# AN ABOUT USING VECTORS IN MATHEMATICAL MODELING IN ECONOMY

**Urdaletova Anarkul Burganakovna[1], Kydyralıev Syrgak Kaparovich[2]**
**Burova Elena Sergeevna[3]**

**ABSTRACT**

A significant part of not only ordinary citizens, but also scientists perceive mathematics as an extremely abstract science that has nothing to do with the surrounding reality. In our opinion, a significant contribution to the creation of this false idea is made by mathematicians themselves, who pay little attention to demonstrating the possibilities of using mathematical tools for understanding the world. This paper provides examples of the use of vectors for the analysis of economic situations. The main attention in the work is paid to the coordinate representation of the vectors.

**Keywords:** Vectors, Analytical geometry, Coordinates, Dot product, revenue, Procurement management, Use of mathematical tools.

**Jel Codes:** C02, C60.

## 1. INTRODUCTION

Approximately 400 years ago, Analytic Geometry was born, using algebraic methods to solve geometric problems (Stillwell, 2004). The main idea behind this science is to use coordinates to describe the location of points. The effectiveness of analytical geometry methods was immediately appreciated by outstanding mathematicians. Among those who made a significant contribution to its development are Newton, Claireau, Euler, Lagrange and many others. It turns out that the coordinate approach is effective not only in solving purely mathematical problems, but also in many other sciences (Mizrahi & Sullivan, 1998; Kydyraliev, Urdaletova & Burova, 2020). Our report provides examples showing the benefits of using Analytical Geometry methods in economics and business.

In order to once again emphasize the usefulness of using mathematical methods, we present an excerpt from the famous book by L. Solovyov "The Story of Hodja Nasreddin".

*This is exactly what little Nasreddin decided: if the Bukharian residents do not know how to be merciful themselves, they must be forced to do so.*

*Having defined the task, he thereby determined the course of his further reflections. They boiled down to the search for a game in which he would have an advantage over the Bukharians. In order not to bother himself with thoughts about the many thousands of hard – hearted Bukhara inhabitants, he found it useful to merge in his imagination all together, into one Big Bukharian.*

*The matter became simpler: it turned out to be much easier to think about one Bukharians, albeit a very large one.*

[1] Kyrgyz-Turkish Manas University, anarkul.urdaletova@manas.edu.kg
[2] American University of Central Asia, kydyraliev_s@auca.kg
[3] American University of Central Asia, burova_e@auca.kg

Little Nasruddin's reasoning is a vivid example of the use of mathematical modeling of a situation, although, most likely, he himself did not know about it.

So, mathematical modeling of a phenomenon is a retelling of this phenomenon in the language of numbers, functions, equations, inequalities, and so on. Just like every retelling, the model can be good or bad. A good model is a model that allows, spending relatively little effort, to learn quite a lot about the aspects of the phenomenon of interest to us.

## 2. THE CONCEPT OF VECTORS

A scalar is a value expressed as a single number. Examples of a scalar are animal weight, air temperature, product price, and so on.

A vector is a quantity that is defined by multiple scalars. In other words, a vector can be defined as a quantity that has a length and a direction.

Mathematicians say that a vector is a directional line. Vectors in the world around us are found everywhere. We use vectors regularly and very often we don't even think about it. For example, the statement that everything will be fine if the desires coincide with the possibilities can be interpreted in vector language as the statement that the sum of two vectors has the greatest length if the directions of the vectors coincide. Vectors are widely used in physics. Let's demonstrate the corresponding problem.

**Problem**

*Zhamilya and Tunzher kicked the ball at the same time. If only Zhamilya had done this, the ball would have flown north at a speed of 14 m / s. If only Tunger had done this, the ball would have flown northwest at a speed of 10 m / s. How far will the ball be in 2 seconds? (To simplify matters, neglect air resistance.)*

**Solution**

If we take the point of impact on the ball as the origin of coordinates and assume that the direction to the north is determined by the OY axis, then the impact on the ball by Zhamilya is expressed by the vector $\bar{a}$(0; 14). The corresponding vector $\bar{b}$ for Tunzher will be obtained if we consider a square with a diagonal of 10 $\sqrt{2}$ . Let's draw a picture illustrating Tunjer's strike.

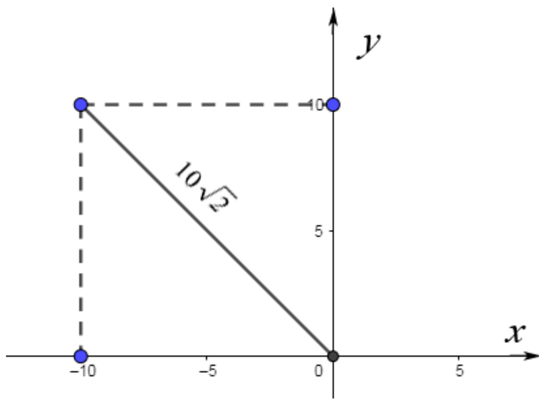


**Figure 1. Coordinate representation of a vector on a plane**

By the Pythagorean theorem, we get the coordinates of the vector $\bar{b}$. They are equal (–10; 10). The result of the simultaneous hitting the ball is determined by the sum of the vectors:

$\bar{a} + \bar{b}$ = (–10; 24). Its length $\sqrt{10^2 + 24^2} = \sqrt{100 + 576}$ = 26 expresses the distance that the ball flew in 1 second. Accordingly, in 2 seconds, excluding air resistance, the ball will fly 52 meters.

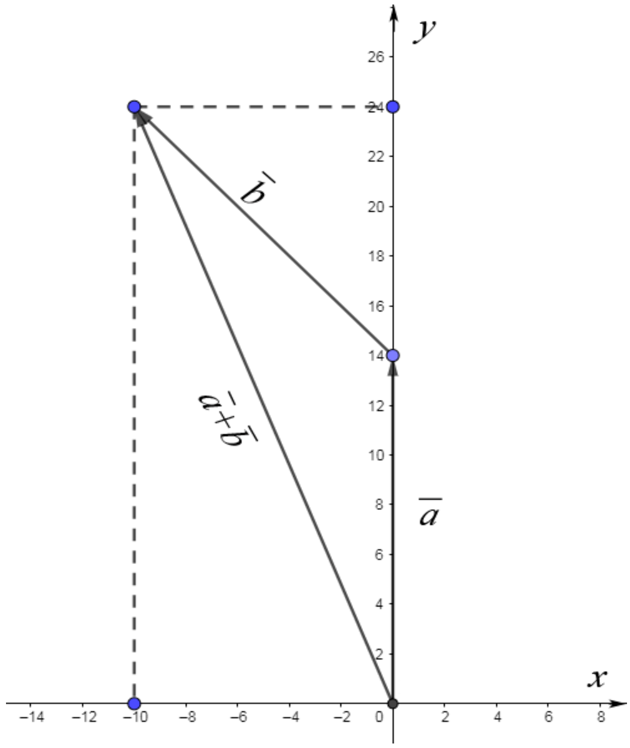


**Figure 2. Vector addition**

## 3. COORDINATE REPRESENTATIONS OF VECTORS

It turns out that using a Cartesian coordinate system makes vectors very useful in other fields of science. Consider a real life situation.

Deniz buys food every Saturday for the next week: 2 kilograms of pasta, 1 kilogram of rice, 3 kilograms of apples, 1.5 kilograms of oranges. For the first time, the price of pasta was 12.5 liras per kg; rice – 9 lire per kg; apples – 5 lire per kg; oranges – 6 lira per kg. As a result, he spent: 2 · 12.5 + 1 · 9 + 3 · 5 + 1.5 · 6 = 58 lire.

At the same time, Deniz hardly thought that this operation could be written in the language of vectors – the coordinate language. So, the volume of purchases can be written as a vector $\bar{a}(2; 1; 3; 1.5)$, and prices as a vector $\overline{p_1}(12.5; 9; 5; 6)$. Then, the amount of money spent on purchases – 58 lira, is the value of the scalar product of vectors:

$(\bar{a}|\overline{p_1})$ = 2 · 12.5 + 1 · 9 + 3 · 5 + 1.5 · 6 = 58.

Suppose that in the following weeks, the volume of purchases remained the same, and the prices in the second week were expressed by the vector $\overline{p_2}(12.8;\ 9;\ 5;\ 5.6)$, in the third – a vector $\overline{p_3}(12;\ 7;\ 4;\ 10)$. We are sure that you understand what prices are in question. The values of the corresponding dot products:

$(\overline{a}|\overline{p_2})$ = 2·12.8 + 1·9 + 3·5 + 1.5·5.6 = 58;

$(\overline{a}|\overline{p_3})$ = 2·12 + 1·7 + 3·4 + 1.5·10 = 58.

So, the amount of money spent on purchases did not change. At the same time, Deniz understands that maintaining the value of costs at a constant level is determined by a small change in prices in the second week, and in the third week there was a decrease in prices for pasta, rice and apples, which was offset by a rather sharp increase in prices for oranges.

Of course, most likely, in such a situation, Deniz does not have to think about any vectors and operations on them. By looking at the receipts for his purchases, he can get a complete picture of what is happening.

Let's look at a similar situation. Ali runs a large trading company that trades in thousands of types of goods. In order to have an idea of the daily activities of his company, he must be guided by the volume of completed trade transactions. But as was demonstrated by the example of Deniz's purchases, the same value, in monetary terms, can be determined by different circumstances. Therefore, it is desirable to have a reliable tool that allows you to navigate the nature of what is happening. We believe that the length of the vector can be used as such a tool.

## 4. USING VECTOR LENGTH IN ECONOMICS

**Problem**

Let the vector be defined by points A (2; –1) and B (14; 4). Find its length.

**Solution**

Figure 3 shows that the length of the vector AB is the length of the hypotenuse of the triangle ABC. Since the legs of the triangle AC and BC are parallel to the coordinate axes, their lengths are equal to the differences of the corresponding coordinates. That is, | AC | = | 14 – 2 | = 12; |BC| = | 4 – (–1) | = 5. Therefore, by the Pythagorean theorem, |AB| = $\sqrt{12^2+5^2}=\sqrt{169}=13.$ It should be noted that the numbers 12 and 15 are the coordinates of the AB vector. Thus, the rule is illustrated:

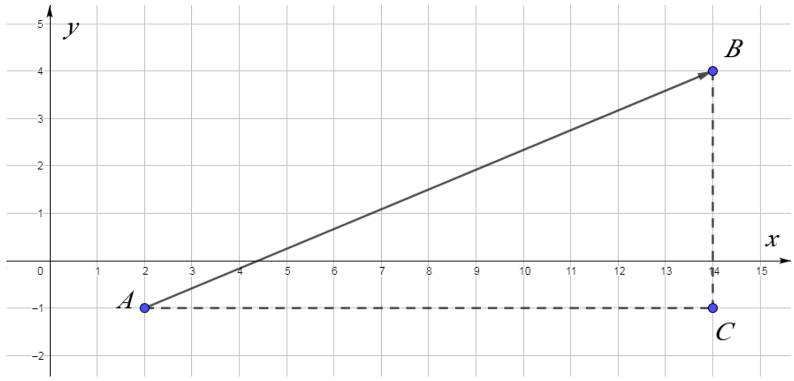


**Figure 3. Vector length**

The length of a vector is equal to the square root of the sum of the squares of its coordinates.

Let's go back to the case of Ali. He needs a reliable market surveillance tool. We propose to use the length of the price change vector for this. Let's demonstrate how this tool works using Deniz's purchases as an example.

The price change between the second and the first week is described by the vector $\overline{p_2 - p_1}(0.3; 0; 0; -0.4)$. Its length: $\left|\overline{p_2 - p_1}\right| = \sqrt{(0.3)^2 + (0)^2 + (0)^2 + (-0.4)^2} = \sqrt{0.25} = 0.5.$

The price change between the third and the second week is described by a vector $\overline{p_3 - p_2}(-0.8; -2; -1; 4.4)$. Its length: $\left|\overline{p_3 - p_2}\right| = \sqrt{(-0.8)^2 + (-2)^2 + (-1)^2 + (4.4)^2} = \sqrt{25} = 5.$

The lengths of the vectors give a noticeable signal about price changes: in the first case, they are insignificant, and between the third and second weeks there was a significant price change.

Of course, Deniz knows about this even without calculating the lengths of the vectors, but in the case of Ali there are thousands of numbers and it is hardly possible to detect significant changes in some of them by direct observation. Therefore, if he uses this tool – the length of the price change vector, he will be able, without spending a lot of effort, to understand at what moments it is necessary to react to a change in the market situation.

## 5. EUCLID'S TREASURE

So, the use of vectors represented in coordinate form is useful when studying economic phenomena. Therefore, it is necessary to improve the method of studying them. It is clear that in order to facilitate the process of studying vectors, it is desirable to use tasks with entertaining plots. One of them is proposed below.

*While dying, Euclid told his children that he had buried 800 gold coins – 100 in each vertex of the square. The first peak can be reached by taking 20 steps to the west and 10 steps to the south from the old apple tree, to the second: by taking 40 steps to the east and 30 steps* *south from the old apple tree. In the 2 indicated places, the children of Euclid actually found 100 coins each, after that they found two more treasures and stopped there, deciding that the father was mistaken – since the square has four peaks, only 400 coins can be buried. Clarify the situation and indicate the points at which the coins are buried.*

In order to simplify the consideration, it is worth using the Cartesian coordinate system, aligning the origin with the old apple tree.

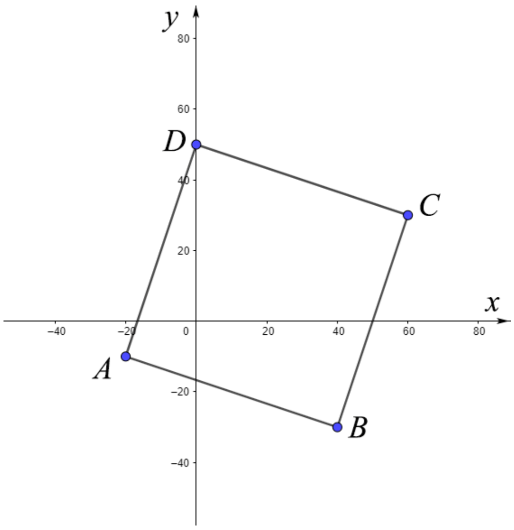


**Figure 4.**

Then, the first two clades are located at points *A (–20; –10)* and *B (40; –30),* the rest are at points *C* and *D*.

Let's define their coordinates.

We find the third treasure from the following considerations: it is located at the vertex *C* of the vector *BC*, which begins at the point *B*; has the same length and is perpendicular to the *BA* vector.

Let's translate these considerations into the language of mathematics.

Let's start with the coordinates of the vector *BA: (–20 – 40; –10 – (–30)) = (–60; 20)*. Accordingly, the length of the vector *BA*: $|BA| = \sqrt{(-60)^2 + 20^2} = \sqrt{4000}$ .

If we denote the coordinates of the point *C* $(x_c; y_c)$, then the vector *BC* has coordinates $(x_c - 40; y_c - (-30))$. The perpendicularity of the vectors means the equality of the dot product to zero: $(x_c - 40)(-60) + (y_c + 30)20 = 0$ .

As a result, there is a system of algebraic equations.

$$\begin{cases} (x_c-40)^2+(y_c+30)^2=4000; \\ (x_c-40)(-60)+(y_c+30)20=0. \end{cases}$$

Apparently the simplest solution to this system is as follows: you need to express $y_C+30$ from the second equation of the system: $y_C+30=3(x_C-40)$, and substitute in the first. Then you get the equation $(x_C-40)^2+[3(x_C-40)]^2=4000$. Hence, $x_C-40=20$ and $x_C-40=-20$. As a result, we get two solutions to the system: *(20; −90)* and *(60; 30)*. Which one is correct? The search for an answer to this question leads to the following answer: both answers are correct, since a square with vertices in *A* and *B* can be located not only to the right and up from them, but also to the left and down.

In order to find the coordinates of points *D* and *F*, you can repeat the process, taking point *A* as the starting point.

But, everything can be done much easier: the *CD* vector has coordinates $(x_D-60;y_D-30)$ and is equal to the *BA* vector. That's why, $(x_D-60;y_D-30)=(-60;20)$. Hence, $x_D=0;y_D=50$. Likewise, you can determine the coordinates of the point *F*: $x_F=-40;y_F=-70$.

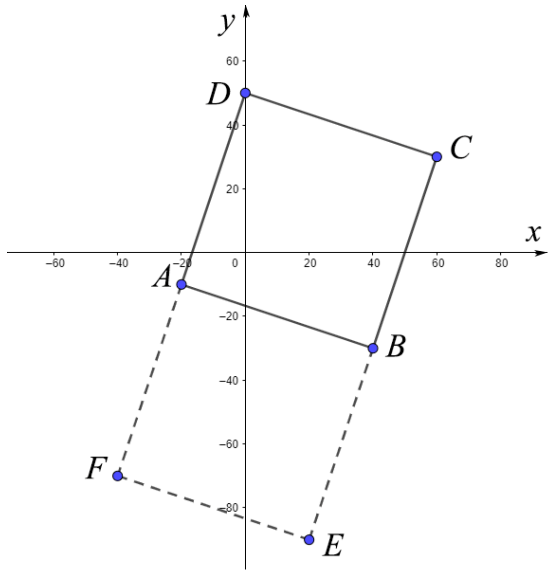


**Figure 5.**

So, we found 6 treasures and their contents are 600 coins. Where are the other 2 treasures? On reflection a little more, you can realize that points *A* and *B* do not have to be adjacent vertices of the square – they can be opposite.

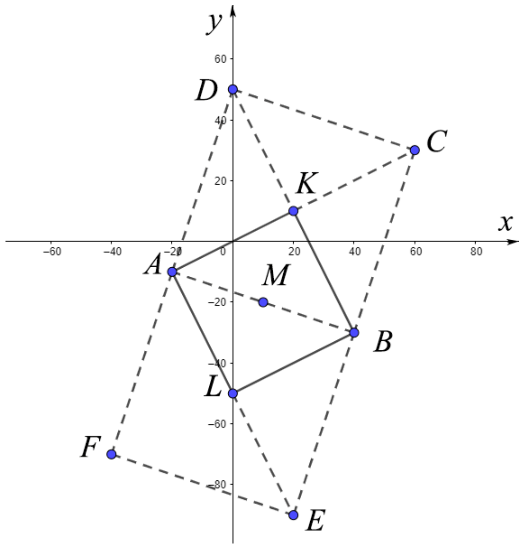


**Figure 6.**

In order to find the coordinates of the two remaining points, you can again repeat the process of solving the system: determine the coordinates of the point *M*, which is the midpoint of the vector *AB*, write the condition for the perpendicularity of the vectors *MK* and *AB*, use the fact that the length of *MK* is equal to half the length of *AB*.

But like last time, there is an easier solution. Since the point *K* is the midpoint of the vector *AC*, its coordinates $\left(\frac{-20+60}{2};\frac{-10+30}{2}\right) = (20;\ 10)$, and the coordinates of the point *L*, from similar considerations, are $(0;\ -50)$.

## 6. CONCLUSION

In order to demonstrate the importance of studying certain scientific aspects, it is very important to use vivid memorable examples. So, for example, in order to introduce the concept of different types of angles in a mathematics textbook for primary schoolchildren, we used a hippopotamus: if you prick him with an obtuse angle, then due to his thick skin, he will not feel it, but it is undesirable to prick him with an acute angle – the hippopotamus can take offense. The example with the hippopotamus turned out to be so extraordinary that it caused a lot of reviews, sometimes even negative ones, in the social networks of Kyrgyzstan. As a similar example, this paper proposes about the Euclidean treasure.

It should be noted that the problem of Euclid's treasure in this work is of an auxiliary nature – it is designed to increase interest in developing skills for working with vectors specified in coordinate form. Such vectors and their properties are useful in describing various economic situations.

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