



Monopole Effect on Isotopes in Sn and Pb Regions

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Abstract. When going toward in description of neutron-rich nuclei, the monopole field plays a determining role in the properties of the quasiparticle states and their interactions. Detailed theoretical in shell model calculations of the characteristics of the neutron-rich isotopes in ^{132}Sn and ^{208}Pb regions are performed using the monopole hamiltonien effect. Some modification mentioned in literatures of effectives interactions are introduced for evaluated the effective single-particle energies. Their effect show a successful and consistent description of excitation energies spectra of these nuclei.

Keywords: Nuclear Structure, Monopole Hamiltonian, Odd-Odd Nuclei, Three body Effects, Similarity, 21.60.Cs; 27.60.+j; 21.30.Fe

1. INTRODUCTION

The structures of the odd-odd nuclides that are adjacent to double-magic nuclides provide the best opportunities to develop and test two-body matrix elements (TBME) for the proton-neutron ($\pi \nu$) interaction [1]. For a long time, shell model calculations using two-body realistic interactions derived from the free nucleon-nucleon force usually fail to reproduce some nuclear properties [2]. This problem can be solved by the consideration of the monopole interaction [3]. The three-body forces have been taken into account to reproduce the shell gaps for neutron rich oxygen and calcium isotopes. Zuker had shown earlier [3,4] that a very simple three body monopole term can solve practically all the spectroscopic problems in the p , sd , and fp shells. Those were earlier assumed to need drastic revisions of the realistic two-body potentials. Recently, Otsuka et al. [5] have also shown that three-body forces are necessary to explain drip line for oxygen isotopes. Therefore, S. Sarkar have incorporated [6] a simple three-body monopole term in CWG as prescribed by Zuker in Ref. [3,4]. The study of neutron-rich nuclei around the magic nuclei provide important information of nuclear structure, particularly the nuclei with few valence nucleons around ^{132}Sn and ^{208}Pb nuclei have a very interesting similar structure. Indeed, Blomqvist [7] discovered that there was some similarity between nuclear spectroscopic properties of the nuclei of these regions. The single particle orbits above and below the shell gap in the two cases are similarly ordered. Every single particle orbit in the ^{132}Sn region has its counterpart in the ^{208}Pb region, with same radial quantum numbers but one unit larger in angular momentum l and j values. As a consequence, effective interactions in the ^{132}Sn region can be estimated from the corresponding well studied effective interactions constructed for nuclei in ^{208}Pb region. This has opened the opportunity to explore the properties of simple states of nuclei of ^{132}Sn region under conditions which are comparable but not identical with those of the ^{208}Pb region. The experimental difficulties around ^{132}Sn relate to the circumstance that these nuclei are very far from the line of β -stability unlike those of ^{208}Pb region. Many works were to study of Sn and Pb regions. A. Korgul and J. Shergur [8, 1] reported a study of the low-spin level structure of ^{134}Sb populated in the β^- decay of ^{134}Sn , in addition, ^{136}Sb had been first observed as a β - n delayed precursor produced in thermal neutron induced fission of ^{235}U , and it has been produced in the projectile fission of ^{138}U at the relativistic energy of 750 MeV/u on ^9Be target by M. N.

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Mineva et G.S. Simpson [9, 10]. Very recently experiments were carried out at the RIKEN Radioactive Isotope Beam Factory (RIBF) [11, 12] to study the neutron rich isotopes of Sn. F. Benrachi and N. Benmicha had performed a similarity between nuclei with two and four valence nucleons in $A = 208$, 132 and 68 Regions [13-15].

The aim of the present study is devoted to determine excitation energies for the odd-odd nuclei with two, four and six valence nucleons near the two very important regions around ^{132}Sn and ^{208}Pb . Basing on three body effects, we carry out some modifications on the *kh5082* [16] and *khpe* [17] original interactions and new interactions named *mkh* and *ekhpe* are introduced.

2. THE MONOPOLE EFFECT

The Hamiltonian can be separated into an unperturbed and a residual parts, $H=H_0+H_r$, where H_0 is one body (1b) single particle field [18]. However, H contains two and three-body (2b and 3b) components, which makes their separation difficult mathematically. Therefore, E. Caurier et al. [18,19], advance another separation to solve this problem.

They propose the following relation :

$$H = H_m + H_M \quad (1)$$

Where H_M and H_m denote respectively the multipole and the monopole Hamiltonians. This later involves single particle energies ε_s and all quadratic and cubic (2b and 3b) forms in the scalar products of fermions operators $a_{rx}^+ \otimes a_{ry}$ [3, 4, 18].

$$H_m = \sum_s \varepsilon_s n_s + \sum_{s \leq t} (a_{st} n_{st} + b_{st} T_{st}) \quad (2)$$

$T_{s/t}$ and $n_{s/t}$ are, respectively, isospin and number operators for s/t orbitals.

The H_m^d diagonal part of this Hamiltonian reproduces the average of *TBME*'s at fixed number of particles and isospin in each orbit (JT representation) [18]. It can be expressed as:

$$H_{mJT}^d = k^d + \sum_{s \leq t} (a_{st} n_{st} + b_{st} T_{st}) + V_m^{d3} \quad (3)$$

The *2b* part easily extended to include the *3b* term V_m^{d3} . Note that a *3b* potential will produce both 1b and 2b terms [3].

These components are given by an angular average over all possible orientations of the two nucleons in s and t orbitals [20]:

$$V_{st}^T = \frac{\sum_{stst} V_{stst}^T (2J+1) (1 - (-1)^{J+T}) \delta_{st}}{\sum_{st} (2J+1) (1 - (-1)^{J+T}) \delta_{st}} \quad (4)$$

$$a_{st} = \frac{1}{4} (3V_{st}^1 + V_{st}^0), \quad b_{st} = \frac{1}{4} (V_{st}^1 - V_{st}^0) \quad (5)$$

$$n_{st} = \frac{1}{1 + \delta_{st}} n_t (n_s - \delta_{st}), \quad T_{st} = \frac{1}{1 + \delta_{st}} \left(T_t T_s - \frac{3}{4} n_{st} \delta_{st} \right) \quad (6)$$

Where the sum over quantum numbers J can be restricted by antisymmetry, and V_{stst}^T stands for the two-body matrix elements [5]. The single particle energy (*SPE*) of the orbital J is effectively shifted by V_{st}^T multiplied by the occupation number of the orbital J . This leads to the change in the *SPE* and determines

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shell structure and the location of the drip-line [21]. The *SPE* and the three body part will transform the realistic interaction (*R*) centroids into [3]:

$$V_{st}^T(\mathbf{R1}) = V_{st}^T(\mathbf{R}) + \chi_{st}'^T \quad (7)$$

$\chi_{st}'^T$ is a corrective term.

The monopole Hamiltonian represents a spherical mean field as extracted from the interacting shell model. The single-particle states in this average potential are called effective single-particle energies (ESPE's). These can be defined as a nucleon separation energy for an unoccupied orbital.

$$ESPE_s = \varepsilon_s + \sum_t (2J_t + 1) V_{st}^T \quad (8)$$

3. RESULTS AND DISCUSSION

In our shell-model calculations, the valence protons and neutrons occupy *jj56pn* and *jj67pn* model spaces [22] for ^{132}Sn and ^{208}Pb neighbors respectively. The microscopic calculations are carried out by means of Oxbash code. The proton and neutron *SPE* are taken from Ref [23, 24]. Basing on three body effects [2], we perform some modifications on the *kh5082*[16] and *khpe* [17] original interactions. The *TBMEs* are modified taking in consideration monopole effect for odd-odd nuclei in the ^{132}Sn and ^{208}Pb regions. These modifications are based on the reduction of a factor 407 keV for elements of the original *kh5082* interaction [16] $\langle gf|V|gf \rangle_{J=0^- \text{ to } 7^-}^{T=1}$. These elements have also been modified using the renormalization factor 0.74. Here *g* and *f* refer to $\pi g_{7/2}$ and $\nu p_{7/2}$ respectively. Noted that the corrected factor is taken from Ref. [2]. These modifications are done for Sn region. In the case of ^{208}Pb region, the elements of the original *khpe* interaction $\langle gh|V|gh \rangle_{J=0^- \text{ to } 9^-}^{T=1}$ are reduced by 250 keV.

We have replaced the single particle energies *SPEs* by the effective ones *ESPEs*. These latter are calculated using the *TBMEs* of the original interaction *khpe*. In which, the used proton and neutron *ESPEs* are evaluated using the relation (8). New interactions named *mkh* and *ekhpe* are constructed.

Fig. 1 shows the neutron and proton single particle energies as a function of the neutron and proton numbers in *Sn* and *Pb* regions. The neutron effective energies in N=83 isotones are shown as a function of the proton valence number varying from 0 to 8. The energy gap between the neutron single particle shell $\nu(2f_{7/2})$ and $\nu(2p_{3/2})$ is 718 keV for ^{133}Sn . The gap corresponding between *ESPEs* decreases to 229 keV for $Z = 58$. For the proton effective single particle energies ($Z=51$), the energy gap between two successive orbital remains below 2 MeV for all states. In the case of neutron effective single particle energies for *khpe* interaction (N=127), we see that the energy differences between the *ESPEs* for neighboring shells are very small in the order of 300 keV, with the exception of those between $1j_{15/2}$ and $1i_{11/2}$, and between $2g_{5/2}$ and $3d_{3/2}$ states, where they became the lowest for $Z > 86$. This difference becomes larger in the order of 1.5 MeV between $3d_{5/2}$ and $1i_{11/2}$ for $Z > 88$ (Fig 1). But in the case of proton effective single particle ($Z=83$), the energy gap reaches a maximum of 2 MeV between $1i_{13/2}$ and $1h_{9/2}$ states.

The comparison between the experimental and the calculated spectra of nuclei on the two regions, can be used to extract the existent similarities between the isotopes of these regions.

For this reason, we have calculated these energies for the nuclei and their counterparts: (^{134}Sb , ^{210}Bi), (^{136}Sb , ^{212}Bi), (^{136}I , ^{212}At) and (^{138}Cs , ^{214}Fr). The Fig. 2 shows the obtained results spectra using *mkh* and *ekhpe* interactions in comparison with the experimental ones. This points out a clear evidence of the striking similarity of the multiplets in the ^{134}Sb arising from the $\pi g_{7/2} \nu f_{7/2}$ configuration together with

those in ^{210}Bi arising from $\pi h_{9/2} \nu g_{9/2}$. We note that the mkh and $ekhpe$ interactions lead to reproduce the energetic sequences for ^{134}Sb and ^{210}Bi and the results are closer to the experiment.

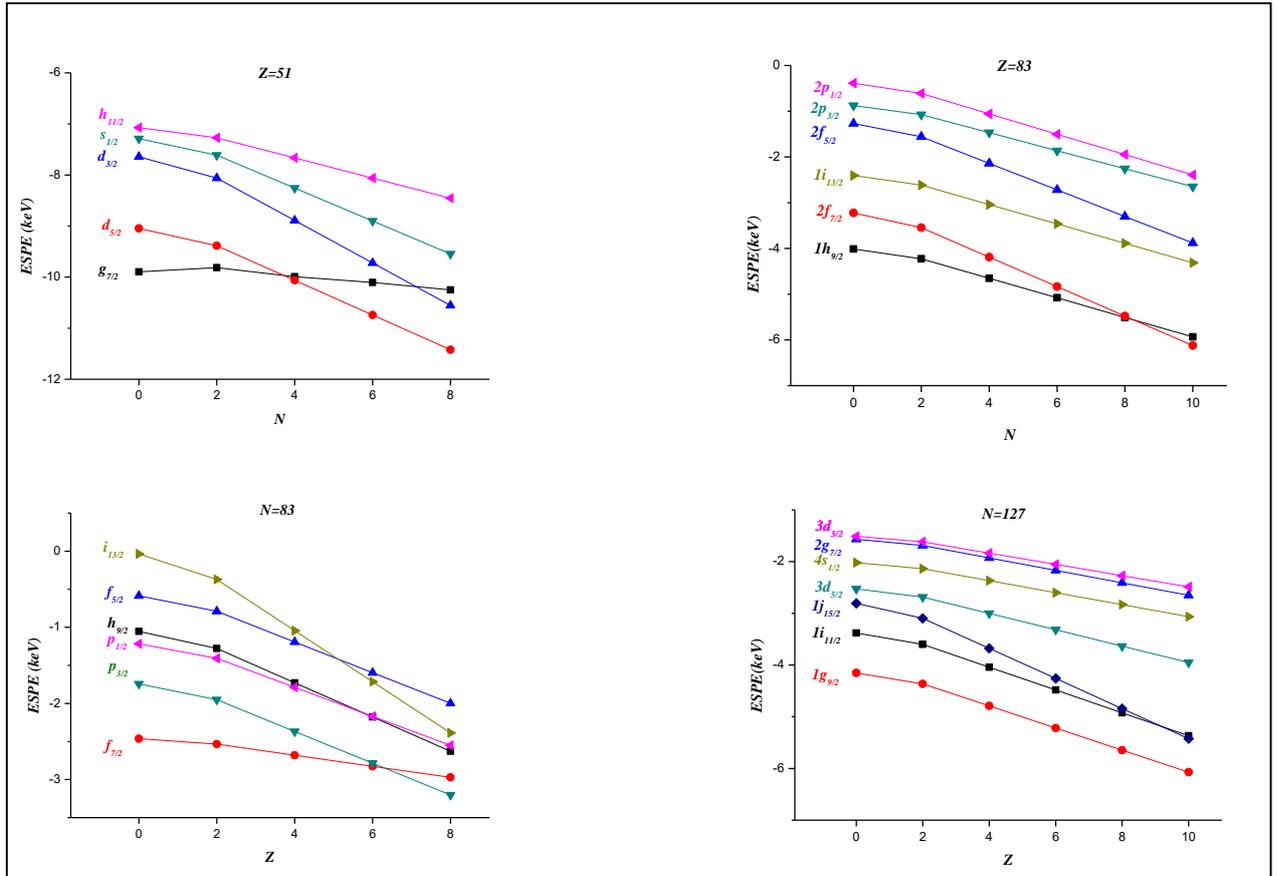


Figure.1: Neutron and proton effective single particle energies as a function of the neutron numbers in the Pb and Sn regions.

The calculated energies of ^{136}I for the lowest states with $J^\pi = 0^-$ to 7^- , are dominated by the $(\pi g_{7/2})^3 \nu f_{7/2}$ configuration, and the energies of the $J^\pi = 0^-$ to 9^- states in ^{212}At , arising from the $(\pi h_{9/2})^3 \nu g_{9/2}$ configuration. Also the energies of the corresponding states in ^{136}Sb and ^{212}Bi are dominated by the $\pi g_{7/2} (\nu f_{7/2})^3$ and $\pi h_{9/2} (\nu g_{9/2})^3$ configurations, respectively. We note that the interaction mkh generally reproduced experimental spectra for ^{136}Sb and ^{136}I nuclei therefore in the case of ^{212}Bi and ^{212}At nuclei with $ekhpe$ interaction, there are small differences between the calculated results and the experimental data available for all states.

For ^{138}Cs and ^{214}Fr nuclei, the calculated excited states are so different from the experimental data. Fig. 3 shows the obtained results spectra using mkh interaction for ^{138}Sb and ^{138}I nuclei. There are no experimental data to allow the comparison with theoretical calculations.

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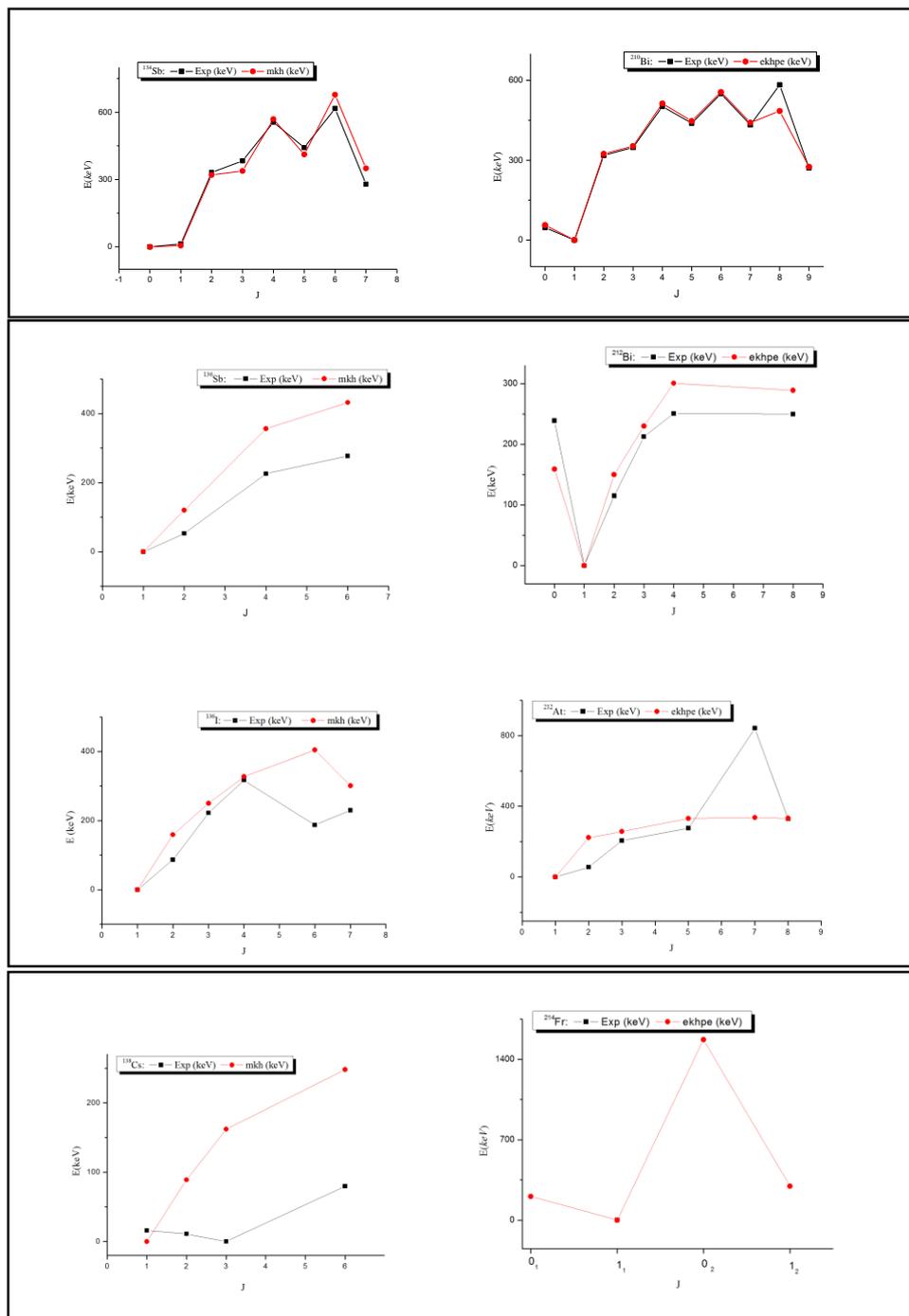


Figure 2. Experimental and calculated energies spectra for the odd-odd nuclei with A = 134,136, 138, 210,212 and 214.

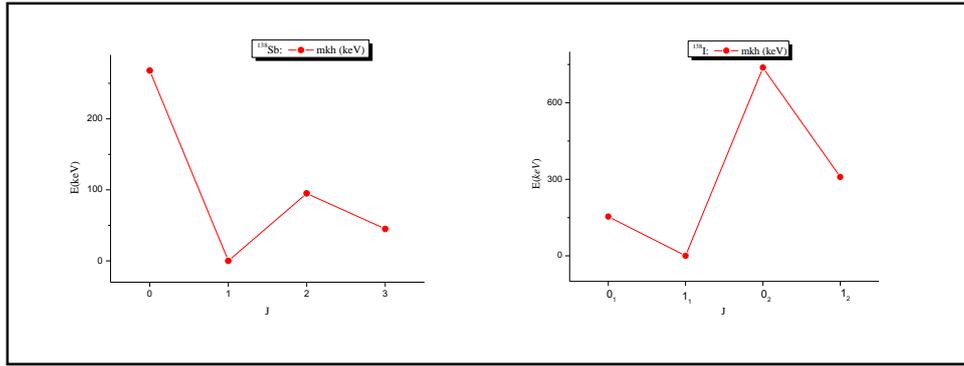


Figure 3: calculated energies spectra for the odd-odd ^{138}Sb and ^{138}I nuclei.

CONCLUSION

In this work, we focused on the study of the monopole effect on the nuclear structure of nuclei far from beta stability. More generally, we have explored in a semi-quantitative manner the nuclear force which lead to modification of shell structure in the ^{132}Sn and ^{208}Pb regions. The properties were derived from two-body effective interactions including monopole effect in the SM framework, by means of *Oxbash* nuclear structure code using the *kh5082* and *khpe* original interactions of the code and carrying out some modifications based on the proton-neutron monopole interaction to get *mkh* and *ekhpe* new interactions. Our calculations, for ^{134}Sb and ^{210}Bi , reproduced the energetic sequences and gave closer energy values to the experimental data for these nuclei and for ^{136}Sb , ^{136}I , ^{212}Bi and ^{212}At nuclei. There are small differences between the calculated results and the experimental data available for all states. However, the calculated excited states for ^{138}Cs are so different from the experimental data. For ^{138}Sb and ^{138}I nuclei, there are no experimental data to allow the comparison with theoretical calculations.

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