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Technology supported teaching implementations designed to improve critical thinking skills

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Highlights

- Teaching transformation geometry acquisitions can be incorporated into lesson plans that emphasize critical thinking skills.
- In the designed lesson plan activities, GeoGebra software enhances students' motivation and boosts the learning environment.
- There are expressions that will enable the development of critical thinking sub-skills in the in-class dialogues of teacher candidates.

Abstract

This research evaluated the influence of technology-based teaching models on the development of critical thinking abilities in elementary school mathematics teacher candidates, as measured using Facione's (1990) criteria. The research was conducted with fourth-grade students studying in the Faculty of Education, Elementary Mathematics Education at a state university in the Marmara Region of Türkiye during the 2019-2020 Fall Semester. A case study research design was used to obtain a qualitative understanding of the participants' experiences. Two pre-service teachers were video-recorded throughout the research implementation process (5 weeks in total, from the 3rd week when the teaching model practices started to the 7th week when it ended). The dialogues in the video recording were analyzed using the descriptive analysis technique. Keywords and sentences describing the critical thinking sub-skills identified by Facione (1990) were observed to be included in the student dialogues, especially in the implementations of the designed teaching model that required the use of GeoGebra. These dialogues concluded that the proposed instructional model programs are consistent with the critical thinking competencies, include activities to use critical thinking skills, and can provide opportunities for developing critical thinking skills.

Article Info: Research Article

Keywords: *Critical Thinking, GeoGebra, Preservice Mathematics Teachers*

1. Introduction

Critical thinking has an important place in the change, development and progress of free societies. Because it is thanks to critical thinking that individuals who make up the society understand life, interpret it, solve the problems they encounter and make decisions (Mete,2021). In the 21st century, in which science and technology are advancing very rapidly, changes and developments in social structures have made it necessary for individuals to change their way of reaching information and thinking (Çatalbaş,

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Sarıtaş, 2022). Today, it is very important to acquire and develop high-level thinking skills such as problem solving, decision making, critical thinking, creative thinking and questioning in educational environments. With the rapid increase in technological developments, the expectations of societies from individuals are also changing. Societies want individuals to become people who have developed high-level skills and solve the problems they face (Karadem, Ongun and Taş, 2023). The proliferation of information and communication technologies has been accompanied by an undeniable shift in ourage. Maintaining up-to-date knowledge of this progression has become indispensable for nations wishing to preserve their strength. Given the increasing complexity of our world, characterized by exponential scientific and technological advancements, the abundance of information, the influence of social media, globalization, and the expeditious transmission of thought, it is essential that educational systems be regularly adapted and updated. In light of this, the goals and objectives of schooling must be constantly revisited to ensure individuals are provided with the necessary skills to be successful in the present day. Associations like UNESCO and the European Council on Education have observed a surge in student participation in the learning process, stressing the importance of personal advancement, ingenuity, self-determination, and the cultivation of critical thinking skills (Maričić, Špijunović & Lazić, 2015).

Examining the explanations of critical thinking, Paul and Elder (2013) posit that it is a form of thought which is honed by rigorously investigating, assessing, and organizing the quality of a person's thinking on any subject matter, topic, or dilemma. Kurnaz (2013) articulated critical thinking as the pursuit of reality in its entirety, including both positive and negative, visible and invisible elements, and forming a conclusion on the matter (p.12). As reported by Halpern (2014), analytical reasoning involves using cognitive skills or strategies to achieve a desired outcome. Eğmir (2016) additionally proposed critical thinking as a structured manner of inquiry by posing analytical questions in the direction of promoting intellectual growth, formulating more powerful concepts and arriving at better judgments. It is clear from the explanations of critical thinking that equipping educational systems with the ability to foster analytic reasoning is essential for forming a world in which citizens possess the ability to think deeply and make sensible choices (Egmir, 2018).

The American Philosophical Association's Delphi panel, made up of 46 specialists in their respective fields, outlined the concept of critical thinking - a key intellectual ability and the focus of our inquiry - during their gathering. Critical thinking was conclusively described at the conclusion of the panel as the act of rationally and objectively considering evidentiary, notional, methodological, analytical and circumstantial factors, with the purpose of forming a reasoned and purposeful judgment, which may lead to interpretation, analysis, evaluation and inference (Facione, 1990). Facione (1990) identified six broad categories of critical thinking skills as criteria: interpretation, analysis, evaluation, inference, explanation, and self-regulation. Facione provides the following descriptions of these abilities:

1. Interpretation: Analyzing the issue, quantifying it systematically, elucidating, classifying, deducing, and classifying; articulating their opinions effectively; distinguishing the primary concepts of a document from subsidiary concepts; determining the author's motive, fundamental concept, and perspective.

2. Analysis: Exploring concepts, identifying and evaluating assertions/claims, noting the distinctions and resemblances between two potential solutions to an issue, determining the central concept of a passage and its relationship to other sentences or sections, specifying undefined statements and visually organizing a composition in an individualized manner.

3. Evaluation: Assessing the validity of claims based on hypotheses; determining the pertinence, utility, or implications of the existing claim to a current situation; articulating an individual's perspective, experience, situation, belief, or judgment and evaluating the veracity of metrics.

4. Inference: Formulating conjectures and suppositions; examining the data in problem, coming to a resolution or making inferences from information, conclusions, axioms, ideas, queries, and elucidations; examining proof, assessing potential options and explaining; generating solutions for solving encountered problems; synthesizing new solutions from existing ones.

5. Explanation: Defining methods and results, assessing processes; presenting the cause of something in the form of logical and consistent results; relying on criteria that form the basis for judgments; creating diagrams that organize key findings; creating charts that reveal relationships between notions and conceptions, presenting analysis of data and the parameters employed to obtain these findings, proposing and defending well-reasoned explanations for events or viewpoints, and determining the data that supports or refutes a specific hypothesis.

6. Self-regulation: Tracking intellectual operations, the elements employed in the processes, and the outcomes attained; self-assessment of decisions, making self-examination and self-correction, particularly emphasizing the capabilities during the evaluation process; assessing the conflicting situation objectively, taking into account personal perspectives, preconceptions, and vested interests; reminding oneself to maintain a clear distinction between one's own thoughts and those of the author when reading or writing.

The importance of cultivating cognitively-adept people is underscored when elementary and secondary education mathematics curriculum objectives are examined (MEB,2018). To attain this objective, emphasizing analytical reasoning in mathematics instruction by incorporating exercises, drills, and assignments aimed at improving integration of critical-thinking strategies into the lesson design process is essential for ensuring students acquire and hone their critical thinking capabilities.

2. Literature

Educating individuals in the practice of critical thought is achievable by employing efficacious approaches (Kökdemir, 2012). Educators face a tremendous amount of accountability. The efficacy of education systems in cultivating critical thinking capabilities is contingent upon the implementation of said skills by pedagogues in instructional activities. Well-trained teachers have a special place in teaching thinking skills (Korkmaz, 2008). Ashton (1988) stated that the biggest obstacle in front of the schools' aim of raising critical thinking individuals is the lack of critical thinking knowledge and skills of teachers (cited in Akbey, 2007). Similarly, according to Wilks (1995), in order for schools to raise students who question well, are more participants, are more open to discussions, determine estimates and priorities, seek alternatives, and make sense of various views, it is necessary to train teachers who will teach the lessons in such a way as to acquire these competencies (cited in Akbey, 2007). According to Norris (1985), educators should facilitate the enhancement of critical thinking skills in their students (cited in Sezer, 2008). Ennis (1991) argued that a key factor in fostering skills related to critical thinking was the instructor. According to Demirci (2000), the capacity of students to develop critical thinking skills is bound up with the training of educators or faculty members in this area (cited in Semerci, 2003). Within this scope the Teacher Training Strategy (2017-2023) published by the Turkish Ministry of National Education specifies the requirement of the efficacy of educational systems is largely dependent upon the aptitude of the teaching personnel and is thus a determining factor in shaping the future of students (MEB, 2017).

Given that each pre-service teacher in the research sample will be a teacher in the future, it is important for them to graduate with strong critical thinking skills. The significance of this research is demonstrated in its capacity to generate a framework for achieving the articulated objectives.

It is essential to incorporate the use of information and communication technologies into curricula in order to meet the demands of modern educational and training requirements. Dynamic geometry software (DGS) environments offer a unique opportunity to harness the power of imagination in mathematics and to pave the way for creativity and discovery. Once the route is established, the student will be more capable of analyzing, inferring, and forming generalizations. Consequently, the student's aptitude for problem-solving will be augmented (Baki, 2001). As reported by Güven and Karataş (2003), students can engage in a process of exploration, conjecture, experimentation, negation, postulation, and elucidation with confidence when they enter the research environment with DGS. Upon conducting a thorough analysis of the literature and employing the use of DGS during the mathematics lesson, it has been stated that their abilities such as formulating, making explanations to the students, making assumptions, making calculations, making proofs, forming generalizations, learning about mistakes/errors by getting feedback, making inferences, showing high-level mental skills, making inductive and deductive inferences, making analysis, developing problem-solving skills, exploring, testing, rejecting skills are affected in a positive way (Baki, 2002; Baydaş, 2010; Borazan,2019; Bintaş & Smart, 2008; Güven & Karataş, 2003; Gonzàlez & Herbst, 2009; Sanders, 1998). In the literature, when the recommended methods, the ways to be followed and the necessary skills are examined in order to increase critical thinking skills in mathematics lessons, making comparison, encountering contradictory situations, induction, generalization, ordering, classification, proving, associating, analyzing, evaluating, understanding concepts, applying, ability to synthesize, solve problems, produce solutions, make assumptions, reason, interpret data, make decisions, reason, pose problems, question, object to decision, analyze assumptions, be open-minded, consider new ideas, skepticism, evidence and activities such as searching for logic, considering alternatives, perseverance, creative use of imagination come to the fore (Budiman,2013; Facione 2011; Doğan-Dolapçıoğlu, 2015; Glazer 2001; Maričić, Špijunović & Lazić, 2015; NCTM, 1999; Rasiman, 2015; Sukmadinata, 2004; Su, Ricci & Mnatsakanian, 2016 Yüksel, Sarı-Uzun & Dost, 2013). Sanders (1998) stated that dynamic mathematics software not only creates a strong teaching and learning environment but also provides a foundation for analysis and deduction, evidence-based thinking, and creativity (Cited by Çörekçioğlu, 2019).

Transformation geometry outcomes are noteworthy among other mathematical topics in which DGS can be used. The National Council of Teachers of Mathematics (2000) recognizes translation, reflection, and rotation as the three essential components of transformation geometry that students should be familiar with (cited by Borazan, 2019). The DGS environment is well-suited for teaching transformation geometry which helps students be creative in math class. Through the study of transformation geometry, students gain an understanding of how to manipulate geometric shapes in two dimensions and explore the characteristics of certain rules and forms (İnce, 2012). Also, transformation geometry is one of the subjects in which reasoning is applied the most intensively in geometry teaching. The fact that dynamic geometry environments provide an experimental environment like a laboratory environment to this reasoning results in an increase in reasoning. Dynamic geometry environments provide an experimental environment like a laboratory environment to this reasoning results in an increase in reasoning. In this context, considering that critical thinking and reasoning are closely related, it is important to examine transformation geometry with critical thinking in dynamic geometry environments (Falcade, 2001).

An examination of the literature on the advantages of Dynamic Geometry Software (DGS) for users, as well as an analysis of Facione's (1990) definitions and descriptions of critical thinking skills suggest that DGS environments may be beneficial in cultivating critical thinking skills.

In that regard, it was aimed to examine the video documentation of technology-supported teaching models designed to promote the growth of Facione (1990) -determined critical thinking abilities and to evaluate the solutions, suggestions, and results obtained by the pre-service teachers during the implementations in terms of the keywords/phrases of the critical thinking sub-skills. Our goal was to provide an example of a teaching model that could encourage the cultivation of analytical reasoning abilities in literary studies. The following problem was sought to be answered within this purpose:

- What is the correlation between pre-service teachers' interactions and critical thinking sub-skills when utilizing a technology-based teaching model designed to enhance critical thinking abilities?

3. Methodology

3.1. Research Design

Qualitative research involves the utilization of qualitative data collection techniques and a systematic approach to uncovering occurrences and opinions within their natural habitat in an accurate and comprehensive manner (Şimşek & Yıldırım, 2008). The study utilized a holistic single case design of the Case Study method, a qualitative research approach, as its model. In the holistic single-case design, the scope of study is limited to a single analysis unit (individual, institution, program, school, etc.) (Aytaçlı, 2012). This approach encompasses all types of data collection tools, including but not limited to case studies, interviews, observations, questionnaires, and documents. Case studies aim to answer the questions "How?", "Why?", and "What?" to illuminate theories (Çepni, 2012). Within this research, a holistic single-case design was employed to provide a framework and guidance for the development of new teaching models aimed at fostering critical thinking skills of elementary school mathematics teacher candidates, based on the analysis of their dialogues during the design process.

3.2. Research Group

This study was conducted during the Fall 2019-2020 academic semester, utilizing two pre-service educators who are participants of the designed teaching model implementations among fourth-graders of the Faculty of Education, Elementary Mathematics Teaching at a state university in the Marmara Region. The criterion sampling technique, a type of purposive sampling, was employed to select the teacher candidates. Criterion sampling enables the selection of observation units from individuals, occurrences, objects, or contexts which possess certain properties (Büyüköztürk et al., 2019). This study employed the following criteria to assess pre-service teachers: no weak academic achievement, ability to express thoughts easily, being volunteer, sociable, and open to sharing. The criterion of not having poor academic achievement was chosen because it was hypothesized that a lack of mathematical knowledge might negatively affect their critical thinking skills. In addition, students with poor success did not meet the voluntary basis. The criterion of being able to express their thoughts easily and being assertive was set to collect as much data as possible in the research.

3.3. Data Collection Tools

Two pre-service teachers were video-recorded for the research during the teaching model implementations, for five weeks, from the third week when the teaching model implementations started to the seventh week when it ended. Since qualitative research is conducted to analyze in-depth and examine events or situations in detail, it is not appropriate to use too many people in qualitative research. Qualitative research can be conducted in small groups with one or two people (Creswell, 2012).

3.4. Analysis of Data

A "descriptive analysis" was conducted to interpret the research data. The descriptive analysis explains situations where the conceptual structure is determined (Çepni, 2012). The data collected during the research were divided into themes based on the critical thinking sub-skills identified by Facione, with the input of two independent researchers. These themes were determined as critical thinking sub-skills of interpretation, analysis, evaluation, inference, explanation, and self-regulation.

To increase the credibility of this research, an environment of mutual trust was tried to be provided with questions to get to know them better during the teaching model implementation process, and a long-term interaction was made throughout the implementation process. Thus, it is aimed to create a favorable environment for the research. At the end of the data collection and analysis process lasted for one year, the data obtained from the novice teachers were examined in detail. During the implementation, the activities of the novice teachers were observed, and when incomprehensible situations were detected and intervened, and it was confirmed what they really meant. Therefore, it is aimed to prevent erroneous evaluations that can occur due to the researcher's misunderstanding of the novice teachers. Purposive sampling method was used to determine the novice teachers that set up the groups to be video recorded. For the consistency of the research, two different instructors were coordinated at each stage of the research, their ideas were consulted and their evaluations were taken into consideration.

3.5. Introduction of The Implementation

At the outset of the program, introductory courses were provided to bolster the technological acumen of teacher candidates (GeoGebra). This spanned 5 lesson hours over a two-week timeframe. Furthermore, at the course, the GeoGebra aspects that will be used regularly were presented and tasks were completed to comprehend and solidify these features. The aim of GeoGebra introductory courses is to equipt pre-service teachers with the knowledge and skills necessary to use GeoGebra accurately and effectively, thus preventing any potential data loss/error/deficiency due to a lack of understanding of the program.

The researcher also prepared an introductory presentation on critical thinking for teacher candidates, which was delivered in one lesson hour under the following headings: What is the concept of thinking? How does critical thinking differ? What are the implications of critical thinking in educational settings? What are the responsibilities of educators in fostering critical thinking skills? What part does critical thinking play in mathematics instruction? Consequently, the requisite steps for implementing the proposed instructional model were fulfilled.

The lesson plans were created to make sure we followed the steps needed to effectively put the teaching model into action. Utilizing the Content-Based Teaching Approach, a critical thinking teaching strategy, these lesson plans were created to enhance the critical thinking abilities of pre-service educators. The outcomes of the lesson plans consisted of the transformation geometry outcomes of the secondary school mathematics course. The usage of these results is meant to foster critical reasoning abilities as outlined by Facione (1990). In secondary school mathematics curricula, transformation geometry is included as an outcome in the 11th and 12th-grade mathematics courses. These outcomes are documented in Table1.

Table 1.

Secondary Education Mathematics Course Transformation Geometry Topics and Outcomes

Grade	No.	Topics	Outcomes
11th	11.3.3.	Transformations of Functions	1. Draws new function graphs from a graph of a function with the help of transformations. a) The symmetry properties of the graph of odd and even functions are emphasized. b) $y = f(x) + b$, $y = f(x - a)$, $y = kf(x)$, $y = f(k\text{x})$, $y = -f(x)$, $y = f(-x)$ the above transformations are graphed using information and communication technologies.
12th	12.4.1.	Basic Transformations on Graphs	1. Determines the coordinates of an image under various transformation types, including translation, rotation, and symmetry, for a given point on a graph. a) The concepts of translation, symmetry, and rotation are reminded. b) The symmetries of a point over; point, the axes, the y-x line, line, and the symmetries of a line over a point are emphasized. The symmetries of a line over line are not included. c) Translation, symmetry, and rotation are handled with the help of information and communication technologies. 2. Solves problems related to basic transformations and their components. a) Modeling studies are included. b) Natural and architectural examples are given.

Outcome-specific worksheets were prepared based on the information provided in Table 1. Each worksheet is intended to achieve a single goal. Table 2 indicates which lesson plan worksheets are designed for which outcomes.

Table 2.

Worksheets And Targeted Outcomes

Lesson plan	Outcome No.	Outcomes
First worksheet	11.3.3.1.a	The symmetry properties of the graph of odd and even functions are emphasized.
Second worksheet	11.3.3.1.b	$y = f(x) + b$, $y = f(x - a)$, $y = kf(x)$, $y = f(k\text{x})$, $y = -f(x)$, $y = f(-x)$ the above transformations are graphed using information and communication technologies.
Third worksheet	12.4.1.1	It is designed to teach the objectives of the outcome for the “ translation transformation”.
Fourth worksheet	12.4.1.1	It is designed to teach the objectives of the outcome towards “ rotational transformation”.
Fifth worksheet	12.4.1.1	It is designed to teach the objectives of outcome towards " symmetry transformation".

In the lecture notes, it was tried to reveal the knowledge that pre-service teachers already have such as definitions, concepts, examples, etc. for the relevant outcome (see Figure 1).

Fonksiyon çeşitlerinden çift ve tek fonksiyon kavramlarını matematiksel biçimde tanımlayınız. Birer örnek veriniz.

Figure 1. An example from the lecture notes

Figure 1. mentions the following:

Define the concepts of even and odd functions in mathematical form. Give an example.

The part of the study shown in Figure 1 aims to increase the students' interpretation and explanation skills, which are among the critical thinking sub-skills identified by Facione (1990).

After the pre-service teachers formed their own definitions, they were given formal mathematical definitions (see Figure 2) and asked to compare these definitions with their own definitions.

Yazalım:	Yazalım:
	$f: R \rightarrow R$, $y = f(x)$ fonksiyonu verilmiş olsun.
Çift fonksiyon:	Çift fonksiyon: $\forall x \in R$ için $f(-x) = f(x)$ ise f fonksiyonu çift fonksiyondur.
Tek Fonksiyon:	Tek Fonksiyon: $\forall x \in R$ için $f(-x) = -f(x)$ ise f fonksiyonu tek fonksiyondur.

Figure 2. An example from the lecture notes

Figure 2. mentions the following:

Let's write:	**Let's write:**
	$f: R \rightarrow R, y = f(x)$
Even function:	Even function: $for\ \forall x \in R\ f(-x) = f(x)$
Odd function:	Odd function: $for\ \forall x \in R\ f(-x) = -f(x)$

As a result of their comparison, they are asked to identify any missing/incorrect parts of their own statements and fill in the gaps given in the course note (see Figure 3). The aim here is to increase the self-regulation skill of pre-service teachers, one of the critical thinking sub-skills.

Varsa;

EKSİKLERİM:

....

HATALARIM:

....

Figure 3. An example from the lecture notes

Figure 3. mentions the following:

If any;

MY SHORTCOMINGS:

....

MY MISTAKES:

....

The subject is reinforced with the questions in the lecture notes (see Figure 4). At the same time, it is aimed to increase the critical thinking sub-skills of the students with the questions.

SORU 1: Bir fonksiyon mutlaka tek veya çift fonksiyon olmak zorunda mıdır? Neden? Düşüncelerinizi birkaç cümle ile ifade ediniz.

Figure 4. An example from the lecture notes

Figure 4. mentions the following:

Question 1: Does a function have to be an even or odd function? Why? Express your thoughts in a few sentences.

With Figure 4, it is aimed to increase interpretation, analysis, evaluation, inference and explanation skills. After the definitions and concepts related to the outcome were formed and reinforced in the lecture notes, it was planned to reach the main goal of the outcome with the support of technology. At this stage, "GeoGebra" programme, one of the dynamic geometry software, was used. GeoGebra applications in the lecture notes were designed to include steps to increase the critical thinking skills of pre-service teachers. In the lecture notes, GeoGebra applications were included as activities. Commands were given in an understandable way within the activity. The pre-service teachers were asked to express the result obtained from the activity in their own words (see Figure 5).

ÖRNEK 1)

$f: R \to R$ olmak üzere;

$f(x) = x^3 - 2x$ olsun.

UYGULAMA 1)

1) **GeoGebra** programı açınız.
2) **Giriş** 'e $\mathbf{x^3 - 2x}$ yazılarak **Enter** tuşuna basınız.
3) **Nokta** ikonuna tıkladıktan sonra grafiğin üzerinde herhangi bir yere tıklayarak **A** noktasını oluşturunuz.
4) **Giriş** bölümüne **yansıt** yazdıktan sonra oluşan satırda **nesne, nokta** yerlerine sırasıyla **A, (0,0)** yazınız ve **Enter** tuşuna basınız. **A'** noktası oluşacaktır.
5) **İmleç** ile **A** noktasını sürüklediğinizde **A'** noktası da hareket edecektir. Bu hareketi gözlemleyiniz. Bu hareketi daha iyi gözlemleyebilmek için **Doğru Parçası** ikonuna, ardından **A** ve **A'** noktalarına tıklayarak doğru parçasını çiziniz. **İmleç** ile **A** noktasını sürükleyerek doğru parçasının hareketini gözlemleyiniz.

Uygulama sonunda elde ettiğim sonuçlar...
....
.... şeklindedir.

Figure 5. An example from the lecture notes

Figure 5 mentions the following:

EXAMPLE 1)

Let $f: R \rightarrow R$ be a function such that $f(x) = x^3 - 2x$.

EXERCISE 1)

1) Open the GeoGebra program.
2) Type $x^3 - 2x$ in input and press Enter.
3) After clicking the Point icon, click anywhere on the graph to create point A.
4) After typing reflect in the input section, write A(0,0) for the object and point inputs, and press the Enter key. A' point will be formed.
5) When you drag point A with the cursor, point A' will also move. Observe this movement. To observe this movement better, click on the Segment icon, then click on points A and A' to draw a segment. Observe the movement of the line segment by dragging point A with the cursor.

The results I got at the end of the exercise are..

Figure 5 aims to increase pre-service teachers' critical thinking sub-skills of interpretation, analysis, evaluation, inference and explanation.

The utilization of GeoGebra, a powerful dynamic geometry implementation, was demonstrated in the implementations. Lesson plans are essential when it comes to implementing teaching models, as they provide a reliable guide for achieving research objectives. The plans were formulated to provide direction to prospective teachers throughout the enrollment phase and to assemble research-focused information from potential educators. The photos of the instructional model platform environment are depicted in Figure 6 and 7 below.

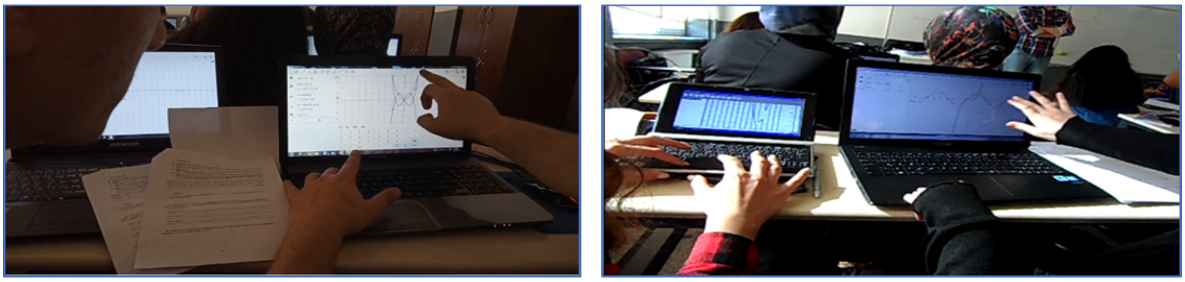

Figure 6. The teacher candidates use GeoGebra

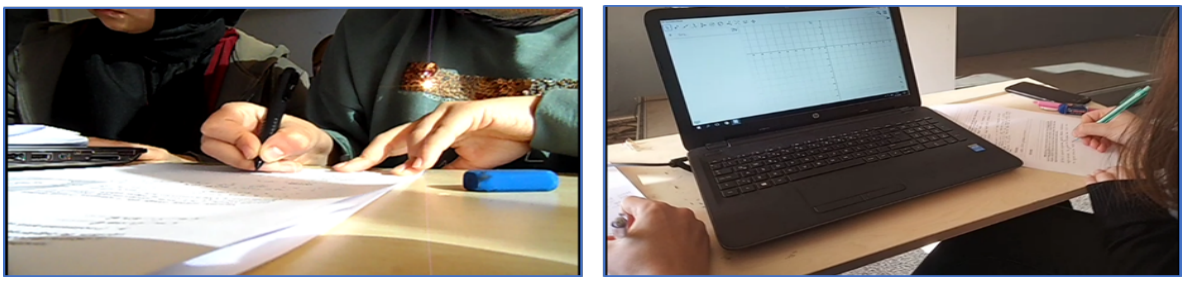

Figure 7. Pre-service teacher fill in the worksheets

Through the worksheets of the lesson plans were conducted on a weekly basis from the third week to the seventh week, the pre service teachers were captured on video in order to analyze the dialogues and behaviors of the pre-service teachers.

4.1. Findings

This section includes the evaluation of critical thinking sub-skills through activities.

a. First Worksheet Implementation 1

ÖRNEK 1)

$f: R \to R$ olmak üzere;

$f(x) = x^3 - 2x$ olsun.

UYGULAMA 1)

1) **GeoGebra** programı açınız.
2) **Giriş** 'e $\mathbf{x^3 - 2x}$ yazılarak **Enter** tuşuna basınız.
3) **Nokta** ikonuna tıkladıktan sonra grafiğin üzerinde herhangi bir yere tıklayarak **A** noktasını oluşturunuz.
4) **Giriş** bölümüne **yansıt** yazdıktan sonra oluşan satırda **nesne, nokta** yerlerine sırasıyla **A, (0,0)** yazınız ve **Enter** tuşuna basınız. **A'** noktası oluşacaktır.
5) **İmleç** ile **A** noktasını sürüklediğinizde **A'** noktası da hareket edecektir. Bu hareketi gözlemleyiniz. Bu hareketi daha iyi gözlemleyebilmek için **Doğru Parçası** ikonuna, ardından **A** ve **A'** noktalarına tıklayarak doğru parçasını çiziniz. **İmleç** ile **A** noktasını sürükleyerek doğru parçasının hareketini gözlemleyiniz.

Uygulama sonunda elde ettiğim sonuçlar..
....
.... şeklindedir.

Figure 8. First Worksheet Implementation 1

Figure 8 mentions the following:

EXAMPLE 1)
Let $f: R \to R$ be a function such that $f(x) = x^3 - 2x$.
EXERCISE 1)
1) Open the GeoGebra program.
2) Type $x^3 - 2x$ in input and press Enter.
3) After clicking the Point icon, click anywhere on the graph to create point A.
4) After typing reflect in the input section, write A(0,0) for the object and point inputs, and press the Enter key. A' point will be formed.
5) When you drag point A with the cursor, point A' will also move. Observe this movement. To observe this movement better, click on the Segment icon, then click on points A and A' to draw a segment. Observe the movement of the line segment by dragging point A with the cursor.

The results I got at the end of the exercise are..

The following sentences from the dialogues of two pre-service teachers, whose videos were analyzed, in the first worksheet for implementation 1 demonstrate that they use their critical thinking skills:

S1: …The same graph will appear again. That is, on the y-axis. Relative to the y-axis in an even function, relative to the x-axis in an odd function…

S1: …Do you know how we can do it? We do $-x$ instead of x. We write $(-3)^3 + 2.(-x)$ and we draw its graph. We draw two graphs...

Interpretation: The dialogues above suggest that pre-service educators are cognizant of the issue, can interpret the data, and express their ideas clearly. Since these indicators include the keywords of "interpretation", one of the sub-skills identified by Facione (1990), it was interpreted that implementation 1 in Figure 8 in the first worksheet contributed to the interpretation skill, one of the critical thinking sub-skills of pre-service teachers.

S1: "…Something wrong came from there. The odd function should be relative to the origin. Wrong. Minus."

Analysis: From S1's sentence, it can be thought that the student was able to compare the disparate approaches presented for resolution of an issue. Since this includes the keywords of the "analysis" skill, one of the sub-skills identified by Facione (1990), it was interpreted that implementation 1 in Figure 8 in the first worksheet would contribute to the analysis skill, one of the critical thinking sub-skills, of pre-service teachers.

S1: … "Look, but we need to examine it from here… Right, relative to the origin."

Evaluation: From S1's sentence, it can be said that the student evaluated the relevance of the existing claim to the present situation. Since this situation includes the keywords associated with the "evaluation" skill, as determined by Facione (1990), it was interpreted that implementation 1 in Figure 8 in the first worksheet would contribute to the development of evaluation skill, one of the critical thinking sub-skills, of pre-service teachers.

S1: … "This is what it means to be an odd function. Did you understand?"

Inference: From the above sentence, it can be stated that after the drawings in GeoGebra, S1 made a conclusion from the data they obtained by telling the result to S2. Since this includes the keywords of "inference", one of the sub-skills identified by Facione (1990), it was interpreted that implementation 1 in Figure 8 in the first worksheet would contribute to the ability of pre-service teachers to make inferences from critical thinking sub-skills.

S2: …"There is a reflection over x in odd functions. Even functions over y."

S1: "I wrote $y = x^2$. (to S2) Wait. He will understand. $y = x^2$this. Let's substitute $-x$ for x. We got the one with minus again $y = x^2$."

Explanation: This dialog indicates that pre-service teachers typically describe consequences and demonstrate the justification for something through compelling outcomes. Since this includes the keywords of "explanation" skill, one of the sub-skills identified by Facione (1990), it was interpreted that implementation 1 in Figure 8 in the first worksheet would contribute to the skill of explanation, one of the critical thinking sub-skills of pre-service teachers.

S1: "I understand. We drew the graph wrong. Right."

Self-regulation: S2 and S1 were able to arrive at the correct conclusion by correcting the error in the function and redrawing the graph. In this sentence, S1 was able to question their own judgment, make self-examination, and self-correction. Since this includes the keywords of "self-regulation", one of the sub-skills identified by Facione (1990), it was interpreted that implementation 1 in Figure 8 in the first worksheet would contribute to self-regulation skill, one of the critical thinking sub-skills of pre-service teachers.

Considering the whole of the analyzed dialogue, it can be concluded that implementation 1 in the first worksheet contributes to all of the critical thinking sub-skills.

b. First Worksheet Question 6

SORU 6:

- $k(x) = \frac{sin^2x}{cosx+1}$
- $m(x) = -x^3 + 3x + sinx$
- $n(x) = \frac{\sqrt[4]{x^2+1}}{x^3-5x}$
- $p(x) = \frac{lnx}{2^x-3x}$
- $r(x) = 5x^2 - 3x + 2$
- $s(x) = \frac{\sqrt{x^2+1}}{x^4}$

Yukarıda verilen fonksiyonların teklik-çiftlik durumlarını GeoGebra kullanarak belirleyiniz ve elde ettiğiniz sonuçları not ediniz.

Figure 9. First worksheet question 6

Figure 9 mentions the following:

QUESTION 6)

- $k(x) = \frac{sin^2x}{cosx+1}$
- $m(x) = -x^3 + 3x + sinx$
- $n(x) = \frac{\sqrt[4]{x^2+1}}{x^3-5x}$
- $p(x) = \frac{lnx}{2^x-3x}$
- $r(x) = 5x^2 - 3x + 2$
- $s(x) = \frac{\sqrt{x^2+1}}{x^4}$

Determine the odd-even states of the above functions using GeoGebra and note your results.

The following sentences from the dialogues of two pre-service teachers, whose videos were analyzed, in the first worksheet for question 6 demonstrate that they use their critical thinking skills:

S1: By showing the graphics they drew on the GeoGebra, "See, these functions are reflected over the origin."

S2: "Exactly."

S1: "Direct origin."

S 2: "Normally, for example, I couldn't see it, frankly."

It is clear from the statement "Normally, for example, I couldn't see it, frankly." that the pre-service teacher is aware of the results obtained in the solution of question 6 in Figure 9 and can make self-examination and self-correction. Since this situation includes the keywords of "self-regulation", one of the sub-skills identified by Facione (1990), it was interpreted that question 6 in Figure 9 would contribute to self-regulation skill, one of the critical thinking sub-skills.

c. Second Worksheet Implementation 3

UYGULAMA 3) $y = k.f(x)$ dönüşümü

➢ $f(x) = x^2 - x$ olsun.

1) GeoGebra programını çalıştırınız sağ tarafta açılan pencereden **Grafik Çizme** seçeneğini seçiniz.
2) **Sürgü** aracını seçiniz. Açılan pencerede **k** isimli bir sürgü oluşturup minimum değeri **-10**, maksimum değeri **10** ve artış miktarını **0.1** olarak tanımlayınız.
3) **Giriş** kısmına $x\text{^}2 - x$ yazıp **Enter** tuşuna basınız.
4) **Giriş** kısmına $k.(x\text{^}2 - x)$ yazıp **Enter** tuşuna basınız.
5) Oluşturduğunuz sürgüdeki noktayı sağa ve sola hareket ettirerek fonksiyonun grafiğindeki değişimleri inceleyiniz.

Uygulama sonunda elde ettiğim sonuçlar.. şeklindedir.

Figure 10. Second Worksheet Implementation 3

Figure 10 mentions the following:

IMPLEMENTATION 3)
$y = k.f(x)$ transformation
➢ Let $f(x) = x^2 — x$.
1) Run the GeoGebra program and select the Graphing option from the window on the right.
2) Select the Slider tool. In the window that opens, create a slider named k and define its minimum value as -10, maximum value as 10, and increment amount as 0.1.
3) Type x^2 — x in the input field and press Enter.
4) Type k.(x^2 — x) in the input field and press Enter.
5) Examine the changes in the graph of the function by moving the point on the slider you created to the right and left.
The results I got at the end of the exercise are ....

When the pre-service teachers came to the part where they had to write the result, they obtained at the end of implementation 3 in Figure 10:

S1: "I cannot put this into a sentence."

S2: "I can't transfer it right now either."

S1: "We couldn't."

The above dialogue took place.

After the result to be obtained at the end of the implementation was reflected on the board by the trainer, one of them expressed the sentence below:

S1: “But I will note it in my mistakes/shortcomings section.”

When the dialogue of exercise 3 in Figure 10 in the second worksheet was examined, it showed that pre-service teachers could make self-examination and self-correction. From this point of view, it was interpreted that the implementation in figure 10 contributed to self-regulation skills.

d. Second Worksheet Implementation 5

UYGULAMA 5) $\boldsymbol{y = -f(x)}$ **dönüşümü**

- $f(x) = x^2 - 2x$ olsun.
- $h(x) = 3x + 5$ olsun.

1) GeoGebra programını çalıştırınız sağ tarafta açılan pencereden Grafik Çizme seçeneğini seçiniz.
2) Girişe $\boldsymbol{x\hat{}2 - 2x}$ yazıp **Enter** tuşuna basınız.
3) **Girişe** $-f$ yazıp **Enter** tuşuna basınız.
4) **Giriş** kısmına $\boldsymbol{3x + 5}$ yazıp **Enter** tuşuna basınız.
5) **Girişe** $-h$ yazıp **Enter** tuşuna basınız.
6) Oluşan grafikleri inceleyiniz.

Uygulama sonunda elde ettiğim sonuçlar..
.... şeklindedir.

Figure 11. Second worksheet implementation 5

Figure 11 mentions the following:

IMPLEMENTATION 5)
$y = -f(x)$ transformation
Let $f(x) = x^2 — 2x$.
Let $h(x) = 3x + 5$.
1) Run the GeoGebra program and select the Graphic Drawing option from the window on the right.
2) Type x^2 — x in the input field and press Enter.
3) Type $-f$ in the input field and press Enter.
4) Type $3x + 5$ in the input field and press Enter.
5) Type $-h$ in the input field and press Enter.
6) Observe the generated graphics.
The results I got at the end of the implementation are ....

Dialogues examined related to the implementation given in Figure 11 are as follows:

S2: “I think there is a reflection about (0,0) again?”

S1: “No. Because $3x + 5$ is neither odd nor even function.”

S2: “I agree.”

S1: “Therefore, when minus “ -“ is written before it, there is no reflection over any axis. Let's look at $x^2 + 2x$. Let's see. It is reflected over x.”

S2: “Yes.”

S1: “Is it reflected?”

S2: “Yes.”

S1: “It is reflected over the origin. Not x.” by showing the graph of the $3x + 5$ function to S2 on the GeoGebra screen (see Figure 12)

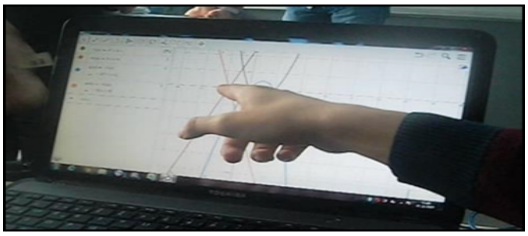

Figure 12. Graphics on the GeoGebra screen

S2: “Here, too, there is a reflection over x.”

S1: “You say that? Of course, it is. What happens when you write the function negative, the y's are negated.”

S2: “So, reflection over x.”

S1: “Exactly.”

S2: “When we multiply the function by minus, we get a reflection over x.”

“None. Because neither odd nor even function.” sentence shows that S1 can categorize even and odd functions. “Therefore, when minus “ -“ is written before it, there is no reflection over any axis.” statement indicates that S1 can present the reason for something in a convincing and consistent way. “You say that? Of course, it is. What happens when you write the function negative, y's are negated.” explanation shows that S1 can draw conclusions and meanings from the data. Therefore, it was interpreted that implementation 5 in Figure 11 in the second worksheet contributed to the skills of interpretation, explanation, and inference.

e. Second Worksheet Question 3

SORU 3:

- $f(x) = sin(x)\ iken\ g(x) = sin(x + a) + b$,
- $k(x) = 2^x\ iken\ l(x) = 2^{x+a} + b,$
- $m(x) = \frac{1}{x}\ iken\ n(x) = \frac{1}{x+a} + b,$
- $s(x) = \sqrt[n]{x}\ iken\ v(x) = \sqrt[n]{x + a} + b,$
- $a(x) = \ln(x)\ iken\ b(x) = \ln(x + a) + b$

Yukarıda $f, k, m, s\ ve\ a$ fonksiyonları ve bu fonksiyonlardan x ekseni üzerinde a ve y ekseni üzerinde b birimlik ötelemeler sonrasında oluşan $g, l, n, v\ ve\ b$ fonksiyonları verilmiştir.

1) Sizce ders içerisinde elde ettiğimiz sonuçlardan olan;

- $f(x - a)$ fonksiyonunda a pozitif ise $f(x)$ fonksiyonun grafiği x ekseni boyunca a birim sağa, a negatif ise $f(x)$ fonksiyonu x ekseni boyunca $|a|$ birim sola ötelenir.

- $f(x) + b$ fonksiyonunun grafiğinde $b > 0$ ise $f(x)$ fonksiyonunun grafiği y ekseni boyunca b birim yukarı, $b < 0$ ise $f(x)$ fonksiyonunun grafiği y ekseni boyunca $|b|$ birim aşağı ötelenerek $y = f(x) + b$ fonksiyonunun grafiği elde edilir.

sonuçları yukarıdaki fonksiyonlarda da sağlanır mı veya hangilerinde sağlanır hangilerinde sağlanmaz? Düşüncelerinizin sebeplerini matematiksel olarak açıklayınız. (Bu soruyu GeoGebra kullanmadan cevaplayınız)

Figure 13. Second worksheet question 3

Figure 13 mentions the following:

QUESTION 3:
$g(x) = sin(x + n) + b$ when $f(x) = sin(x)$,
$l(x) = 2^{x+a} + b$ when $k(x) = 2^x$,
$n(x) = \frac{1}{x+a} + b$ when $m(x) = \frac{1}{x}$,
$v(x) = \sqrt[n]{x + n + b}$ when $s(x) = \sqrt[n]{x}$,
$b(x) = ln(x + a) + b$ when $a(x) = ln(x)$
Above are the f, k, m, s and a functions and the g, l, n, v and b functions formed after translated b unit on the x-axis and a unit on the y-axis.

1) If a is positive in f(x - a), the graph of f(x) is translated a unit right along the x-axis, and if a is negative, the graph of f(x) is translated |a| unit left along the x-axis.

- In the graph of the function f(x) + b, if b > 0, the graph of the function f(x) is translated b unit up along the y-axis, and if b < 0, the graph of the function f(x) is translated |b| unit down along the y-axis. So the graph of the function y = f(x) + b is obtained.

- Do you think that the results we obtained in the course are also provided in the above functions, or in which of them they are provided and which ones are not? Explain the reasons for your thoughts mathematically. (Answer this question without using GeoGebra)

Below are the dialogues that took place while question 3 in Figure 13 in the second worksheet was being solved by the pre-service teachers.

S1: “Is this valid for $sin(x)$ now? $sin(x + a)$. For example, let $x = 20$. Plus 3. In this case it is cancelled. It doesn't work here. Because it becomes $sin23$. It would be b unit though. Because it's independent. For example , $sin(30 + 5)$. $sin\ 30 = 1/2$ part b of the question is ok. There is no problem with that.”

S2: “Why did you say that “the $f(x)$ function is not possible” ?”

S1: “Because look. For example, let $sin\ (x)$ be $x = 20$.”

S2: “Okay.”

S1: “When I give $a = 3$, it should be translated 3 units to the right between sin 20 and sin 23, but there is no such thing.”

S2: “Exactly”

S1: “0.01 unit maybe. But it is correct for b unit. Since b is independent, it provides the value of b. it does not provide for a.”

In the ongoing dialogue, they continued with $l(x) = 2^{x+a} + b$ when $k(x) = 2^x$ from the same question.

S1: “ a is still not possible when 2^x, but b works.”

S2: “b is ok, but why is a not working?”

S1: Look, now let $x = 2$. It's going to be 4. Let $a = 1$ it becomes 2^3 and 8. It increases 4 units.

S2: “But there is a case of going up - down and left - right on the graph. I think it will work on the graph. Draw the chart. (speaking of GeoGebra) Let's see, with the app.”

Among the expressions in the dialogue of question 3 in Figure 13, “… But it is correct for b unit. Since b is independent.” shows that the participant presents the cause of something through the demonstration of compelling, consistent outcomes. It is seen that generate visualizations demonstrating the associations between concepts and ideas “But there is a case of going up - down and left - right on the graph. I think it will work on the graph. Draw on the graph.”. Since these dialogues contain the keywords of " explanation " skill, one of the sub-skills identified by Facione (1990), it was interpreted that question 3 in Figure 5 would contribute to teacher candidates' skill of explanation, one of the sub-skills of critical thinking.

SORU 4: Analitik düzlemde, $A(x, y)$ noktasının $P(1, 5)$ noktasına göre simetri dönüşümü altındaki görüntüsü $A'(5, -2)$ noktasıdır. Buna göre x ve y değerlerini GeoGebra programı yardımıyla bulunuz.

Figure 14. Fifth worksheet question 4

Figure 14 mentions the following:

QUESTION 4: On the plane, the image of the point A(x,y) under the symmetry transformation with respect to the point P(1,5) is the point A'(5, - 2). Accordingly, find the x and y values with the help of GeoGebra program.

S1: After reading the question, showed hand gestures and said, "If a point is reflected about a point, this point is the reflection of that point respect to that point."

S2: “Exactly. If the reflection of point A with respect to point B is point C, the reflection of point C with respect to point B is point A.”

The dialogue above reveals that question 4 in Figure 14 helped improve interpretation skill, since S1 and S2 were able to deduce meaning from it and explain it in their own words.

4.2. Synthesis

The descriptive analysis method was utilized to analyze the video recording data and the verity on the effects of the designed teaching model process on critical thinking skills were included. The synthesis of the obtained findings is expressed in the following tables.

Table 3.

The Synthesis Of The Obtained Findings

Theme	Data Place	Code	Teacher candidate dialogues
Interpretation	First Worksheet Implementation 1	Recognizing the problem. Interpreting the data. Expressing their ideas in their own words.	S1: *...Do you know how we can do it? We do $-x$ instead of x. We write $(-3)^3+2.(-x)$ and we draw its graph. We draw two graphs...* S1: *...The same graph will appear again. That is, on the y-axis. Relative to the y-axis in an even function, relative to the x-axis in an odd function…*
	Second worksheet Implementation 5	Drawing conclusions or meaning	S1: *You say that? Of course, it is. What happens when you write the function negative, the y's are negated.*
	Fifth worksheet question 4	Inferring and coding	*S1: If a point is reflected about a point, this point is the reflection of that point respect to that point..* *S2: Exactly. Point A respect to point B*
Analysis	First Worksheet Implementation 1	Revealing the similarities and differences between two different approaches given as a solution to a problem.	S1: *Something wrong came from there. The odd function should be relative to the origin. Wrong. Minus.*
Evaluation	First Worksheet Implementation 1	Relevance of existing claim to current situation.	*S1: Look, but we need to examine it from here… Right, relative to the origin.*
Inference	First Worksheet Implementation 1	Drawing conclusions or meaning from data, concepts, questions, and definitions.	S1: *This is what it means to be an odd function. Did you understand?*
Explanation	First Worksheet Implementation 1	Presenting reasons in a form of convincing and consistent results.	T2: *There is a reflection over x in odd functions. Even functions over y.* T 1: *... I wrote $y = x^2$. (to S2) Wait. He will understand.*
	Second worksheet Implementation 5	Presenting reasons in a form of convincing and consistent results.	S1: *Therefore, when minus “ - “ is written before it, there is no reflection over any axis.*

	Second worksheet question 3	Presenting reasons in a form of convincing and consistent results	S1: … but it becomes b unit. Because it is independent.
	Second worksheet question 3	Creating graphs that show connections between concepts and ideas	*S2: But there is a case of going up - down and left - right on the graph. I think it will work on the graph. Draw on the graph…*
Self-regulation	First Worksheet Implementation 1	Questioning their own judgments; self-examination and self-correction.	S1: *I understand. We drew the graph wrong. Right.*
	First worksheet question 6	Monitoring the achieved results with awareness.	S2: *Normally, for example, I couldn't see it, frankly.*
	Second worksheet implementation 3	Sensitively examining issues such as personal bias and interest	*S1: We could not make it … But I will note it in my mistakes/shortcomings section.*

The video recordings were analyzed within the context of the above dialogues, which were considered to be related to critical thinking skills. The dialogues were assessed according to the keywords/sentences of critical thinking sub-skills, and conclusions were drawn about the teaching model implementations.

5. Conclusion and Discussion

Upon analyzing the video recordings of the pre-service teachers, it was noted that they used; all of the critical thinking sub-skills in the first worksheet implementation 1, self-regulation in the first worksheet question 6, self-regulation in the second worksheet implementation 3, inference and explanation in the second worksheet implementation 5, interpretation in the fifth worksheet question 4, and inference in the fifth worksheet question 6. The designed teaching model implementations appear to incorporate activities that facilitate critical thinking, foster the utilization of critical thinking abilities among pre-service teachers, and as a result, support the improvement of critical thinking abilities.

This research found that similarly to Suh (2010), the use of technology can create environments that are conducive to the advancement of students' analytical thinking abilities in mathematics. In Suh's study, data were obtained from fifth and sixth grade students. In addition, the study was conducted on a subject that the student group had not been taught before. In this respect, this study can be applied to younger age groups or to the same pre-service teachers on subjects they have not learned before.

The results of this investigation were consistent with those of Peter (2012), who found that educators should demonstrate the thought process, ask pertinent questions, and guide learners in their critical thinking to enhance their ability to think critically. As a situation that does not overlap with Peter's study, it can be said that the pre-service teachers in this study stated that they started the training with prejudice, but they enjoyed it afterwards.

Our research corroborates discoveries of Obay (2009), who utilized problem-solving as a technique to facilitate the learning of critical thinking. Obay (2009) also worked with pre-service elementary mathematics teachers in his study. The observation of the development of critical thinking skills in pre-service teachers using a different method from this study can be considered as an indicator that critical thinking skills can be taught.

The research conducted echoes the results of Basri and As'ari (2018), which demonstrated that using a teaching approach that incorporates carefully constructed tasks (questions) can help to sharpen critical thinking skills among high school students. At the end of their study, Basri and As'ari (2018) advised lecturers, teachers and researchers to give assignments to students to improve their critical thinking skills. Since this study aims to train teachers who are conscious about critical thinking, it is a study that aims to reach the results obtained by Basri and As'ari.

6. Suggestions and Limitations

It was noted that pre-service teachers exhibited a tendency to converse with their desk companions and friends who are in front of or behind them, solicit their opinions, and discuss with them during the implementations. In conclusion, the designed lesson plans are more suitable for group work.

As the focus of this study is on developing critical thinking skills, it is expected to provide new insights and ideas for future studies in this field.

Analysis of the instructional materials indicates that the lesson plans are effective in fostering higher-order cognitive abilities, such as critical thinking and therefore future studies can be performed to assess the implications of these and analogous implementations on differing higher order cognitive abilities such as problem-solving and creative thinking in mathematics. It is believed to be imperative to apply this research to high school students and document the findings in order to contrast them. Because it is vital to illustrate the development of critical thinking skills through a lesson plan that is commensurate with or exceeds the student's proficiency.

In the first semester of the 2019-2020 academic year, the education faculty of a state university in the Marmara Region is limited to the 4th grade teacher candidates in the elementary mathematics teaching program. It is limited to the transformation geometry acquisitions in the secondary education mathematics curriculum of the Ministry of National Education.

According to Richard Paul (2007) in the related literature, becoming a fully professional critical thinker requires ten years of preparation. When evaluated from this point of view and considering that there are different perspectives in teaching critical thinking, specializing in such a subject requires a significant process in terms of time and effort. In this study, it may be among the limitations that the researcher is not fully equipped in critical thinking education.

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