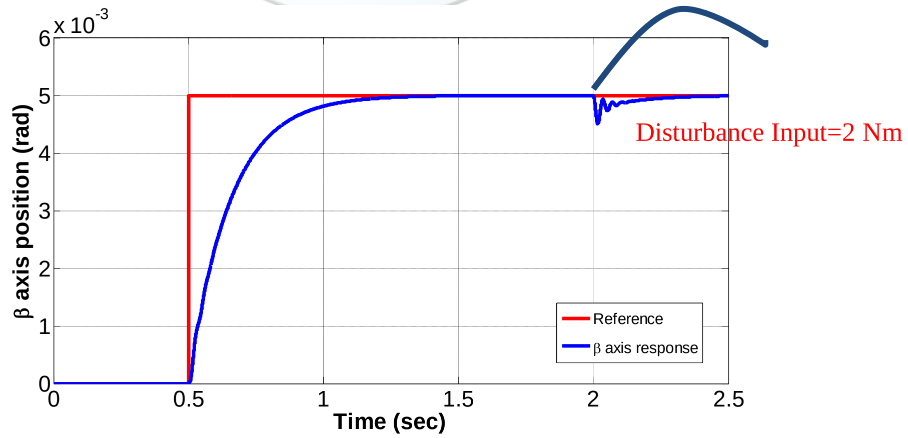

Figure 9: β-axes response for the step reference input and disturbance.

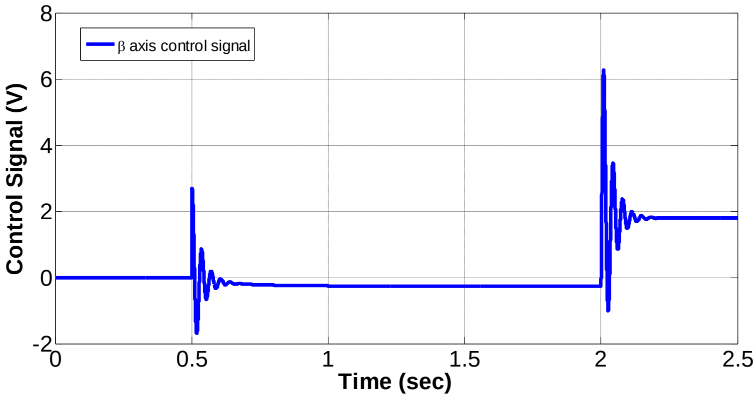

Figure 10. Control signal for β axes response.

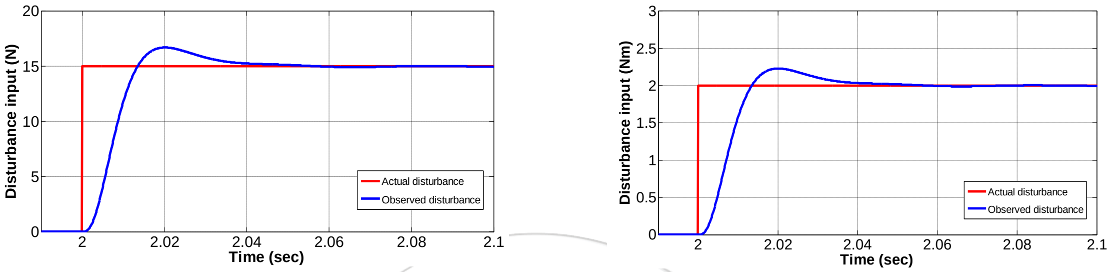

Figure 11. Actual and observed disturbance value for z and β axis.

5. CONCLUSION AND FUTURE WORK

In this paper, firstly, fundamentals of modeling of 4-pole hybrid electromagnet have been given and control methods of 4-pole hybrid electromagnet were explained by using virtual axis currents. Then, designing of a sliding mode controller and disturbance observer have been outlined. To clarify the effectiveness of the proposed design approach simulation studies was conducted in MATLAB environment. In the near future, we are planning to implement the controller on the experimental setup.

ACKNOWLEDGMENT

This study is supported by the Scientific and Technological Research Council of Turkey (TUBITAK) by project number 112M210.

REFERENCES

[1]. Y. Yakushi, T. Koseki and S. Sone, "3 degree-of-freedom zero power magnetic levitation control by a 4-pole type electromagnet", International Power Electronics Conference,vol.4, pp.2136-2141, April 3-7, 2000, Tokyo, Japan.

[2]. J. Liu, T. Koseki ,“Robust Control of a 4-Pole Electromagnet in Semi-Zero-Power Levitation Scheme with a Disturbance Observer” The Transactions of I.E.E. Japan, *A Publication of Industry Applications Society* Vol. 122-D, No. 1, pp. 7-15, January 2002

[3]. K. Erkan, B. Okur, T. Koseki, and F. Yigit, “Experimental evaluation of zero-power levitation control by transfer function approach for a 4-pole hybrid electromagnet,” 2011 IEEE International Conference on Mechatronics, pp. 23–28, Apr. 2011.

[4]. Vadim Utkin, Juergen Guldner, Jingxin Shi, Sliding Mode Control in Electro-Mechanical Systems, Second Edition, CRC Press, 2009.

[5]. Katsuhiko Ogata, Modern Control Engineering, 5th Edition, Prentice Hall, 2009.

[6]. Shtessel, Y., Edwards, C., Fridman L., Levant A., “Sliding Mode Control and Observation (Control Engineering)”,2013th Edition

[7]. J.-H. Lee, P. E. Allaire, G. Tao, and X. Zhang, “Integral sliding-mode control of a magnetically suspended balance beam: analysis, simulation, and experiment,” IEEE/ASME Transactions on Mechatronics, vol. 6, no. 3, pp. 338–346, 2001.

[8]. Lee, J.-H., “A Design of Integral Sliding Mode Suspension Controller to Reject the Disturbance Force Acting on the Suspension System in the Magnetically Levitated Train System” , Transactions of the Korean Society for Noise and Vibration Engineering, Volume 17, Issue 12 2007, pp.1152-1160

[9]. Zhen Gang Sun; Cheung N.C; Shi Wei Zhao; Wai-Chuen Gan, Integral sliding mode control with integral switching gain for magnetic levitation apparatus, 2009. PESA 2009. 3rd International Conference on Power Electronics Systems and Applications, 2009.

[10].Manabe, S.“Coefficient diagram method”, 14th IFAC Symposium on Automatic Control in Aerospace, Aug. 24-28, 1998, Seoul, Korea, 199- 210. (1998).