

Research Article

Solving Multidimensional Knapsack Problem with Bayesian Multiploid Genetic Algorithm

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ARTICLE INFO	ABSTRACT
Article history: Received Revised Accepted Keywords: Bayesian Optimization Algorithm MKP Genetic Algorithm Optimization	Solving optimization problems is still a big challenge in the area of optimization algorithms. Many proposed algorithms in the literature don't consider the relations between the variables of the nature of the problem. However, a recently published algorithm, called "Bayesian Multiploid Genetic Algorithm" exploits the relations between the variables and then solves the given problem. It also uses more than one genotype unlike the simple Genetic Algorithm (GA) and it acts like an implicit memory in order to remember the old but good solutions. In this work, the well-known Multidimensional Knapsack Problem (MKP) is solved by the Bayesian Multiploid Genetic Algorithm. And the results show that exploiting relations between the variables gets a huge advantage in solving the given problem.

1. Introduction

Constraint Optimization Problems (COP) are still a big challenge in the area of computer science. One of them is the Multidimensional Knapsack Problem (MKP). The MKP is an extended version of the standard 0-1 Knapsack Problem (KP). While standard KP has only one resource, the MKP can has more than one resource. MKP is actually can be considered a real-world problem. Many real-world problems can be solved by MKP, such as cutting stock[1], loading problems [2], resource allocation for distributed computing[3], project selection[4], etc. MKP is still a current benchmark problem that continues to be solved with different approaches in recent years[5]–[8].

In the past ten years, many metaheuristic (MH) techniques are applied to solve different NP-Hard global optimization problems. Some of them are the following: Estimation of Distribution Algorithms (EDA)[9], Artificial Bee Colony (ABC)[10], Harmony Search (HS)[11], Ant Colony Optimization

(ACO)[12], Whale Optimization Algorithm (WOA)[13], Bat Algorithm (BA)[14], etc. However, all of them mentioned before don't consider the interactions between the problem's variables. In an ecosystem, all the objects have connected some way and they affect each other.

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For this reason, it is important to exploit interactions between the variables and then use them to solve the problem effectively.

In this work, the Bayesian Multiploid Genetic Algorithm (BMGA)[15] is used, which has both an implicit memory scheme (to remember old solutions) and a Bayesian Network (to exploit interactions between variables) in order to solve the well-known Multidimensional Knapsack Problem (MKP) considering as a real-world benchmark problem.

To evaluate the BMGA's performance, six algorithms are selected as comparative algorithms. Note that, the results for the six algorithms listed below are already taken from the [16] and compared

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to the BMGA's results.

- Moth Search Algorithm (MS)[17, s.]: A new optimization algorithm that is inspired by the Levy flights and the phototaxis of the months. In this method, the fittest individual is considered the light source. The moths close to the fittest one fly in the form of Levy flights. On the other hand, because of the phototaxis, the moths far from the fittest one fly to the fittest one with big steps. These two different behaviors are the exploration and the exploitation of any other optimization method.
- Self-Learning MS (SLMS)[16]: In regular MS, each individual update their positions according to the fittest one. But it may cause it to fall into the local optimum. Authors, introduce a new selflearning strategy to enable individuals to update their positions not only according to the fittest one but also the closer individuals which have fitter than themselves.
- Modified multi-verse optimization algorithm (MMVO)[18]: In this algorithm, authors are inspired by the popular multi-verse theory which is based on three concepts: the wormhole, the black hole, and the white hole. In the algorithm, each universe is considered a solution candidate. If an individual gets close to the white hole, means it is getting better fitness values, and vice-versa (closing to the black hole), it is getting worse fitness values. And the wormholes are used as a diversification operator in the algorithm to maintain the diversity in the population.
- Binary Gravitational Search Algorithm (BGSA)[19]: This is the binary version of GSA which is based on Newton's laws of gravity and motion: The gravitational force affects the objects and makes them attract each other. In the algorithm, each object is considered an agent, and each one of them has its mass. And their masses define their fitness values. As time passes, objects are attracted by the fitter masses which leads them to get a better fitness value.
- Binary Hybrid Topology Particle Swarm Optimization (BHTPSO)[19]: In the simple PSO[20], the particles may fall into local optimum if their velocities are zero. Because zero velocity means the particle's fitness value is good and shouldn't be changed. To overcome this, a small value is added to their velocities for

Algorithm 1 Pseudocode of BMGA

$g \leftarrow number \ of \ genotypes$
$pop \leftarrow init(popSize, g)$
randomly generate a probVec[]
while a termination condition not met do
genotype2phenotype(pop,probVec[])
$best \leftarrow evaluate(pop)$
$BN \leftarrow constructBN(pop)$
$auxPop \leftarrow sampleBN(BN)$
$probVec[] \leftarrow constructProbVec(auxPop)$
$pop \leftarrow tournamentSelection(pop)$
$pop \leftarrow uniformCrossover(pop)$
$pop \leftarrow bitwiseMutation(pop)$
return best

acceleration and enables them to escape from the local optimum.

• Binary Hybrid Topology Particle Swarm Optimization Quadratic Interpolation (BHTPSO-QI)[19]: This is the form of BHTPSO which is incorporated with a quadratic crossover operator.

This paper continues as follows: In Section 2, the BMGA is explained. In Section 3, MKP and its datasets are explained. In Section 4, the results are shown and finally, Section 5 concludes the paper.

2. Bayesian Multiploid Genetic Algorithm

The BMGA is constructed on the simple GA. However, it differs from simple GA in many ways.

First, the individuals in the simple GA have only one chromosome to represent a candidate solution. Both genetic operators and fitness calculations are done over this chromosome for each individual. However, in BMGA, each individual has two different structures: The genotypes and a phenotype. As we know, in nature, all living things have genotypes and phenotypes. Genotypes are inherited from the parents. However, the phenotype is the one that decides a living thing will look like to what.

The number of genotypes in an individual can be more than one. This feature provides an implicit memory scheme to the algorithm. Because all the genetic operators (crossover – mutation) are executed on the genotypes and the fitness calculations are executed on the phenotype of the individual. In this way, genotypes act like a memory. On the other hand, each individual has only one phenotype and its fitness value is calculated using that phenotype. That is, no matter how many genotypes an individual has, only one phenotype determines its fitness value. The structure of an individual is illustrated in Figure 1.

Second, BMGA uses a well-known Bayesian

Optimization Algorithm (BOA)[21] in it, in order to exploit the interactions between the variables. The BOA starts with a randomly generated population and creates a Bayesian Network (BN) by using this population. Since there is no prior information, BOA uses a greedy algorithm to form different BNs and then measure their quality using a special metric. After the algorithm finds the most suitable BN, then samples new individuals using that final BN. This procedure continues until a termination condition is met.

	0	1	2	3	4	5
genotype #1	1	1	0	1	1	0
genotype #2	0	0	0	1	0	0
genotype						
genotype #k	1	1	0	1	0	0
Prob. Vector	0.5	0.3	0.7	0.1	0.8	0.9
phenotype	?	?	0	1	?	0

Figure 1 Illustration of an individual

In BMGA, the BOA is used in order to form a probability vector to determine the phenotypes of the individuals in the population. This works like this: First, the GA part of the BMGA randomly generates a population. In this generating part, only the genotypes of the individuals are generated. For instance, if an individual has four genotypes, and the size of the population is 100, then 400 genotypes are generated. Next, a probability vector is randomly formed to determine the phenotypes of the individuals for the first iteration. The probability vector is formed via genotypes of the best k% individuals of the population by calculating the probability of being 1 for each gene using Equation 1. After the probability of being 1 is calculated the Equation 1, for each gene in the phenotype, a random real number is generated between 0 and 1. This generated value is then compared to the corresponding value in the probability vector to determine the value of the phenotype. Assuming that number of genotypes is g, this comparison is held by using Equation 2 and Equation 3.

$$ProbVector_{i} = \frac{number of ones for the ith gene}{size of the given population} \quad (1)$$

$$val = (rand() < ProbVector_i)? 1:0$$
(2)

$$pheno_{i} = \begin{cases} 0, & \sum^{g} geno_{i} = 0\\ 1, & \sum^{g} geno_{i} = g\\ val, & other \end{cases}$$
(3)

Once the phenotypes are determined, the fitness values of all individuals are calculated. After this first iteration, for the latter iterations, a new BN is constructed by using the same rule, and then a second population is sampled via BN. This second population is named auxiliary population, in short, "aux-pop". The aux-pop has only one chromosome like in the simple GA. For the latter iterations, the probability vector is formed by the aux-pop, again, using Equation 1.

After explaining the key points of the BMGA, now we can explain it in general. The general pseudocode of the BMGA is given in Algorithm 1. After the BOA parts of the BMGA are executed, next, the GA part of the BMGA starts to run.

The standard tournament selection (size of n) is applied in the BMGA's selection phase: Randomly chosen n solution candidates are compared and the fittest one is passed on to the next generation. This operation is performed until the size of the next generation is satisfied. Without recalculating its fitness value, the elite solution candidate from the last iteration is passed on to the current generation (elitism).

The uniform crossover approach is employed for crossover operation. It begins by randomly generating a mask vector in binary and then selecting two individuals, say i_1 , and i_2 , at each iteration. For each variable j (genes in chromosome), if the mask vector's corresponding value is 1, i_1 's 1st genotype's j^{th} variable is swapped with i_2 's 2nd genotype's j^{th} variable with a probability of p_c . This procedure is performed on each pair of genotypes in each solution candidate separately.

The simple bitwise mutation is employed as a mutation tool. The genotypes of each solution candidate are inverted with a probability of pm in the bitwise mutation process.

3. Preparing Test Environment

3.1. Multidimensional Knapsack Problem

MKP is a well-known benchmark problem for optimization algorithms. The goal of the problem finding a subset of given a number of items that obtain the optimal profit value while satisfying the given constraints. In this problem, there are n items, m resources, and $n \times m$ constraints. The formulation of the problem is given in Equation 4 and Equation 5.

$$\max \qquad \sum_{j=1}^{n} p_j \times x_j \qquad (4)$$

subject to $\sum_{j=1}^{n} r_{ij} \times x_j \le c_i, \ \forall_i \in \{1, 2, \dots, m\}$ (5)

where *n* is the number of items, *m* is the number of resources (knapsacks), x_j denotes the whether the item *j* is collected, p_j is the profit of the item *j*, r_{ij} is the consumption of the *j*th item at the *i*th resource and finally c_i is the capacity of the *i*th resource.

In order to test BMGA for MKPs, the first five problems are taken from the six different datasets which are provided on the well-known OR-LIB[22]. There are 30 different problems in that dataset and each has a different problem size. All problems have a tightness ratio of 0.25, however, their number of items and number of resources is changing. Also, they are encoded as "cbX-Y" in the result table, which means "Chu-Beasley, dataset X, problem Y". All the necessary information is given in Table 2.

In this work, all 30 datasets are solved by BMGA. Then, the results are compared to the results obtained from [16]

3.2. Experimental Studies

For the experimental tests, first, some parameters of BMGA are set. These settings can be seen in Table 1.

Table 1 Parameter settings						
Parameter	Value					
number of generations	1000					
Size of the population	100					
Probability of crossover	1.0					
Probability of mutation	0.03					
Tournament size	4					
number of genotypes	4					
Population rate to form BN	0.1					

Note that, each value in Table 1 is obtained by conducting a number of preliminary sensitivity tests[15].

For each test, with the same set of seeds, 30 independent runs were performed. For each run, best-of-generation (BOG) is saved and the overall performance is calculated as shown in Equation 6.

$$\bar{F}_{BOG} = \frac{1}{G} \sum_{i=1}^{G} \left(\frac{1}{N} \sum_{j=1}^{N} F_{BOG_{ij}} \right) \tag{6}$$

where *G* is the number of generations, *N* is the number of runs, $F_{BOG_{ij}}$ is the BOG of the *j*th run's *i*th generation. Last, \overline{F}_{BOG} is the overall offline performance.

4. Results

After setting the parameters in Table 1, the BMGA is tested for 30 different MKP datasets. For each

problem, 30 independent runs are executed. The results can be seen in Table 2. Also, the charts of the first problems of each group can be seen in Figure 2.

In the table, "Optimum" is the best-known optimum value so far provided in [22], "Best" is the best solution found during each test, and "Mean" is the average performance for the particular problem. In the "Prob. size" column, the size of the problem is given. For example, "5x100-0.25" means that the problem has 5 resources, 100 items, and a tightness ratio of 0.25.

Results show that BMGA is capable to solve MKP, is competitive, and obtains better solutions in terms of #Mean and #Best for most of the problem instances because of its Bayesian probability vector. More clearly, both in terms of #Best and #Mean, BMGA has got better results in 20 of 30 problems.

Although BMGA performs better than the other algorithms, for the test instances with 500 items, it performs slightly worse than the other test instances. That is because having more items means a bigger BN. And since BOA's BN is constructed with a 1-incoming edge rule, it is getting hard to exploit relations between the variables.

5. Conclusion

In this paper, the well-known optimization problem, the MKP, is solved by a recently proposed algorithm, the BMGA.

BMGA combines the powers of EDA and MS. By saying EDA, we mean BOA and by saying MS, we mean GA. While the MS part is responsible for the optimization process, the EDA part is responsible for exploiting relations between variables. Since it is a recently proposed algorithm, it has become mandatory to solve well-known optimization benchmark problems such as MKP.

For testing, BMGA is used to solve the most popular optimization dataset library's MKP instances. Then, the results are compared to the most recent paper. To get a fair comparison, the same number of fitness value calculations were done. The results showed that BMGA outperforms even the latest proposed algorithms.

This work tells us that exploiting the relations between the problem variables is important and useful while solving global optimization problems.

Prob. size	Prob.	Optimum	Profit	BMGA	MS	SLMS	MMVO	BGSA	BHTPSO	BHTPSO-QI
	cb1-1	24381	Best	24311	24253	24231	24192	24152	24169	24301
		27301	Mean	24072	24004	24015	24050	23835	23822	23821
	cb1-2	24274	Best	24274	24258	24274	24274	23986	24109	23944
			Mean	24225	23934	24145	24274	23536	23657	23688
5x100-0.25	ch1-3	23551	Best	23247	23538	23538	23538	23386	23435	23418
54100 0.25	001 5	20001	Mean	23175	23272	23440	23520	23041	23072	23073
	ch1-4	23534	Best	23330	23256	23330	23288	23172	23253	23192
	001 4		Mean	23289	23024	23156	23120	22863	22928	22923
	ch1-5	23001	Best	23952	23845	23947	23947	23755	23815	23774
	001 5	23771	Mean	23901	23567	23800	23900	23459	23473	23527
	cb2-1	59312	Best	59203	58084	59107	58473	57565	57814	57800
			Mean	58980	57369	58736	58240	56554	56874	56685
	ch2-2	61472	Best	61227	60248	61280	60692	60057	59982	59767
	002 2	01472	Mean	61185	59386	61041	60390	58613	58588	58680
5x250-0.25	ch2-3	62130	Best	61831	61212	61787	61702	59936	60630	60524
58250 0.25	002 0	02150	Mean	61684	59922	61476	61330	58975	59234	59186
	ch2-4	59463	Best	59167	58386	59101	58441	57970	57736	57884
	002 4	57405	Mean	58777	57752	58787	58300	56744	56773	56584
	ch2-5	58951	Best	58753	57755	58485	58082	56959	57378	57550
	002 3	56751	Mean	58566	56929	58097	58300	55961	56129	56361
	ch3-1	120148	Best	119992	116296	119914	119978	111206	114493	114438
	005-1	120140	Mean	119921	115444	119625	119900	108930	111017	111469
	ch3-2	117879	Best	116722	113732	117362	115634	108522	112821	112147
	005-2	117072	Mean	115888	112257	116858	115400	106631	109276	109247
5x500-0.25	ch3-3	121131	Best	117859	117666	120888	119156	111271	114774	116099
5x500-0.25	005-5	121151	Mean	117083	116367	120711	118900	109430	112035	112001
	ah2 1	120804	Best	120501	116454	120030	119124	111283	115828	114327
	003-4	120004	Mean	119662	115396	119644	118900	109062	112200	111671
	ah 2 5	122210	Best	119059	117900	121907	121141	112391	115889	117242
	003-5	122319	Mean	118284	116767	121512	120800	110564	112253	113364
	cb4-1	23064	Best	22917	22753	22835	22805	22836	22905	22876
	04-1	23004	Mean	22694	22459	22604	22700	22334	22425	22449
	ch4-2	22801	Best	22836	22611	22650	22630	22441	22573	22408
	04-2	22001	Mean	22541	22255	22432	22480	21991	22047	22017
10x100_0 25	ch/ 3	22131	Best	22012	21886	21962	22131	21849	21797	21949
10x100-0.25	04-3	22131	Mean	21662	21466	21632	21720	21313	21342	21461
	ch4-4	22772	Best	22420	22319	22463	22347	22325	22418	22376
	04-4	22112	Mean	22391	21992	22233	22160	21961	22037	22029
	ah 1 5	22751	Best	22312	22440	22619	22417	22168	22215	22254
	004-5	22751	Mean	22182	22132	22279	22290	21840	21822	21903
10x250-0.25	cb5-1 59187 Best 58820 57 Mean 58812 56	57757	58725	58476	56928	57530	57036			
		57107	Mean	58812	56708	58148	58310	55759	55854	55960
	ch5 2	58781	Best	58339	57363	58321	57937	56337	56568	56490
	005-2	J-2 J0/01	Mean	58267	56793	58074	57790	55455	55443	55708
	ch5 2	58007	Best	57804	56690	57764	57062	55573	56426	55982
	005-5	50077	Mean	57626	56024	57372	56960	54638	54793	54727
	ch5 /	61000	Best	60597	59930	60597	60326	58595	59030	59077
	005-4	01000	Mean	60460	58934	60194	60030	57766	58057	57721
	ah 5 5	5 58092	Best	57567	56863	57233	56276	56186	56217	56204
	005-5		Mean	57559	56066	56961	56060	54850	54941	54872
	-h (1	117821	Best	117371	113362	117287	-	108487	110996	111669
	000-1		Mean	116882	112541	116830	_	105760	107698	108367
	cb6-2	119249	Best	118730	115022	118737	_	109569	114262	113001
			Mean	118250	114250	118385	_	106775	108648	109197
	1 < 2	119215	Best	118905	115419	118488	_	109705	113987	112419
10x500-0.25	cb6-3		Mean	118402	114372	118003	_	106853	108576	109004
	cb6-4	1100	Best	117354	115038	118116	_	108628	112476	112198
		118829	Mean	116593	113444	117714	_	105679	107692	107796
		11 (720)	Best	115334	112971	116530	_	106972	109567	109287
	cb6-5	116530	Mean	114447	111707	115301	_	104509	106217	106212

Table 2 Results for the test sets



Figure 2 Results for the first dataset of each group of problem size

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