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Araştırma Makalesi — Research Article

# A NEW MATHEMATICAL MODEL FOR PARALLEL ASSEMBLY LINE BALANCING PROBLEM WITH ERGONOMIC CONSTRAINTS: ERGOPALBP

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**Anahtar Kelimeler**

*Montaj Hattı Dengeleme,*
*Paralel Montaj Hattı,*
*Ergonomi,*
*Matematiksel Model,*
*Gerçek Hayat Uygulaması.*

**Öz**

Paralel montaj hattı (PMH) sistemleri, gerçek hayattaki uygulamalarda yüksek hacimli ve seri üretim için en çok tercih edilen montaj hattı (MH) tiplerinden biridir. PMH'lerde, bir paralel montaj hattı dengeleme problemi (PAMHDP) temel olarak belirli öncelik ilişkilerine ve görev işleme sürelerine göre çözülür. Ancak, akademik araştırmalar genellikle istasyonlarda çalışanların maruz kaldığı ergonomik baskıları hesaba katmaz. Gerçek hayattaki PMHDP'lerde ergonomik yönleri dikkate almak daha doğru bir yaklaşım olacaktır. Bu çalışmada ergonomik kısıtlı PMHDP (ergoPMHDP) ele alınmaktadır. Buna göre, klasik PMHDP matematiksel modeli, ergonomik kısıtlamalar eklenerek değiştirilir. ergoPMHDP için önerilen matematiksel model, gerçek hayat uygulaması olarak bir PMH sisteminde uygulanmıştır. Klasik PMHDP için kullanılan ve ergoPMHDP için önerilen matematiksel modellerle elde edilen ergonomik risk faktörleri ve toplam istasyon işlem süreleri karşılaştırılmıştır. Elde edilen sonuçlara göre ergoPMHDP için önerilen matematiksel model başarılı bir performans göstermektedir.

# ERGONOMIK KISITLI PARALEL MONTAJ HATTI DENGELEME PROBLEMI İÇİN YENİ BİR MATEMATİKSEL MODEL: ERGOPMHDP

**Keywords**

*Assembly Line Balancing,*
*Parallel Assembly Line,*
*Ergonomics,*
*Mathematical Model,*
*Real-life Application.*

**Abstract**

Parallel assembly line (PAL) systems are one of the most preferred assembly line (AL) types for high volume and mass production in real-life applications. In the PALs, a parallel assembly line balancing problem (PALBP) is basically solved according to certain priority relations and task processing times. However, academic research generally does not take into account the ergonomic strains exposed by workers at stations. It would be a more accurate approach to consider ergonomic aspects in real-life PALBPs. In this study, the ergonomic-constrained PALBP (ergoPALBP) is discussed. Accordingly, the classical PALBP mathematical model is modified by adding ergonomic constraints. The mathematical model proposed for ergoPALBP has been implemented in a PAL system as a real-life application. Both ergonomic risk factors and total station operation times obtained with mathematical models used for classical PALBP and proposed for ergoPALBP were compared. According to the results obtained, the mathematical model proposed for the ergoPALBP shows a successful performance.

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# A NEW MATHEMATICAL MODEL FOR PARALLEL ASSEMBLY LINE BALANCING PROBLEM WITH ERGONOMIC CONSTRAINTS: ERGOPALBP

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## Highlights

- Ergonomic-constrained parallel assembly line balancing problem is introduced.
- A new mathematical model is presented.
- Rapid entire body assessment method is integrated into the proposed model.
- A real-life application is implemented.

## Graphical Abstract

**Table.** The summary of the results for the classical PALBP and the proposed ergoPALBP models

| | **PALBP** | **ergoPALBP** |
|---|---|---|
| **Processing time deviation** | 6.8% | 6.8% |
| **Ergonomic risk factor (ERF) deviation** | 18% | 5.2% |
| **The sum of the ERF difference among stations** | 114 | 34 |

**Purpose and Scope**
This study aims to present a new approach considering ergonomic aspects in real-life PALBPs.

**Design/methodology/approach**
A mathematical model is presented for ergoPALBP. A classical PALBP model is modified by adding ergonomic constraints. The proposed model is implemented in a PAL system as a real-life application.

**Findings**
Both ergonomic risk factors and total station operation times obtained with mathematical models used for classical PALBP and proposed for ergoPALBP are compared. The results show that ERF deviation is reduced by about 13% for the related real-life application. Processing times between stations are not adversely affected.

**Research limitations/implications**
Since ergoPALBP is NP-hard in nature, the proposed mathematical model may be forced for solving the large-sized problem sets. Therefore, the meta-heuristic approaches can be improved.

**Practical implications**
Considering ergonomic constraints in real life PALBP will balance the workload per worker in terms of both time and work strains.

**Social Implications (if applicable)**
Adding ergonomic constraints in assembly line balancing problems will prevent musculoskeletal disorders on workers. It will be ensured that workers can live a healthier and more comfortable life. It will also contribute to occupational health and safety.

**Originality**
There is almost no study considering ergonomic constraints in PALBPs. Accordingly, the classical PALBP mathematical model is modified by adding ergonomic constraints. This paper will contribute to real-life PALBP applications.

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## 1. Introduction

At the beginning of the twenty century, assembly lines (ALs) implemented by Henry Ford and his engineering team became essential mass and high-volume manufacturing systems. Since then, ALs have been used in many industries, from automotive to food (Küçükkoç and Zhang, 2017; Zhang et al, 2020). In this process, the ALs have shown many developments and their many types and problems have been addressed according to the production characteristics such as U-shaped, two-sided, line balancing (e.g., cycle time and number of stations), and work assignment. One of these is parallel ALs (PALs). A PAL system consists of at least two adjacent ALs parallel to each other. (Özcan et al. 2010a). Özcan et al. (2010a) described the main advantages of PAL systems as follows;

- In the facility layout, shorter ALs can be designed.
- The number of operators/workers can be reduced by establishing joint station(s).
- Even if any station on any line stops, other lines continue production. This situation provides production effectiveness.
- The line efficiency is increased by reducing the idle durations.
- The ALs with joint or different cycle times can work together.

Classic ALs problems, which are handled with certain constraints and objectives, are generally called AL balancing problems (ALBPs). ALBP has restrictions such as cycle time, the number of stations, and precedence relationships. These restrictions are also valid for PALs.

In fact, one of the most important constraints in real-life parallel ALBPs (PALBPs) is the ergonomic aspects of the tasks, as in other ALBP types. Considering ergonomic conditions is one of the issues that companies should be sensitive about in order to care about worker health and to prevent work accidents. In the PALs, even if the durations of the tasks assigned to the same station may be the same, their processing difficulty may be different. Accordingly, tasks should be assigned to stations not only by taking into account the durations but also by considering the ergonomic characteristics of the tasks.

The PALs were first introduced by Gökçen et al. (2006), and they proposed a mathematical model for the PALBP. Since then, many papers have been published on the PALBPs. Scholl and Boysen (2009) presented a binary linear programming model and a branch bound algorithm to minimize the number of the station(s) in the PALs. Esmaeilian et al. (2009) proposed a single-pass heuristic algorithm in order to minimize the cycle time in the PALs with mixed-model (MMPALs). Özcan et al. (2010b) improved a simulated annealing algorithm in order to minimize the number of station(s) in the MMPALs. In addition, Özcan et al. (2010a) proposed a solution approach based on tabu search algorithm for the tow-sided PALBP (TSPALBP). Küçükkoç et al. (2013) presented an ant colony optimization algorithm in order to minimize the number of the station(s) in the TSPALBP. Araújo et al. (2015) considered line balancing and worker assignment problems in the PALs (PALWABP). They proposed mixed-model linear programming model, tabu search, and biased random-key genetic algorithm in order to separately minimize the cycle time in the PALWABP. Küçükkoç and Zhang (2015) proposed a single-pass heuristic algorithm to minimize the station's number in the u-shaped PALBPs (UPALBPs). In addition, Küçükkoç and Zhang (2017) improved a mixed-model parallel u-line heuristic in order to minimize the number of stations(s) in the u-shaped MMPALBPs (UMMPALBPs). Özcan et al. (2018) presented a chance-constrained, piecewise linear, mixed-integer programming model and a tabu search algorithm for minimizing the stations' number in the PALBP considering stochastic task durations (SPALBP). Özcan et al. (2022) improved a new binary linear programming model and proposed a new artificial bee colony-based solution approach in order to minimize the cycle time in the PALWABP. For a detailed literature survey of the PALBP, the review paper published by Aguilar et al. (2020), Bakar et al. (2020), Jiao et al. (2021), and Boysen et al. (2021) can also be viewed.

In recent years, ergonomic aspects have been extensively discussed in the AL literature. Güner and Hasgül (2012) proposed integer programming model for the u-shaped ALBP (UALBP) with ergonomic factors. Battini et al. (2016) presented a multi-objective model for ALBP considering energy expenditure-based ergonomics Şahin and Kaya (2018) proposed a goal programming model for the ALBP under the ergonomic constraints. Kahya et al. (2018) developed a new ALBP model with ergonomic risk factors by using COMSOAL algorithm. Polat et al. (2018) developed a goal programming model to minimize the cycle time of the ALBP under ergonomic workload constraints. Kahya and Yetkin (2019) proposed a new model considering REBA method for an ergonomic ALBP. Akyol and Baykasoğlu (2019) introduced ALBP with the worker assignment (ALWABP) considering ergonomic risks. Xu et al. (2019) proposed a multi-objective particle swarm optimization algorithm to minimize the number of station(s) in the ALBP considering the fatigue balance of workers. They used the REBA method to calculate the posture risk of each task. Zhang et al. (2020) presented a multi-objective approach in order to minimize ergonomic risk and cycle time for the u-shaped ALWABP (UALWABP). Ozdemir et al. (2021) proposed a fuzzy multi-objective model for the ALBP with ergonomic risks. As mentioned above, although there are many studies considering ergonomic risks on the ALs in the literature, to the best of the authors' knowledge, there is only one published paper that considered ergonomic aspects on the PALBP. In the paper published by Mokhtarzadeh et al. (2021), a mixed-integer non-linear programming model, constraint programming, and a heuristic algorithm were presented. In addition, ergonomic risks were calculated with NIOSH, REBA, OCRA, EAWS, and COPSOQ methods, after the tasks are classified as easy, medium, and hard by using the ELECTRE TRI method. Yetkin and Kahya (2022) developed a bi-objective ergonomic ALBP model with a conic scalarization method. They preferred the REBA method to calculate the physical workload caused by the tasks.

Developing mathematical models with ergonomic constraints for ALBPs is quite difficult due to the complexity of nonlinearity. Ergonomic risk-calculating methods widely preferred in the literature include nonlinear aggregation functions (Otto and Scholl, 2011). This study proposes a linear mathematical model in order to solve the PALBP under the ergonomic constraints (ergoPALBP). The widely used REBA method to consider ergonomic risks in ALs is adapted to the mathematical model. Using the proposed mathematical model, three different problems, which are commonly used in the literature, are solved, respectively, small-sized, medium-sized, and large-sized.

This paper is organised as follows: Section 2 presents the problem definition, REBA method, and mathematical model under the methodology topic. Section 3 is about a real-life application results and discussion. Finally, conclusions and future works are given in Section 4.

## 2. Material and Method

### 2.1. Problem Definition

The definitions of the ergoPALBP addressed in this study are given below;

- The PAL systems consist of at least adjacent two ALs.
- Each line $l$ has a set of tasks ($l = 1, \dots, L, L \geq 2$).
- The stations of each line $l$ do not have to be different. That is, any station $s$ can be jointly installed between the adjacent PALs ($s = 1, \dots, S$).
- The tasks in the non-adjacent ALs cannot be assigned to the same station while the joint stations can be installed on adjacent ALs.
- Each task $t$ in the PALs must be assigned to only one station ($t = 1, \dots, T_l$).
- The processing time ($pt_{lt}$) is known in advance for each task $t$ in each AL $l$.
- The worker strain values (trunk, load/force, wrist, etc.) are predetermined for each task.
- A precedence diagram ($P$) is available among the tasks of each line. If there is a priority condition between tasks $t$ and $k$ in a line, these tasks are included in the related set ($t, k \in P(t, k)$).
- The cycle time ($CT$) of each AL is the same and already known. The total processing durations of tasks assigned to the stations cannot exceed the $CT$.
- The part transportation and walking durations between the lines are negligible.
- The main aim is to minimize the total number of stations in the PAL systems by balancing processing durations and ergonomic risk levels among the stations.

### 2.2. REBA (Rapid Entire Body Assessment) Method

The workers' body, i.e., musculoskeletal postures, should be considered in order to analyze the ergonomic risks in the ALs. A worker is subjected to certain strains during his/her every move (e.g., holding, coupling, bending, lifting, turning, etc.). In order to detect these strains and convert them to numerical data, many methodologies are used in the literature such as EAWS, NIOSH, OCRA, and REBA. While some of these methods enable detailed analysis, they are only suitable for a few sectors. On the other hand, although other methods can be applied to many sectors, they do not allow for detailed analysis. In order to overcome these disadvantages, the REBA method can be used in the analysis of working postures.

The REBA method, which is widely used in the AL literature, was first introduced by Hignett and McAtamney (2000). The REBA method basically considers the trunk, neck, legs, upper arms, lower arms, and wrists. The ergonomic risk value called REBA score is calculated by including the load, force, and coupling to the degrees of strain of these limbs. The tables introduced by Hignett and McAtamney (2000) are used to determine the value of each parameter. The score table called 'Table A' is used for the trunk, neck, and legs (see., Table 1). The score table called 'Table B' is used for the upper arms, lower arms, and wrists (see., Table 2). Then, A and B scores are calculated by considering load, force, and coupling parameters (see., Table 3). According to these scores, the C score is calculated by using Table C (see., Table 4). Finally, the REBA score is obtained by adding the activity score to the C score. This process is performed for each task in the ALs. The REBA scores of the tasks assigned to the stations are summed in order to determine the ergonomic risks of the related stations. A general REBA score sheet is available in Figure 1. In addition, the REBA score levels are given in Table 5. For detailed body diagrams of the REBA method, refer to Hignett and McAtamney (2000).

**Table 1.** Table A for REBA method

| **TABLE A** | | **NECK** | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | ***1*** | | | | ***2*** | | | | ***3*** | | | |
| | | **LEGS** | | | | | | | | | | | |
| | | ***1*** | ***2*** | ***3*** | ***4*** | ***1*** | ***2*** | ***3*** | ***4*** | ***1*** | ***2*** | ***3*** | ***4*** |
| **TRUNK** | ***1*** | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 3 | 3 | 5 | 6 |
| | ***2*** | 2 | 3 | 4 | 5 | 3 | 4* | 5 | 6 | 4 | 5 | 6 | 7 |
| | ***3*** | 2 | 4 | 5 | 6 | 4 | 5 | 6 | 7 | 5 | 6 | 7 | 8 |
| | ***4*** | 3 | 5 | 6 | 7 | 5 | 6 | 7 | 8 | 6 | 7 | 8 | 9 |
| | ***5*** | 4 | 6 | 7 | 8 | 6 | 7 | 8 | 9 | 7 | 8 | 9 | 9 |

**Table 2.** Table B for REBA method

| **TABLE B** | | **LOWER ARM** | | | | | |
|---|---|---|---|---|---|---|---|
| | | ***1*** | | | ***2*** | | |
| | | **WRIST** | | | | | |
| | | ***1*** | ***2*** | ***3*** | ***1*** | ***2*** | ***3*** |
| **UPPER ARM** | ***1*** | 1 | 2 | 3 | 1 | 2 | 3 |
| | ***2*** | 1 | 2 | 3 | 2 | 3 | 4* |
| | ***3*** | 3 | 4 | 5 | 4 | 5 | 5 |
| | ***4*** | 4 | 5 | 5 | 5 | 6 | 7 |
| | ***5*** | 6 | 7 | 8 | 7 | 8 | 8 |
| | ***6*** | 7 | 8 | 8 | 8 | 9 | 9 |

**Table 3.** Load/force and coupling scores for REBA method

| **LOAD/FORCE** | <5 kg | 5-10 kg | >10 kg | Shock or rapid buildup of force |
|---|---|---|---|---|
| **SCORE** | 0 | 1* | 2 | +1 |
| **COUPLING** | Good | Fair | Poor | Unacceptable |
| **SCORE** | 0* | 1 | 2 | 3 |

**Table 5.** REBA ergonomic risk levels

| **DEGREE** | **REBA SCORE** | **RISK LEVEL** | **ACTION** |
|---|---|---|---|
| 0 | 1 | Negligible | Non necessary |
| 1 | 2-3 | Low | May be necessary |
| 2 | 4-7 | Medium | Necessary |
| 3 | 8-10 | High | Necessary soon |
| 4 | 11-15 | Very high | Necessary urgent |

## 2.3. Mathematical Model

In this study, the mathematical model presented by Gökçen et al. (2006) is utilized to minimize the number of stations. In addition, this model is modified by adding new constraints to adapt the REBA method. After the number of stations is determined, the alternative line balances are generated to obtain lower REBA scores and to balance ergonomic risks among the stations according to the available number of stations. The notations of the models are given below;

***Indices:***
$l$: Line number
$s, m$: Station number
$t, k$: Task number

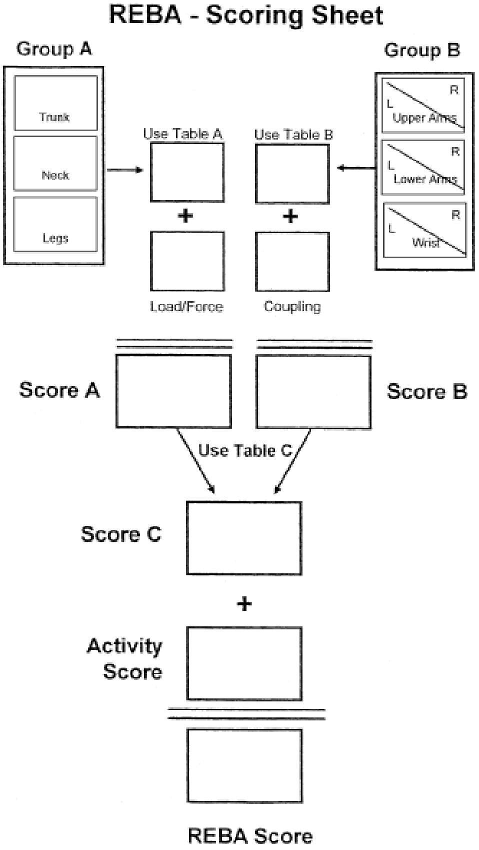

**Figure 1.** REBA assessment sheet (Hignett and McAtamney, 2000)

***Parameters:***

$pt_{lt}$: Processing time of each task $t$ in each AL $l$
$S_{max}$: Number of potential stations
$T_l$: Number of the tasks in each AL $l$
$L$: Number of the lines
$P(t,k)$: The set of tasks $k$ that includes the successor of task $t$
$CT$: Joint cycle time
$M$: A large number
$erf_{lt}$: Ergonomic risk factor of task $t$ in each AL $l$

***Decision variables:***

$$\mathrm{x_{lts}} = \begin{cases} 1, \text{if station } s \text{ is established for task t in the line } l \\ 0, \qquad\qquad\qquad\qquad\qquad \text{otherwise} \end{cases}$$

$$\mathrm{U_{ls}} = \begin{cases} 1, \text{if station } s \text{ is established for the line } l \\ 0, \qquad\qquad\qquad\qquad \text{otherwise} \end{cases}$$

$$\mathrm{z_s} = \begin{cases} 1, \text{if station s is established} \\ 0, \qquad\qquad \text{otherwise} \end{cases}$$

S = Number of the stations minimized
$ERF_s$ = Ergonomic risk factor of the station $s$
$w_{sm}$ = Unrestricted auxiliary decision variable

### 2.3.1. Classic PALBP Model

The classic PALBP mathematical model is given below;

$$min\, S = \sum_{s=1}^{S_{max}} z_s \tag{1}$$

**Subject to:**

$$\sum_{s=1}^{S_{max}} x_{lts} = 1 \text{ for } \forall\, l = 1,\dots,L \text{ and } t = 1,\dots,T_l \tag{2}$$

$$\sum_{t=1}^{T_l} pt_{lt} \cdot x_{lts} + \sum_{t=1}^{T_{l+1}} pt_{(l+1)t} \cdot x_{(l+1)ts} \leq CT \text{ for } \forall \text{ l} = 1, \dots, L-1 \text{ and s} = 1, \dots, S_{max} \quad (3)$$

$$\sum_{t=1}^{T_l} x_{lts} \leq M \cdot U_{ls} \text{ for } \forall\, l = 1, \dots, L \text{ and } s = 1, \dots, S_{max} \quad (4)$$

$$U_{ls} \leq z_s \text{ for } \forall\, l = 1, \dots, L \text{ and } s = 1, \dots, S_{max} \quad (5)$$

$$U_{ls} + U_{(l+a)s} = 1 \text{ for } \forall\, l = 1, \dots, L-2, a = 2, \dots, L-l, \text{and } s = 1, \dots, S_{max} \quad (6)$$

$$\sum_{s=1}^{S_{max}} (S_{max} - s + 1) \cdot (x_{lts} - x_{lks}) \geq 0 \text{ for } \forall\, t, k \in P(t,k) \text{ and } l = 1, \dots, L \quad (7)$$

$$x_{lts}, U_{ls}, z_s \in \{0,1\}\ and\ S \geq 0 \text{ for } \forall\, l = 1, \dots, L, t = 1, \dots, T_l, \text{and } s = 1, \dots, S_{max} \quad (8)$$

Equation 1 is the objective function that minimizes the number of stations in PALs. Equation 2 ensures that any task in any PAL is assigned to only one station. According to Equation 3, the total processing durations of the tasks assigned to the same station from the adjacent two AL cannot exceed the $CT$. Equation 4 means that if a task $t$ on line $l$ is assigned to station $s$, the station $s$ serves line $l$. Equation 5 ensures that if station $s$ serves any line $l$, station s is established. Equation 6 avoids assigning the same station to non-adjacent ALs. Equation 7 provides precedence relationships amongst the tasks. Equation 8 restricts the decision variables.

### 2.3.2. Proposed ergoPALBP Model

After minimizing the number of stations according to the mathematical model described above, the REBA method is added to the mathematical model, and the PALBP model is modified as the ergoPALBP model. Since the number of stations is determined in the previous model, the objective function is no longer the minimization of the number of stations in the ergoPALBP model. The decision variable S is the parameter now, not the decision variable of the ergoPALBP model. The ergoPALBP mathematical model is given below;

$$min \sum_{s=1}^{S} \sum_{m=1}^{S} |ERF_s - ERF_m| \quad s < m \quad (9)$$

**Subject to:**

Equations (2)-(7) and $S_{max}$ is changed as $S\ (s = 1, \dots, S)$

$$\sum_{l=1}^{L} \sum_{t=1}^{T_l} erf_{lt} \cdot x_{lts} = ERF_s \text{ for } \forall\, s = 1, \dots, S \quad (10)$$

$$x_{lts}, U_{ls}, z_s \in \{0,1\} \text{ and } ERF_s \geq 0 \text{ for } \forall\, l = 1, \dots, L, t = 1, \dots, T_l, \text{and } s = 1, \dots, S \quad (11)$$

Equation 10 calculates the total REBA score of all tasks assigned to station $s$. Equation 11 defines the updated decision variables. Here, the $erf_{lt}$ values, which is the ergonomic risk factor of each task $t$ in each line $l$, are actually the auxiliary decision parameters. Note that, $erf_{lt}$ can be separately calculated from the mathematical model. Ergonomic risk values can be easily calculated since the REBA table values of all tasks are known in advance. Objective function 9 is an absolute function. That is, it is a non-linear equation. Therefore, it should be transformed into a linear form to solve as a linear programming model the problem. Accordingly, the new model with updated objective function and the added constraints are given below;

$$min \sum_{s=1}^{S} \sum_{m=1}^{S} w_{sm} \quad s < m \quad (12)$$

**Subject to:**

Equations (2)-(7), (10)-(11) and $S_{max}$ is changed as $S\ (s = 1, \dots, S)$

$$ERF_s - ERF_m \leq w_{sm} \text{ for } \forall\, s, m = 1, \dots, S \text{ and } s < m \quad (13)$$

$$ERF_m - ERF_s \leq w_{sm} \text{ for } \forall\, s, m = 1, \dots, S \text{ and } s < m \quad (14)$$

Accordingly, the absolute value expression in objective function 9 is equalized to $w_{sm}$, which is the unrestricted auxiliary decision variable. The new objective function is Equation 12. Since objective function 9 is an absolute value function, objective function 12 must always take a positive value. Therefore, Equations 13-14 ensure that the $w_{sm}$ is always positive.

## 3. A Real-life Application for Proposed Mathematical Model

In this section, a real-life application is performed for the proposed ergoPALBP mathematical model. Thus, it is aimed to explain the proposed mathematical model better and analyze its applicability. Accordingly, the young bed assembly line of a furniture production facility in Turkey is addressed. Since the time and method studies have been performed in advance on the existing AL, the task processing times and precedence relationships among the tasks are already known. In addition, ergonomic risk levels arising from various body postures of the operators/workers are known in advance with ergonomic measurements. The REBA method is used for analyzing ergonomic measurements. A PAL is designed using the information of the related single AL. That is, the real-life application consists of adjacent and the same two ALs. Both of them have seventeen tasks.

First, the PALBP application is solved by using the model presented by Gökçen et al. (2006). Then, it is solved for the ergoPALBP by considering ergonomic risk factor measurements. Finally, the line balancing and risk factor distribution results of the two models are compared. Summary data on line balancing and ergonomic conditions are given in Table 6.

**Table 6.** Line balancing data and ergonomic conditions of the tasks

| | **LINE BALANCING DATA** | | **ERGONOMIC DATA** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | **TABLE A** | | | **TABLE B** | | | **AUXILIARY** | | |
| **Task no** | ***Immediate predecessor(s)*** | ***Task durations*** | ***Trunk*** | ***Neck*** | ***Legs*** | ***Upper arms*** | ***Lower arms*** | ***Wrists*** | ***Load/ force*** | ***Coupling*** | ***Activity score*** |
| 1 | -- | 4 | 2 | 2 | 2 | 2 | 2 | 3 | 1 | 0 | 0 |
| 2 | 1 | 3 | 1 | 2 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 3 | 1 | 9 | 1 | 1 | 2 | 2 | 2 | 3 | 0 | 1 | 1 |
| 4 | 2, 3 | 5 | 1 | 3 | 2 | 3 | 2 | 3 | 0 | 0 | 0 |
| 5 | 4 | 9 | 1 | 3 | 2 | 1 | 2 | 1 | 1 | 0 | 0 |
| 6 | 5 | 4 | 5 | 1 | 1 | 2 | 1 | 3 | 1 | 0 | 0 |
| 7 | 5 | 8 | 4 | 1 | 1 | 2 | 2 | 3 | 0 | 0 | 0 |
| 8 | 6, 7 | 7 | 4 | 1 | 1 | 2 | 1 | 2 | 0 | 0 | 0 |
| 9 | 8 | 5 | 1 | 2 | 2 | 1 | 1 | 1 | 0 | 0 | 0 |
| 10 | 9 | 1 | 2 | 2 | 2 | 3 | 2 | 1 | 0 | 1 | 0 |
| 11 | 9 | 3 | 1 | 1 | 2 | 1 | 1 | 1 | 0 | 0 | 0 |
| 12 | 7 | 1 | 1 | 1 | 2 | 2 | 1 | 2 | 0 | 0 | 0 |
| 13 | 9 | 5 | 3 | 1 | 1 | 4 | 2 | 1 | 0 | 2 | 1 |
| 14 | 12 | 3 | 1 | 1 | 1 | 3 | 2 | 1 | 0 | 0 | 0 |
| 15 | 10, 11, 13 | 5 | 3 | 1 | 2 | 1 | 1 | 2 | 1 | 0 | 0 |
| 16 | 15 | 3 | 1 | 3 | 2 | 1 | 1 | 3 | 0 | 0 | 0 |
| 17 | 13, 14, 16 | 13 | 2 | 2 | 1 | 3 | 1 | 2 | 1 | 0 | 1 |

Both models are implemented in Cplex Optimization Studio 12.8. Computer specifications are 12th Generation Intel® Core(TM) i5-12400F 2.50 GHz processor and 16 GB memory.

In this PAL system performed production with the joint $CT$, the $CT$ is 21. According to the mathematical model applied for the PALBP, the minimum number of stations is 9. Stations 1, 2, 4, 5, and 7 are common stations for the two ALs. Stations 3 and 8 serve the first AL, and Stations 6 and 9 serve the second AL. Ergonomic measurements given in Table 6 are taken into account to calculate the $ERF$ values. For example, the ERF values of Task 1 at each line ($erf_{11}$ and $erf_{21}$) are calculated as follows; first, Table A value in Table 1 is calculated according to trunk (2), neck (2), and legs (2) measurements (Table A=4). Then, the Table B value in Table 2 is determined according to the upper arms (2), lower arms (2), and wrists (3) measurements (Table B=4). A score is obtained by adding the load/force measurement (1) to Table A value according to Table 3 (Score A=5). In addition, the B score is calculated by adding the coupling measurement (0) to Table B value according to Table 3, (Score B=4). The Score C value in Table 4 is obtained by using the Score A and Score B values (Score C=5). Finally, the REBA score of Task 1 is calculated by adding the activity score (0) to Score C ($erf_{11}$=$erf_{21}$=5). Station 7 has the maximum $ERF$ value ($ERF$=17). Stations 2, 3, and 5 have the minimum $ERF$ value ($ERF$=10). The values in the REBA tables for task 1 are indicated by the mark ‘*’. The tasks assigned to stations, processing times, and ergonomic risk factors for PALBP are given in Table 7. Since the minimum number of the stations obtained by the mathematical model applied for the PALBP is 9, the number of the station is considered as 9 in the mathematical model proposed for the ergoPALBP. Accordingly, the Stations 2, 3, 5, 6, and 8 are common stations for the two ALs. Stations 1 and 4 serve the first AL, and Stations 7 and 9 serve the second AL. The tasks assigned to stations, processing times, and ergonomic risk factors for PALBP are given in Table 8.

**Table 7.** The application results for the PALBP model

| STATIONS | LINE-I TASKS | LINE-II TASKS | STATION PROCESSING TIMES | ERF |
|---|---|---|---|---|
| 1 | 1,2 | 1,3 | 20 | 16 |
| 2 | 3,4 | 2 | 17 | 10 |
| 3 | 5,6,7 | -- | 21 | 10 |
| 4 | 8,9,10 | 4 | 21 | 14 |
| 5 | 12,14 | 5,7 | 21 | 10 |
| 6 | -- | 6,8,9,13 | 21 | 14 |
| 7 | 15,16 | 10,11,15 | 17 | 17 |
| 8 | 13,17 | -- | 18 | 11 |
| 9 | -- | 12,14,16,17 | 20 | 12 |

**Table 8.** The application results for the ergoPALBP model

| STATIONS | LINE-I TASKS | LINE-II TASKS | STATION PROCESSING TIMES | ERF |
|---|---|---|---|---|
| 1 | 1,2,3 | -- | 16 | 11 |
| 2 | 4,5 | 1,2 | 21 | 13 |
| 3 | 6,7 | 3 | 21 | 12 |
| 4 | 8,9,11,12,13 | -- | 21 | 13 |
| 5 | 10 | 4,5 | 15 | 12 |
| 6 | 14,15 | 6,7 | 20 | 13 |
| 7 | -- | 8,9,10,11,15 | 21 | 14 |
| 8 | 16,17 | 12,16 | 20 | 13 |
| 9 | -- | 13,14,17 | 21 | 13 |

As can be seen in Tables 7 and 8, assignment of tasks to stations, station processing times and ergonomic risk factors have changed for both models. In the PALBP model, the processing time deviation (*PTD*) calculated with formula 23 was calculated as 6.8%. In addition, this level is maintained in the ergoPALBP model. In the PALBP model, the *ERF* deviation (*ERFD*) calculated with formula 24 is approximately 18%. In addition, this deviation is approximately 5% in the ergoPALBP model. The sum of the *ERF* difference between stations (*SERF*) calculated by formula 25 is 114 in the PALBP model, while it is 34 in the ergoPALBP model. The results summaries for the two models are given in Table 9.

Table 9 shows that although the variability of task times among the stations does not change, lower and balanced ERF values are obtained. In other words, both the line balance on the basis of time is preserved and the risk factors are balanced in terms of ergonomic strains. Among alternative line balances, i.e., different task assignments, for an available number of stations and cycle time, a more suitable PAL design can be established in terms of the ergonomic conditions. It can be said that this situation is an essential indicator for balancing the ergonomic strains of workers in practice.

$$PTD(\%) = \frac{\sum_{s=1}^{S}|PT_s - CT|}{CT \cdot S} \tag{15}$$

$$ERFD(\%) = \frac{\sum_{s=1}^{S}\sum_{m=1}^{S}|ERF_s - ERF_m|}{ERF_{average} \cdot S} \quad s < m \tag{16}$$

$$SERF = \sum_{s=1}^{S}\sum_{m=1}^{S}|ERF_s - ERF_m| \quad s < m \tag{17}$$

**Table 9.** The summary of the results for the PALBP and ergoPALBP models

| | PALBP | ergoPALBP |
|---|---|---|
| **PTD** | 6.8% | 6.8% |
| **ERFD** | 18% | 5.2% |
| **SERF** | 114 | 34 |

## 4. Conclusions and Future Studies

One of the AL types widely used in real-life AL applications is the PALs. In PALs, considering ergonomic conditions instead of line balancing by considering only processing times will provide an important perspective for real-life applications. In this study, ergonomic constraints in the PALBPs are discussed. The mathematical model presented by Gökçen et al. (2006) for classical PALBP has been updated by adding ergonomic constraints. The REBA method, which is widely used in the ALBP literature, is used to include ergonomic conditions. The proposed mathematical model is used in an existing real-life ALBP. A PAL system is designed by considering the priority relationships, task times, and ergonomic conditions of the exiting ALBP. The designed PAL is solved for both the classical PALBP model and the proposed ergoPALBP model. According to the ergonomic risk factors of the stations, approximately 13% improvement is achieved with the ergoPALBP model compared to the classical PALBP model. According to the results obtained, both the economic pressures that the workers are exposed to in the PALs are balanced and the total processing times of the stations are not adversely affected.

The following aspects may be considered for the ergoPALBP problem in the future:

- State-of-the-art approaches such as swarm intelligence-based algorithms (ant colony optimization, particle swarm optimization, artificial be colony, etc.) and evolutionary algorithms (genetic algorithm, genetic programming, etc.) may be improved to solve large-sized problems in real-life.
- Multi-objective PALs, mixed-model PALs, U-shaped PALs, two-sided PALs considering ergonomic risk factors may be a good topic for future studies.
- More robust and realistic perspectives can be obtained in practice by addressing worker-oriented different perspectives that cause variability (worker assignment) according to the capabilities of the operators/workers and uncertainty (interval task times or stochastic task times) along with ergonomic constraints.
- Different ergonomic methods (e.g., NIOSH, OCRA, EAWS, COPSOQ, etc.) may be considered in the ergoPALBPs.

## Conflict of Interest

No conflict of interest was declared by the authors.

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