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Modeling of Contingent Capital Under a Double Exponential Jump-Diffusion Model with Switching Regime

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Abstract: This paper discusses a theoretical explanation that relies on investment within the framework of a regime-switching structural model whose investment cost is financed by equity and CoCos. The unexpected return of the project is governed by a continuous and temporal Markov chain. Explicit solutions have been proposed under a regime-switching structural model when the value of the cash flows generated by the firm follows a double-exponential step-distribution diffusion process. The equilibrium price theory under the jump diffusion model was developed using the structural model introduced by Leland (1994) and later extended by Kou (2002) and Chen and Kou (2009). The study focused on the influence of contingent convertibles on investment and financing policies and the inefficiencies related to debt overhang and asset substitution in the presence of an investment option.

Keywords: Contingent capital, Jump-diffusion model, Credit risk, Real option.

Introduction

Credit risk is still a major concern in both asset pricing and corporate finance. There are two basic approaches to modeling credit risk: the reduced form model and the structural model. The first model directly allocates credit with an intensity process. The default time in this model is obtained at the first jump times. This model specifies the firm value process and models the balance sheet components as contingent claims on the firm value process. However, the first jump time in structural models is specified by which, the enterprise value falls below a barrier level. Unlike intensity models, structural models are more popular for examining capital structures because they provide information about the components of the balance sheet. One of the earliest structural models dates back to Merton (1974). Later, Black and Cox (1976) extend this model by allowing default to occur before debt maturity. In this context, Longstaf and Schwartz (1995) extend the Black and Cox model by introducing interest rate risk.

In the field of corporate finance, it is well known that the capital structure depends optimally on the financial conditions of the company such as the level of cash flow generated by the company or the value of the unlevered company. Generally, financial conditions change randomly all the time, and more often than not the optimal capital structure that has been established will quickly become obsolete. In order to keep the optimal capital structure stable, one must update the capital structure dynamically and continuously. For example, if a firm is in financial distress, it must retire or issue debt to dynamically adjust the firm's leverage, see Titman and Tsyplakov (2007). Typically, such an adjustment would entail considerable adjustment costs in many situations. For example, if the firm is a small and medium-sized enterprise, the adjustment is very difficult and the adjustment costs are very high (Yang & Zhang, 2013). On the other hand, if a firm has introduced CoCo into its capital structure, to some extent, the adjustment is done automatically and usually does not involve any additional costs (Song & Yan, 2016). Goncharenko et al.Rauf (2021) provided evidence that banks are less likely to issue CoCo bonds when their asset values are volatile. In this context, Avdjiev (2020) showed that larger and better capitalized banks are more likely to issue CoCo bonds.

This paper mainly focuses on the business cycle whose value of the firm's cash flows follows an exponential double jump diffusion process. In this sense, Chen (2010) and Bhamra et al. (2010) incorporated the economic cycle into a consumption-based asset pricing model. Guo et al. (2005) addressed an irreversible investment problem related to regime change for a firm that is fully funded by equity. Barucci and Del Viva (2012a) explored the prices of CoCos, direct equity and debt, and the capital structure of the firm with a two-period model.

Yang and Zhao (2015) developed a new form of CoCo called contingent convertible securities when the value of the issuing firm follows a diffusion process with double exponential distribution jumps. However, their research does not account for the change in economic cycle. Pengfei et al. (2017) evaluated a standard model of real options when the cash flow follows an exponential diffusion process with expected return determined by a continuous-time two-state Markov chain. They examined explicit solutions and evaluated the investment option with complete information using the "estimate and verify" method. Also, they provided a free boundary partial differential equation to evaluate the option with partial information. To this end, they solved an optimal stopping problem with a bi-variate Markov process by applying the filtering techniques of Liptser and Shiryayev (1977) and finite difference methods.

Incorporating jumps into the firm value dynamics can solve several problems, to have the uncertainty of triggering a conversion or default event because it creates non-zero credit spreads for short maturities, as Pleger (2012) points out. On the other hand, following Duffie and Lando (2001), incorporating jumps in the firm value process could cause sudden information release. Therefore, it can reduce the conflict-of-interest problem (information asymmetry) between shareholders and bondholders. These jumps can be reflected on significant events, such as the introduction of new products, technical innovations, changes in laws or government policies, and tax or interest rate adjustments. To insert the timing and size of these jumps into the valuation model, it is assumed that the value of the asset follows a diffusion process with jumps of double exponential distribution. This allows us to evaluate an explicit equation for the value of the firm's securities. Based on Merton (1976), the jumps are described by a fish-counting process.

The dynamics of asset value are divided into two processes: a continuous process capturing new information that has a marginal effect on the firm and a diffusion process with jumps capturing instantaneous new information that has a significant impact on the firm. In fact, these jumps are described by two distributions, the first fish distribution is characterized by the intensity of the process and the second exponential distribution is specified by the importance of the jumps. Thus, the value of the company will depend on the importance of each of the jumps as well as the number of jumps during a given period. We note that the different sources of hazards, size of jumps and number of jumps are independent of each other. Jumps in the general model Kou (2002) and Kou and Wang (2003) can be positive or negative. The jump size follows a specific exponential distribution according to the sign of the jump, consequently the name of double exponential function. Recently, Siamak et al. (2022) modeled contingent capital based on a market trigger in double jump diffusion processes for asset values and stock prices. They showed that designing a contingent capital contract with a predetermined and pre-specified conversion ratio is not feasible to maintain a single equilibrium state. The conversion ratio depends on the jump in equity and the conditional expectation of contingent capital at the time of conversion; therefore, it is a random variable at the time of conversion and cannot be assigned a predefined value.

In addition, there is sometimes a sudden change in the project's performance with its continuous variation and therefore a diffusion process with jumps will be more appropriate for the cash flows. The double exponential distribution provides analytical solutions for debt and equity values. Note that the analysis of these solutions is difficult in the standard jump diffusion process. The diffusion process with double exponential distribution jumps is first introduced by Kou (2002). It is a special case of Levy processes. Moreover, the double exponential distribution has a memoryless function that facilitates the computation of expected means and variance terms. The main objective of this paper is to examine the value of the firm in a regime-switching model when the firm's cash flow follows a diffusion process with jumps of double exponential distribution. We attempt to explore the value of the investment option and the timing of the option exercise in a structural model that combines jump risk and regime switching. Indeed, we develop the theory of equilibrium prices under a diffusion process with jumps using a structural model introduced by Leland (1994) and later extended by Kou (2002) and Chen and Kou (2009). We also try in this paper to model the investment decision (behavior) of a firm as a real option and study the optimal policy of the firm in maximizing its equity value.

Regime-Switching Model of a Company with an Investment Option

Consider a probability space (Ω, F, P) on which B is a standard Brownian motion, L represents the two-state continuous-time Markov chain. It is right-continuous with values $\{0,1\}$ without loss of generality and defines bad and good economic regime states, respectively. Let a standard poisson process N of intensity λ. We assume that the process L starts from $i \in \{0,1\}$, for an exponential time distribution with its parameter λ_i until reaching a jump to state $j \in \{0,1\}$, $(j \neq i)$, i.e., λ_i representing the jump intensity from state i to state j. The inter-regime times that are exponentially distributed are independent. Similarly, they are independent of the Brownian motion B and the poisson process N. We consider a firm with no assets in place but initially has a perpetual option to invest at any time in a project by incurring a fixed sunk cost $I > 0$. We also assume that the state variable of our model (cash flow) follows a diffusion process with double exponential distribution jumps, which leads to a more sophisticated and realistic model. For any time t, we obtain a continuous stochastic cash flow denoted x. Time is continuous and infinite, and defined by t where $t \in (0, \infty)$. We note that τ is the constant tax rate that the firm should pay on their revenues, $\tau \in \{0,1\}$. We assume that the firm finances its investment cost with the contingent convertible debt and that the value of the project is equal to the value of the unlevered firm.

After exercising the investment option, the firm's capital structure has three hybrids, equity, risky debt and CoCos. To determine them, we only need to specify their cash flows. Indeed, we assume that the issued debts are perpetual and we specify the coupon rate of the risky debt by c_s (constant) and the coupon rate of the contingent convertible debt by c_c (constant). We denote by x_0^c and x_1^c the two CoCo conversion thresholds and by x_0^b and x_1^b the two risky debt default thresholds that relate to the two regime states of the firm. We assume that 0 and 1 represent the recession and boom regimes of the economy respectively. The indices 0 and 1 express the states of the economic regime $l = 0$ and $l = 1$ respectively. Due to the homogeneity of time in the model, we assume that the CoCo default thresholds and conversion thresholds are time independent. We note that the conversion thresholds are determined exogenously by a financial regulator while the default thresholds are specified endogenously by the firm's shareholders to maximize the value of equity. If a conversion trigger event occurs, shareholders distribute to CoCo holders a fraction noted, β_l, of the equity; where $0 \leq \beta_l \leq 1$ represents the conversion ratio under the regime-switching model. If the firm defaults, a fraction noted, α, of the value of future cash flows will be lost due to bankruptcy costs; where $0 \leq \alpha \leq 1$ is a constant representing the loss rate or bankruptcy rate.

Model Setup

In this paper, we try to determine the prices of the company's securities and the optimal investment problem if the company's cash flow follows a diffusion process with double exponential distribution jumps. To do this, we apply the risk-neutral price theory, the equilibrium price theory under the jump diffusion model and the reverse induction method. Certainly, the objective of the investor is to choose the optimal investment time to maximize the return of the return of the project. For this purpose, we assume that the dynamics of the firm's cash flow follow a diffusion process with double exponential distribution jumps which is determined by:

$$\frac{dX_t}{X_{t^-}} = \mu(L_t)dt + \sigma(L_t)dB_t + d\left(\sum_{i=1}^{N_t} Z_i - 1\right) - \lambda\varepsilon\, dt, \quad (1)$$

with $\mu(L_t)$ and $\sigma(L_t)$ representing the risk-adjusted average and volatility parameters respectively determined by the state L of the economic environment (recession or boom), ε denotes the average percentage of the size jump that is equal to $\mathbb{E}(Z_i - 1)$ where Z_i represents identically independent distributed random jumps with the same non-negative laws defined on (Ω, F, P) and the density of its logarithm follows a double exponential distribution which is expressed as follows:

$$f(y) = p\eta_1 e^{-\eta_1 y} 1_{\{y \geq 0\}} + q\eta_2 e^{\eta_2 y} 1_{\{y<0\}} \quad \eta_1 > 1, \quad \eta_2 > 0 \quad (2)$$

with $p\,,q \geq 0$ and $p + q = 1$ representing the jump probabilities up and down respectively, $1/\eta_1$ and $1/\eta_2$ determine the means of the two distributions, respectively. Therefore, the average percentage jump size ε is expressed by the following formula:

$$\varepsilon = \mathbb{E}(Z_i - 1) = \frac{p\eta_1}{\eta_1 - 1} + \frac{q\eta_2}{\eta_2 + 1} - 1 \quad (3)$$

We assume that all processes B, N, L and $\{Z_i\}$ are mutually independent and $Y_i = \ln(Z_i)$. The unique solution of equation (1) is expressed by:

$$X_t = xe^{A_t} \quad (4)$$

Where $A_t = \int_0^t \mu(L_s)\, ds - \int_0^t \left(\frac{\sigma(L_s)^2}{2} + \lambda\varepsilon\right) ds + \int_0^t \sigma(L_s)\, dB_s + \sum_{i=1}^{N_t} Y_i$

Note that $\mu_0 \equiv \mu(0)$, $\mu_1 \equiv \mu(1)$, $\sigma_0 \equiv \sigma(0)$ and $\sigma_1 \equiv \sigma(1)$. Obviously, we assume that $\mu(0) < \mu(1)$, i.e. the expected return to cash flow in a bad economic regime (recession) is low and the expected return to cash flow in a good economic regime (boom) is high. Process A is a Levy process, i.e. a process with stationary and independent increments. According to Pengfei et al. (2017), the Laplace transform of A is the function $g_l(\cdot)$ such that:

$$g_l(\beta) = r + \lambda_l - \frac{1}{2}\sigma_l^2\beta^2 - \left(\mu_l - \frac{1}{2}\sigma_l^2 - \lambda\varepsilon\right)\beta - \lambda\left(\frac{p\eta_1}{\eta_1 - \beta} + \frac{q\eta_2}{\eta_2 + \beta} - 1\right), l \in \{0,1\} \quad (5)$$

To examine the optimal investment behavior, we follow the same approach as Pengfei et al. (2017) except that our study considers the value of the non-leveraged firm as the value of the project. Certainly, any investor aims to maximize the return on the future cash flow of the company by choosing an optimal investment time. Thus, the objective function (the value of the investment option) can be expressed as follows:

$$f(X_t, L_t) = \max_{T_D \in \mathcal{T}} \mathbb{E}\left[\int_{T_D}^{\infty} e^{-r(s-t)} X_s ds - e^{-r(T_D - t)} I |\ \mathcal{F}_t\right], \quad (6)$$

where $r > 0$ denotes the risk-free interest rate, $\mathcal{T}$ denotes a set of all stopping times, $\mathcal{F}_t \equiv \mathcal{F}_t^{L,B,N,X} = \sigma\{L_s, B_s, N_s, X_s | s \leq t\}$ represents the σ-algebra over Ω generated by observations of L, B, N, X up to time t. Note that the value of the option in equation (5) is defined before the triggering of the fault time T_D.
Let $V(X_t;\ L_t)$ be the present value of the future cash flow at time t, i.e. the present value of the project. Therefore, we have:

$$V(X_t;\ L_t) = \mathbb{E}\left[\int_{T_D}^{\infty} e^{-r(s-t)} X_s ds \mid \mathcal{F}_t\right] \quad (7)$$

Using the project present value expression above and the conditional expectation property, the value of the investment option is calculated as follows:

$$f(X_t, L_t) = \max_{T_D \in \mathcal{T}} \mathbb{E}\left[\int_{T_D}^{\infty} e^{-r(T_D - t)} \left(V\left(X_{T_D}, L_{T_D}\right) - I\right) | \mathcal{F}_t\right] \quad (8)$$

Firm Value under a Double Exponential Jump-Diffusion Model with the Regime Switching

Project Value

A European call option is considered as an investment option with a maturity t, payoff f and whose underlying is given by equation (8) (the discounted risk-neutral expectation of its future flow). For a current cash flow level x and a current economic regime state $l \in \{0,1\}$, we note that $V_l(x)$ is the equilibrium price of the project value, which is a function of $(x\ , l)$, i.e. $V_l(x) = V(x, l)$. Since our model is homogeneous, then $V_l(x)$ is independent of time. Thus, applying standard risk-neutral pricing theory, the project value under the regime-switching model must satisfy the following system:

$$\begin{cases} rV_0(x) = (\mu_0 - \lambda\varepsilon)xV_0(x)' + \dfrac{\sigma^2}{2}\, x^2V_0(x)'' + \lambda_0\big(V_1(x) - V_0(x)\big) \\ \qquad\qquad +\lambda\mathbb{E}\big(V_0(xe^{Y_i}) - V_0(x)\big) + \xi(x) \\ rV_1(x) = (\mu_0 - \lambda\varepsilon)xV_1(x)' + \dfrac{\sigma^2}{2}\, x^2V_1(x)'' + \lambda_1\big(V_0(x) - V_1(x)\big) \\ \qquad\qquad +\lambda\mathbb{E}\big(V_1(xe^{Y_i}) - V_1(x)\big) + \xi(x) \end{cases} \quad (9)$$

Where $\xi(x_t) = ax_t + b, t \in [0,T_D)$ is an always linear function of the cash rate x up to a stopping time $T_D = inf\{\, t \geq 0 : x_t \notin D\}$, corresponding at the time of the first passage of x from the domain D. At this instant, the company's assets generate a lump sum dividend of the value of the unlevered firm, which can be expressed by the function $(.)$. With a and b are constants to be determined. Hence, the values of $V_0(x)$ and $V_1(x)$ must satisfy the following ordinary differential equations:

$$\begin{cases} rV_0(x) = (\mu_0 - \lambda\varepsilon)xV_0(x)' + \dfrac{\sigma^2}{2}\, x^2V_0(x)'' + \lambda_0\big(V_1(x) - V_0(x)\big) + \lambda\mathbb{E}\big(V_0(xe^{Y_i}) - V_0(x)\big) \\ \qquad\qquad +(1-\tau)x \\ rV_1(x) = (\mu_1 - \lambda\varepsilon)xV_1(x)' + \dfrac{\sigma^2}{2}\, x^2V_1(x)'' + \lambda_1\big(V_0(x) - V_1(x)\big) + \lambda\mathbb{E}\big(V_1(xe^{Y_i}) - V_1(x)\big) \\ \qquad\qquad +(1-\tau)x \end{cases} \quad (10)$$

The first term in equation (10) denotes the marginal increase in the value of the unleveraged firm if the cash flow increases by one unit. The second determines the effects of cash flow volatility. The third represents the effects of the stochastic transition of the growth rate on the expected change in the value of the unleveraged firm. The last term indicates the effects of the stochastic transition of the jumps on the expected variation of $V_l(x)$.
Certainly, the return of cash flows (after the option is exercised) is constant. Hence, for a given cash flow level x, and a regime state $l \in \{0,1\}$, the value of the project is expressed as:

$$V_l(x) = q_l x \ , \quad l \in \{0,1\} \quad (11)$$

Where q_l is a constant to be determined. If we substitute equation (11) into (10), we obtain the constant q_l as follows:

$$q_l = \frac{(1-\tau)(r + \lambda_0 + \lambda_1 - \mu_0 - \mu_1 + \mu_l)}{(r + \lambda_1 - \mu_1)(r + \lambda_0 - \mu_0) - \lambda_0\lambda_1}\ , \quad l \in \{0,1\} \quad (12)$$

From the results obtained, we deduce that the value of the investment project is independent of the project risk and it does not depend on jumps. These findings can be attributed to the risk adjustment of the parameter in our model. Furthermore, jumps do not change the project return on average because we have $\sum_{i=1}^{N_t}(Z_i - 1) - \lambda\varepsilon_t$ is a martingale with zero mean.

The Equity Value after Conversion

According to the risk-neutral pricing method, the value of equity after CoCo conversion, $E_l^c(x)$, for $x \in (x_0^b, +\infty)$ is expressed as:

$$\begin{cases} rE_0^c(x) = (\mu_0 - \lambda\varepsilon)xE_0^c(x)' + \dfrac{\sigma^2}{2}\, x^2E_0^c(x)'' + \lambda_0\big(E_1^c(x) - E_0^c(x)\big) + \lambda\mathbb{E}\big(E_0^c(xe^{Y_i}) - E_0^c(x)\big) \\ \qquad\qquad +(1-\tau)(x - c_s) \\ rE_1^c(x) = (\mu_1 - \lambda\varepsilon)xE_1^c(x)' + \dfrac{\sigma^2}{2}\, x^2E_1^c(x)'' + \lambda_1\big(E_0^c(x) - E_1^c(x)\big) + \lambda\mathbb{E}\big(E_1^c(xe^{Y_i}) - E_1^c(x)\big) \\ \qquad\qquad +(1-\tau)(x - c_s) \end{cases} \quad (13)$$

For $x \in (x_1^b, x_0^b]$,

$$\begin{cases} E_0^c(x) = 0 \\ rE_1^c(x) = (\mu_0 - \lambda\varepsilon)xE_1^c(x)' + \dfrac{\sigma^2}{2}x^2E_1^c(x)'' + \lambda_0\big(0 - E_1^c(x)\big) + \lambda\mathbb{E}\big(E_1^c(xe^{Y_i}) - E_1^c(x)\big) \\ +(1-\tau)(x - c_s) \end{cases} \quad (14)$$

And for $x \in (0, x_1^b\,]$,

$E_0^c(x) = E_1^c(x) = 0 \quad (15)$

According to Gua et al. (2005), we define the set $[x_l^b, \infty)$ by the continuity region, $(0, x_l^b)$ by the stopping region and $[x_1^b, x_0^b]$ by the transition region. Subsequently, we impose the following smooth-pasting conditions: $E_0^c(x_0^b)' = 0$, $E_1^c(x_1^b)' = 0$ which ensure the continuity of the slopes at the endogenous fault thresholds. We also have, $\lim_{x\to x_0^{b+}} E_1^c(x) = \lim_{x\to x_0^{b-}} E_1^c(x)$.

The solution of equation (13) and (14) is based on equation (5) of Yang and Zhao (2015) and the guess-and-verify method. Thus, the general solutions are expressed as:

$$\begin{cases} E_0^c(x) = \sum_{i=1}^{4} A_i x^{\beta_i} + q_0 x - \dfrac{(1-\tau)c_s}{r} & if\ x > x_0^b \\ E_1^c(x) = \sum_{i=1}^{4} B_i x^{\beta_i} + q_1 x - \dfrac{(1-\tau)c_s}{r} & if\ x > x_0^b \\ E_1^c(x) = \sum_{i=1}^{4} C_i x^{\gamma_i} + \dfrac{(1-\tau)x}{(r+\lambda_1-\mu_1)} - \dfrac{(1-\tau)c_s}{r+\lambda_1} & if\ x_1^b \le x \le x_0^b \end{cases} \quad (16)$$

- For $x \in (x_0^b, +\infty)$

To determine the value of $E_0^c(x)$ for $x \in (x_0^b, +\infty)$, we impose the following boundary conditions: $E_0^c(x) = q_0 x - \frac{(1-\tau)c_s}{r}$ if $x > x_0^b$ and $E_0^c(x) = 0$ if $x \le x_0^b$. Subsequently, substituting equation the boundary conditions into equation (14), we obtain the solution of the equity value as follows:

$$\frac{\sum_{i=1}^{4} A_i (x_0^b)^{\beta_i}}{\eta_2 + \beta_i} + \frac{q_0 x_0^b}{1+\eta_2} - \frac{(1-\tau)c_s}{r\eta_2} = 0 \quad (17)$$

- For $x \in (x_1^b, x_0^b\,]$

The boundary conditions suitable for the transition region $(x_1^b, x_0^b\,]$ are as follows: $E_1^c(x) = q_1 x - \frac{(1-\tau)c_s}{r}$ if $x > x_0^b$, $E_1^c(x) = \frac{(1-\tau)x}{(r+\lambda_1-\mu_1)} - \frac{(1-\tau)c_s}{r+\lambda_1}$ if $x_1^b \le x \le x_0^b$ and $E_1^c(x) = 0$ if $x < x_1^b$. Substituting the boundary conditions into (14) and (15), we obtain the solutions of the equity value for this region:

$$\frac{\sum_{i=1}^{4} C_i (x_1^b)^{\gamma_i}}{\eta_2 + \gamma_i} + \frac{(1-\tau)}{(r+\lambda_1-\mu_1)}\frac{x_1^b}{1+\eta_2} - \frac{c_s(1-\tau)}{(r+\lambda_1)\eta_2} = 0 \quad (18)$$

$$\frac{\sum_{i=1}^{4} C_i (x_0^b)^{\gamma_i}}{\gamma_i - \eta_1} + \frac{(1-\tau)}{(r+\lambda_1-\mu_1)}\frac{x_0^b}{(1-\eta_1)} - \frac{c_s(1-\tau)}{(r+\lambda_1)(-\eta_1)} - \left(\frac{\sum_{i=1}^{4} B_i (x_0^b)^{\beta_i}}{\beta_i - \eta_1} - \frac{q_1 x_0^b}{1-\eta_1} + \frac{(1-\tau)c_s}{r(-\eta_1)}\right) = 0 \quad (19)$$

To conclude, the value of equity after conversion for different situations and for different states is determined by the following system:

$$E_0^c(x) = \begin{cases} \dfrac{\sum_{i=1}^{4} A_i x^{\beta_i}}{\eta_2 + \beta_i} + \dfrac{q_0 x}{1+\eta_2} - \dfrac{(1-\tau)c_s}{r\eta_2}, & if\ x > x_0^b \\ 0 \qquad , & if\ x \le x_0^b \end{cases} \quad (20)$$

$$E_1^c(x) = \begin{cases} \dfrac{\sum_{i=1}^4 B_i x^{\beta_i}}{\beta_i - \eta_1} + \dfrac{q_1 x}{1-\eta_1} - \dfrac{(1-\tau)c_s}{r(-\eta_1)}, & if\ \ x > x_0^b \\ \dfrac{\sum_{i=1}^4 C_i x^{\gamma_i}}{\eta_2 + \gamma_i} + \dfrac{(1-\tau)}{(r+\lambda_1-\mu_1)}\dfrac{x}{1+\eta_2} - \dfrac{c_s(1-\tau)}{(r+\lambda_1)\eta_2}, & if\ \ x_1^b \le x \le x_0^b \\ 0 \qquad , & if\ \ x < x_1^b \end{cases} \quad (21)$$

The equity value $E_l^c(x)$ in the continuity regime $\left[x_l^b, \infty\right)$ is specified by the following two terms: $q_l x - \frac{(1-\tau)c_s}{r}$ which denotes the value of equity in the absence of default and $\sum_{i=1}^4 A_i x^{\beta_i}$ which offers the change in the value of $E_l^c(x)$ when there is a regime change or when the cash flow level x crosses the default threshold x_l^b. Similarly, in the transient regime $[x_1^b, x_0^b]$, the value of equity is also represented by two terms: the first $\frac{(1-\tau)\lambda_1 x}{(r+\lambda_1-\mu_1)} - \frac{(1-\tau)c_s}{r}$ which determines equity if there is no default, while the second captures the change in $E_l^c(x)$ at the time of default or at the time of the regime change.

We note that;

$$A_i = \frac{\lambda_0}{g_0(\beta_i)} B_i \text{ and } B_i = \frac{\lambda_1}{g_1(\beta_i)} A_i \quad (22)$$

With $i = 1,2,3,4$ and the functions $g_0()$ and $g_1()$ are determined from the equation $g_l(\beta)$. Using equation (22), we obtain the following expression:

$$g_0(\beta) g_1(\beta) = \lambda_0 \lambda_1 \quad (23)$$

The above equation has 8 distinct roots of which four are positive. Based on the study of Pengfei et al. (2017), we deduce that the function $J(\beta) = 0$ has eight real roots and only four of them are positive, where $J(\beta) = (g_0(\beta) g_1(\beta) - \lambda_0\lambda_1)(\eta_1 - \beta)^2(\eta_2 + \beta)^2$. Similarly, we note that $\{\gamma_i\}$ are four roots of the equation $g_0(\gamma_i) = 0$ with $i = 1,2,3,4$ i.e., the parameters to be determined are only the roots of the equation $g_0(\gamma) = 0$.

From the smooth-pasting conditions at the optimal fault thresholds x_0^b and x_1^b, we deduce that the optimal fault thresholds x_0^b and x_1^b must satisfy the system of the following equation:

$$\begin{cases} \sum_{i=1}^{4} A_i (x_0^b)^{\beta_i} \beta_i + q_0 x_0^b = 0 \\ \sum_{i=1}^{4} C_i (x_1^b)^{\gamma_i} \gamma_i + \dfrac{(1-\tau)}{r+\lambda_1-\mu_1} x_1^b = 0 \end{cases} \quad (24)$$

Risky Debt Value under Different Regions

To determine the value of risky debt after the exercise of the investment option under the regime-switching model with jumps, we first consider that there are three different regions. Indeed, for $x \in (x_0^b, +\infty)$, the value $D_l^s(x)$ is expressed as:

$$\begin{cases} rD_0^s(x) = (\mu_0 - \lambda\varepsilon) x D_0^s(x)' + \dfrac{\sigma^2}{2} x^2 D_0^s(x)'' + \lambda_0\big(D_1^s(x) - D_0^s(x)\big) \\ \qquad + \lambda \mathbb{E}\big(D_0^s(xe^{Y_i}) - D_0^s(x)\big) + c_s \\ rD_1^s(x) = (\mu_1 - \lambda\varepsilon) x D_1^s(x)' + \dfrac{\sigma^2}{2} x^2 D_1^s(x)'' + \lambda_1\big(D_0^s(x) - D_1^s(x)\big) \\ \qquad + \lambda \mathbb{E}\big(D_1^s(xe^{Y_i}) - D_1^s(x)\big) + c_s \end{cases} \quad (25)$$

For $x \in (x_1^b, x_0^b)$,

$$\begin{cases} D_0^s(x) = (1-\alpha)q_0 x & (26) \\ rD_1^s(x) = (\mu_1 - \lambda\varepsilon)xD_1^s(x)' + \dfrac{\sigma^2}{2}x^2 D_1^s(x)'' + \lambda_1\big((1-\alpha)q_0 x - D_1^s(x)\big) \\ \qquad + \lambda\mathbb{E}\big(D_1^s(xe^{Y_i}) - D_1^s(x)\big) + c_s \end{cases}$$

And For $x \in (0, x_1^b)$,

$$\begin{cases} D_0^s(x) = (1-\alpha)q_0 x \\ D_1^s(x) = (1-\alpha)q_1 x \end{cases} \quad (27)$$

According to Yang and Zhao (2015) and the guess-and-verify method, the general solutions of the stochastic differential equations of (25) and (26) are expressed as:

$$\begin{cases} D_0^s(x) = \sum_{i=1}^{4} A_{4+i}\, x^{\beta_i} + \dfrac{c_s}{r} & if \;\; x > x_0^b \\ D_1^s(x) = \sum_{i=1}^{4} B_{4+i} x^{\beta_i} + \dfrac{c_s}{r} & if \;\; x < x_1^b \\ D_1^s(x) = \sum_{i=1}^{4} C_{4+i}\, x^{\gamma_i} + \dfrac{(1-\alpha)\lambda_1 q_0 x}{(r+\lambda_1-\mu_1)} + \dfrac{c_s}{r+\lambda_1} & if \;\; x_1^b < x < x_0^b \end{cases} \quad (28)$$

- For $x \in (x_0^b, +\infty)$

To examine the value of the risky debt if $x \in (x_0^b, +\infty)$, we impose the following boundary conditions: $D_0^s(x) = \frac{c_s}{r}$ if $x > x_0^b$ and $D_0^s(x) = (1-\alpha)q_0 x$ if $x \le x_0^b$. Substituting equation the boundary conditions into equation (25), we obtain the value of risky debt after the investment, which is represented by the following expression:

$$\frac{(1-\alpha)q_0 x_0^b}{1+\eta_2} - \frac{\sum_{i=1}^{4} A_{4+i}\, {x_0^b}^{\beta_i}}{\eta_2+\beta_i} - \frac{c_s}{r\eta_2} = 0 \quad (29)$$

- For $x \in (x_1^b, x_0^b)$

Referring to Luo and Yang (2017), we impose the following boundary conditions on risky debt: $D_1^s(x) = \frac{c_s}{r}$ if $x > x_0^b$, $D_1^s(x) = \frac{(1-\alpha)\lambda_1 q_0 x}{(r+\lambda_1-\mu_1)} + \frac{c_s}{r+\lambda_1}$ if $x_1^b \le x \le x_0^b$ and $D_1^s(x) = (1-\alpha)q_1 x$ if $x < x_1^b$. If we substitute the boundary conditions in the ODE (25) and (26), then the value of the risky debt is defined by the following solutions:

$$\frac{(1-\alpha)q_1 x_1^b}{1+\eta_2} - \left(\frac{\sum_{i=1}^{4} C_{i+4}(x_1^b)^{\gamma_i}}{\eta_2+\gamma_i} + \frac{(1-\alpha)\lambda_1 q_0}{(r+\lambda_1-\mu_1)}\frac{x_1^b}{1+\eta_2} + \frac{c_s}{(r+\lambda_1)\eta_2}\right) = 0 \quad (30)$$

$$\frac{\sum_{i=1}^{4} C_{i+4}(x_0^b)^{\gamma_i}}{\gamma_i-\eta_1} + \frac{(1-\alpha)\lambda_1 q_0}{(r+\lambda_1-\mu_1)}\frac{x_0^b}{1-\eta_1} + \frac{c_s}{(r+\lambda_1)(-\eta_1)} - \frac{\sum_{i=1}^{4} B_{i+4}(x_0^b)^{\gamma_i}}{\beta_i-\eta_1} - \frac{c_s}{r(-\eta_1)} = 0 \quad (31)$$

The results obtained for the two regimes $l = \{0,1\}$ are summarized by the following system of equation:

$$D_0^s(x) = \begin{cases} \dfrac{\sum_{i=1}^{4} A_{i+4} x^{\beta_i}}{\eta_2+\beta_i} + \dfrac{c_s}{r\eta_2}\,, & if \; x > x_0^b \\ \dfrac{(1-\alpha)q_0 x}{1+\eta_2} \,, & if \; x \le x_0^b \end{cases} \quad (32)$$

$$D_1^s(x) = \begin{cases} \frac{\sum_{i=1}^4 B_{i+4} x^{\beta_i}}{\beta_i - \eta_1} + \frac{c_s}{r(-\eta_1)} & , \quad if\ x > x_0^b \\ \frac{\sum_{i=1}^4 C_{i+4} x^{\gamma_i}}{\eta_2 + \gamma_i} + \frac{(1-\alpha)\lambda_1 q_0}{(r + \lambda_1 - \mu_1)} \frac{x}{1+\eta_2} + \frac{c_s}{(r+\lambda_1)\eta_2} & , \quad if\ x_1^b \leq x \leq x_0^b \\ \frac{(1-\alpha) q_1 x}{1+\eta_2} & \quad x < x_1^b \end{cases} \quad (33)$$

The value of the risky debt is equal to the value of the perpetual coupons c_s plus the change in the value of the risky debt at a regime change or default time of the firm T_D (the state variable x reaches the default threshold x_l^b).

The Equity Value before Conversion

In order to obtain the value of equity after the exercise of the investment option prior to the conversion of the contingent convertible debt, $E_l(x)$, we must first identify the conversion time and subsequently determine the value of the conversion threshold. Indeed, following Glasserman and Nouri (2012), we assume that the conversion event is triggered at the moment when the value of the unlevered firm is less than or equal to the value of the levered firm. Therefore, we can define the conversion time by the following expression:

$$T_l^c = inf\left\{t \geq 0 : \psi V_l(x_l^c) \leq \frac{c_s + c_c}{r}\right\} \quad l \in \{0,1\} \quad (34)$$

And therefore, the conversion threshold in a regime-switching model is expressed as:

$$x_l^c = \frac{1}{\psi q_l} \frac{c_s + c_c}{r} \qquad \psi \in (0,1) \quad (35)$$

As $E_l^c(x)$ determines the value of equity after conversion, x_l^c represents the conversion barrier and c_c denotes the coupon rate of the CoCo debt paid continuously, hence referring to Barucci and Del Viva (2013), the conversion rate in a regime-switching model is defined by, $\beta_l = min\left(\frac{\frac{c_c}{r}}{E_l^c(x_l^c)}, 1\right)$, where $l \in \{0,1\}$.

We now represent the value of equity before CoCo conversion for the three regions we specified earlier. In fact, for $x \in (x_0^c, \infty)$, the equity value must satisfy the following equation:

$$\begin{cases} rE_0(x) = (\mu_0 - \lambda\varepsilon) x E_0(x)' + \frac{\sigma^2}{2} x^2 E_0(x)'' + \lambda_0\big(E_1(x) - E_0(x)\big) + \lambda \mathbb{E}\big(E_0(xe^{Y_i}) - E_0(x)\big) \\ \qquad + (1-\tau)(x - c_s - c_c) \\ rE_1(x) = (\mu_1 - \lambda\varepsilon) x D_1^s(x)' + \frac{\sigma^2}{2} x^2 E_1(x)'' + \lambda_1\big(E_0(x) - E_1(x)\big) + \lambda \mathbb{E}\big(E_1(xe^{Y_i}) - E_1(x)\big) \\ \qquad + (1-\tau)(x - c_s - c_c) \end{cases} \quad (36)$$

For $x \in (x_1^c, x_0^c)$,

$$\begin{cases} E_0(x) = (1-\beta)\left(\sum_{i=1}^4 A_i x^{\beta_i} + q_0 x - \frac{(1-\tau)c_s}{r}\right) \quad (37) \\ rE_1(x) = (\mu_1 - \lambda\varepsilon) x E_1(x)' + \frac{\sigma^2}{2} x^2 E_1(x)'' + \lambda_1\left((1-\beta)\left(\sum_{i=1}^4 A_i x^{\beta_i} + q_0 x - \frac{(1-\tau)c_s}{r}\right) - E_1(x)\right) \\ \qquad + \lambda \mathbb{E}\big(E_1(xe^{Y_i}) - E_1(x)\big) + (1-\tau)(x - c_s - c_c) \end{cases}$$

And for $x \in (0, x_1^c)$,

$$\begin{cases} E_0(x) = (1-\beta)\left(\sum_{i=1}^{4} A_i x^{\beta_i} + q_0 x - \frac{(1-\tau)c_s}{r}\right) \\ E_1(x) = (1-\beta)\left(\sum_{i=1}^{4} B_i x^{\beta_i} + q_1 x - \frac{(1-\tau)c_s}{r}\right) \end{cases} \quad (38)$$

Similarly, referring to Yang and Zhao (2015) and the guess-and-verify method to solve the ODE of (37) and (38). Thus, these solutions must satisfy the following equations:

$$\begin{cases} E_0(x) = \sum_{i=1}^{4} A_{i+8} x^{\beta_i} + q_0 x - \frac{(1-\tau)(c_s+c_c)}{r} & if\ x > x_0^c \\ E_1(x) = \sum_{i=1}^{4} B_{i+8} x^{\beta_i} + q_1 x - \frac{(1-\tau)(c_s+c_c)}{r} & if\ x > x_0^c \\ E_1(x) = \sum_{i=1}^{4} C_{i+8} x^{\gamma_i} + \frac{(1-\tau)\lambda_1 x}{(r+\lambda_1-\mu_1)} - \frac{(1-\tau)(c_s+c_c)}{r+\lambda_1} & if\ x_1^c \le x \le x_0^c \end{cases} \quad (39)$$

➢ For $x \in (x_0^c, \infty)$:

The equity value for this region is determined in two steps, first we impose the boundary conditions for $l = 0$: $E_0(x) = q_0 x - \frac{(1-\tau)(c_s+c_c)}{r}$ if $x > x_0^c$ and $E_0(x) = (1-\beta)\left(\sum_{i=1}^{4} A_i x^{\beta_i} + q_0 x - \frac{(1-\tau)c_s}{r}\right)$ if $x \le x_0^c$. Second, we substitute equation the boundary conditions into (36). Thus, the solutions of the equity value $E_0(x)$ is determined by:

$$(1-\beta)\left(\frac{\sum_{i=1}^{4} A_{i+8}(x_0^c)^{\beta_i}}{\eta_2+\beta_i} + \frac{q_0 x_0^c}{1+\eta_2} - \frac{(1-\tau)c_s}{r\eta_2}\right) - \left(\frac{\sum_{i=1}^{4} A_{i+8}(x_0^c)^{\beta_i}}{\eta_2+\beta_i} + \frac{q_0 x_0^c}{1+\eta_2} - \frac{(1-\tau)(c_s+c_c)}{r\eta_2}\right) = 0 \quad (40)$$

➢ For $x \in (x_1^c, x_0^c)$

Then, we impose the following boundary conditions in the transitional regime $[x_1^c, x_0^c]$: $E_1(x) = q_1 x - \frac{(1-\tau)(c_s+c_c)}{r}$ if $x > x_0^c$, $E_1(x) = \frac{(1-\tau)\lambda_1 x}{(r+\lambda_1-\mu_1)} - \frac{(1-\tau)(c_s+c_c)}{r+\lambda_1} + (1-\beta)\left(\sum_{i=1}^{4} B_i x^{\beta_i} + \frac{q_0 x \lambda_1}{(r+\lambda_1-\mu_1)} - \frac{\lambda_1(1-\tau)c_s}{(r+\lambda_1)r}\right)$ if $x_1^c \le x \le x_0^c$ and $E_1(x) = (1-\beta)\left(\sum_{i=1}^{4} B_i x^{\beta_i} + q_1 x - \frac{(1-\tau)c_s}{r}\right)$ if $x < x_1^c$. We integrate the boundary conditions into equation (37), we find that the value of the equity $E_1(x)$ in the region where $x \in (x_1^c, x_0^c)$ is represented by the following solutions:

$$\begin{aligned} &(1-\beta)\left(\frac{\sum_{i=1}^{4} B_i (x_1^c)^{\beta_i}}{\eta_2+\beta_i} + \frac{q_1 x_1^c}{1+\eta_2} - \frac{(1-\tau)c_s}{r\eta_2}\right) \\ &\quad - \left[\frac{\sum_{i=1}^{4} C_{i+8}(x_1^c)^{\gamma_i}}{\eta_2+\gamma_i} + \frac{(1-\tau)}{(r+\lambda_1-\mu_1)}\frac{x_1^c}{1+\eta_2} - \frac{(1-\tau)(c_s+c_c)}{(r+\lambda_1)\eta_2} \right. \\ &\quad \left. + (1-\beta)\left(\frac{\sum_{i=1}^{4} B_i (x_1^c)^{\beta_i}}{\eta_2+\beta_i} + \frac{q_0\lambda_1}{(r+\lambda_1-\mu_1)}\frac{x_1^c}{1+\eta_2} - \frac{\lambda_1(1-\tau)c_s}{(r+\lambda_1)r\eta_2}\right)\right] = 0 \quad (41) \end{aligned}$$

$$\begin{aligned} &\frac{\sum_{i=1}^{4} C_{i+8}(x_0^c)^{\gamma_i}}{\gamma_i-\eta_1} + \frac{(1-\tau)}{(r+\lambda_1-\mu_1)}\frac{x_0^c}{1-\eta_1} - \frac{(1-\tau)(c_s+c_c)}{r+\lambda_1} \\ &\quad + (1-\beta)\left(\frac{\sum_{i=1}^{4} B_i (x_0^c)^{\beta_i}}{\beta_i-\eta_1} + \frac{q_0\lambda_1}{(r+\lambda_1-\mu_1)}\frac{x_0^c}{1-\eta_1} - \frac{\lambda_1(1-\tau)c_s}{(r+\lambda_1)r(-\eta_1)}\right) \\ &\quad - \left[\frac{\sum_{i=1}^{4} B_{i+8}(x_0^c)^{\gamma_i}}{\beta_i-\eta_1} + \frac{q_1 x_0^c}{1-\eta_1} - \frac{(1-\tau)(c_s+c_c)}{r(-\eta_1)}\right] = 0 \quad (42) \end{aligned}$$

The value of equity before conversion in different situations is summarized by the following equation:

$$E_0(x)=\begin{cases}\dfrac{\sum_{i=1}^{4}A_{i+8}x^{\beta_i}}{\eta_2+\beta_i}+\dfrac{q_0x}{1+\eta_2}-\dfrac{(1-\tau)(c_s+c_c)}{r\eta_2}, & if\ x>x_0^c\\(1-\beta)\left(\dfrac{\sum_{i=1}^{4}A_ix^{\beta_i}}{\eta_2+\beta_i}+\dfrac{q_0x}{1+\eta_2}-\dfrac{(1-\tau)c_s}{r\eta_2}\right), & if\ x\le x_0^c\end{cases} \quad (43)$$

$$E_1(x)=\begin{cases}\dfrac{\sum_{i=1}^{4}B_{i+8}x^{\beta_i}}{\beta_i-\eta_1}+\dfrac{q_1x}{1-\eta_1}-\dfrac{(1-\tau)(c_s+c_c)}{r(-\eta_1)}, & if\ x>x_0^c\\ \dfrac{\sum_{i=1}^{4}C_{i+8}x^{\gamma_i}}{\gamma_i-\eta_1}+\dfrac{(1-\tau)}{(r+\lambda_1-\mu_1)}\dfrac{x}{1-\eta_1}-\dfrac{(1-\tau)(c_s+c_c)}{r+\lambda_1}+ & \\ (1-\beta)\left(\dfrac{\sum_{i=1}^{4}B_ix^{\beta_i}}{\beta_i-\eta_1}+\dfrac{q_0\lambda_1}{(r+\lambda_1-\mu_1)}\dfrac{x}{1-\eta_1}-\dfrac{\lambda_1(1-\tau)c_s}{(r+\lambda_1)r(-\eta_1)}\right), & if\ x_0^c\le x\le x_1^c\\(1-\beta)\left(\dfrac{\sum_{i=1}^{4}B_ix^{\beta_i}}{\eta_2+\beta_i}+\dfrac{q_1x}{1+\eta_2}-\dfrac{(1-\tau)c_s}{r\eta_2}\right), & if\ x<x_1^c\end{cases} \quad (44)$$

The equity value $E_l(x)$ in the continuity regime $[x_l^b,\infty)$ is composed by the following two components: the first $q_lx-\frac{(1-\tau)(c_s+c_c)}{r}$ determines the equity value if there is no conversion event and the second captures the change in the $E_l(x)$ value at the time of the regime change or if a conversion trigger event occurs. Equivalently, the equity value in the transitional regime $[x_1^b,x_0^b]$ is composed by two the following two terms: the first $\frac{(1-\tau)\lambda_1x}{(r+\lambda_1-\mu_1)}-\frac{(1-\tau)(c_s+c_c)}{r+\lambda_1}$ specifies the value $E_l(x)$ if there is no conversion and the second defines the change in the equity value $E_l(x)$ at the time of conversion or at the time of regime change.

The Value of the Contingent Convertible Debt CoCo

The value of equity after the conversion of CoCo $D_l^c(x)$ for $x\in(x_0^c,+\infty)$ is expressed as:

$$\begin{cases}rD_0^c(x)=(\mu_0-\lambda\varepsilon)xD_0^c(x)'+\dfrac{\sigma^2}{2}x^2D_0^c(x)''+\lambda_0\left(D_1^c(x)-D_0^c(x)\right)\\ \qquad +\lambda\mathbb{E}\left(D_0^c(xe^{Y_i})-D_0^c(x)\right)+c_c\\ rD_1^c(x)=(\mu_0-\lambda\varepsilon)xD_1^c(x)'+\dfrac{\sigma^2}{2}x^2D_1^c(x)''+\lambda_1\left(D_0^c(x)-D_1^c(x)\right)\\ \qquad +\lambda\mathbb{E}\left(D_1^c(xe^{Y_i})-D_1^c(x)\right)+c_c\end{cases} \quad (45)$$

For $x\in(x_1^c,x_0^c]$,

$$\begin{cases}D_0^c(x)=\beta\left(\sum_{i=1}^{4}A_ix^{\beta_i}+q_0x-\dfrac{(1-\tau)c_s}{r}\right) \quad (46)\\ rD_1^c(x)=(\mu_0-\lambda\varepsilon)xD_1^c(x)'+\dfrac{\sigma^2}{2}x^2D_1^c(x)''+\lambda_0\left(\beta\left(\sum_{i=1}^{4}A_ix^{\beta_i}+q_0x-\dfrac{(1-\tau)c_s}{r}\right)-D_1^c(x)\right)\\ \qquad +\lambda\mathbb{E}\left(D_1^c(xe^{Y_i})-D_1^c(x)\right)+c_c\end{cases}$$

And for $x\in(0,x_1^c\,]$,

$$\begin{cases}D_0^c(x)=\beta\left(\sum_{i=1}^{4}A_ix^{\beta_i}+q_0x-\dfrac{(1-\tau)c_s}{r}\right)\\ D_1^c(x)=\beta\left(\sum_{i=1}^{4}B_ix^{\beta_i}+q_1x-\dfrac{(1-\tau)c_s}{r}\right)\end{cases} \quad (47)$$

The ODE solution of (45) and (46) is based on the solution of equation (5) in Yang and Zhao (2015). Thus, the solutions of these differential equations are represented by:

$$\begin{cases} D_0^c(x) = \sum_{i=1}^{4} A_{i+12} x^{\beta_i} + \frac{c_c}{r} & if\ x > x_0^c \\ D_1^c(x) = \sum_{i=1}^{4} B_{i+12} x^{\beta_i} + \frac{c_c}{r} & if\ x > x_0^c \\ D_1^c(x) = \sum_{i=1}^{4} C_{i+12} x^{\gamma_i} + \frac{c_c}{r+\lambda_1} + & \\ \beta\left(\sum_{i=1}^{4} B_i x^{\beta_i} + \frac{q_0 x \lambda_1}{(r+\lambda_1-\mu_1)} - \frac{\lambda_1(1-\tau)c_s}{(r+\lambda_1)r}\right) & if\ x_1^c \leq x \leq x_0^c \end{cases} \quad (48)$$

- For $x \in (x_0^c, +\infty)$

In order to obtain the CoCo value $D_0^c(x)$ for $x \in (x_0^c, +\infty)$, we impose the following boundary conditions: $D_0^c(x) = \frac{c_c}{r}$ if $x > x_0^c$ and $D_0^c(x) = \beta\left(\sum_{i=1}^{4} A_i x^{\beta_i} + q_0 x - \frac{(1-\tau)c_s}{r}\right)$ if $x \leq x_0^c$. We integrate the boundary conditions into equations (45), we obtain the CoCo value is expressed as follows:

$$\beta\left(\frac{\sum_{i=1}^{4} A_i (x_0^c)^{\beta_i}}{\eta_2+\beta_i} + \frac{q_0 x_0^c}{1+\eta_2} - \frac{(1-\tau)c_s}{r\eta_2}\right) - \left(\frac{\sum_{i=1}^{4} A_{i+12} (x_0^c)^{\beta_i}}{\eta_2+\beta_i} + \frac{c_c}{r\eta_2}\right) = 0 \quad (49)$$

- For $x \in (x_1^c, x_0^c)$

Similarly, to obtain the value of the CoCo debt in the region (x_1^c, x_0^c), we impose the boundary conditions as follows: $D_1^c(x) = \frac{c_c}{r}$ if $x > x_0^c$, $D_1^c(x) = \beta\left(\sum_{i=1}^{4} B_i x^{\beta_i} + \frac{q_0 x \lambda_1}{(r+\lambda_1-\mu_1)} - \frac{\lambda_1(1-\tau)c_s}{(r+\lambda_1)r}\right) + \frac{c_c}{r+\lambda_1}$ if $x_1^c \leq x \leq x_0^c$ and $D_1^c(x) = \beta\left(\sum_{i=1}^{4} B_i x^{\beta_i} + q_1 x - \frac{(1-\tau)c_s}{r}\right)$ if $x < x_1^c$. We now substitute the boundary conditions into equation (46), from which the value of the CoCo debt is determined by the following solutions:

$$\beta\left(\frac{\sum_{i=1}^{4} B_i (x_1^c)^{\beta_i}\eta_2}{\eta_2+\beta_i} + \frac{q_1 x_1^c}{1+\eta_2} - \frac{(1-\tau)c_s}{r\eta_2}\right) - \left(\frac{\sum_{i=1}^{4} C_{i+12} (x_1^c)^{\gamma_i}}{\eta_2+\gamma_i} + \frac{c_c}{(r+\lambda_1)\eta_2} + \beta\left(\frac{\sum_{i=1}^{4} B_i (x_1^c)^{\beta_i}\eta_2}{\eta_2+\beta_i} + \frac{q_0\lambda_1}{(r+\lambda_1-\mu_1)}\frac{x_1^c}{1+\eta_2} - \frac{\lambda_1(1-\tau)c_s}{(r+\lambda_1)r\eta_2}\right)\right) = 0 \quad (50)$$

$$\frac{\sum_{i=1}^{4} C_{i+12} (x_0^c)^{\gamma_i}}{\gamma_i-\eta_1} + \frac{c_c}{(r+\lambda_1)(-\eta_1)} + \beta\left(\frac{\sum_{i=1}^{4} B_i (x_0^c)^{\beta_i}}{\beta_i-\eta_1} + \frac{q_0\lambda_1}{(r+\lambda_1-\mu_1)}\frac{x_0^c}{1-\eta_1} - \frac{\lambda_1(1-\tau)c_s}{(r+\lambda_1)r(-\eta_1)}\right) - \left(\frac{\sum_{i=1}^{4} B_{i+12} (x_0^c)^{\beta_i}}{\beta_i-\eta_1} + \frac{c_c}{r(-\eta_1)}\right) = 0 \quad (51)$$

The value of CoCo can be summarized by the following system:

$$D_0^c(x) = \begin{cases} \sum_{i=1}^{4} A_{i+12} \frac{(x_0^c)^{\beta_i}}{\eta_2+\beta_i} + \frac{c_c}{r\eta_2}, & if\ x > x_0^c \\ \beta\left(\frac{\sum_{i=1}^{4} A_i (x_0^c)^{\beta_i}}{\eta_2+\beta_i} + \frac{q_0 x_0^c}{1+\eta_2} - \frac{(1-\tau)c_s}{r\eta_2}\right) & if\ x \leq x_0^c \end{cases} \quad (52)$$

$$D_1^c(x) = \begin{cases} \dfrac{\sum_{i=1}^4 B_{i+12}(x_0^c)^{\beta_i}}{\beta_i - \eta_1} + \dfrac{c_c}{r(-\eta_1)} & , \quad if\ x > x_0^c \\ \dfrac{\sum_{i=1}^4 C_{i+12}(x_0^c)^{\gamma_i}}{\gamma_i - \eta_1} + \dfrac{c_c}{(r+\lambda_1)(-\eta_1)} + \beta\left(\begin{array}{c} \dfrac{\sum_{i=1}^4 B_i(x_0^c)^{\beta_i}}{\beta_i - \eta_1} + \dfrac{q_0\lambda_1}{(r+\lambda_1-\mu_1)}\dfrac{x_0^c}{(1-\eta_1)} \\ -\dfrac{\lambda_1(1-\tau)c_s}{(r+\lambda_1)r(-\eta_1)} \end{array}\right) \\ \qquad if\ x_1^c \le x \le x_0^c \\ \beta\left(\dfrac{\sum_{i=1}^4 B_i(x_1^c)^{\beta_i}\eta_2}{\eta_2+\beta_i} + \dfrac{q_1 x_1^c}{1+\eta_2} - \dfrac{(1-\tau)c_s}{r\eta_2}\right) & if\ \ x < x_1^c \end{cases} \quad (53)$$

The value of the contingent convertible debt is determined by the value of the perpetual coupon payments c_c plus the change in the value of the convertible debt CoCo at the time of the regime change or at conversion, i.e. when the state variable x crosses the conversion threshold x_l^c.

The total value of the firm before conversion $V_l^T(x)$, is determined by the sum of the value of risky debt before conversion, the value of equity and the value of CoCo. It is expressed as:

$$V_0^T(x) = D_0^c(x) + D_0^s(x) + E_0(x)$$

$$V_0^T(x) = \frac{\sum_{i=1}^4 A_{i+12}x^{\beta_i}}{\eta_2+\beta_i} + \frac{\sum_{i=1}^4 A_{i+4}x^{\beta_i}}{\eta_2+\beta_i} + \frac{\sum_{i=1}^4 A_{i+8}x^{\beta_i}}{\eta_2+\beta_i} + \frac{q_0 x}{1+\eta_2} + \frac{\tau(c_s+c_c)}{r\eta_2} \quad (54)$$

$$V_1^T(x) = D_1^c(x) + D_1^s(x) + E_1(x)$$

$$V_1^T(x) = \frac{\sum_{i=1}^4 B_{i+12}x^{\beta_i}}{\eta_2+\beta_i} + \frac{\sum_{i=1}^4 B_{i+4}x^{\beta_i}}{\eta_2+\beta_i} + \frac{\sum_{i=1}^4 B_{i+8}x^{\beta_i}}{\eta_2+\beta_i} + \frac{q_1 x}{1+\eta_2} + \frac{\tau(c_s+c_c)}{r\eta_2} \quad (55)$$

The Optimal Capital Structure and the Agency Cost of Debt

Timing and Pricing of the Investment Option

We examine the price and time of the investment option with reference to Yang and Zhao (2015) and Pengfei et al. (2017). Since the model is time-homogeneous, the optimal investment decision making can be evaluated as a threshold policy. Let $x^i(l),\ l \in \{0,1\}$ be the optimal investment threshold with T_l^i corresponding to the optimal investment time which is determined by $T_l^i = inf\{t \ge 0: X_t \ge x^i(l)\}$. Thus, the exercise of the investment option occurs at the time when the cash flow level x crosses the threshold $x^i(l)$, with the current economic regime state being $l \in \{0,1\}$. Let $x_0^i \equiv x^i(0)$ and $x_1^i \equiv x^i(1)$ exist. Obviously, we have $x_0^i > x_1^i$.

For any $t,\ l_t = 0\ (l_t = 1)$ and $X_t \ge x_0^i\ (X_t \ge x_1^i)$, the irreversible investment option must be exercised immediately by the investor to obtain a perpetual stochastic cash flow x. Let $f_0(x) = f(x,0)$ and $f_1(x) = f(x,1)$ correspond to the investment option functions for a bad economic regime and a good economic regime, respectively. Indeed, according to the optimality principle, the option value for $x \in (0, x_1^i]$ must satisfy the following equation:

$$\begin{cases} rf_0(x) = (\mu_0 - \lambda\varepsilon)xf_0(x)' + \dfrac{\sigma^2}{2}x^2 f_0(x)'' + \lambda_0\big(f_1(x) - f_0(x)\big) + \lambda\mathbb{E}\big(f_0(xe^{Y_i}) - f_0(x)\big) \\ rf_1(x) = (\mu_1 - \lambda\varepsilon)xf_1(x)' + \dfrac{\sigma^2}{2}x^2 f_1(x)'' + \lambda_1\big(f_0(x) - f_1(x)\big) + \lambda\mathbb{E}\big(f_1(xe^{Y_i}) - f_1(x)\big) \end{cases} \quad (56)$$

For $x \in (x_1^i, x_0^i]$

$$\begin{cases} rf_0(x) = (\mu_0 - \lambda\varepsilon)xf_0(x)' + \dfrac{\sigma^2}{2}x^2 f_0(x)'' + \lambda_0\big((V_1^T(x) - I) - f_0(x)\big) + \lambda\mathbb{E}\big(f_0(xe^{Y_i}) - f_0(x)\big) \\ f_1(x) = V_1^T(x) - I \end{cases} \quad (57)$$

And for, $x \in (x_0^i, \infty)$

$$\begin{cases} f_0(x) = V_0^T(x) - I \\ f_1(x) = V_1^T(x) - I \end{cases}, (58)$$

Where $q_0 \equiv q(0)$ and $q_1 \equiv q(1)$ are determined in the equation (11).

For $x \in (0, x_1^i]$

Motivated by the relevant results obtained by Yang and Zhao (2015) and Pengfei et al. (2017), the general solution of $f_l(x)$ for $x \in (0, x_1^i]$ is determined by:

$$f_0(x) = \sum_{i=1}^{8} A_{i+16} x^{\beta_i} \quad and \quad f_1(x) = \sum_{i=1}^{8} B_{i+16} x^{\beta_i} \quad (59)$$

Since 0 is an absorbing barrier of the cash flow process, the first condition on the option boundary is equal to $f_0(0) = f_1(0) = 0$ and the parameters β_i must be positive. Hence,

$$f_0(x) = \sum_{i=1}^{4} A_{i+16} x^{\beta_i} \quad et \quad f_1(x) = \sum_{i=1}^{4} B_{i+16} x^{\beta_i} \quad (60)$$

Similarly, based on equation (5) of Yang and Zhao (2015) and the guess-and-verify method, the resolution of equation (56) and (57) is expressed as follows:

$$\begin{cases} f_0(x) = \sum_{i=1}^{4} A_{i+16} x^{\beta_i} & if\ x < x_1^i \\ f_0(x) = \sum_{i=1}^{4} C_{i+16} x^{\gamma_i} + \frac{\lambda_0 q_1 x}{(r+\lambda_0-\mu_0)(1+\eta_2)} + \frac{\tau\lambda_0(c_s+c_c)}{r(r+\lambda_0)\eta_2} - \frac{\lambda_0 I}{r+\lambda_0} \frac{\sum_{i=1}^{4} B_{i+12} x^{\beta_i}}{\eta_2+\beta_i} + & \\ \quad \frac{\sum_{i=1}^{4} B_{i+4} x^{\beta_i}}{\eta_2+\beta_i} + \frac{\sum_{i=1}^{4} B_{i+8} x^{\beta_i}}{\eta_2+\beta_i} & if\ x_1^i \le x \le x_0^i \\ f_1(x) = \sum_{i=1}^{4} B_{i+16} x^{\beta_i} & if\ x < x_1^i \end{cases} \quad (61)$$

If we substitute equation the boundary conditions into (56), then the option value for the region $(l = 1)$ is expressed by the following solution:

$$\frac{\sum_{i=1}^{4} B_{i+16}(x_1^i)^{\beta_i}}{\beta_i-\eta_1} + \frac{\sum_{i=1}^{4} B_{i+12}(x_1^i)^{\beta_i}}{(\beta_i-\eta_1)(\eta_2+\beta_i)} + \frac{\sum_{i=1}^{4} B_{i+4}(x_1^i)^{\beta_i}}{(\beta_i-\eta_1)(\eta_2+\beta_i)} + \frac{\sum_{i=1}^{4} B_{i+8}(x_1^i)^{\beta_i}}{(\beta_i-\eta_1)(\eta_2+\beta_i)} + \frac{q_1 x_1^i}{(1-\eta_1)(1+\eta_2)} + \frac{I}{\eta_1} + \frac{\tau(c_s+c_c)}{r\eta_2(-\eta_1)} = 0 \quad (62)$$

- For $x \in (x_1^i, x_0^i]$

Substituting the above equation into equation (57), the option value if $x \in (x_1^i, x_0^i]$ is determined by the following expressions:

$$\frac{\sum_{i=1}^{4} A_{i+16}(x_1^i)^{\beta_i}}{\eta_2+\beta_i} + \frac{\sum_{i=1}^{4} C_{i+16}(x_1^i)^{\gamma_i}}{\eta_2+\gamma_i} + \frac{q_1\lambda_0 x_1^i}{(r+\lambda_0-\mu_0)(1+\eta_2)^2} - \frac{I\lambda_0}{(r+\lambda_0)\eta_2} + \frac{\lambda_0\tau(c_s+c_c)}{(\eta_2)^2 r(r+\lambda_0)} + \frac{\sum_{i=1}^{4} B_{i+12}(x_1^i)^{\beta_i}}{(\eta_2+\beta_i)^2} + \frac{\sum_{i=1}^{4} B_{i+4}(x_1^i)^{\beta_i}}{(\eta_2+\beta_i)^2} + \frac{\sum_{i=1}^{4} B_{i+8}(x_1^i)^{\beta_i}}{(\eta_2+\beta_i)^2} = 0 \quad (63)$$

$$\frac{\sum_{i=1}^{4} A_{i+12}\left(x_0^i\right)^{\beta_i}}{(\beta_i-\eta_1)(\eta_2+\beta_i)}+\frac{\sum_{i=1}^{4} A_{i+4}\left(x_0^i\right)^{\beta_i}}{(\beta_i-\eta_1)(\eta_2+\beta_i)}+\frac{\sum_{i=1}^{4} A_{i+8}\left(x_0^i\right)^{\beta_i}}{(\beta_i-\eta_1)(\eta_2+\beta_i)}+\frac{q_0x_0^i}{(1-\eta_1)(1+\eta_2)}+\frac{I}{\eta_1}+\frac{\tau(c_s+c_c)}{r\eta_2(-\eta_1)}$$
$$-\frac{\sum_{i=1}^{4} C_{i+16}\left(x_0^i\right)^{\gamma_i}}{(\gamma_i-\eta_1)}+\frac{\lambda_0q_1x_0^i}{(r+\lambda_0-\mu_0)(1-\eta_1)}+\frac{\tau\lambda_0(c_s+c_c)}{r(r+\lambda_0)(-\eta_1)}-\frac{\lambda_0 I}{(r+\lambda_0)(-\eta_1)}=0 \quad (64)$$

Where $g_0(\gamma_i)=0$, with $i=1,2,3,4$. In other words, the parameters γ_i are determined by solving $g_0(\gamma)=0$. The value of the investment option can be summarized by the following equation:

$$f_0=\begin{cases}\dfrac{\sum_{i=1}^{4} A_{i+16}x^{\beta_i}\eta_2}{\eta_2+\beta_i}, & if\ x\le x_1^i\\ \dfrac{\sum_{i=1}^{4} B_{i+12}x^{\beta_i}}{\eta_2+\beta_i}+\dfrac{\sum_{i=1}^{4} B_{i+4}x^{\beta_i}}{\eta_2+\beta_i}+\dfrac{\sum_{i=1}^{4} B_{i+8}x^{\beta_i}}{\eta_2+\beta_i}+\dfrac{\sum_{i=1}^{4} C_{i+16}\left(x_1^i\right)^{\gamma_i}}{\eta_2+\gamma_i} & \\ +\dfrac{q_1\lambda_0x_1^i}{(r+\lambda_0-\mu_0)(1+\eta_2)}-\dfrac{I\lambda_0}{(r+\lambda_0)\eta_2}+\dfrac{\lambda_0\tau(c_s+c_c)}{\eta_2 r(r+\lambda_0)} & if\quad x_1^i\le x\le x_0^i\\ V_0^T(x)-I & if\ x>x_0^i\end{cases} \quad (65)$$

$$f_1=\begin{cases}\dfrac{\sum_{i=1}^{4} B_{i+16}x^{\beta_i}}{\beta_i-\eta_1} & if\ x<x_1^i\\ V_1^T(x)-I & if\ x\ge x_1^i\end{cases} \quad (66)$$

After assessing the value of the firm and the value of the investment option, we can specify the solutions of the optimal investment thresholds and the agency cost of debt in order to study the impact of contingent convertible bonds on the debt overhang problem, asset substitution and firm value in a regime-switching model and a diffusion process with exponential distribution jump.

Optimal Investment Thresholds and the Agency Cost of Debt

Our objective in this section is to examine the optimal firm investment policy that maximizes the equity value $E_l(x)$ and the total firm value $V_l(x)$. Referring to Mauer and Sarkar (2005) and Song and Yang (2015), we take the investment threshold x_l^i as a decision variable for given coupon rates. Since shareholders optimally choose the investment threshold x_l^i, we consider two different investment policies that maximize the equity value $E_l(x)$ and the firm value $V_l(x)$. Indeed, we assume that there are two types of investment thresholds "the first-best thresholds" denoted, x_l^{iF}, which maximizes firm value and "the second-best thresholds" denoted, x_l^{iS}, which maximizes firm value. Therefore, the last smoth-pasting condition at the investment thresholds $x_l^{iF}$and x_l^{iS} must satisfy the following formulas respectively:

$$f_l'\left(x_l^{iF}\right)=V_l'\left(x_l^{iF}\right) \quad (67)$$

$$f_l'\left(x_l^{iS}\right)=E_l'\left(x_l^{iS}\right) \quad (68)$$

Since equations (67) and (68) determine the option's investment time and equation (58) presents the option's exercise region and its continuation, then the optimal investment threshold, $x_l^{iF}$where $l\in\{0,1\}$, satisfies the following system of equations:

$$\begin{cases}\dfrac{\sum_{i=1}^{4} B_{i+16}\beta_i\left(x_1^{iF}\right)^{\beta_i}}{\beta_i-\eta_1}=\dfrac{\sum_{i=1}^{4} B_{i+12}\beta_i\left(x_1^{iF}\right)^{\beta_i}}{\eta_2+\beta_i}+\dfrac{\sum_{i=1}^{4} B_{i+4}\beta_i\left(x_1^{iF}\right)^{\beta_i}}{\eta_2+\beta_i}+\dfrac{\sum_{i=1}^{4} B_{i+8}\beta_i\left(x_1^{iF}\right)^{\beta_i}}{\eta_2+\beta_i}+\dfrac{q_1x_1^{iF}}{1+\eta_2}\\ \dfrac{\sum_{i=1}^{4} C_{i+16}\gamma_i\left(x_0^{iF}\right)^{\gamma_i}}{\eta_2+\gamma_i}+\dfrac{q_1\lambda_0x_0^{iF}}{(r+\lambda_0-\mu_0)(1+\eta_2)}=\dfrac{q_0x_0^{iF}}{1+\eta_2}\end{cases} \quad (69)$$

And the optimal investment threshold x_l^{iS}, $l\in\{0,1\}$ satisfies:

$$\begin{cases} \dfrac{\sum_{i=1}^{4} B_{i+16}\beta_i\left(x_1^{iS}\right)^{\beta_i}}{\beta_i-\eta_1} = \dfrac{\sum_{i=1}^{4} B_{i+8}\beta_i\left(x_1^{iS}\right)^{\beta_i}}{\beta_i-\eta_1} + \dfrac{q_1 x_1^{iS}}{1-\eta_1} \\ \dfrac{\sum_{i=1}^{4} C_{i+16}\gamma_i\left(x_0^{iS}\right)^{\gamma_i}}{\eta_2+\gamma_i} + \dfrac{q_1\lambda_0 x_0^{iS}}{(r+\lambda_0-\mu_0)(1+\eta_2)} = \dfrac{\sum_{i=1}^{4} A_{i+8}\,\beta_i\left(x_0^{iS}\right)^{\beta_i}}{\eta_2+\beta_i} + \dfrac{q_0 x_0^{iS}}{1+\eta_2} \end{cases} \quad (70)$$

After the determination of the optimal investment thresholds, we can calculate the agency cost of debt by subtracting the value of the firm under the investment threshold x_l^{iF} from the value of the firm under the investment threshold x_l^{iS}, see Mauer and sarkar (2005). Thus, following Luo and Yang (2017), the agency cost of debt is expressed as:

$$AC = \frac{f_l^F - f_l^S}{f_l^S}, \qquad l \in \{0,1\} \ (71)$$

Where f_l^F and f_l^S denote the value of the firm below the investment threshold x_l^{iF} and x_l^{iS} respectively.

The Optimal Capital Structure of the Company

Based on the previous conclusions, we discuss in this part the evolution of the value of the company with the coupon rate of the risky debt and the coupon rate of the CoCo debt. In addition, we discuss the optimal capital structure. For this, we assume that the level of cash flow x at the time of investment and debts are valued at a reasonable price due to the standard assumption that bondholders are rational. Therefore, we choose the optimal coupon rates c_s of SBs and c_c of CoCos to maximize the firm value $V_0^T(x)$ determined by (54) if the current economy is in recession and maximize the value of company $V_1^T(x)$ specified by (55) if the current economy is expanding.

From the above, we note that the two coupon rates correspond to the following nonlinear programming problems:

$$V_l^{T^*}(x) = \sup_{c_s \geq 0, c_c \geq 0} V_l^T(x; c_s, c_c), \qquad l \in \{0,1\} \quad (72)$$

We denote $V_0^T(x)$ by $V_0^T(x; c_s, c_c)$ to emphasize that the value of the firm depends on the coupon rates. If we solve equation (72) for $l = 0$, we obtain the optimal coupon rate c_s^* of the risky debt and the optimal coupon rate c_c^* of the CoCos.

Numerical Analysis and Discussion

To clarify the impact of contingent convertible bonds on firm valuation under the regime-switching model if the state variable follows a diffusion process with doubly exponential distribution jumps, we first select the values of basic parameters that are related to the business as follows: current cash flow level $x_0 = 1$, annualized risk-free interest rate $r = 0.06$, rate of return for $(l = 0)$ $\mu_0 = 0.01$, the rate of return $(l = 1)$ $\mu_1 = 0.04$, the volatility $\sigma_0 = \sigma_1 = 0.25$, the effective tax rate $\tau = 0.35$, the cost of bankruptcy $\alpha = 0.25$ and the investment cost $I = 200$. The conversion ratio $\beta = 0.4$ according to Koziol and Lawrenz (2012) and the capital adequacy ratio $\phi = 0.05$ according to Glasserman and Nouri (2012). The coupon rate of the risky debt $c_s = 1.1$ and the coupon rate of CoCos $c_c = 0.5$ are determined by maximizing the value of the total enterprise. Second, we choose the variables which are related to jumps referring to Kou and Yang (2003), $q = 0.7$ defines the probability of jumping +up, $p = 0.3$ denotes the probability of jumping down $\frac{1}{\eta_1} = 0.02$ and $\frac{1}{\eta_2} = 0.03$ represent the means of two distributions respectively. According to Pengfei et al. (2017), we take the intensity of the jump $\lambda_0 = 0.3$ and $\lambda_1 = 0.1$.

Debt Overhang Effect

To solve the problem of debt overhang, as Pennacchi et al. (2014), we calculate the net increase in the value of equity when the value of firms without debt (or value of assets) adds one unit, i.e. $\partial E/\partial A - 1$, of which one negative value means that the amount that the shareholders collect is ultimately less than what they originally invested and therefore indicates a distortion of debt overhang.

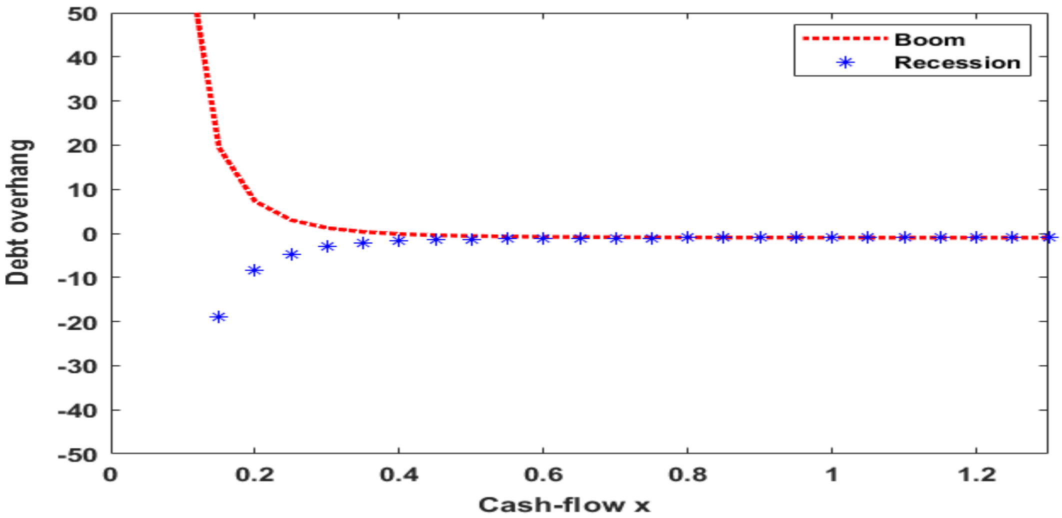

Figure 1. Debt overhang effects vs cash-flowx

Figure 1 graphically represents the effect of cash level and economic regime on over-indebtedness. In general, the inefficiency linked to debt-overhang decreases with the level of cash and even gradually disappears if the level is high enough, as expected. Inefficiency is less evident in times of expansion than in times of recession. These conclusions are easy to understand. Also, we find that there is almost no debt overhang during booms and busts if CoCos debt is issued in the capital structure of the firm, for a high level of cash-flow. Moreover, Figure 1 reveals that the closer the level of cash-flow is to the conversion threshold, the closer the debt overhang problem is to 0. In fact, the more the level of cash-flow tend towards the conversion threshold, the more the he incentive for shareholders to inject equity is important to avoid any conversion, because conversion is very costly for them.

Asset Substitution Effect

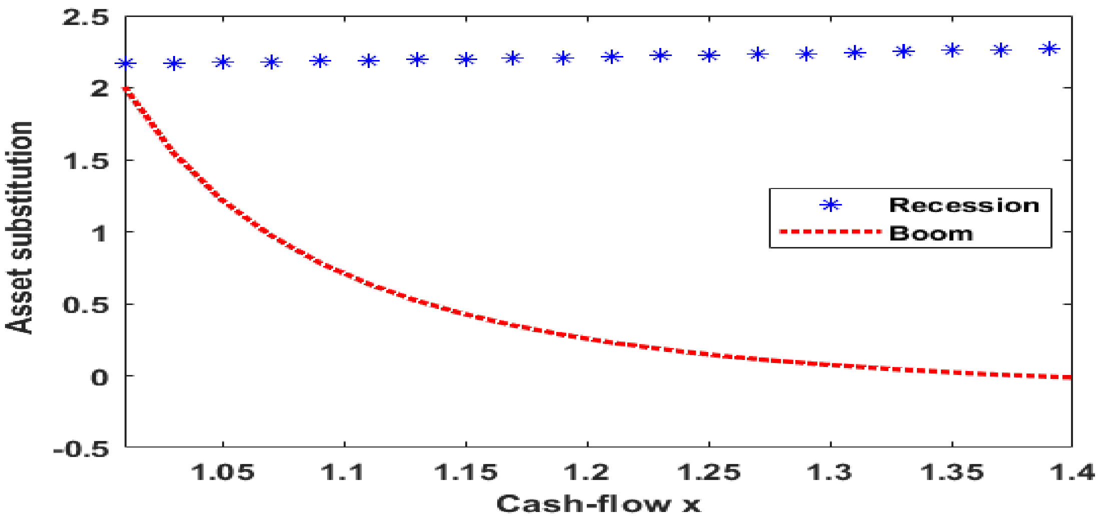

Figure 2. Asset substitution effect vs cash

To measure the incentive to transfer risk, following Pennacchi et al. (2014), we take the value of equity as a function of the volatility rate and calculate its derivative, namely $\partial E/\partial\sigma$. If positive, an increase in business risk leads to an increase in equity value and therefore shareholders have an incentive to transfer the risk to debt holders. Naturally, the higher the value, the stronger the incentive. As expected, Figure 2 indicates that there is a stronger incentive for risk transfer during recessions than during booms. Figure 2 shows that if the current cash level is close to the conversion threshold during boom times, the incentive to transfer risk decreases and even it may be disappears altogether and becomes close to 0. In general, the lower the level cash-flow, the weaker the incentive to transfer risk.

Intuitively, the inefficiencies resulting from asset substitution and o debt overhang should decrease with the conversion rate of the CoCos. Indeed, a higher conversion rate means a more severe penalty in case of conversion and, therefore, shareholders have less incentive to invest in a high-risk period and to inject funds into the company more actively. Indeed, Figures 1 and 2 further show that inefficiencies are much larger in recession than in boom times and during boom times, if the conversion rate is high enough, around $\beta = 0{:}4$, the two inefficiencies disappear.

Conclusion

In this paper, we consider a company without assets in place with a perpetual option in an investment project whose cost of exercising this option is sunk. The unexpected return of the project is governed by a continuous and temporal Markov chain. We present explicit expressions for the pricing of the company's securities and we evaluate the value of the investment option as well as the optimal investment time.

Closed-form solutions have been examined in a regime-switching structural model when the value of cash flows generated by the firm follows a diffusion process with double exponential distribution jumps. This makes the model proposed in this paper more complicated and realistic. The equilibrium price theory under the jump diffusion model was developed based on a structural model introduced by Leland (1994) and later extended by Kou (2002) and Chen and Kou (2009).

This paper presents a theoretical explanation that is based in particular on investing in a regime-switching model for a firm's capital structure composed of CoCos, equity and risky debt. The modeling of a firm's investment decision is determined as a real option and the optimal policy of the firm is obtained by maximizing the equity value and the value of the firm. Additionally, we examine the impact of CoCo contingent capital as a financing instrument on the inefficiencies resulting from over-indebtedness and asset substitution under a double exponential Jump-diffusion model with switching regime.

Scientific Ethics Declaration

The authors declare that the scientific ethical and legal responsibility of this article published in EPESS journal belongs to the authors.

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