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Theoretical Description of Energy Spectra and QUADRUPOLE Transition Probabilities of ^{192} Pt

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Abstract. In this paper, we have studied the energy spectra and quadrupole transition probabilities of 192Pt nucleus with emphasis on shape coexistence. A transitional Interacting Boson Model Hamiltonian which are based on affine SU(1,1) lie algebra is used to consider the coexistence of spherical and axial symmetry shapes in this nuclei. Parameter free (up to overall scale factors) predictions for theoretical predictions are found to be in good agreement with experimental counterparts. Our results offer a combination of spherical and deformed shapes for this nucleus which is expected to be an excellent example for SO(6)dynamical limit.

Keywords: Quadrupole transition; shape coexistence; boson model; affine lie algebra; deformed shapes

1. INTRODUCTION

The interacting boson model (IBM) which describes the nuclear structure of even–even nuclei the IBM Hamiltonian was written from the beginning in second quantization form in terms of the generators of the unitary Lei algebra U(6) The model assumes that low-lying collective excitations of the nucleus can be described in terms of the number N of s and d bosons. The bosons correspond to pairs of nucleons in valance shell, coupled to angular momentum (j=0) s boson (j=2) d boson; N is constant for a given nucleus and equal to half its number of valance nucleons. Lei algebra U(6) subtended by s and d bosons . The model present three special limits that can be solved easily these three limits are $U(5)$, $SU(3)$ and $O(6)$ dynamical symmetry appropriate for an harmonic vibrator , axial deformed rotor and γ- unstable deformed rotor .When the numbers of protons (or neutrons) are modified, the energy levels and electromagnetic transition rates of atomic nuclei change too and suggest a transition from one kind of the collective behavior to another [1-3]. The quantum shape phase transitions have been studied 25 years ago with using the classical limits of the Interacting Boson Model (IBM) [4-10] These descriptions point out that there is a first order shape phase transition between $U(5)$ and $SU(3)$limits and a second order shape phase transition between $U(5)$ and $O(6)$ limits. The analytic description of nuclear structure at the critical point of phase transitions has attracted extensive interest in the recent decades. One has to employ some complicated numerical methods to diagonalize the transitional Hamiltonian in these situations but Pan *et al* in Refs.[11-12] have been proposed a new solution which was based on affine $SU(1,1)$ algebraic technique and explores the properties of nuclei have classified in the $U(5) \leftrightarrow SO(6)$ transitional region of IBM. ^{192} Pt isotope are expected to lie in this transitional region. the ^{192} Pt isotope were good examples of the $U(6)$ nuclei [13-36]. However, during the last few years, new experimental data and calculations have led to a modified picture on these nuclei. By using the collective models in describing the structure of^{192} Pt isotope [13]. These mean ^{192}Pt isotope appear to evolve from the $O(6)$ to $U(5)$-like structure in IBM classification. On the other hand, this isotope can be described via *O(6)* limit where one has to use

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the *U(5)* limit predictions for the intruder states. These levels [15]. All these new experiments and theoretical calculations have provided new insights for these nuclei which is helpful to understand their structures [17-25].

In this study, we have focused on the ^{192}Pt isotope with emphasis on the energy levels and quadrupole transition probabilities. We have used the transitional Hamiltonian [35] to consider the evolution of these isotope between spherical and gamma unstable shapes. Different energy levels and quadrupole transition probabilities are determined in the IBM-1 frameworks and compared with experimental counterparts [15-35]. Recently great analyses has been performed to describe them. Iachello in Refs.[1-2] have established a new set of dynamical symmetries, i.e. *E*(5) and *X*(5), for nuclei which are located at the critical point of transitional regions. The *E*(5) symmetry describes a second order phase transition which corresponds to the transitional states in the region from *U*(5) to *O*(6) symmetries in the IBM. Different analyses which have carried in the investigation of this transitional region [13-35]. An algebraic solution has been proposed by Pan *et al* [11-12] which was based on the affine $SU(1,1)$ Lie algebra to exhibits the properties of nuclei which are located in the *U*(5)↔*SO*(6) transitional region.

2. THEORETICAL FRAMEWORK

2.1. Transitional Hamiltonian based on affine $SU(1,1)$ algebra

The $SU(1,1)$ Algebra has been described in detail in Refs.[11-12]. Here, we briefly outline the basic ansatz and summarize the results. The Lie algebra corresponds to the $SU(1,1)$ group is generated by S^{ν}, $\nu = 0$ and $\pm$, which satisfies the following commutation relations

$$[S^0, S^{\pm}] = \pm S^{\pm} \qquad , \qquad [S^+, S^-] = -2S^0 \tag{1}$$

The Casimir operator of $SU(1,1)$ group can be written as

$$\hat{C}_2 = S^0(S^0 - 1) - S^+S^- \qquad , \tag{2}$$

Representations of $SU(1,1)$ are determined by a single number κ, thus the representation of Hilbert space is spanned by orthonormal basis $|\kappa\mu\rangle$ where κ can be any positive number and $\mu = \kappa, \kappa+1, \ldots$. Therefore,

$$\hat{C}_2(SU(1,1))|\kappa\mu\rangle = \kappa(\kappa-1)|\kappa\mu\rangle \qquad , \qquad S^0|\kappa\mu\rangle = \mu|\kappa\mu\rangle \tag{3}$$

In IBM, the generators of $d-$ boson pairing algebra is created by

$$S^+(d) = \frac{1}{2}(d^{\dagger}.\, d^{\dagger}) \quad , \quad S^-(d) = \frac{1}{2}(\tilde{d}.\, \tilde{d}) \quad , \quad S^0(d) = \frac{1}{4}\sum_{\nu}(d_{\nu}^{\dagger}d_{\nu} + d_{\nu}d_{\nu}^{\dagger}) \tag{4}$$

Similarly, $s-$ boson pairing algebra forms another $SU^s(1,1)$ algebra which is generated by

$$S^+(s) = \frac{1}{2}s^{\dagger 2} \quad , \quad S^-(s) = \frac{1}{2}s^2 \quad , \quad S^0(s) = \frac{1}{4}(s^{\dagger}s + ss^{\dagger}) \tag{5}$$

On the other hand, the infinite dimensional $SU(1,1)$ algebra is generated by using of [11-12]

$$S_n^{\pm} = c_s^{2n+1} S^{\pm}(s) + c_d^{2n+1} S^{\pm}(d) \quad , \quad S_n^0 = c_s^{2n} S^0(s) + c_d^{2n} S^0(d) \qquad (6)$$

Where c_s and c_d are real parameters and n can be $0, \pm 1, \pm 2, \ldots$. These generators satisfy the commutation relations,

$$[S_m^0, S_n^{\pm}] = \pm S_{m+n}^{\pm} \quad , \quad [S_m^+, S_n^-] = -2S_{m+n+1}^0 \qquad (7)$$

Then, $\{S_m^{\mu}, \mu = 0, +, -; \pm 1, \pm 2, \ldots\}$ generates an affine Lie algebra $SU(1,1)$ without central extension. By employing the generators of $SU(1,1)$ Algebra, the following Hamiltonian is constructed for the transitional region between $U(5) \leftrightarrow SO(6)$ limits [11-12]

$$\hat{H} = g\, S_0^+ S_0^- + \varepsilon\, S_1^0 + \gamma\, \hat{C}_2(SO(5)) + \delta\, \hat{C}_2(SO(3)) \qquad (8)$$

g, ε, γ and δ are real parameters where $\hat{C}_2(SO(3))$ and $\hat{C}_2(SO(5))$ denote the Casimir operators of these groups. It can be seen that Hamiltonian (8) would be equivalent with $SO(6)$ Hamiltonian if $c_s = c_d$ and with $U(5)$ Hamiltonian when $c_s = 0$ & $c_d \neq 0$. Therefore, the $c_s \neq c_d \neq 0$ requirement just corresponds to the $U(5) \leftrightarrow SO(6)$ transitional region. In our calculation we take c_d (=1) constant value and c_s vary between 0 and c_d.

Eigenstates of Hamiltonian (8) can obtain with using the Fourier-Laurent expansion of eigenstates and $SU(1,1)$ generators in terms of unknown $c-$number parameters x_i with $i = 1, 2, \ldots, k$. It means, one can consider the eigenstates as [11-12]

$$\left|k; \nu_s \nu n_\Delta LM\right\rangle = \sum_{n_i \in Z} a_{n_1} a_{n_2} \ldots a_{n_k} x_1^{n_1} x_2^{n_2} \ldots x_k^{n_k} S_{n_1}^+ S_{n_2}^+ \ldots S_{n_k}^+ \left|lw\right\rangle \quad , \qquad (9)$$

Due to the analytical behavior of wave functions, it suffices to consider x_i near zero. With using the commutation relations between the generators of $SU(1,1)$ Algebra, i.e. Eq.(7), wave functions can be considered as:

$$\left|k; \nu_s \nu n_\Delta LM\right\rangle = N S_{x_1}^+ S_{x_2}^+ \ldots S_{x_k}^+ \left|lw\right\rangle \quad , \qquad (10)$$

where N is the normalization factor and

$$S_{x_i}^+ = \frac{c_s}{1 - c_s^2 x_i} S^+(s) + \frac{c_d}{1 - c_d^2 x_i} S^+(d) \quad , \qquad (11)$$

The c-numbers x_i are determined through the following set of equations

$$\frac{\epsilon}{x_i} = \frac{g c_s^2 (\nu_s + \frac{1}{2})}{1 - c_s^2 x_i} + \frac{g c_d^2 (\nu + \frac{5}{2})}{1 - c_d^2 x_i} - \sum_{i \neq j} \frac{2}{x_i - x_j} \qquad \text{for i=1,2,...,k} \qquad (12)$$

Eigenvalues of Hamiltonian (8), i.e. $E^{(k)}$, can be expressed as [11-12]

$$E^{(k)} = h^{(k)} + \gamma\nu(\nu+3) + \delta L(L+1) + \varepsilon\Lambda_1^0 \qquad , \qquad \Lambda_1^0 = \frac{1}{2}[c_s^2(\nu_s + \frac{1}{2}) + c_d^2(\nu + \frac{5}{2})] \tag{13}$$

Which

$$h^{(k)} = \sum_{i=1}^{k} \frac{\varepsilon}{x_i} \quad , \tag{14}$$

The quantum number *k*, is related to total boson number *N*, by

$$N = 2k + \nu_s + \nu$$

To obtain the numerical results for $E^{(k)}$, we have followed the prescriptions have introduced in Refs.[7-8], namely a set of non-linear Bethe-Ansatz equations (BAE) with $k-$ unknowns for $k-$ pair excitations must be solved. To this aim we have changed the variables as

$$\grave{o} = \frac{\varepsilon}{g} (g = 1 \ \ kev \ [11\text{-}12]) \qquad \mathrm{c} = \frac{c_s}{c_d} \le 1 \qquad \mathrm{y}_i = c_d^2 x_i$$

so, the new form of Eq.(12) would be

$$\frac{\grave{o}}{y_i} = \frac{c^2(\nu_s + \frac{1}{2})}{1 - c^2 y_i} + \frac{(\nu + \frac{5}{2})}{1 - y_i} - \sum_{i \neq j} \frac{2}{y_i - y_j} \qquad \text{for i=1,2,...,k} \tag{15}$$

We have solved Eq. (15) with definite values of c and ε for $i = 1$ to determine the roots of Beth-Ansatz equations (BAE) with specified values of ν_s and ν , similar to procedure which have done in Refs.[7-8]. Then, we have used "Find root" in the Maple13 to get all $y_j^{'}$s. We carry out this procedure with different values of c and ε to provide energy spectra (after inserting γ and δ) with minimum variation as compared to the experimental counterparts;

$$\sigma = \left(\frac{1}{N_{tot}} \sum_{i,\, tot} \left| E_{\exp}(i) - E_{cal}(i) \right|^2\right)^{1/2}$$

Which N_{tot} is the number of energy levels where are included in extraction processes. We have extracted the best set of Hamiltonian's parameters, i.e. γ and δ , via the available experimental data [27-29] for excitation energies of selected states, $0_1^+, 2_1^+, 4_1^+, 0_2^+, 2_2^+, 4_2^+$ and *etc*, e.g. 12 levels up to 2_4^+, or two neutron separation energies for nuclei which are considered in this study. In summary, we have extracted γ and δ externally from empirical evidences and other quantities of Hamiltonian, e.g. c and ε would determine through the minimization of s .

2.2. $B(E2)$ Transition

The reduced electric quadrupole transition probabilities, $B(E2)$, are considered as the observables which as well as quadrupole moment ratios within the low-lying state bands prepare more information about the nuclear structure. The *E2* transition operator must be a Hermitian tensor of rank two and consequently, number of bosons must be conserved. With these constraints, there are two operators possible in the lowest order, therefore the electric quadrupole transition operator employed in this study is defined as [7],

$$\hat{T}^{(E2)}_{\mu} = q_2\left[[\hat{d}^\dagger \times \tilde{s} + \hat{s}^\dagger \times \tilde{d}]^{(2)}_{\mu} + q_2' \, [\hat{d}^\dagger \times \tilde{d}]^{(2)}_{\mu}\right] \quad , \qquad (16)$$

Where q_2 is the effective quadrupole charge, q_2' is a dimensionless coefficient and $s^\dagger(d^\dagger)$ represent the creation operator of $s(d)$ boson. Reduced electric quadrupole transition rate between $I_i \to I_f$ states is given by [3]

$$B(E2; I_i \to I_f) = \frac{\left|\langle I_f \| T(E2) \| I_i \rangle\right|^2}{2I_i + 1} \quad , \qquad (17)$$

3. NUMERICAL RESULT

3.1. Energy levels

Investigations of experimental energy spectra which have been done in Refs.[13-34], suggest ^{192}Pt isotopes as the empirical evidences for $U(5) \leftrightarrow SO(6)$ transitional region. Consequently, the transitional Hamiltonians, Eq.(8) in IBM-1 framework, can be considered in the determination of energy spectra. There are 12 levels up to the 2_4^+ level for c_s=0.8 displayed in Figures 1 for IBM-1. The best fits for IBM-1 Hamiltonian's parameters, namely ε, δ and γ which are extracted from experimental data, by similar method has been explained in Refs.[15-18], we have determined the c_s values which all of them are presented in Table 1. These quantities described the best agreement between the calculated energy levels in this study and their experimental counterparts taken from Refs.[35], i.e. minimum values for σ .

Our results which suggest a combination of the vibrational and gamma unstable limits in ^{192}Pt isotope, our result which suggests a shape coexistence in the ^{192}Pt.

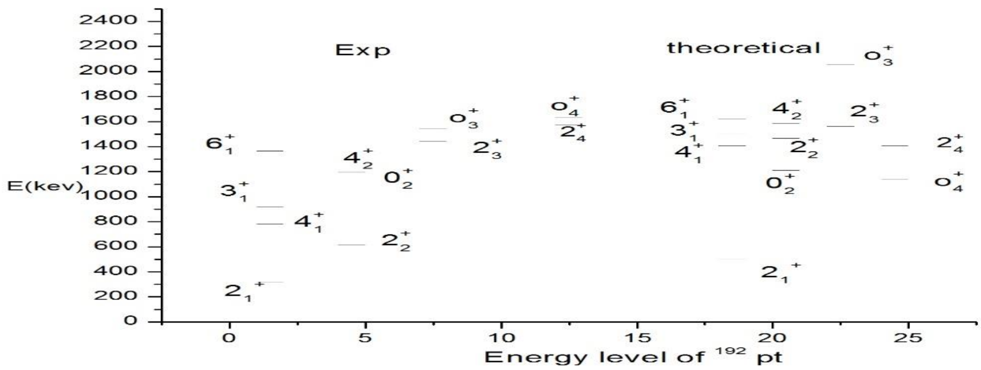

Figure 1. Energy spectra of ^{192}Pt nucleus which are determined by transitional Hamiltonian and the experimental values are taken from Refs.[8]for cs=0.8

Table 1. Parameters of IBM-1 Hamiltonian are showed for ^{192} Pt.

C_s	δ	γ	α	σ
0.4	−4.12	−44.12	800	326
0.5	−4.19	−45.50	800	328
0.7	−4.37	− 49.19	800	322
0.8	−4.49	−51.50	800	276

3.2.B(E2) Transition probabilities

Computation of electromagnetic transition is a sign of good test for nuclear model wave functions. To determine boson effective charges We have extracted these quantities from the empirical B(E2) values via Least square technique The parameters of Eqs. 16, B(E2) values have been presented in Tables 2 and Figure 2 for IBM-1

Table 2. Transition probability

Nucleus	Transition probability		
192pt	$\dfrac{B(E2;\ 2_2^+ \rightarrow 2_1^+)}{Exp. = 109.7\ ;\ The. = 74}$	$\dfrac{B(E2;\ 2_2^+ \rightarrow 0_1^+)}{Exp. = 0.54\ ;\ The. = 0.26}$	$\dfrac{B(E2;\ 3_1^+ \rightarrow 2_2^+)}{Exp. = 102.9\ ;\ The. = 62}$
	$\dfrac{B(E2;\ 3_1^+ \rightarrow 2_1^+)}{Exp. = 0.68\ ;\ The. = 0.5}$	$\dfrac{B(E2;\ 3_1^+ \rightarrow 4_1^+)}{Exp. = 38.9\ ;\ The. = 24}$	$\dfrac{B(E2;\ 6_1^+ \rightarrow 4_1^+)}{Exp. = 70.30\ ;\ The. = 87}$
	$\dfrac{B(E2;\ 4_1^+ \rightarrow 2_1^+)}{Exp. = 89.5\ ;\ The. = 79}$	$\dfrac{B(E2;\ 2_1^+ \rightarrow 0_1^+)}{Exp. = 57.1\ ;\ The. = 57}$	

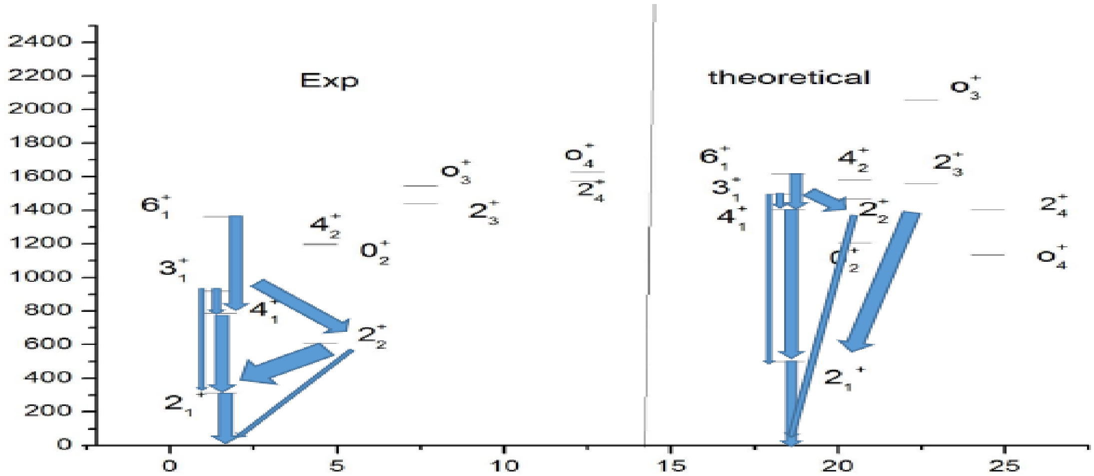

Figure 2. B(E2) transition probabilities (in w.u.)

4. CONCLUSIONS

A su(1,1) based transitional Hamiltonian is used to determine energy spectra and transitional rates of 192 Pt results suggest a combination of spherical shape together deformed one in the structure of nucleus . The control parameter of Hamiltonian has a mixing role which show the coexistence of these shapes

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