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INTUITIONISTIC FUZZY SOFT TOPOLOGY VIA NEIGHBORHOOD

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Abstract. In this paper, we bring out some properties of intuitionistic fuzzy soft topology by using the concept of neighborhood. Firstly, we introduce the concept of intuitionistic fuzzy soft point differently from the previous published papers which is related to intuitonistic fuzzy soft point and study some of its basic properties. We also generate an intuitonistic fuzzy soft topology by using the systems of neighborhood and discuss certain properties of intuitonistic fuzzy soft topology including continuous mapping.

1. Intoduction

The concept of fuzzy sets which is the most suitable theory for dealing with uncertainties defined by Zadeh [24] in 1965. After the concept of fuzzy sets several researches studied on the generalizations of fuzzy sets. For instance, intuitonistic fuzzy set [2], interval-valued fuzzy set [25], interval-valued intuitonistic fuzzy set [3],rough set [10], bipolar fuzzy set [26], pythagorean fuzzy set [22] and etc. All these theories are successful to some grade in dealing with problem arising from the uncertainty. In these sets there are difficulties how to set the membership function in each particular case, possibly due to the inadequacy of the parameterization part in them. As a new approach for modeling uncertainty, the concept of soft set theory is introduced by Molodtsov [9]. He presented the basic results of the new theory and pointed out several directions for the applications of soft sets. Extensions of soft set have been proposed recently such as fuzzy soft set [7], intuitionistic fuzzy soft set [8], vague soft set [21], bipolar fuzzy soft set [1]. Topological structures of soft set ([4], [5], [11], [18], [20], [27]) fuzzy soft set ([12],[13], [14],[19]), intuitonistic fuzzy soft set([6], [16], [17]) were defined by many researches.

Neighborhood structure is very important to study in a topological space. Therefore, we aim to study in the present paper, first of all, we define the intuitonistic fuzzy soft point in order to establish neighborhood structure in intuitonistic fuzzy soft topological space and study some basic properties of them. We investigate

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the necessary conditions for a family of intuitonistic fuzzy soft sets to generate an intuitionistic fuzzy soft topology.

2. Intuitionistic fuzzy soft set theory

With this section, we give some definitions and several results on intuitionistic fuzzy soft set theory. From now on, let $X \neq \emptyset$ be a universe, T be a set of all parameters for X and $A \subseteq T$.

Definition 2.1 [2] $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle x \in X\}$ is called an intuitionistic fuzzy set (namely, if- set) where $\mu_A :\rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denote membership and nonmembership functions respectively. Here, $\mu_A(x)$ and $\nu_A(x)$ are membership and nonmembership degree of each $x \in X$ to the intuitionistic fuzzy set A and $\mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

$IF(X)$ refers to the set of all intuitonistic fuzzy sets on X.

If $A,\ B \in IF(X)$ and $(A_i)_{i\in J} \subseteq IF(X)$. Then:

(1) $A \subseteq B \Leftrightarrow \mu_A(x) \leqslant \mu_B(x)$ *and* $\nu_A \geqslant \nu_B(x)$, for all $x \in X$.
(2) $A = B \Leftrightarrow A \subseteq B,\ B \subseteq A$
(3) $\bigcup_{i\in J} A_i = \{\langle x, \bigvee_{i\in J} \mu_{A_i}(x), \bigwedge_{i\in J} \nu_{A_i}(x)\rangle : x \in X\}$
(4) $\bigcap_{i\in J} A_i = \{\langle x, \bigwedge_{i\in J} \mu_{A_i}(x), \bigvee_{i\in J} \nu_{A_i}(x)\rangle : x \in X\}$
(5) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$
(6) $\widetilde{1} = \{\langle x, 1, 0\rangle : x \in X\}$ and $\widetilde{0} = \{\langle x, 0, 1\rangle : x \in X\}$

Definition 2.2 [15] An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ of X is an if- set on X, where $\alpha, \beta \in [0,1]$ and $\alpha + \beta \leq 1$, defined by

$$x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha,\beta), & \text{if } x = y; \\ (0,1), & \text{otherwise.} \end{cases} \text{ for each } y \in X.$$

Let $A \in IF(X)$. We say that intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is contained in A if $\mu_A(x) \geqslant \alpha$ and $\nu_A \leq \beta$ and denoted by $x_{(\alpha,\beta)} \in A$.

An intuitionistic fuzzy point can be written $x_{(\alpha,\beta)} = (x_\alpha, 1 - x_{1-\beta})$ as ordered pair of fuzzy points. [15]

Theorem 2.1. [15] *Let $A \in IF(X)$. Then $x_{(\alpha,\beta)} \in A \Leftrightarrow x_\alpha \in \mu_A$ and $x_{1-\beta} \in 1 - \nu_A$.*

Theorem 2.2. [15] *Let $A, B \in IF(X)$. $A \subseteq B \Leftrightarrow x_{(\alpha,\beta)} \in A$ implies $x_{(\alpha,\beta)} \in B$ for any $x_{(\alpha,\beta)}$ in X.*

Definition 2.3 [9] F_A is called a soft set on X if and only if F_A is a mapping of A into $\mathcal{P}(X)$, i.e., $F_A : A \rightarrow \mathcal{P}(X)$

Definition 2.4 [8] F_A is called an intuitionistic fuzzy soft set on X if and only if F_A is a mapping of A into $IF(X)$, i.e., $F_A : A \rightarrow IF(X)$

For every $a \in A$, $F_A(a)$ is an if- set of X and it is said to be intuitionistic fuzzy value set of parameter a.

Serkan et. al.[17], redefined the intuitionistic fuzzy soft set as follow:

Definition 2.5 [17] F is called an intuitionistic fuzzy soft set (namely, ifs-set) on X if and only if F is a mapping of T into $IF(X)$, i.e., $F : T \rightarrow IF(X)$.

It may be written as follows:

$$F = \{(t, \{\langle x, \mu_{F(t)}(x), \nu_{F(t)}(x)\rangle : x \in X\}) : t \in T\}$$

The element $(t, F(t))$ is not appeared in F if $F(t) = \widetilde{0}$.

$F(t) = (\mu_{F(t)}, \nu_{F(t)})$ is an ifs-set over X. For simplicity, we denote $\mu_{F(t)}, \nu_{F(t)}$ by f_t and $f_t^{'}$, respectively.

$IFS(X,T)$ refers to the set of all intuitonistic fuzzy soft sets over X.

Example 2.1 Let F represent the age of the people with respect to the given parameters. Let the set of people under consideration be $X = \{x_1, x_2, x_3\}$. Let $E = \{young(t_1), middle\ age(t_2)\}$ and define

$F(t_1) = \{\frac{x_1}{[0.3,0.7]}, \frac{x_2}{[0.5,0.1]}, \frac{x_3}{[0.3,0.2]}\}$, $F(t_2) = \{\frac{x_1}{[0.4,0.3]}, \frac{x_2}{[0.9,0.1]}, \frac{x_3}{[0.2,0.5]}\}$.

The family $\{F(t_1), F(t_2)\}$ of if- sets is an ifs set.

In other word, an ifs set is a parametrized family of if-set of X.

Definition 2.6 [17] Let $F, G \in IFS(X,T)$. Then:

(1) F is subset of G if $F(t) \subseteq G(t),\ \forall t \in T$, i.e., $f_t(x) \leq g_t(x)$ *and* $f_t^{'}(x) \geq g_t^{'}(x),\ x \in X$. It is symbolized by $F \widetilde{\subseteq} G$.

(2) F and G are equal if $F \subseteq G,\ G \subseteq F$. It is symbolized by $F = G$.

(3) The intersection of F and G is an ifs-set H defined by $H(t) = F(t) \cap G(t),\ \forall t \in T$, i.e., $h_t(x) = (f_t(x) \wedge g_t(x), f_t^{'}(x) \vee g_t^{'}(x)),\ x \in X$. H is symbolized by $F \widetilde{\cap} G$.

(4) The union of F and G is an ifs-set H defined by $H(t) = F(t) \cup G(t),\ \forall t \in T$, i.e., $h_t(x) = (f_t(x) \vee g_t(x), f_t^{'}(x) \wedge g_t^{'}(x)),\ x \in X$. H is symbolized by $F \widetilde{\cup} G$.

(5) The complement of F is an ifs-set H defined by $H(t) = (F(t))^c,\ \forall t \in T$, i.e., $h_t(x) = (f_t^{'}(x), f_t(x)),\ x \in X$. H is symbolized by F^c.

(6) F is called a null ifs-set, denoted by Φ, if $F(t) = \widetilde{0}$ for all $t \in T$.

(7) F is called universal ifs-set, denoted by $\widetilde{X}$, if $F(t) = \widetilde{1}$, for all $t \in T$.

$(\widetilde{X})^c = \Phi$ and $\Phi^c = \widetilde{X}$.

Proposition 2.3. [17] *Let $F, G \in IFS(X,T)$ and $(G_i)_{i \in J} \subseteq IFS(X,T)$. Then:*

(1) $F \widetilde{\cap} \left(\widetilde{\bigcup}_{i \in J}(G_i)\right) = \widetilde{\bigcup}_{i \in J}\left(F \widetilde{\cap} (G_i)\right),\ F \widetilde{\cup} \left(\widetilde{\bigcap}_{i \in J}(G_i)\right) = \widetilde{\bigcap}_{i \in J}\left(F \widetilde{\cup} (G_i)\right)$

(2) $\Phi \widetilde{\subseteq} F \widetilde{\subseteq} \widetilde{X}$

(3) $\left(\widetilde{\bigcap}_{i \in J}(G_i)\right)^c = \widetilde{\bigcup}_{i \in J}(G_i)^c,\ \left(\widetilde{\bigcap}_{i \in J}(G_i)\right)^c = \widetilde{\bigcup}_{i \in J}(G_i)^c$

(4) $F \widetilde{\cup} F^c = \widetilde{X}$ *and* $(G^c)^c = G$

(5) *If* $F \widetilde{\subseteq} G$*, then* $(G)^c \widetilde{\subseteq} (F)^c$.

Definition 2.7 ([16], [23]) Let $IFS(X,T)$ and $IFS(Y,K)$ be two families of all ifs- sets over X and Y, respectively. Let $\varphi : X \to Y$ and $\psi : T \to K$ be two maps. Then, the pair $(\varphi, \psi) : IFS(X,T) \to IFS(Y,K)$ is called an intuitionistic fuzzy soft map for which:

(1) If $F \in IFS(X,T)$, then the image of F under (φ, ψ), denoted by $(\varphi, \psi)(F)$, is the ifs-set over Y given by

$$\varphi(f)_k(y) = \begin{cases} \bigvee_{\varphi(x)=y} \bigvee_{\psi(t)=k} f_t(x), & \text{if } x \in \varphi^{-1}(y); \\ 0, & \text{otherwise.} \end{cases} \quad \forall k \in \psi(T),\ \forall y \in Y$$

and

$$\varphi(f^{'})_k(y) = \begin{cases} \bigwedge_{\varphi(x)=y} \bigwedge_{\psi(t)=k} f_t^{'}(x), & \text{if } x \in \varphi^{-1}(y); \\ 1, & \text{otherwise.} \end{cases} \quad \forall k \in \psi(T),\ \forall y \in Y$$

(2) If $G \in IFS(Y,K)$, then the preimage of G under (φ, ψ), denoted by $(\varphi, \psi)^{-1}(G)$, is the ifs-set over X, given by

$\varphi^{-1}(g_t)(x) = g_{\psi}(t)(\varphi(x))\ \forall t \in \psi^{-1}(K),\ \forall x \in X$ and

$\varphi^{-1}(g_t^{'})(x) = g_{\psi}^{'}(t)(\varphi(x))\ \forall t \in \psi^{-1}(K),\ \forall x \in X$.

The ifs-map (φ, ψ) is called injective (onto), if φ and ψ are injective (onto).

Proposition 2.4. [23] *Let $F \in IFS(X,T)$, $(F_i)_{i\in J} \subset IFS(X,T)$, $G \in IFS(Y,K)$ and $(G_i)_{i\in J} \subset IFS(Y,K)$. Then,*

(1) *If $F_1 \widetilde{\subseteq} F_2$, then $(\varphi,\psi)(F_1) \widetilde{\subseteq} (\varphi,\psi)(F_2)$.*
(2) *If $G_1 \widetilde{\subseteq} G_2$, then $(\varphi,\psi)^{-1}(G_1) \widetilde{\subseteq} (\varphi,\psi)^{-1}(G_2)$.*
(3) *$F \widetilde{\subseteq} (\varphi,\psi)^{-1}((\varphi,\psi)(F))$. If (φ,ψ) is injective, then the equality holds.*
(4) *$(\varphi,\psi)\left((\varphi,\psi)^{-1}(G)\right) \widetilde{\subseteq} G$. If (φ,ψ) is surjective, then the equality holds.*
(5) *$(\varphi,\psi)\left(\widetilde{\bigcup}_{i\in J} F_i\right) = \widetilde{\bigcup}_{i\in J}(\varphi,\psi)(F_i)$.*
(6) *$(\varphi,\psi)\left(\widetilde{\bigcap}_{i\in J} F_i\right) \widetilde{\subseteq} \widetilde{\bigcap}_{i\in J}(\varphi,\psi)(F_i)$.*
(7) *$(\varphi,\psi)^{-1}\left(\widetilde{\bigcup}_{i\in J} G_i\right) = \widetilde{\bigcup}_{i\in J}(\varphi,\psi)^{-1}(G_i)$.*
(8) *$(\varphi,\psi)^{-1}\left(\widetilde{\bigcap}_{i\in J} G_i\right) = \widetilde{\bigcap}_{i\in J}(\varphi,\psi)^{-1}(G_i)$*
(9) *$(\varphi,\psi)^{-1}(G^c) = \left((\varphi,\psi)^{-1}(G)\right)^c$.*
(10) *$((\varphi,\psi)(F))^c \widetilde{\subseteq} (\varphi,\psi)(F^c)$.*
(11) *If (φ,ψ) is surjective, then $(\varphi,\psi)\left(\widetilde{X}\right) = \widetilde{Y}$.*
(12) *$(\varphi,\psi)^{-1}\left(\widetilde{Y}_K\right) = \widetilde{X}$ $(\varphi,\psi)^{-1}(\Phi) = \Phi$.*
(13) *$(\varphi,\psi)(\Phi) = \Phi$.*

Definition 2.8 [6] Let $X \neq \emptyset$ and $\mathcal{T}$ be a family of ifs- sets over X. An ifs-topological space is a pair $(X,\mathcal{T})$ satisfying the following properties:

(T1) $\Phi, \widetilde{X} \in \mathcal{T}$
(T2) If $F,\ G \in \mathcal{T}$, then $F \widetilde{\cap} G \in \mathcal{T}$
(T3) If $F_i \in \mathcal{T}$, then $\widetilde{\bigcup}_{i\in J}(F_i) \in \mathcal{T}$, $\forall i \in J$.

$\mathcal{T}$ is called a topology of ifs- sets on X. Every member of $\mathcal{T}$ is called open ifs-set. If $G^c \in \mathcal{T}$, G is said to be closed ifs- set in $(X,\mathcal{T})$

$\mathcal{T}^0 = \{\Phi, \widetilde{X}\}$ and $\mathcal{T}^1 = IFS(X,T)$ are ifs- topologies on X.

The intersection of any family of ifs- topologies on X is also a ifs- topology on X.

Definition 2.9 [16] Let $(X,\mathcal{T}_1)$ and $(Y,\mathcal{T}_2)$ be two ifs- topological spaces. $(\varphi,\psi) : (X,\mathcal{T}_1) \to (Y,\mathcal{T}_2)$ is called continuous if $(\varphi,\psi)^{-1}(G) \in \mathcal{T}_1,\ \forall G \in \mathcal{T}_2$.

We see that (φ,ψ) is continuous iff the preimage of every closed fuzzy soft set is closed.

If $(\varphi,\psi) : (X,\mathcal{T}_1) \to (Y,\mathcal{T}_2)$ and $(\varphi',\psi') : (Y,\mathcal{T}_2) \to (Z,\mathcal{T}_3)$ are fuzzy soft continuous, $(\varphi',\psi') \circ (\varphi,\psi)$ is also fuzzy soft continuous.

3. Neighborhood Structures of Intuitionistic Fuzzy Soft Topological Spaces

Definition 3.1 Let $F \in IFS(X,T)$ and $\alpha,\ \beta : T \to [0,1]$ be mappings such that $\alpha(t) = 0, \forall t \in T \setminus A,\ \beta(t) = 1, \forall t \in T \setminus A$ and $\alpha(t) + \beta(t) \leq 1$. The intuitionistic fuzzy soft set F is called ifs- point if $F(t) = x_{(\alpha(t),\beta(t))},\ \forall t \in T$.

Here, $F(t) = x_{(\alpha(t),\beta(t))}$ is an intuitionistic fuzzy point for all $t \in T$.

Obviously, $F(t)(y) = \begin{cases} (\alpha(t),\beta(t)), & \text{if } x = y; \\ (0,1), & \text{otherwise} \end{cases}$

The ifs- point is denoted by $x^{(\alpha,\beta)}$ and x is called its support.

$\alpha(t)$ is called value of $x^{(\alpha,\beta)}$ and $\beta(t)$ is called non-value of $x^{(\alpha,\beta)}$, $\forall t \in T$.

An ifs-point $x^{(\alpha,\beta)}$ is called belong to an ifs-set G, denoted by $x^{(\alpha,\beta)}\widetilde{\in}G$, if $x^{(\alpha,\beta)}\widetilde{\subseteq}G$ (equivalently, $\alpha(t) \leq g_t(x)$ and $\beta(t) \geq g_t^{'}(x),\ for\ x \in X$).

An ifs- point may be seen as a parameterized family of intuitionistic fuzzy point.

$IFSP(X,T)$ refers to the family of all ifs- points on X.

Two ifs- points $x^{(\alpha,\beta)}$, $y^{(\lambda,\mu)}$ are called distinct if their supports are distinct.

Theorem 3.1. *Let $G, H \in IFS(X,T)$. Then $G\widetilde{\subseteq}H \Leftrightarrow x^{(\alpha,\beta)}\widetilde{\in}G$ implies $x^{(\alpha,\beta)}\widetilde{\in}H$ for any ifs-point $x^{(\alpha,\beta)}$.*

Proof. Let $G\widetilde{\subseteq}H$ and $x^{(\alpha,\beta)}\widetilde{\in}G$. Then $\alpha(t) \leq g_t(x) \leq h_t(x)$ and $\beta(t) \geq g_t^{'}(x) \geq h_t^{'}(x),\ \forall t \in T,\ x \in X$. Thus, $x^{(\alpha,\beta)}\widetilde{\in}H$.

Conversely, let $\alpha(t) = g_t(x)$ and $\beta(t) = g_t^{'}(x),\ \forall t \in T,\ x \in X$. Then $x^{(\alpha,\beta)} \in IFSP(X,T)$ and $x^{(\alpha,\beta)}\widetilde{\in}G$. From the hypothesis $x^{(\alpha,\beta)}\widetilde{\in}H$. Thus, $g_t(x) = \alpha(t) \geq h_t(x)$ and $g_t^{'}(x) = \beta(t) \geq h_t^{'}(x)$. Hence, $G\widetilde{\subseteq}H$.

Theorem 3.2. *Let $G \in IFS(X,T)$. $G = \widetilde{\bigcup}_{x^{(\alpha,\beta)}\widetilde{\in}G} x^{(\alpha,\beta)}$, i.e., G is the union of all its ifs-points.*

Proof. Let $G \neq \Phi$. Let $y \in X$. $x_{(\alpha(t),\beta(t))} = (x_{\alpha(t)}, 1 - x_{1-\beta(t)})$ for each $t \in T$.

$$(\widetilde{\bigcup}_{x^{(\alpha,\beta)}\widetilde{\in}G} x^{(\alpha,\beta)})(t)(y) = \left(\bigvee_{x_{\alpha(t)}\in g_t} x_{\alpha(t)}(y), \bigwedge_{x_{1-\beta(t)}\in(1-g_t^{'})} 1 - x_{1-\beta(t)}(y)\right)$$

Here, $\bigvee_{x_{\alpha(t)}\in g_t} x_{\alpha(t)}(y) = g_t(y)$ and $\bigvee_{x_{1-\beta(t)}\in(1-g_t^{'})} x_{1-\beta(t)}(y) = (1 - g_t^{'})(y)$.

So, $g_t^{'}(y) = 1 - \bigvee_{x_{1-\beta(t)}\in(1-g_t^{'})} x_{1-\beta(t)}(y) = \bigwedge_{x_{1-\beta(t)}\in(1-g_t^{'})} 1 - x_{1-\beta(t)}(y)$.

Hence, $\widetilde{\bigcup}_{x^{(\alpha,\beta)}\widetilde{\in}G} x^{(\alpha,\beta)} = G$.

Proposition 3.3. *Let (φ,ψ) be a soft map between X and Y.*

(1) *If $x^{(\alpha,\beta)} \in IFSP(X,T)$, then $(\varphi,\psi)(x^{(\alpha,\beta)} \in IFSP(Y,K)$ and $(\varphi,\psi)(x^{(\alpha,\beta)}) = x^{\varphi(x^{(\alpha,\beta)})} = x^{\varphi(x)_{(\alpha,\beta)}}$.*

(2) *If $x^{(\alpha,\beta)}\widetilde{\in}G$, then $(\varphi,\psi)(x^{(\alpha,\beta)})\widetilde{\in}(\varphi,\psi)(G)$, where $G \in IFS(X,T)$.*

(3) *If $x^{(\alpha,\beta)}\widetilde{\in}(\varphi,\psi)^{-1}(H)$, then $(\varphi,\psi)(x^{(\alpha,\beta)})\widetilde{\in}H$, where $H \in IFS(Y,K)$.*

Proof. (1) For all $k \in \psi(T)$ and $y \in Y$,

$$\varphi(x^{(\alpha,\beta)})(k)(y) =$$

$$\left(\begin{cases} \bigvee_{\varphi(z)=y}\bigvee_{\varphi(t)=k} x_{\alpha(e)}(z), & \text{if } z \in \varphi^{-1}(y) \\ 0, & \text{otherwise} \end{cases}, \begin{cases} \bigwedge_{\varphi(z)=y}\bigwedge_{\varphi(t)=k} x_{\alpha(t)}(z), & \text{if } z \in \varphi^{-1}(y) \\ 1, & \text{otherwise} \end{cases}\right)$$

$$= \left(\begin{cases} \alpha(t), & \text{if } y = \varphi(x),\ \psi(t) \in \psi(T) \\ 0, & \text{otherwise} \end{cases}, \begin{cases} \beta(t), & \text{if } y = \varphi(x),\ \psi(t) \in \psi(T) \\ 1, & \text{otherwise} \end{cases}\right)$$

$$= x^{\varphi(x)_{(\alpha,\beta)}}(k)(y)$$

$$= x^{\varphi(x^{(\alpha,\beta)})}$$

Hence, $(\varphi,\psi)(x^{(\alpha,\beta)}) = x^{\varphi(x)_{(\alpha,\beta)}}$.

(2) Now let $x^{(\alpha,\beta)}\widetilde{\in}G$, then $x_{(\alpha(t),\beta(t))} \in G(t)$ for each $t \in T$ and $\alpha(t) \leq g_t(x),\ \beta(t) \geq g_t^{'}(x),\ x \in X$.

$$\begin{aligned}\alpha(t) \leqslant g_t(x) &\leq \bigvee_{\varphi(x)=\varphi(z)} \bigvee_{\psi(t)=k} g_t(z)\\ &= \varphi\left(g_t)\right)(\varphi(x))\\ &= \varphi(g)\psi(t)\varphi(x) \text{ for each } t \in A \subseteq B.\\ \beta(t) \geq g_t^{'}(x) &\geq \bigwedge_{\varphi(x)=\varphi(z)} \bigwedge_{\psi(t)=k} g_t^{'}(z)\\ &= \varphi\left(g_t^{'})\right)(\varphi(x))\\ &= \varphi(g^{'})\psi(t)\varphi(x) \text{ for each } t \in A \subseteq B.\end{aligned}$$

Therefore, we get $x^{\varphi(x)_{(\alpha,\beta)}} \in (\varphi,\psi)(G)$.

(3) Let $x^{(\alpha,\beta)}\widetilde{\in}(\varphi,\psi)^{-1}(H)$. By (2) and since $(\varphi,\psi)\left((\varphi,\psi)^{-1}(H)\right)\widetilde{\subseteq}H$, we obtain $(\varphi,\psi)(x^{(\alpha,\beta)})\widetilde{\in}H$.

Proposition 3.4. *Let $(G_i)_{i\in J} \subset IFS(X,T)$ and $x^{(\alpha,\beta)} \in IFSP(X,T)$. If $x^{(\alpha,\beta)}\widetilde{\in}\widetilde{\bigcap}_{i\in J}G_i$, then $x^{(\alpha,\beta)}\widetilde{\in}G_i$ for each $i \in J$.*

Proof. Let $x^{(\alpha,\beta)}\widetilde{\in}\widetilde{\bigcap}_{i\in J}G_i$, then we have $x_{(\alpha(t),\beta(t))} \in \bigwedge\limits_{i\in J} G_i(t)$ and

$\alpha(t) \leq \bigwedge\limits_{i\in J}(g_i)_t(x) \leq (g_i)_t(x)$ and $\beta(t) \geq \bigvee\limits_{i\in J}(g_i)_t^{'}(x) \geq (g_i)_t^{'}(x)$ for each $t \in T,\ i \in J,\ x \in X$.

Consequently, $x^{(\alpha,\beta)}\widetilde{\in}(G_i)$ for each $i \in J$.

Proposition 3.5. *Let $x^{(\alpha,\beta)},\ x^{(\lambda,\mu)} \in IFSP(X,T)$. Then $x^{(\alpha,\beta)}\widetilde{\cup}x^{(\lambda,\mu)} = x^{(\alpha\vee\lambda,\beta\vee\mu)}$.*

Proof. Straightforward.

Remark 3.1 $x^{(\alpha,\beta)}\widetilde{\in}G\widetilde{\cup}H$ does not imply $x^{(\alpha,\beta)}\widetilde{\in}G$ or $x^{(\alpha,\beta)}\widetilde{\in}H$. By the following example we see the result.

Example 3.2 Let $T = \{t_1,t_2,t_3\}$ and $X = \{x_1,x_2,x_3,x_4\}$ and define the soft sets G,H and absolute fuzzy soft point x_T^{λ}, where

$g(t_1) = \{\frac{x_1}{1},\frac{x_2}{1},\frac{x_3}{0},\frac{x_4}{0}\},\ g(t_2) = \{\frac{x_1}{1},\frac{x_2}{1},\frac{x_3}{1},\frac{x_4}{0}\},\ g(t_3) = \{\frac{x_1}{0},\frac{x_2}{1},\frac{x_3}{1},\frac{x_4}{1}\},$
$g^{'}(t_1) = \{\frac{x_1}{0},\frac{x_2}{0},\frac{x_3}{1},\frac{x_4}{1}\},\ g^{'}(t_2) = \{\frac{x_1}{0},\frac{x_2}{0},\frac{x_3}{0},\frac{x_4}{1}\},\ g^{'}(t_3) = \{\frac{x_1}{1},\frac{x_2}{0},\frac{x_3}{0},\frac{x_4}{0}\},$
$h(t_1) = \{\frac{x_1}{0},\frac{x_2}{0},\frac{x_3}{1},\frac{x_4}{1}\},\ h(t_2) = \{\frac{x_1}{1},\frac{x_2}{1},\frac{x_3}{1},\frac{x_4}{1}\},\ h(t_3) = \{\frac{x_1}{1},\frac{x_2}{1},\frac{x_3}{0},\frac{x_4}{0}\}.$
$h^{'}(t_1) = \{\frac{x_1}{1},\frac{x_2}{1},\frac{x_3}{0},\frac{x_4}{0}\},\ h^{'}(t_2) = \{\frac{x_1}{0},\frac{x_2}{0},\frac{x_3}{0},\frac{x_4}{0}\},\ h^{'}(t_3) = \{\frac{x_1}{0},\frac{x_2}{0},\frac{x_3}{1},\frac{x_4}{1}\}.$

Then $G\widetilde{\cup}H = \widetilde{X}$ and $x^{(\alpha,\beta)}\widetilde{\in}g\widetilde{\cup}H$, but $x^{(\alpha,\beta)}\widetilde{\notin}G$ and $x^{(\alpha,\beta)}\widetilde{\notin}H$.

Theorem 3.6. *Let $x^{(\alpha,\beta)} \in IFSP(X,T)$ and $G,H \in IFS(X,T)$. If $x^{(\alpha,\beta)}\widetilde{\in}G\widetilde{\cup}H$, then there exist $x^{(\alpha_1,\beta_1)}\widetilde{\in}G$ and $x^{(\alpha_2,\beta_2)}\widetilde{\in}H$ such that $x^{(\alpha,\beta)} = x^{(\alpha_1,\beta_1)}\widetilde{\cup}x^{(\alpha_2,\beta_2)}$.*

Proof. Let $x^{(\alpha,\beta)}\widetilde{\in}G\widetilde{\cup}H$. Then $x_{(\alpha(t),\beta(t))} \in G(t)\cup H(t)$ for each $t \in T$ and $\alpha(t) \leq g_t(x)\vee h_t(x),\ \beta(t) \geq g_t^{'}(x)\wedge h_t^{'}(x)$ for each $x \in X$.

Therefore, $\alpha(t) \leq g_t(x)$ for some $t \in T$, $\alpha(t) \leqslant h_t(x)$ for some $t \in T$ and $\beta(t) \geq g_t^{'}(x)$ for some $t \in T$, $\beta(e) \geq h_t^{'}(x)$ for some $t \in T$.

Now choose $A_1 = \{t \in T : \alpha(t) \leq g_t(x),\ \beta(t) \geq g_t^{'}(x)\}$ and $A_2 = \{t \in T : \alpha(t) \leqslant h_t(x),\ \beta(t) \geq h_t^{'}(x)\}$ and choose

$\alpha_1 : T \to [0,1]$ by $\alpha_1(t) = \begin{cases}\alpha(t), & \text{if } t \in A_1;\\ 0, & \text{otherwise.}\end{cases}$

$\beta_1 : T \to [0,1]$ by $\beta_1(t) = \begin{cases}\beta(t), & \text{if } t \in A_1;\\ 1, & \text{otherwise.}\end{cases}$

and $\alpha_2 : T \to [0,1]$ by $\alpha_2(t) = \begin{cases} \alpha(t), & \text{if } t \in A_2; \\ 0, & \text{otherwise.} \end{cases}$

$\beta_2 : E \to [0,1]$ by $\beta_2(t) = \begin{cases} \beta(t), & \text{if } t \in A_2; \\ 1, & \text{otherwise.} \end{cases}$

Since $\alpha_1(t) \leq g_e(x)$ and $\beta_1(e) \geq h_e^{'}(x)$ for each $t \in A_1$, we get $x^{(\alpha_1,\beta_1)} \widetilde{\in} G$.
Since $\alpha_2(t) \leq h_t(x)$ and $\beta_2(t) \geq h_t^{'}(x)$ for each $t \in A_2$, we get $x^{(\alpha_2,\beta_2)} \widetilde{\in} H$.
Therefore, $x^{(\alpha,\beta)} = x^{(\alpha_1,\beta_1)} \widetilde{\cup} x^{(\alpha_2,\beta_2)}$.

Definition 3.2 Let $x^{(\alpha,\beta)} \in IFSP(X,T)$. An ifs-set G in $(X,\mathcal{T})$ is called a neighborhood of $x^{(\alpha,\beta)}$ if there exists $H \in \mathcal{T}$ such that $x^{(\alpha,\beta)} \widetilde{\in} H \widetilde{\subseteq} G$.

Ifs-set G is called open neighborhood of $x^{(\alpha,\beta)}$ if it is open.

The family consisting of all neighborhoods of $x^{(\alpha,\beta)}$ is said to be the system of neighborhood of $x^{(\alpha,\beta)}$ and denoted by $\mathcal{U}_{\mathcal{T}}(x^{(\alpha,\beta)})$.

Theorem 3.7. *Let $(X,\mathcal{T})$ be an ifs-topological space. $G \in \mathcal{T}$ if and only if G is a neighborhood of each of its ifs-points.*

Proof. Straightforward.

Definition 3.3 Let $\varepsilon,\ \omega : T \to [0,1]$ be a mapping and $x^{(\alpha,\beta)} \in IFSP(X,T)$. ε is called compatible with α (ω is called compatible with β) if ε provides that $0 \leq \varepsilon(t) \leq \alpha(t)$ (ω provides that $1 \geq \omega(t) \geq \beta(t)$) for each $t \in T$.

Theorem 3.8. *Let $(X,\mathcal{T})$ be an ifs-topological space and $\mathcal{U}(x^{(\alpha,\beta)})$ be the neighborhood system of ifs-point $x^{(\alpha,\beta)}$. Then we have the followings:*

$(N1)$ $\widetilde{X} \in \mathcal{U}(x^{(\alpha,\beta)})$ and if $G \in \mathcal{U}(x^{(\alpha,\beta)})$, then $x^{(\alpha,\beta)} \widetilde{\in} G$.

$(N2)$ If $G,\ H \in \mathcal{U}(x^{(\alpha,\beta)})$, then $G \widetilde{\cap} H \in \mathcal{U}(x^{(\alpha,\beta)})$.

$(N3)$ If $G \widetilde{\subseteq} H$ and $G \in \mathcal{U}(x^{(\alpha,\beta)})$, then $H \in \mathcal{U}(x^{(\alpha,\beta)})$.

$(N4)$ If $G \in \mathcal{U}(x^{(\alpha-\varepsilon,\beta-\omega)})$ for all ε compatible with α and ω compatible with β, then $G \in \mathcal{U}(x^{(\alpha,\beta)})$.

$(N5)$ If $G \in \mathcal{U}(x^{(\alpha_1,\beta_1)})$ and $H \in \mathcal{U}(x^{(\alpha_2,\beta_2)})$, then $G \widetilde{\cup} H \in \mathcal{U}(x^{(\alpha_1,\beta_1)} \widetilde{\cup} x^{(\alpha_2,\beta_2)})$.

$(N6)$ If $G \in \mathcal{U}(x^{(\alpha,\beta)})$, then there exists $H \in \mathcal{U}(x^{(\alpha,\beta)})$ such that $H \widetilde{\subseteq} G$ and $H \in \mathcal{U}(y^{(\lambda,\mu)}),\ \forall y^{(\lambda,\mu)} \widetilde{\in} H$.

Conversely, let for each ifs-point $x^{(\alpha,\beta)}$ there exists a nonempty collection $\mathcal{U}(x^{(\alpha,\beta)})$ of ifs-sets on X satisfying (N1)-(N6). Then the family $\mathcal{T} = \{G \in IFS(X,T) : G \in \mathcal{U}(y^{(\lambda,\mu)}),\ \forall y^{(\lambda,\mu)} \widetilde{\in} G\}$ is an ifs-topology on X such that $\mathcal{U}(x^{(\alpha,\beta)})$ is the family of all neighborhoods of $x^{(\alpha,\beta)}$ in $(X,\mathcal{T})$.

Proof. (N1)-(N3) are omitted.

$(N4)$ Let $G \in \mathcal{U}(x^{(\alpha-\varepsilon,\beta-\omega)})$ for all ε compatible with α and ω compatible with β. Then there exists $H_{(\varepsilon,\omega)} \in \mathcal{T}$ such that $x^{(\lambda-\varepsilon,\beta-\omega)} \widetilde{\in} H_{(\varepsilon,\omega)} \widetilde{\subseteq} G$.

Let $H := \widetilde{\bigcup}_{(\varepsilon,\omega)} H_{(\varepsilon,\omega)}$, then $H \in \mathcal{T}$ and $H \widetilde{\subseteq} G$. By the Theorem 3.6 and since $Sup_{\varepsilon(t)}\{\alpha(t)-\varepsilon(t)\} = \alpha(t)$ and $Inf_{\omega(t)}\{\beta(t)-\omega(t)\} = \beta(t)$, then $\widetilde{\bigcup}_{(\varepsilon,\omega)} x^{(\alpha-\varepsilon,\beta-\omega)} = x^{(\alpha,\beta)} \widetilde{\subseteq} \widetilde{\bigcup}_{(\varepsilon,\omega)} H_{(\varepsilon,\omega)} = H \widetilde{\subseteq} G$ for all compatible $\varepsilon(t) > 0$ and $\omega(1) < 1$.

So we have $x^{(\alpha,\beta)} \widetilde{\in} H \widetilde{\subseteq} G$, i.e., $G \in \mathcal{U}(x_A^{(\alpha,\beta)})$.

$(N5)$ Let $G \in \mathcal{U}(x^{(\alpha_1,\beta_1)})$ and $H \in \mathcal{U}(x^{(\alpha_2,\beta_2)})$. Then there exists $M \in \mathcal{T}$ such that $x^{(\alpha_1,\beta_1)} \widetilde{\in} M \widetilde{\subseteq} G$ and there exists $N \in \mathcal{T}$ such that $x^{(\alpha_2,\beta_2)} \widetilde{\in} N \widetilde{\subseteq} H$.

Since $x^{(\alpha_1,\beta_1)}\widetilde{\in}M$, then $x_{(\alpha_1(t),\beta_1(t))}) \in M(t)$ for each $t \in T$. From here, we obtain $\alpha_1(t) \leq m_t(x)$ and $\beta_1(t) \geq m_t^{'}(x)$, $x \in X$.

Since $x^{(\alpha_2,\beta_2)}\widetilde{\in}N$, then $x_{(\alpha_2(t),\beta_2(t))}) \in N(t)$ for each $t \in T$. From here, we obtain $\alpha_2(t) \leq n_t(x)$ and $\beta_2(t) \geq n_t^{'}(x)$, $x \in X$.

Then, we get $m_t(x) \vee n_t(x) \geq \alpha_1 \vee \alpha_2$ and $m_t^{'}(x) \wedge n_t^{'}(x) \leq \beta_1 \wedge \beta_2$ for each $t \in T$, $x \in X$.

So, $x^{(\alpha_1,\beta_1)}\widetilde{\cup}x^{(\alpha_2,\beta_2)} = x^{(\alpha_1\vee\alpha_2,\beta_1\wedge\beta_2)}\widetilde{\in}M\widetilde{\cup}N$ and $M\widetilde{\cup}N \in \mathcal{T}$ and $M\widetilde{\cup}N\widetilde{\subseteq}G\widetilde{\cup}H$.

Hence, $G\widetilde{\cup}H \in \mathcal{U}(x^{(\alpha_1,\beta_1)}\widetilde{\cup}x^{(\alpha_2,\beta_2)})$.

$(N6)$ Let $G \in \mathcal{U}(x^{(\alpha,\beta)})$. Then $\exists H \in \mathcal{T} : x^{(\alpha,\beta)}\widetilde{\in}H\widetilde{\subseteq}G$. Since $H \in \mathcal{T}$, H is a neighborhood of its points, i.e., $H \in \mathcal{U}(y^{(\lambda,\mu)})$, $\forall y^{(\lambda,\mu)}$. Since $x^{(\alpha,\beta)} \in H$, we have $H \in \mathcal{U}(x^{(\alpha,\beta)})$,. Therefore, since $G \in \mathcal{U}(x^{(\alpha,\beta)})$ there exists $H \in \mathcal{U}(x^{(\alpha,\beta)})$ such that $H\widetilde{\subseteq}G$ and $H \in \mathcal{U}(y^{(\lambda,\mu)})$, $\forall y^{(\lambda,\mu)}$.

Conversely, let the subfamily $\mathcal{U}(x^{(\alpha,\beta)}) \subset IFSP(X,T)$ satisfies the above six conditions for each $x^{(\alpha,\beta)}$ and let $\mathcal{T} = \{G \in IFS(X,T) : G \in \mathcal{U}(y^{(\lambda,\mu)}),\ \forall y^{(\lambda,\mu)}\widetilde{\in}G\}$.

We show that the family $\mathcal{T}$ is an ifs-topology on X.

$(T1)$ By (N1), $\widetilde{X} \in \mathcal{T}$ and $\Phi \in \mathcal{T}$ is vacuously satisfied.

$(T2)$ Let $G,\ H \in \mathcal{T}$ and $y^{(\lambda,\mu)}\widetilde{\in}(G\widetilde{\cap}H)$. By Proposition 3.4 and the construction of $\mathcal{T}$, G and $H \in \mathcal{U}(y^{(\lambda,\mu)})$. By (N2), we have $G\widetilde{\cap}H \in \mathcal{U}(y^{(\lambda,\mu)})$. Hence, $G\widetilde{\cap}H \in \mathcal{T}$.

$(T3)$ Let $G = \widetilde{\bigcup}_{i\in J}G_i$, where $G_i \in \mathcal{T},\ \forall i \in J$ and $y^{(\lambda,\mu)}\widetilde{\in}G$.

Let $x \in X$ and assume that for each $\bigvee_{i\in J}(g_i)(t)(x) = \alpha(t)$ and $\bigwedge_{i\in J}(g_i)^{'}(t)(x) = \beta(t)$ for each $t \in T$.

Then $\alpha,\ \beta : T \to [0,1]$ be a mappings and $x^{(\alpha,\beta)} \in G$.

Now choose an ε compatible with α and a ω compatible with β.

For the ifs-point $x^{(\alpha-\varepsilon,\beta-\omega)}$ and $e \in E$ there exists $i_t \in J$ such that $(g)_{i_t}(t)(x) > \alpha(t) - \varepsilon(t)$ and $(g)_{i_t}^{'}(t)(x) < \beta(t) - \omega(t)$.

Hence, $x^{(\alpha-\varepsilon,\beta-\omega)}\widetilde{\in}\widetilde{\bigcup}_{i_e\in J}G_{i_t}$.

Let $H := \widetilde{\bigcap}_{i_t\in J}G_{i_t}$ and let $y^{(\lambda,\mu)}\widetilde{\in}H$. Then, $Sup(g)_{i_t}(t)(y) \geq \lambda(t)$ and $Inf(g)_{i_t}^{'}(t)(y) \leq \mu(t)$.

From here, $\forall\gamma(t) > 0\ \exists i_{t_0} \in J :\ (g)_{i_{t_0}}(t)(y) > \lambda(t) - \gamma(t)$ and $\forall\gamma^{'}(t) > 0\ \exists i_{t_0} \in J :\ (g)_{i_{t_0}}^{'}(t)(y) < \mu(t) - \gamma^{'}(t)$.

Without loss of generality, let $\lambda(t) - \gamma(t) < (g)_{i_t}(t)(y)$ and let $\mu(t) - \gamma > (g)_{i_{t_0}}^{'}(t)(y)$.

Then, $y^{(\overline{\alpha}-\gamma,\overline{\mu}-\gamma^{'})}\widetilde{\in}(G)_{i_t}$, where

$$\overline{\lambda}(k) = \begin{cases} \lambda(e), & \text{if } k = e \in E; \\ 0, & \text{otherwise} \end{cases}$$

$$\overline{\mu}(k) = \begin{cases} \mu(e), & \text{if } k = e \in E; \\ 1, & \text{otherwise} \end{cases}$$

By (N5), $\widetilde{\bigcup}_{t\in T}y^{(\overline{\alpha}(t)-\gamma(t),\overline{\mu}(t)-\gamma^{'}(t))}\widetilde{\in}\widetilde{\bigcup}_{i_t\in J}(G)_{i_t}$, then since $\vee\overline{\lambda} = \lambda$ and $\wedge\overline{\mu} = \mu$, we get $\widetilde{\bigcup}_{i_t\in J}(G)_{i_t} \in \mathcal{U}(y^{(\lambda,\gamma,\mu-\gamma^{'})}$. By (N4), we get $\widetilde{\bigcup}_{i_t\in J}(G)_{i_t} \in \mathcal{U}(y^{(\lambda,\mu)})$.

Then, for $x^{(\alpha-\varepsilon,\beta-\omega)}$ there exists H such that $H \in \mathcal{U}(y^{(\lambda,\mu)})$, $\forall y^{(\lambda,\mu)} \widetilde{\in} H$ and $x^{(\alpha-\varepsilon,\beta-\omega)} \widetilde{\in} H \widetilde{\subseteq} G$.

Hence $G \in \mathcal{U}(x^{(\alpha-\varepsilon,\beta-\omega)}$, $\forall \varepsilon > 0$, $\omega < 1$ and then by (N4) $\widetilde{\bigcup}_{i\in J}(G)_i \in \mathcal{U}(x_A^{(\alpha,\beta)})$.

Thus, $\mathcal{T}$ is an ifs-topology on X such that $\mathcal{U}(x^{(\alpha,\beta)})$ is the family of all neighborhoods of $x^{(\alpha,\beta)}$ in $(X, \mathcal{T})$.

Theorem 3.9. *Let $\mathcal{T}$ and $\mathcal{T}^*$ be two ifs- topologies over X and Y, respectively and (φ, ψ) be a soft mapping from X to Y, then we obtain following equivalent statements:*

(1) (φ, ψ) is continuous.

(2) for each ifs-point $x^{(\alpha,\beta)}$ on X the inverse of every neighborhood of $(\varphi, \psi)(x^{(\alpha,\beta)})$ under (φ, ψ) is a neighborhood of $x^{(\alpha,\beta)}$.

(3) for each ifs-point $x^{(\alpha,\beta)}$ on X and neighborhood G of $(\varphi, \psi)(x^{(\alpha,\beta)})$, there exists a neighborhood H of $x^{(\alpha,\beta)}$ such that $(\varphi, \psi)(H) \widetilde{\subseteq} G$.

Proof. (1) $\Rightarrow$ (2) Let $G \in \mathcal{U}_{\mathcal{T}^*}\left((\varphi, \psi)(x^{(\alpha,\beta)})\right)$. Then there exists $H \in \mathcal{T}^*$ such that $(\varphi, \psi)(x^{(\alpha,\beta)}) \widetilde{\in} H \widetilde{\subseteq} G$. Since (φ, ψ) is continuous, $(\varphi, \psi)^{-1}(H) \in \mathcal{T}$ and we get $x^{(\alpha,\beta)} \widetilde{\in} (\varphi, \psi)^{-1}(H) \widetilde{\subseteq} (\varphi, \psi)^{-1}(G)$.

(2) $\Rightarrow$ (3) Let $G \in \mathcal{U}_{\mathcal{T}^*}\left((\varphi, \psi)(x^{(\alpha,\beta)})\right)$. By hypothesis $(\varphi, \psi)^{-1}(G) \in \mathcal{U}_{\mathcal{T}}(x^{(\alpha,\beta)})$. Let choose

$H := (\varphi, \psi)^{-1}(G) \in \mathcal{U}_{\mathcal{T}}(x^{(\alpha,\beta)})$. Hence, we get $(\varphi, \psi)(H) = (\varphi, \psi)\left((\varphi, \psi)^{-1}(G)\right) \widetilde{\subseteq} G$.

(3) $\Rightarrow$ (1) Let $G \in \mathcal{T}^*$. Let $x^{(\alpha,\beta)} \widetilde{\in} (\varphi, \psi)^{-1}(G)$. Then $(\varphi, \psi)(x^{(\alpha,\beta)}) \widetilde{\in} G$ and since $G \in \mathcal{T}^*$, we obtain $G \in \mathcal{U}_{\mathcal{T}^*}\left((\varphi, \psi)(x^{(\alpha,\beta)})\right)$. By hypothesis there exists $H \in \mathcal{U}(x^{(\alpha,\beta)})$ such that $(\varphi, \psi)(H) \widetilde{\subseteq} G$. Then $H \widetilde{\subseteq} (\varphi, \psi)^{-1}\left((\varphi, \psi)(H)\right) \widetilde{\subseteq} (\varphi, \psi)^{-1}(G)$, for $H \in \mathcal{U}(x^{(\alpha,\beta)})$.

Therefore, $H \widetilde{\subseteq} (\varphi, \psi)^{-1}(G)$, for $H \in \mathcal{U}(x^{(\alpha,\beta)})$.

So, $(\varphi, \psi)^{-1}(G) \in \mathcal{T}$.

4. Conclusion

With this work, we generate an intuitionistic fuzzy soft topology by using the system of intuitionistic fuzzy soft neighborhood. For this goal, firstly we have defined the intuitonistic fuzzy soft point as a generalized of intutionistic fuzzy point. Furthermore, one could study some structures in intuitionistic fuzzy soft topological spaces related to neighborhood structures.

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