Research Article

The Effect of the Conversion Coefficients of Platinum-Based Resistance Thermometers on the Uncertainty Estimation

*F. M. Patan Alper^D

Department of Physics, Yeditepe University, İstanbul, Turkey E-mail: mpatan@yeditepe.edu.tr

Received 16 December 2022, Revised 20 March 2023, Accepted 20 May 2023

Abstract

Different types of thermometers (resistance thermometers, thermocouples, liquid in glass thermometers, radiation thermometers, etc.) are used in temperature measurements. Resistance thermometers are among the most reliable types of sensors used for sensitive temperature measurements. The traceability, accuracy and precision of the measurement results are important for the reliability of the measurements. There are many parameters that affect the uncertainty estimation in measurements made with resistance thermometers. One of the parameters to be considered in the uncertainty estimation is the interpolation error in converting the resistance value to temperature. Different methods (ITS-90, Calendar Van Dusen CVD, Polynomial equation) can be used to convert the resistance value to temperature. The problem is that there are differences in the temperature values read using the coefficients obtained by different methods. In this study, the effect of errors from CVD and polynomial equation methods on measurement uncertainty was investigated.

Keywords: Resistance thermometer, Calendar van dusen, ITS-90, uncertainty estimation.

1. Introduction

Standard platinum resistance thermometers (SPRTs) are the most precise and reproducible temperature sensors, which is why they are the instruments specified for use in the ITS-90 over a wide range of temperature [1,2]. They are based on the positive temperature coefficient of electrical resistance of platinum, which is about 0.4%/°C at 0 °C. The sensing resistor is made using a fine platinum wire of diameter <0.1 mm carefully wound in a strain-free manner on a silica (quartz) glass former. It is enclosed in a long silica tube (sheath) at the head of which the connecting leads are joined to a length of flexible cable. The resistance at 0 °C is normally about 25.5 Ω , which gives a sensitivity of about 0.1 Ω /°C. SPRTs are very fragile and pure platinum is very soft. Therefore secondary standards and industrial thermometers are usually made with more rugged (stronger) sensors, using less pure platinum wires or printed platinum films. The resistance is generally 100 Ω at 0 °C, so the sensitivity is about 0.4 $\Omega/^{\circ}C$, and they are protected by stainless-steel metal sheaths. They are often called Resistance Temperature (RTDs). Industrial Platinum Detectors Resistance Thermometer (IPRT) or Pt-100s. Platinum is almost invariably used as the sensing resistor in resistance thermometry, because it has a good temperature coefficient, it can operate to very high temperatures (in special applications up to 962 °C) and it is resistant to chemical attack, particularly oxidation. However, it is expensive and sometimes copper or nickel are used, for reasons of cost [3,4].

BIPM, IEC, ISO, and some other bodies [11]. The EA guide EA-4/02 • Expression of the Uncertainty of Measurement in Calibration is adapted from the GUM with

According to ITS 90, platinum resistance thermometers are calibrated at fixed points in their temperature range. Calibration results are converted from resistance to temperature for user convenience. When the calibration is done at fixed points, unit conversion can be done by using the ITS-90 calculation. Alternatively the transformation can be realized using Calendar Van Dusen (CVD) or Polynomial equation and the calibration is usually done with comparison method in less costly baths and ovens instead of fixed points [5,6]. In the literature, the conversion coefficients obtained from the measurement results are evaluated by different methods. In a group of studies, interpolation results from polynomial equation results were evaluated [7,8]. In CVD evaluation method, the results of the matrix method are used for uncertainty estimation [9,10].

It is important to recognize that uncertainties are estimated, rather than calculated. The estimate is made using experimental and other available information, but in practice the evaluation usually includes significant elements of subjective assessment. The result is always open to question, by the individual, colleagues, managers, assessors, customers, etc. Nevertheless, the uncertainty budget demonstrates the extent to which the measurement process has been critically analyzed and understood. It also shows clearly which sources of uncertainty are most significant, and hence should be improved if the uncertainty is to be reduced. In recent years uncertainty estimation and analysis has been based on the 'GUM', the ISO Guide to the Expression of Uncertainty in Measurement, which was issued in 1993 by reference to calibration and includes several worked examples [12].

The level of uncertainty depends on the equipment and procedures used in the measurement and calibration. All components of uncertainty should be evaluated and combined, to reach an overall calibration uncertainty at each temperature. In this study, the contribution from the conversion coefficients obtained from different methods that influence the uncertainty will be evaluated in order to reduce the measurement uncertainty.

2. Experiment

This study investigates the effect on the uncertainty of using the different methods for deriving the conversion coefficients for resistance thermometer calibration data in the temperature range -40 to 500 $^{\circ}$ C.

2.1 Apparatus

Platinum resistance thermometer (Pt-100, Fluke 5626), Super thermometer (Hart scientific 1595A), Alcohol Bath (Fluke 7341), Water Bath (Hart Scientific 7037), Oil Bath (Hart Scientific 6022), Salt Bath (Hart Scientific 6055)

2.2 Method

The calibration method of a thermometer is based on comparison measurement. The process of comparison method: thermometers are calibrated with reference or standard thermometers, in thermally stabilized liquid baths or furnace. The temperature sensors are connected to the Model 1575 A bridge which is used for the measurements. The ice point is prepared and before calibration starts, the value of ice point of test thermometer is measured. The calibration is done from the lowest temperature towards 0 °C, and then from the highest temperature value down towards 0 °C. The thermometers are immersed at least 25 cm into liquid. Because the thermometers' immersion depth is important, it is noted in the certificate. The use of metal blocks in liquid baths increases homogeneity and decreases the uncertainty. At each temperature, when the bath is stable, the reference and test thermometers are measured alternately. At least 10 values are taken with 10 second intervals for every measurement point. The order of measurement is as reference - test - reference, etc. After taking measurements at all the calibration temperatures, the ice point value is again measured.

The difference between ice point values that are taken at the start and end of calibration is used to estimate the uncertainty components for the thermometer hysteresis and stability.

In the calibration of platinum resistance thermometers (PRT) by comparison measurement in baths, 3 methods can be used to construct the resistance curve against temperature: CVD (A, B, C coefficients), CVD (Alpha, beta, delta coefficients) and Polynomial equations coefficients (fitting a polynomial equation to the calibration data using a least-squares regression routine: i.e. curve-fitting, which minimizes the sum of the squares of the residuals (differences between the curve and the data). The RMS (root mean square) deviation decreases as the order of the fit increases.

In this study, the calculation of the CVD A,B,C and polynomial coefficients and the effect of the interpolation on the measurement uncertainty estimation will be discussed.

The CVD equations are widely used with industrial PRTs shown below.

For t > 0 °C:

$$R_t = R_\theta \left(1 + At + Bt^2 \right) \tag{1}$$

For t < 0 °C:

$$R_t = R_0 \left(1 + At + Bt^2 + C \left(t - 100 \right) t^3 \right)$$
(2)

The other method to be compared with the CVD method is the polynomial equation. Using polynomial equations, a temperature-resistance curve or table can be created for temperature sensors. In this method, better results can be obtained by increasing the degree of the equation. Choosing the optimum degree of fit is a compromise between reducing the residuals sufficiently, but not over-fitting the data, which happens if the degree approaches the number of points (the curve begins to fit the errors – i.e. oscillate so as to pass more closely though the points.

$$t(R)[^{\circ}C] = A_0 + A_1R + A_2R^2 + A_3R^3 + \dots + A_5R^5$$
(3)

3. Measurements and Results

Before calculating the conversion coefficients, the calibration of the thermometer to be tested was carried out with a traceable reference thermometer in accordance with international calibration procedure as specified in the method section. The measurement results are given in Table 1.

Calibration outputs are degrees Celsius (°C) for the reference thermometer, resistance (Ω) for the test thermometer. The ice point value of the test thermometer will be recorded as R_o, and the test thermometer value at any temperature point will be recorded as R_t.

The first method to find the coefficients is CVD method, coefficient calculation (A,B,C) was obtained using Matlab software. The second method was to find the CVD coefficients with the Matrix method using the Excel program. Finally, the coefficients (A₀, A₁, A₂, A₃, A₄, A₅) were calculated with the 5th degree polynomial equation. The coefficients obtained by the 3 methods are given in Table 2.

Table 1. Calibration Result of Test Thermometer.

Reference Therm./ °C	Test Therm. / $\boldsymbol{\Omega}$
-39.82	84.3475
-19.81	92.2530
0.00	100.0200
50.17	119.5132
90.18	134.8346
150.04	157.3988
235.01	188.7495
349.69	229.7272
449.51	264.1700

Table 2. Resistance-Celsius conversion coefficients.

CVD Matlab	CVD Excel	Polynomial
A: 3.91160E-03	A: 3.91183E-03	A0:1.0003E+02
B: -5.83748E-07	B: -5.84033E-07	A1:3.9131E-01
C: 2.55447E-14	C: 2.53484E-14	A2:-6.0826E-05
		A3:1.5665E-08
		A4:-3.5604E-11
		A5:3.0860E-14

The coefficients were put into the CVD and polynomial equations and the interpolation residuals were estimated from the differences between the resistance values calculated from the coefficients and the measured resistance values. Measured test thermometer value, calculated test thermometer value and difference values to be used as interpolation error for each method used are given in the Tables 3,4,5.

Table 3. CVD Results from MATLAB data.

Reference	Test	Test Therm.	Residual
Therm./ °C	Therm./\2	Matlab/ Ω	m°C
-39.82	84.3475	84.3473	1
-19.81	92.2530	92.2452	20
0.00	100.0200	100.0200	0
50.17	119.5132	119.5032	25
90.18	134.8346	134.8259	22
150.04	157.3988	157.4056	17
235.01	188.7495	188.7437	14
349.69	229.7272	229.7212	15
449.51	264.1700	264.1676	6
499.33	280.9500	280.9457	11

Table 4. CVD Results from Excel program matrix method.

	, , , , , , , , , , , , , , , , , , ,		
Reference Therm./ °C	Test Therm./Ω	Test Therm. Excel/ Ω	Residual m°C
-39.82	84.3475	84.3463	3
-19.81	92.2530	92.2447	21
0.00	100.0200	100.0200	0
50.17	119.5132	119.5043	22
90.18	134.8346	134.8277	17
150.04	157.3988	157.4084	24
235.01	188.7495	188.7476	5
349.69	229.7272	229.7256	4
449.51	264.1700	264.1717	4
499.33	280.9500	280.9493	2

Table 5. Results from 5th degree polynomial equation.

Reference	Test	Test Therm.	Residual
Therm./ °C	Therm./Ω	Excel/ Ω	m°C
-39.82	84.3475	84.3467	2
-19.81	92.2530	92.2500	7
0.00	100.0200	100.0274	19
50.17	119.5132	119.5100	8
90.18	134.8346	134.8295	13
150.04	157.3988	157.4060	18
235.01	188.7495	188.7463	8
349.69	229.7272	229.7279	2
449.51	264.1700	264.1700	0
499.33	280.9500	280.9500	0

3.1 Uncertainties

International standards (the 'GUM', the ISO Guide to the Expression of Uncertainty in Measurement and the EA guide EA-4/02 Expression of the Uncertainty of Measurement in Calibration) were used to calculate the measurement uncertainty. All parameters affecting the measurement were analysed, given in Table 6. Uncertainties may vary depending on the calibration method and system used. The error from interpolation, one of the uncertainty contributions, was evaluated separately and its effect on the measurement uncertainty was analysed. In the estimation of uncertainty,

the coverage factor is usually k = 2 for the 95% confidence interval.

Table 6. Uncertainty estimation for Pt-100.

Uncertainty Component	Uncertainty	Statistical Distribution	Standard Uncertainty (°C)
Ref. Therm. Drift	0.00002 Ω	Rectangular	1.16E-04
Ref. Therm. Uncertainty	0.002 °C	Normal	0.001
Ref. Therm. Stability	0.00025 °C	Normal	0.00025
Indicator Uncertainty	$0.00004 \ \Omega$	Normal	0.0002
Indicator Resolution	0.00001 °C	Rectangular	5.78E-06
Indicator Drift	1.35E-04 Ω	Rectangular	0.00135
External Resistance	0.000012Ω	Normal	5.50E-05
Self-Heating Bath Stability	0.0000015 Ω 0.001 °C	Rectangular Normal	8.67E-06 0.001
Bath Uniformity	0.001 °C	Rectangular	0.001156
Ice Point	0.0044 °C	Normal	0.0022
Test Therm. Resolution	0.000001 Ω	Rectangular	5.78E-06
Test Therm. Stability	0.0001 Ω	Normal	0.001
Test Therm. Hysteresis	0.0002 Ω	Rectangular	0.001156
Interpolation	0.002 °C	Normal	0.002
Total Uncertainty		0.0037	
Expanded Uncertainty (k=2 %95 confidence level)		0.007	

The effects from interpolation were considered in the measurement uncertainty calculation, and it was evaluated how the uncertainty values changed when different programs were used. It has been concluded that the uncertainty value can be underestimated with the data obtained from which program.

Using a mathematical package such as Excel or Matlab, no significant difference was observed between these calculations as they used the same curve-fitting algorithms. However, making the coefficient calculation with the help of a more practical and fast polynomial equation provides significant convenience to the user. When the results obtained by the CVD method and the Polynomial equation are compared with each other, it is seen that the error from the curve fitting algorithm does not significantly increase the measurement uncertainty. In addition to the studies, the results obtained using the 5th polynomial equation wererepeated with the 4th degree polynomial equation. The results obtained from the 4th degree polynomial equation are closer to the measured values, but far from the results obtained from the CVD equation, as it calculates the test value with a more rough approximation.

The studies were carried out with different resistance thermometers in order to see the error from the reproducibility parameter. In the calculations made for different thermometers, similar uncertainty values were calculated between the methods. It has been observed that if the stability and reproducibility of the thermometers are good, the uncertainty from the equations decreases.

The temperature points selected in the measurements were chosen randomly and the number of measurement points was chosen above the number of measurements sufficient to calculate the coefficients from the equations. After the coefficients were entered into the display, the thermometers were recalibrated and the measurements were taken in degrees Celsius. The measurement results at different temperatures were compared with the calculated values. These tests took place for approximately 10 thermometers.

4. Conclusion

Precise and accurate measurements in temperature measurements depend on certain conditions. It is important to analyze the measurement uncertainty completely and to carry out the measurements with the most ideal method. In this study, measurement uncertainty has been studied precisely and focused on obtaining a low interpolation error from the parameters affecting the measurement uncertainty. Resistance-Celcius conversion coefficients were calculated with 3 methods and were taken into account in the uncertainty estimation. Of the methods, MATLAB and EXCEL gave almost the same results. The lowest uncertainty is 0.007 °C, while the highest uncertainty is 0.030 °C. The lowest uncertainty values were obtained by the third method, the polynomial equation method, varying from 0.007 °C to 0.023 °C. In summary, although the polynomial equation method is not widely used compared to the CVD method, it has been determined as the method to obtain a lower uncertainty value in cases where measurement uncertainty is important.

Acknowledgements:

I wish to thank Dr. Richard Rusby from National Physics Laboratory (NPL) for help and comments. I would also like to thank Maria Aiordachioaiei for help during this study.

Nomenclature

t	temperature (°C)
R_t	resistance at temperature t (Ω)
R_0	resistance at $0 \ ^{\circ}C(\Omega)$
А, В, С	constants in CVD method
$A_{0}, A_{1}, A_{2}, A_{3}, A_{4}, A_{5}$	constants in polynomial method

References:

- [1] H. Preston-Thomas, "The International Temperature Seale of 1990 (ITS-90)," *Metrologia*, 27, 3, 1990.
- [2] J.J. Connolly, "Platinum Resistance Thermometry," Monograph II:NMI Technology Transfer Series, 2004.
- [3] T.J. Quinn, "Temperature". 2nd ed. Academic Press; pp. 1-23, 1990.
- [4] R. Rusby, "Good Practice Guide No. 125 Introduction to Temperature Measurement", NPL, 2016.
- [5] P.R.N. Childs, "6 Resistance temperature detectors," in Practical Temperature Measurement, Oxford:Butterworth-Heinemann, pp. 145-193, 2001.
- [6] Bureau International des Poids et Mesures, BIPM, Guide to the Realization of the ITS-90, Platinum Resistance Thermometry, 2018.
- [7] D.R. White, P. Saunders, "Determination of the Uncertainties for ITS-90 Realization by SPRTs Between Fixed Points," *Metrologia*, 43, 327, 2006.
- [8] D.R. White, P. Saunders, "The propagation of uncertainty with calibration equations," *Meas. Sci. Technol.* 18, 2157, 2007.
- [9] S. Dyurish, R. Palenchar, "Reduction of Measurement Uncertainty by Taking into Account Correlation in Measurements and Temperature Scale," *Realization Meas. Tech.* 49, 689, 2006.
- [10] P. Rosenkranz, "Uncertainty Propagation for Platinum Resistance Thermometers Calibrated According to ITS-90," *Int. J. Thermophys*, 32: 106-119, 2011.
- [11] Joint Committee for Guides in Metrology (JCGM), JCGM 100, Evaluation of Measurement Data-Guide to the Expression of Uncertainty in Measurement (GUM), GUM 1995 with minor corrections (JCGM, BIPM, Paris, 2008).
- [12] EA-4/02 rev.01 Expression of the uncertainty of measurement in calibration, 2013.