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Pointwise Quasi Hemi-Slant Submanifolds of Cosymplectic Manifolds

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Abstract: The object of this manuscript is to investigate related to the geometry of distributions on pointwise quasi hemi-slant submanifolds (abbr. PQHS) in cosymplectic manifolds. In this context, the preconditions for such distributions to be integrable, totally geodesic foliation, totally geodesic and mixed totally geodesic are obtained. In addition, we are going to present several examples to guarantee these new types of submanifolds in cosymplectic manifolds.

Kosimplektik Manifoldların Noktasal Yarı-Eğimli Alt Manifoldları

Anahtar Kelimeler
Noktasal yarı eğimli, kosimplektik manifold, Tamamen jeodezik yapraklanma

Öz: Bu makalenin amacı, kosimplektik manifoldlarda noktasal yarı-eğimli alt manifoldlar (kısaltılmış PQHS) üzerindeki dağılımların geometrisiyle ilgili araştırma yapmaktır. Bu bağlamda, bu tür dağılımların integrallenebilir olması, tamamen jeodezik yapraklanma, tamamen jeodezik ve karışık tamamen jeodezik olması için ön koşullar elde edilmektedir. Ek olarak, kosimplektik manifoldlarda bu yeni alt manifold tiplerini garanti etmek için birkaç örnek sunacağız.

1. INTRODUCTION

Differential geometry has been one of the most outstanding branches of mathematics and physics since the earliest times. Among the most outstanding topics in the field of differential geometry in recent years is the contact geometry. Contact geometry has a very important place in physical and other mathematical structure. Sophus Lie first mentioned contact structures in his work on partial differential equations [1]. In recent years, the geometry of contact Riemannian manifolds has received great attention. In contact geometry, there have been many classes of manifolds considered as odd-dimensional analogs of Kähler spaces, the most important ones being cosymplectic and Sasakian spaces. An odd-dimensional equivalent of a Kähler manifold can be presented by a cosymplectic manifold, locally a product of a Kähler manifold having a line or a circle [2]. An obvious instance of a cosymplectic manifold can be presented with the product of 1-dimensional manifold with (2n)-dimensional Kähler manifold.

On the other side, submanifold theory has got outstanding characteristics in Mathematical, Mechanics and Physics. In the recent twenty years, Kähler manifold applications are widely known (in particular, in the target spaces for non-linear σ -models having supersymmetry). Today, submanifolds theory has an important place in computer design, image processings, economic modelling. Submanifolds geometry concept has started with the concept of the extrinsic geometry of the surface and it is developed for ambient space with time. In this context, the submanifolds of a cosymplectic manifold have been studied by G. D. Ludden [3]. Later on, A. Cabras et al. [4] has given the proof of the fact that in a cosymplectic manifold there is not ant extrinsic sphere which is tangent to the structure vector fields. In 1990, Chen has put forward the notion of slant submanifold, which totally real submanifolds and generalizes holomorphic [5]. Then, the theory of submanifolds is investigated by many geometers like [6-14]. As a generalization of slant submanifolds; semi-slant submanifolds, hemi-slant submanifolds, bi-slant submanifolds, quasi bi-slant submanifolds, quasi hemi-slant submanifolds, pointwise quasi bi-slant submanifolds, PQHS submanifolds [15-29] and many others. In 2013, B. Şahin defined the concept of pointwise semi-slant submanifolds [30]. In 2014, K. S. Park has given the concept of pointwise almost h-semi- slant submanifolds and pointwise almost h-slant submanifolds in an almost quaternionic Hermitian manifold [31-32]. In 2020, Akyol et al. [33] initiated the study of quasi bi-slant submanifolds of an almost contact metric manifold by generalizing slant, semi-slant, hemi-slant and bi-slant submanifolds (See also: [34]). Motivated by all those work in the present article, we are going to investigate PQHS submanifolds of cosymplectic manifolds.

The layout of the manuscript can be given as follows: In the second section, the fundamental descriptions and formulae about cosymplectic manifolds and the geometry of submanifolds are given. The third section gives the definition of PQHS submanifolds of cosymplectic manifolds and we obtained some results for the next sections. In the fourth section, we deals with main theorems related to the geometry of distributions. Finally in the last section, we proved two examples of such submanifolds.

2. SOME BASIC CONCEPTS

This section presents a cosymplectic manifold definition and some fundamentals about submanifolds theory.

An almost contact structure (φ,ξ,η) on a $(2m+1)$-dimensional manifold N is defined by a (1,1) tensor field φ, a vector field ξ and a 1-form η satisfying the following conditions:

$$\varphi^2=-I+\eta\otimes\xi,\ \ \eta(\xi)=1,\ \ \eta o\varphi=0,\ \ \varphi\xi=0. \quad (1)$$

One can always find a Riemannian metric $<,>$ on an almost contact manifold N which satisfies the conditions given below

$$\begin{gathered}<\varphi U,\varphi V>=<U,V>-\eta(U)\eta(V)\\ \eta(U)=<U,\xi>,\end{gathered} \quad (2)$$

where U,V are vector fields on N.

It is said that an almost contact structure (φ,ξ,η) is normal when the almost complex structure J on the product manifold $N\times\mathbb{R}$ is given by

$$J(U,f\tfrac{d}{dt})=(\varphi U-f\xi,\eta(U)\tfrac{d}{dt}),$$

in which f is a C^∞-function on $N\times\mathbb{R}$ without torsion i.e., J is integrable. The condition for normality in terms of φ,ξ and η is $[\varphi,\varphi]+2d\eta\otimes\xi=0$ on N, in which $[\varphi,\varphi]$ is the Nijenhuis tensor of φ. Lastly, the fundamental two-form Φ is defined $\Phi(U,V)=<U,\varphi V>$.

If the almost contact structure $(\varphi,\xi,\eta,<,>)$ is normal and both Φ and η are closed, then this structure can be expressed as cosymplectic [3, 35-37]. From the point of the covariant derivative of φ, the cosymplectic condition can be specified by

$$(\nabla_U\varphi)V=0, \quad (3)$$

for every U,V tangent to N, in which ∇ stands for the Riemannian connection of the metric $<,>$ on N. Furthermore, for cosymplectic manifold

$$\nabla_U\xi=0. \quad (4)$$

Let N be a Riemannian manifold isometrically immersed in $\widetilde{N}$ and induced Riemannian metric on N is described by the $<,>$ throughout this manuscript. Let h and $\mathcal{A}$ denote second fundamental form and the shape operator, respectively, of immersion of N into $\widetilde{N}$. If ∇ is the induced Riemannian connection on N, then the Gauss and Weingarten formulae are presented by [5]

$$\widetilde{\nabla}_UV=\nabla_UV+h(U,V) \quad (5)$$

and

$$\widetilde{\nabla}_UV=-\mathcal{A}_VU+\nabla_U^{\perp}V, \quad (6)$$

for any $U,V\in\Gamma(TN)$, $V\in\Gamma(T^{\perp}N)$ and $\nabla^{\perp}$ stands for the connection on the normal bundle $T^{\perp}N$ of N.

If N is totally geodesic, then $h(U,V)=0$ for all $U,V\in\Gamma(TN)$.

At this point, one has the following description from [38]:

Definition 2.1 A submanifold N of an almost Hermitian manifold $\widetilde{N}$ is known pointwise slant if, for every point $p\in N$, the Wirtinger angle $\theta(U)$ is independent of the selection of nonzero vector $U\in T_p^*N$, where T_p^*N is the tangent space of nonzero vectors. Under these conditions, θ is known slant function of N.

Definition 2.2 A submanifold N is known (i) $(\mathfrak{D}_1,\mathfrak{D}_2)$-mixed totally geodesic if $h(Z,W)=0$, for any $Z\in\Gamma(\mathfrak{D}_1)$ and $W\in\Gamma(\mathfrak{D}_2)$ (ii) $\mathfrak{D}$-totally geodesic if it is $(\mathfrak{D},\mathfrak{D})$-mixed totally geodesic [34].

3. POINTWISE QUASI HEMI-SLANT SUBMANIFOLDS OF COSYMPLECTIC MANIFOLDS

In this section, we are going to present basic definitions and lemmas related to PQHS submanifolds of cosymplectic manifolds.

Definition 3.1 A submanifold N of cosymplectic manifolds $(\widetilde{N},\varphi,\xi,\eta,<,>)$ is known PQHS if there exist distributions $\mathfrak{D}$, $\mathfrak{D}_\theta$ and $\mathfrak{D}^{\perp}$ such that

(i) $TN=\mathfrak{D}\oplus\mathfrak{D}_\theta\oplus\mathfrak{D}^{\perp}\oplus<\xi>$.

(ii) The distribution $\mathfrak{D}$ is invariant, i.e. $\varphi\mathfrak{D}=\mathfrak{D}$.

(iii) For a vector field which is different from zero $U\in(\mathfrak{D}_\theta)_p$, $p\in N$, the angle θ between φU and $(\mathfrak{D}_\theta)_p$ is slant function and is independent of the choice of the point p and U in $(\mathfrak{D}_\theta)_p$.

(iv) The distribution $\mathfrak{D}^{\perp}$ is anti-invariant, i.e., $\varphi\mathfrak{D}^{\perp} \subseteq T^{\perp}N$.
The θ is known as a PQHS angle of N. A PQHS submanifold N is known proper if its pointwise-slant function satisfies $\theta \neq 0, \frac{\pi}{2}$, and θ is not constant on N.

If we represent by k_1, k_2 and k_3 the dimension of $\mathfrak{D}$, $\mathfrak{D}_\theta$ and $\mathfrak{D}^{\perp}$, respectively, thus with the usage of generalized PQHS submanifold definition, one can easily see the following particular cases;

(i) N is pointwise hemi-slant submanifold when $k_1 = 0$,

(ii) N is semi-invariant submanifold when $k_2 = 0$,

(iii) N is pointwise semi-slant submanifold when $k_3 = 0$.

Let N be a PQHS submanifold of a cosymplectic manifold $\tilde{N}$. Thus, for any $U \in \Gamma(TN)$, one has

$$U = PU + QU + RU + \eta(U)\xi, \tag{7}$$

in which P, Q and R stands for the projections on the distributions $\mathfrak{D}$, $\mathfrak{D}_\theta$ and $\mathfrak{D}^{\perp}$, respectively.

$$\varphi U = TU + FU, \tag{8}$$

where FU and TU are normal and tangential components on N, respectively. By using (7) and (8), we get immediately

$$\varphi U = TPU + FPU + TQU + FQU + TRU + FRU,$$

in which due to the fact that $\varphi\mathfrak{D} = \mathfrak{D}$, one has $FPU = 0$. Therefore, one gets

$$\varphi(TN) = \mathfrak{D} \oplus T\mathfrak{D}_\theta \oplus F\mathfrak{D}_\theta \oplus \varphi\mathfrak{D}^{\perp}$$

and

$$T^{\perp}N = F\mathfrak{D}_\theta \oplus \varphi\mathfrak{D}^{\perp} \oplus \mu,$$

in which μ stands for the orthogonal complement of $F\mathfrak{D}_\theta \oplus \varphi\mathfrak{D}^{\perp}$ in $T^{\perp}N$ and $\varphi\mu = \mu$. At the same time, for every $Z \in T^{\perp}N$, one has

$$\varphi Z = BZ + CZ, \tag{9}$$

in which $BZ \in \Gamma(\mathfrak{D}_\theta \oplus \mathfrak{D}^{\perp})$ and $CZ \in \Gamma(\mu)$.

When the condition (iii) given in Definition 3.1 is used together with (8) and (9), one obtains the followings:

$$T\mathfrak{D} = \mathfrak{D}, \quad T\mathfrak{D}_\theta = \mathfrak{D}_\theta, \quad T\mathfrak{D}^{\perp} = \{0\}, \quad BF\mathfrak{D}_\theta = \mathfrak{D}_\theta, \quad BF\mathfrak{D}^{\perp} = \mathfrak{D}^{\perp}.$$

When Eqs. (8) and (9) are used, one obtains the following Lemma.

Lemma 3.2 Let N be a PQHS submanifold of an almost contact metric manifold Ñ. Therefore, one has

$$(a)\ T^2U = -(\cos^2\theta)U, \quad (b)\ BFU = -(\sin^2\theta)U,$$

$$(c)\ T^2U + BFU = -U, \quad (d)\ FTU + CFU = 0,$$

for any $U \in \mathfrak{D}_\theta$.

With the help of (3), (8) and (9) and Definition 3.1, one obtains the following Lemma.

Lemma 3.3 Let N be a PQHS submanifold of an almost contact metric manifold Ñ. Then, we have

$$\text{(i)} < TU, TV >= (\cos^2\theta) < U, V >,$$

$$\text{(ii)} < FU, FV >= (\sin^2\theta) < U, V >,$$

for any $U, V \in \Gamma(\mathfrak{D}_\theta)$.

Proof. One can follow a similar way presented in Proposition 2.8 of [38].

When Eqs. (3), (5), (6), (8) and (9) are used and the normal and tangential components are compared, one has the following:

Lemma 3.4 Let N be a PQHS submanifold of a cosymplectic manifold Ñ. Therefore, one obtains

$$\nabla_U TV - A_{FV}U - T\nabla_U V - Bh(U,V) = 0$$

and

$$h(U, TV) + \nabla_U^{\perp} FV - F(\nabla_U V) - Ch(U,V) = 0,$$

for all $U, V \in \Gamma(TN)$.

Lemma 3.5 Let N be a PQHS submanifold of a cosymplectic manifold Ñ. Thus, one has

$$(\tilde{\nabla}_U T)V = A_{FV}U + Bh(U,V),$$

$$(\tilde{\nabla}_U F)V = Ch(U,V) - h(U,TV),$$

for any $U, V \in \Gamma(TN)$.

Lemma 3.6 N be PQHS submanifold of a cosymplectic manifold Ñ. Thus, one has

$$T([U,V]) = A_{\varphi V}U - A_{\varphi U}V$$

and

$$F([U,V]) = \nabla_U^{\perp}\varphi V - \nabla_V^{\perp}\varphi U,$$

for any $U, V \in \mathfrak{D}^{\perp}$.

Proof. Let $U, V \in \Gamma(\mathfrak{D}^{\perp})$, then

$$(\tilde{\nabla}_U\varphi)V = \tilde{\nabla}_U\varphi V - \varphi(\tilde{\nabla}_U V).$$

Taking into account of (3) in the above equation, we have

$$-A_{\varphi V}U + \nabla_U^{\perp}\varphi V - T\nabla_U V - F\nabla_U V - Bh(U,V) - Ch(U,V) = 0.$$

When the normal and tangential parts are compared in the equation given above, one obtains

$$-A_{\varphi V}U - T\nabla_U V - Bh(U,V) = 0 \tag{10}$$

and

$$\nabla_U^{\perp}\varphi V - F\nabla_U V - Ch(U,V) = 0. \tag{11}$$

From equations (10) and (11), one may conclude the the statement of Lemma 3.6.

Lemma 3.7 Let N be a PQHS submanifold of a cosymplectic manifold $\tilde{N}$. Under these assumptions, we have

(i) $< [U,V], \xi >= 0,$

(ii) $< \tilde{\nabla}_U V, \xi >= 0,$

for all $U,V \in (\mathfrak{D} \oplus \mathfrak{D}_\theta \oplus \mathfrak{D}^{\perp})$.

4. BASIC RESULTS

Theorem 4.1 Let N be a PQHS submanifold of a cosymplectic manifold $\tilde{N}$. Then, $\mathfrak{D}$ is integrable if and only if

$$< h(V,TU), FQZ > - < h(U,TV), FZ >$$
$$= < \nabla_U TV - \nabla_V TU, TQZ >$$
$$+ < T\nabla_V TU + Bh(V,TU), RZ >,$$

where $U,V \in \Gamma(\mathfrak{D}), Z = QZ + RZ \in \Gamma(\mathfrak{D}_\theta \oplus \mathfrak{D}^{\perp})$.

Proof. The distribution $\mathfrak{D}$ is integrable on N if and only if

$$< [U,V], \xi >= 0 \quad and \quad < [U,V], Z >,$$

for all $U,V \in \Gamma(\mathfrak{D})$, $Z = QZ + RZ \in \Gamma(\mathfrak{D}_\theta \oplus \mathfrak{D}^{\perp})$. For any $V \in \Gamma(\mathfrak{D})$, one has $< V, \xi >= 0$. Taking the covariant derivative of (4) along U, one has

$$< \tilde{\nabla}_U V, \xi > + < V, \tilde{\nabla}_U \xi >= 0. \tag{12}$$

From the equations (4) and (12), we obtain

$$< [U,V], \xi >=< \tilde{\nabla}_U V, \xi > - < V, \tilde{\nabla}_U \xi, >= 0.$$

Next, for every $U,V \in \Gamma(\mathfrak{D})$ and $Z = QZ + RZ \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}_\theta)$. Using (5), (8) and $FV = 0$ for all $V \in \Gamma(\mathfrak{D})$, we get

$$< [U,V], Z >=< \tilde{\nabla}_U \varphi V, \varphi Z > - < \tilde{\nabla}_V \varphi U, \varphi Z >$$
$$=< \tilde{\nabla}_U TV, TQZ + FQZ > + < \tilde{\nabla}_U TV, FRZ >$$
$$- < \tilde{\nabla}_V TU, \varphi QZ + \varphi RZ >.$$

By using (9) in the above equation, we have

$$< [U,V], Z >=< \nabla_U TV, TQZ > + < h(U,TV), FQZ >$$
$$+ < h(U,TV), FRZ > \quad + < \varphi(\tilde{\nabla}_V TU), RZ >$$
$$- < \tilde{\nabla}_V TU, TQZ + FQZ >$$
$$=< \nabla_U TV - \nabla_V TU, TQZ > + < h(U,TV), FZ >$$
$$+ < T\nabla_V TU + Bh(V,TU), RZ >$$
$$- < h(V,TU), FQZ > \tag{13}$$

The proof comes from (13).

Theorem 4.2 Let N be a PQHS submanifold of a cosymplectic manifold $\tilde{N}$. Then, $\mathfrak{D}_\theta$ is integrable if and only if

$$\sin(2\theta)\, Z(\theta) < U,V > -cos^2\theta < \nabla_Z U, V >$$
$$=< A_{CFU}V - \nabla_V BFU, Z >$$
$$+ < \nabla_V TU, TPZ) - < \nabla_Z BFU, V >,$$

where $U,V \in \Gamma(\mathfrak{D}_\theta), Z = PZ + RZ \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}^{\perp})$.

Proof. For every $U,V \in \Gamma(\mathfrak{D}_\theta), Z = PZ + RZ \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}^{\perp})$, utilizing (2), (3), (8) and (9), one has

$$< [U,V], Z >=< \tilde{\nabla}_U V, Z > - < \tilde{\nabla}_V U, Z >$$
$$= - < \tilde{\nabla}_Z \varphi U, \varphi V > - < [U,Z], V >$$
$$- < \tilde{\nabla}_V \varphi U, \varphi Z >$$
$$=< \tilde{\nabla}_Z T^2 U, V > + < \tilde{\nabla}_Z FTU, V >$$
$$+ < \tilde{\nabla}_Z BFU + CFU, V > - < [U,Z], V >$$
$$- < \tilde{\nabla}_V TU, \varphi Z > - < \tilde{\nabla}_V FU, \varphi Z >. \tag{14}$$

On the other hand, taking into account of Lemmma 3.2, using (5), (6), equation (14)

$$< [U,V], Z > = \sin(2\theta)\, Z(\theta) < U,V >$$
$$+ \cos^2\theta \quad < \tilde{\nabla}_U V, \mathrm{Z} > -\sin^2\theta < [U,Z], V >$$
$$+ < \nabla_Z BFU, V > \; - < \nabla_V TU, TPZ >$$
$$+ < \nabla_V BFU - A_{CFU}V, Z >$$
$$= \sin(2\theta) Z(\theta) < U,V > -cos^2\theta < \nabla_Z U, V >$$
$$+ < \nabla_Z BFU, V > - < \nabla_V TU, TPZ >$$
$$+ < \nabla_V BFU - A_{CFU}V, Z >. \tag{15}$$

The proof comes from (15).

Theorem 4.3 Let N be a PQHS submanifold of a cosymplectic manifold $\tilde{N}$. Then, $\mathfrak{D}^{\perp}$ is integrable if and onl if

$$< T([U,V]), TPZ - QZ >=< BF([U,V]), QZ >,$$

where $U,V \in \Gamma(\mathfrak{D}^{\perp})$, $Z = PZ + QZ \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}_\theta)$.

Proof. For any $U,V \in \Gamma(\mathfrak{D}^{\perp})$, $Z = PZ + QZ \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}_\theta)$, by using (2), (3), (6), one obtains

$$< [U,V], Z >=< \tilde{\nabla}_U \varphi V, \varphi Z > - < \tilde{\nabla}_V \varphi U, \varphi Z >$$
$$=< -A_{\varphi V}U + \nabla_U^{\perp}\varphi V, \varphi PZ >$$

$$+< \varphi(A_{\varphi V}U - \nabla^{\perp}_{U}\varphi V), QZ >$$

$$+< A_{\varphi U}V - \nabla^{\perp}_{V}\varphi U, TPZ >$$

$$+< \varphi(\nabla^{\perp}_{V}\varphi U - A_{\varphi U}V), QZ >. \quad (16)$$

By virtue of (8) and (9), equation (16)

$$< [U,V], Z >=< A_{\varphi U}V - A_{\varphi V}U, TPZ >$$

$$+< T(A_{\varphi V}U - A_{\varphi U}V), QZ >$$

$$+< B(\nabla^{\perp}_{V}\varphi U - \nabla^{\perp}_{U}\varphi V), QZ >.$$

From Lemma 3.6, one has

$$< [U,V], Z > =< T([U,V]), TPZ - QZ >$$

$$-< BF([U,V]), QZ >. \quad (17)$$

Hence the proof follows from (17).

Theorem 4.4 Let N be a PQHS submanifold of a cosymplectic manifold $\tilde{\mathrm{N}}$. Then, $\mathfrak{D}$ defines a totally geodesic foliation on N if and only if

$$< T\nabla_U TV + Bh(U,TV), QZ >=< h(U,TV), FRZ >$$

and

$$< \nabla_U V, TBW >=< F\nabla_U V + Ch(U,V), CW >,$$

where $U,V \in \Gamma(\mathfrak{D})$, $Z = QZ + RZ \in \Gamma(\mathfrak{D}_\theta \oplus \mathfrak{D}^\perp)$ and $W \in \Gamma(TN)^\perp$.

Proof. For every $U,V \in \Gamma(\mathfrak{D})$, $Z = QZ + RZ \in \Gamma(\mathfrak{D}_\theta \oplus \mathfrak{D}^\perp)$, utilizing (2), (5), (8) and (9), one has

$$< \tilde{\nabla}_U V, Z >=< \tilde{\nabla}_U \varphi V, \varphi Z >$$

$$=< \tilde{\nabla}_U TV, \varphi QZ > +< \tilde{\nabla}_U TV, \varphi RZ >$$

$$= -< \varphi(\nabla_U TV + h(U,TV)), QZ >$$

$$+< h(U,TV), \varphi RZ >$$

$$= -< T\nabla_U TV + Bh(U,TV), QZ >$$

$$+< h(U,TV), FRZ >. \quad (18)$$

Now, for all $W \in \Gamma(TN)^\perp$ and $U,V \in \Gamma(\mathfrak{D})$, we get

$$< \tilde{\nabla}_U V, W >=< \tilde{\nabla}_U \varphi V, BW + CW >$$

$$= -< \tilde{\nabla}_U V, \varphi BW + \varphi CW >$$

$$= -< \nabla_U V, TBW > +< \varphi\tilde{\nabla}_U V, CW >$$

$$= -< \nabla_U V, TBW >$$

$$+< F\nabla_U V + Ch(U,V), CW >. \quad (19)$$

Thus from (18) and (19), which achieves the proof.

Theorem 4.5 Let N be a PQHS submanifold of a cosymplectic manifold $\tilde{\mathrm{N}}$. Then, $\mathfrak{D}_\theta$ defines a totally geodesic foliation on N if and only if

$$\cos^2\theta < [U,Z], V > + \sin(2\theta)\, Z(\theta) < U,V >$$

$$=< T\nabla_Z TU + Bh(Z,TU), V >$$

and

$$< FA_{FV}U, W >=< C\nabla^{\perp}_{U}FV + \nabla^{\perp}_{U}FTV, W >,$$

where $U,V \in \Gamma(\mathfrak{D}_\theta)$, $Z = PZ + RZ \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}^\perp)$ and $W \in \Gamma(TN)^\perp$.

Proof. For every $U,V \in \Gamma(\mathfrak{D}_\theta)$, $Z = PZ + RZ \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}^\perp)$, utilizing (2), (5), (8) and (9), we have

$$< \tilde{\nabla}_U V, Z >= U < V, Z > -< V, \tilde{\nabla}_U Z >$$

$$= -< [U,Z], V > -< \tilde{\nabla}_Z \varphi U, \varphi V >$$

$$-< \tilde{\nabla}_Z FU, \varphi V >$$

$$= -< [U,Z], V > +< T\nabla_Z TU, V >$$

$$+< Bh(Z,TU), V > +< \tilde{\nabla}_Z BFU, V >$$

$$+< \tilde{\nabla}_Z CFU, V >.$$

Then from (6), Lemma 3.2 and using the property of slant function, we get

$$< \tilde{\nabla}_U V, Z >= -< [U,Z] + T\nabla_Z TU + Bh(Z,TU), V >$$

$$-< \tilde{\nabla}_Z \sin^2\theta U - A_{CFU}Z, V >$$

$$= -\sin(2\theta)Z(\theta) < U,V > -\sin^2\theta < \tilde{\nabla}_Z U, V >$$

$$-< [U,Z] + T\nabla_Z TU + Bh(Z,TU), V >. \quad (20)$$

From (20), we obtain

$$\cos^2\theta < \tilde{\nabla}_U V, Z >= -\cos^2\theta < [U,Z], V >$$

$$-\sin(2\theta)Z(\theta) < U,V >$$

$$+< T\nabla_Z TU + Bh(Z,TU), V >. \quad (21)$$

Now, for every $W \in \Gamma(TN)^\perp$, with the help of (2), (6), (8) and Lemma 3.2, we have

$$< \tilde{\nabla}_U V, W >=< \tilde{\nabla}_U \varphi V, \varphi W >$$

$$= -< \tilde{\nabla}_U T^2 V - \tilde{\nabla}_U FTV, W >$$

$$-< \varphi(-A_{FV}U + \nabla^{\perp}_{U}FV), W >,$$

which gives

$$\sin^2\theta < \tilde{\nabla}_U V, W >=< FA_{FV}U - \nabla^{\perp}_{U}FTV - C\nabla^{\perp}_{U}FV, W >. \quad (22)$$

Thus from (21) and (22), which achieves the proof .

Theorem 4.6 Let N be a PQHS submanifold of a cosymplectic manifold $\tilde{N}$. Then, $\mathfrak{D}^{\perp}$ defines a totally geodesic foliation on N if and only if

$$< A_{FV}U, TQZ > = < \nabla^{\perp}_{U}FV, FQZ >$$

and

$$< A_{FV}U, BW > = < \nabla^{\perp}_{U}FV, CW >,$$

where $U, V \in \Gamma(\mathfrak{D}^{\perp})$, $Z = PZ + QZ \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}_{\theta})$ and $W \in \Gamma(TN)^{\perp}$.

Proof. For all $U, V \in \Gamma(\mathfrak{D}^{\perp})$, $Z = PZ + QZ \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}_{\theta})$, using (2), (6) and (8), one has

$$\begin{aligned} < \tilde{\nabla}_U V, Z > &= < \tilde{\nabla}_U \varphi V, \varphi Z > \\ &= < \tilde{\nabla}_U \varphi V, \varphi PZ + \varphi QZ > \\ &= < -A_{\varphi V}U + \nabla^{\perp}_{U}\varphi V, FPZ > \\ &+ < -A_{FV}U + \nabla^{\perp}_{U}FV, TQZ + FQZ > \\ &= -< A_{FV}U, TQZ > + < \nabla^{\perp}_{U}FV, FQZ >. \quad (23) \end{aligned}$$

Now, for every $U, V \in \Gamma(\mathfrak{D}^{\perp})$ and $W \in \Gamma(TN)^{\perp}$, utilizing (2), (6), (9), we get

$$\begin{aligned} < \tilde{\nabla}_U V, W > &= < \tilde{\nabla}_U FV, BW + CW > \\ &= -< A_{FV}U, BW > + < \nabla^{\perp}_{U}FV, CW >. \quad (24) \end{aligned}$$

The proof comes from (23) and (24).

Theorem 4.7 Let N be a PQHS submanifold of a cosymplectic manifold $\tilde{N}$. Then, $\mathfrak{D}$ is totally geodesic if and only if

$$\begin{aligned} &< T\nabla_U V + Bh(U,V), BW > \\ &= -< F\nabla_U V + Ch(U,V), CW >, \end{aligned}$$

where $U, V \in \Gamma(\mathfrak{D})$ and $W \in \Gamma(TN)^{\perp}$.

Proof. For any $U, V \in \Gamma(\mathfrak{D})$ and $W \in \Gamma(TN)^{\perp}$, by using (2), (5) , (8) and (9), we have

$$\begin{aligned} < (h(U,V), W > &= < \tilde{\nabla}_U \varphi V, \varphi W > \\ &= < \varphi\tilde{\nabla}_U V, BW > + < \varphi\tilde{\nabla}_U V, CW > \\ &= < T\nabla_U V + Bh(U,V), BW > \\ &+ < F\nabla_U V + Ch(U,V), CW >. \quad (25) \end{aligned}$$

The proof comes from (25).

Theorem 4.8 Let N be a PQHS submanifold of a cosymplectic manifold $\tilde{N}$. Then, $\mathfrak{D}_{\theta}$ is totally geodesic if and only if

$$< A_W U, BFV > = < \nabla^{\perp}_{U}W, CFV + FTV >,$$

where $U, V \in \Gamma(\mathfrak{D}_{\theta})$ and $W \in \Gamma(TN)^{\perp}$.

Proof. For any $U, V \in \Gamma(\mathfrak{D}_{\theta})$ and $W \in \Gamma(TN)^{\perp}$, by using (2) and (8), we get

$$\begin{aligned} < h(U,V), W > &= -< \tilde{\nabla}_U \varphi W, \varphi V > \\ &= < \tilde{\nabla}_U W, \varphi TV > + < \tilde{\nabla}_U W, \varphi FV > \\ &= < \tilde{\nabla}_U W, T^2V + FTV > \\ &+ < \tilde{\nabla}_U W, BFV + CFV >. \quad (26) \end{aligned}$$

Taking into account of (6) and from Lemma 3.2, equation (26)

$$\begin{aligned} < h(U,V), W > &= \cos^2\theta < \tilde{\nabla}_U V, W > \\ &+ < \nabla^{\perp}_{U}W, FTV + CFV > \\ &- < A_W U, BFV >. \quad (27) \end{aligned}$$

From (27), we obtain

$$\begin{aligned} \sin^2\theta < \tilde{\nabla}_U V, W > &= < \nabla^{\perp}_{U}W, FTV + CFV > \\ &- < A_W U, BFV >. \end{aligned}$$

This implies

$$\begin{aligned} < h(U,V), W > &= \csc^2\theta\{< \nabla^{\perp}_{U}W, FTV + CFV > \\ &- < A_W U, BFV >\}. \quad (28) \end{aligned}$$

Thus from (28), which achieves the proof.

Theorem 4.9 Let N be a PQHS submanifold of a cosymplectic manifold $\tilde{N}$. Then, $\mathfrak{D}^{\perp}$ is totally geodesic if and only if

$$< B\nabla^{\perp}_{U}W - TA_W U, TV > = < FA_W V - C\nabla^{\perp}_{U}W, FV >,$$

where $U, V \in \Gamma(\mathfrak{D}^{\perp})$ and $W \in \Gamma(TN)^{\perp}$.

Proof. For any $U, V \in \Gamma(\mathfrak{D}^{\perp})$ and $W \in \Gamma(TN)^{\perp}$, by using (2), (6), (8) and (9), we have

$$\begin{aligned} < h(U,V), W > &= -< \tilde{\nabla}_U \varphi W, \varphi V > \\ &= -< \varphi(-A_W U + \nabla^{\perp}_{U}W), \varphi V > \\ &= < TA_W U - B\nabla^{\perp}_{U}W, TV > \\ &+ < FA_W V - C\nabla^{\perp}_{U}W, FV >. \quad (29) \end{aligned}$$

The proof comes from (29).

Theorem 4.10 Let N be a PQHS submanifold of a cosymplectic manifold $\tilde{N}$. Then, $\mathfrak{D} - \mathfrak{D}_{\theta}$ mixed totally geodesic if and only if

$$< FA_{FV}U - \nabla^{\perp}_{U}FTV - C\nabla^{\perp}_{U}FV, W > = 0,$$

where $U \in \Gamma(\mathfrak{D})$, $V \in \Gamma(\mathfrak{D}_{\theta})$ and $W \in \Gamma(TN)^{\perp}$.

Proof. For any $U \in \Gamma(\mathfrak{D})$, $V \in \Gamma(\mathfrak{D}_\theta)$ and $W \in \Gamma(TN)^\perp$, by using (2), (5), (6), (8), (9) and from Lemma 3.2, one can obtain

$$\begin{aligned} < h(U,V),W > &=< \widetilde{\nabla}_U \varphi V, \varphi W > \\ &=< \widetilde{\nabla}_U TV, \varphi W > +< \widetilde{\nabla}_U FV, \varphi W > \\ &=< \widetilde{\nabla}_U T^2 V + FTV, W > -< \varphi\widetilde{\nabla}_U FV, W > \\ &= \cos^2\theta < \widetilde{\nabla}_U V, W > -< \nabla_U^\perp FTV, W > \\ &-< \varphi\widetilde{\nabla}_U FV, W >. \end{aligned} \tag{30}$$

From equation (30), we have

$$\sin^2\theta < h(U,V),W >$$
$$=< FA_T VU - \nabla_U^\perp FTV - C\nabla_U^\perp FV, W >,$$

which gives

$$< h(U,V),W >$$
$$= \csc^2\theta\{< FA_{FV}U - \nabla_U^\perp FTV - C\nabla_U^\perp FV, W >\},$$

which completes the proof.

Theorem 4.11 Let N be a PQHS submanifold of a cosymplectic manifold $\widetilde{\mathrm{N}}$. Then, $\mathfrak{D} - \mathfrak{D}^\perp$ mixed totally geodesic if and only if

$$\nabla_V BW - A_{CW}V \in \Gamma(\mathfrak{D}^\perp),$$

where $U \in \Gamma(\mathfrak{D})$, $V \in \Gamma(\mathfrak{D}^\perp)$ and $W \in \Gamma(TN)^\perp$.

Proof. For every $U \in \Gamma(\mathfrak{D})$, $V \in \Gamma(\mathfrak{D}^\perp)$ and $W \in \Gamma(TN)^\perp$, by using (2), (5), (6), (8) and (9), we get

$$\begin{aligned} < h(U,V),W > &= -< \widetilde{\nabla}_V \varphi W, \varphi U > \\ &= -< \widetilde{\nabla}_V BW + CW, TU > \\ &= -< \nabla_V BW - A_{CW}V, TU >. \end{aligned} \tag{31}$$

The proof comes from (31).

Theorem 4.12 Let N be a PQHS submanifold of a cosymplectic manifold $\widetilde{\mathrm{N}}$. Then, $\mathfrak{D}_\theta - \mathfrak{D}^\perp$ mixed totally geodesic if and only if

$$< A_{FV}U, BW >=< \nabla_U^\perp FV, CW >,$$

where $U \in \Gamma(\mathfrak{D}_\theta)$, $V \in \Gamma(\mathfrak{D}^\perp)$ and $W \in \Gamma(TN)^\perp$.

Proof. For every $U \in \Gamma(\mathfrak{D}_\theta)$, $V \in \Gamma(\mathfrak{D}^\perp)$ and $W \in \Gamma(TN)^\perp$, by using (2), (5), (6), (8) and (9), one has

$$\begin{aligned} < h(U,V),W > &=< \widetilde{\nabla}_U \varphi V, \varphi W > \\ &=< \widetilde{\nabla}_U FV, BW + CW > \\ &=< \nabla_U^\perp FV, CW > -< A_{FV}U, BW >. \end{aligned} \tag{32}$$

The proof comes from (32).

Finally, we mention the following examples.

5. EXAMPLES

Example 5.1 For $\theta \in (0, \frac{\pi}{2})$, consider a submanifold N of a cosymplectic manifold $\widetilde{\mathrm{N}}$ described by immersion f as follows:

$$f(\theta,u,w,s,m,n,z) = \left(\frac{\sqrt{3}}{2}w, 0, 0, -\frac{w}{2}, u^2, -\sin\theta, u^2, \cos\theta, s, 0, u^2, -\sin\theta, u^2, \cos\theta, m, n, z\right).$$

By aid of simple calculations, one can easily control that the tangent bundle of N is spanned by the set $\{X_1, X_2, X_3, X_4, X_5, X_6, X_7\}$, where

$$X_1 = -\cos\theta\frac{\partial}{\partial y_3} - \sin\theta\frac{\partial}{\partial y_4} - \cos\theta\frac{\partial}{\partial y_6} - \sin\theta\frac{\partial}{\partial y_7},$$

$$X_2 = 2u\frac{\partial}{\partial x_3} + 2u\frac{\partial}{\partial x_4} + 2u\frac{\partial}{\partial x_6} + 2u\frac{\partial}{\partial x_7},$$

$$X_3 = \frac{\sqrt{3}}{2}\frac{\partial}{\partial x_1} - \frac{1}{2}\frac{\partial}{\partial y_2}, \qquad X_4 = \frac{\partial}{\partial x_5},$$

$$X_5 = \frac{\partial}{\partial x_8}, \qquad X_6 = \frac{\partial}{\partial y_8}, \qquad X_7 = \frac{\partial}{\partial z}.$$

φ be the (1,1) tensor field defined by

$$\varphi\left(\frac{\partial}{\partial x_i}\right) = -\frac{\partial}{\partial y_i}, \quad \varphi\left(\frac{\partial}{\partial y_j}\right) = \frac{\partial}{\partial x_j},$$

$$\varphi(\frac{\partial}{\partial z}) = 0, \quad 1 \le i,j \le 8.$$

If the linearity of φ and $<,>$ is used, one has

$$\varphi^2 = -I + \eta \otimes \xi, \qquad \varphi\xi = 0, \qquad \eta(\xi) = 1,$$
$$< \varphi U, \varphi V >=< U,V > -\eta(U)\eta(V),$$

for every $U,V \in \Gamma(T\widetilde{N})$. Hence $(\widetilde{N}, \varphi, \xi, \eta, <,>)$ is almost contact metric manifold. At the same time, one can easily illustrate that $(\widetilde{N}, \varphi, \xi, \eta, <,>)$ is a cosymplectic manifold of dimension 17. Thus we have

$$\varphi X_1 = -\cos\theta\frac{\partial}{\partial x_3} - \sin\theta\frac{\partial}{\partial x_4} - \cos\theta\frac{\partial}{\partial x_6} - \sin\theta\frac{\partial}{\partial x_7},$$

$$\varphi X_2 = -2u\frac{\partial}{\partial y_3} - 2u\frac{\partial}{\partial y_4} - 2u\frac{\partial}{\partial y_6} - 2u\frac{\partial}{\partial y_7},$$

$$\varphi X_3 = -\frac{\sqrt{3}}{2}\frac{\partial}{\partial y_1} - \frac{1}{2}\frac{\partial}{\partial x_2}, \qquad \varphi X_4 = -\frac{\partial}{\partial y_5},$$

$$\varphi X_5 = -\frac{\partial}{\partial y_8}, \qquad \varphi X_6 = \frac{\partial}{\partial x_8}, \qquad \varphi X_7 = 0.$$

With simple computations, one can obtain $\mathfrak{D} = Span\{X_5, X_6\}$ is an invariant, $\mathfrak{D}_\theta = Span\{X_1, X_2\}$ is a pointwise slant with slant function $-\cos^{-1}(\frac{\sin 2\theta}{\sqrt{2}})$ and

$\mathfrak{D}^{\perp} = Span\{X_3, X_4\}$ is anti-invariant. Thus f defines a proper 7 -dimensional PQHS submanifold in cosymplectic manifold $\tilde{N}$.

Example 5.2 For $\theta \in (0, \frac{\pi}{2})$ and $k \in R$ consider a submanifold N of a cosymplectic manifold $\tilde{N}$ described by immersion γ as follows:

$$\gamma(u,v,w,\theta,s,t,q) = (u,w,0,\frac{s}{\sqrt{2}},0,\frac{t}{\sqrt{2}},0,v,$$

$$\cos(\theta+k),-\sin(\theta+k),0,\frac{s}{\sqrt{2}},0,\frac{t}{\sqrt{2}},q).$$

One can obviously observe the fact that the tangent bundle of N is spanned by the tangent vectors

$$Z_1 = \frac{\partial}{\partial x_1}, \qquad Z_2 = \frac{\partial}{\partial y_1}, \qquad Z_3 = \frac{\partial}{\partial x_2},$$

$$Z_4 = \cos(\theta+k)\frac{\partial}{\partial y_2} - \sin(\theta+k)\frac{\partial}{\partial y_3},$$

$$Z_5 = \frac{1}{\sqrt{2}}(\frac{\partial}{\partial x_4} + \frac{\partial}{\partial y_5}), \quad Z_6 = \frac{1}{\sqrt{2}}(\frac{\partial}{\partial x_6} + \frac{\partial}{\partial y_7}),$$

$$Z_7 = \frac{\partial}{\partial z}.$$

One can describe (1,1)-tensor field φ as

$$\varphi(\frac{\partial}{\partial x_i}) = \frac{\partial}{\partial y_i}, \quad \varphi(\frac{\partial}{\partial y_j}) = -\frac{\partial}{\partial x_j}, \quad \forall i,j = 1,\ldots,7.$$

When the linearity of φ and $<,>$, one has

$$\varphi^2 = -I + \eta \otimes \xi, \qquad \varphi\xi = 0, \qquad \eta(\xi) = 1,$$

$$< \varphi U, \varphi V > = < U, V > - \eta(U)\eta(V),$$

for every $U, V \in \Gamma(T\tilde{N})$. Hence $(\tilde{N}, \varphi, \xi, \eta, <,>)$ is almost contact metric manifold. At the same time, it can be obviously seen that $(\tilde{N}, \varphi, \xi, \eta, <,>)$ is a cosymplectic manifold of dimension 15. Thus we have

$$\varphi Z_1 = \frac{\partial}{\partial y_1}, \qquad \varphi Z_2 = -\frac{\partial}{\partial x_1}, \qquad \varphi Z_3 = \frac{\partial}{\partial y_2},$$

$$\varphi Z_4 = -\cos(\theta+k)\frac{\partial}{\partial x_2} + \sin(\theta+k)\frac{\partial}{\partial x_3},$$

$$\varphi Z_5 = \frac{1}{2}(\frac{\partial}{\partial y_4} - \frac{\partial}{\partial x_5}), \quad \varphi Z_6 = \frac{1}{2}(\frac{\partial}{\partial y_6} - \frac{\partial}{\partial x_7}),$$

$$\varphi Z_7 = 0.$$

Now, let the distributions $\mathfrak{D} = Span\{Z_1, Z_2\}$, $\mathfrak{D}_\theta = Span\{Z_3, Z_4\}$, $\mathfrak{D}^{\perp} = Span\{Z_5, Z_6\}$. Then obviously $\mathfrak{D}$, $\mathfrak{D}_\theta$ and $\mathfrak{D}^{\perp}$ satisfy the definition of pointwise quasi hemi slant of a cosymplectic manifold. Thus γ defines a proper 7-dimensional PQHS submanifold of R^{15} with pointwise slant function $(\theta + k)$.

6. CONCLUSION

In this paper, we have presented a novel class of submanifolds of cosymplectic manifolds that may be seen as a generalization of quasi hemi-slant, hemi-slant, slant etc. submanifolds. Moreover, conditions for such distributions to be integrable, totally geodesic foliation, totally geodesic and mixed totally geodesic are obtained.

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