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Research Article

On moving average based location charts under modified successive sampling

Tahir Mahmood*[1], Mehvish Hyder[2], Syed Muhammad Muslim Raza[2], Muhammad Moeen Butt[2], Muhammad Riaz[3]

[1] *School of Computing, Engineering and Physical Sciences, University of the West of Scotland, Paisley, PA1 2BE, United Kingdom*
[2] *Department of Economics and Statistics, Dr Hasan Murad School of Management (HSM), University of Management and Technology, Lahore, Pakistan*
[3] *Department of Mathematics, King Fahd University of Petroleum and Minerals, Dhahran, 31261, Saudi Arabia*

Abstract

Ceramics are made up of water, clay, and powders. These are categorized as non-metallic and inorganic materials. It is revealed in the literature that Longquan celadon glaze had irregular cracks in glaze layers due to the relatively high content of Na_2O. Therefore, it is necessary to monitor the influence of Na_2O in the ceramic process. Control charts are a possible tool to monitor the changes in the ceramic process. For single event issues, simple random sampling strategy is utilized; however, modified successive sampling is preferred as the favored sampling strategy at regular intervals of time when the quality of any product is evaluated. Hence, this paper is designed to propose moving average $MA_{MSS(S)}$ and double moving average $DMA_{MSS(S)}$ based control charts to detect small to moderate location shifts using the modified successive sampling technique. We have highlighted the performance evaluations of designed control charts with respect to run-length metrics, and their comparison has been made with the existing $Shewhart_{MSS(S)}$ control chart. The results revealed that the $DMA_{MSS(S)}$ performs more efficiently as compared to the $Shewhart_{MSS(S)}$ and $MA_{MSS(S)}$ control charts. Further, to demonstrate the application of the designed charts, a dataset of the chemical composition of the ceramic is also utilized.

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*Corresponding Author.
Email addresses: tahir.mahmood@uws.ac.uk (T. Mahmood), s.mehvish108@gmail.com (M. Hyder), muslim.raza@umt.edu.pk (S.M.M. Raza), muhammad.moeen@umt.edu.pk (M.M. Butt), riazm@kfupm.edu.sa (M. Riaz)
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1. Introduction

A ceramic is a non-metallic inorganic solid made of metallic or non-metallic compounds essential to everyday life, like glass, plates, tiles, toilets, and bricks [13]. Ceramics are made up of mixtures of earth elements, clay, water, and powders, molding them into desired shapes. After shaping it, it is fired in a high-temperature oven called a kiln. Often, ceramics are coated with waterproof and paint-like decorative materials called glazes. It has caught much attention from researchers with its wide variety and long history. The Longquan and Jingdezhen kilns in the Ming dynasty are the most famous kilns in Chinese ceramic history. Jingdezhen kiln was trying to adopt Longquan celadon due to its good glaze features and unique color, pattern, and style quality. The firing technology and production process of imitated Longquan celadon in Jingdezhen have reached a relatively high level. The contents of micro and macro chemical elements in the ceramic glaze and body depend upon firing technology and raw material.

The melting temperature variation is lower in glaze, and the viscosity rate is higher in the celadon body in Na_2O contents. By comparing the local area raw materials, both kilns are not entirely the same as each other because the contents of Na_2O in Jingdezhen differed by 1.01% from those in Longquan. Longquan celadon glaze has irregular cracks in glazes layers due to relatively high contents of Na_2O. Therefore, it is necessary to identify the effects of Na_2O in the ceramic process, which can be achieved using control charts. Control charts are a possible tool to monitor the changes in the ceramic process. Generally, control charts are a well-known statistical process control tool used for real-time monitoring [11, 36]. It comprises two control limits known as lower/upper control limits and a center line, abbreviated as LCL, UCL, and CL, indicating the process's stability, whether in-control (IC) or out-of-control (OOC).

With respect to structure, control charts are classified into memory-less and memory-type control charts. Memory-less control charts are designed only on the current information, while memory-type control charts are specifically designed to monitor current information and take into account the history of the process [25]. Shewhart control charts are the type of memory-less and efficient for the detection of large shifts in the process, while exponentially weighted moving average (EWMA), cumulative sum (CUSUM), moving average (MA), and double moving average (DMA) are the memory-type control charts, which are sensitive and require to detect moderate and small shifts [27]. Many memory-less and memory-type control charts are available in the literature for monitoring scale and location parameters [8]. Some recent literature on Shewhart charts is shown in studies [14, 23, 24, 41], Shangchen and Mohammad [38] also discussed the memoryless chart and their control limits. Alevizakos et al. [7] developed memory-type MA and DMA control charts for the efficient monitoring of moderate to small shifts, respectively. A comparison between different control charts (moving average, geometric moving average, and CUSUM) procedures is presented in [34]. The study is based on different tests (Girshick and Rubins optimal tests, run sum tests). For the calculation of ARL of geometric moving average charts by using simulation techniques, a numerical method is carried out by [35]. The control chart tests based on geometric moving averages properties are compared with different tests based on normal moving averages evaluated in [33]. Zhang et al. [43] defined the formula for ARL of the moving average control chart. Khoo and Wong [22] proposed a modified form of the MA chart named as double moving DMA chart, which improves the performance of the MA chart to detect small to moderate shifts in the mean. Amir et al. [9] used a DMA chart for the monitoring of location parameter using auxiliary information. To get the optimal parameters of the DMA chart, a zero-inflated Binomial process based on the DMA scheme was developed by [10].

For the estimation of population parameters, sampling is an effective procedure [6]. Using different sampling schemes, we can estimate the population parameters more precisely. Therefore, there exists wide-ranging literature on quality control using different sampling schemes. Salazar and Sinha [37] designed a Shewhart chart by replacing simple random sampling (SRS) with ranked set sampling (RSS) scheme. Muttlak and Al-Sabah [28] used two RSS schemes (median RSS and extreme RSS) to develop new Shewhart control charts to monitor the processs location. Abujiya et al. [4] used median RSS scheme to propose an EWMA control chart for monitoring location parameter. A CUSUM control chart was designed by using the RSS scheme [5]. The improvements in linear-profiles monitoring based on RSS can be seen in studies [1, 2, 32, 40]. To monitor shifts in mean for a normal process, Nawaz et al. [30] and Nawaz and Han [29] designed the Shewhart, CUSUM, EWMA, and HWMA charts based on the neoteric RSS scheme. Hussain et al. [16] evaluated different median-based Shewhart, CUSUM, and EWMA charts for location monitoring under RSS and NRSS schemes. Tekşen and Anagün [39] introduced interval type-2 fuzzy c-control charts by using the method of ranking. To monitor the location parameter, Karagöz [21] proposed a robust control chart for the monitoring of skewed and contaminated processes. Czabak-Górska et al. [12] proposed a hybrid control chart for location and dispersion monitoring based on the classic and robust estimators. Mehmood et al. [26] performed a comparative analysis between FAR and ARL based control charts using run rules. The sampling scheme plays an important role in SPC because cost efficiency and process stability are achieved if the defective product is detected on time. For a single occasion, the SRS scheme is preferred in most surveys. But for real-time surveillance at regular intervals of time, we are required to assess the quality of the product. In these cases, successive sampling is preferable for detecting the changes in the process on different occasions. While using sampling intervals, the selection of samples from an ordered population is called systematic sampling, and the sampling procedure of systematic sampling is quite different from modified successive sampling (MSS). Yaqub et al. [42] presented a new cost-efficient sampling scheme named MSS based on the Shewhart chart. The constructed chart performs more efficiently than traditional Shewhart control charts based on the SRS. Abbas et al. [3] designed an improved S2 chart to detect variability in the process under the MSS scheme. Riaz et al. [31] showed that the linear profile monitoring methods under the MSS scheme outperformed the SRS schemes linear profile monitoring methods. Recently, Hyder et al. [17] introduced new moving average control charts based on the MSS scheme for monitoring the processs dispersion parameter.

The aforementioned literature highlights that for monitoring location parameter using MSS, only a single study is designed for the Shewhart charting structure [26]. Furthermore, as pre-defined, the Shewhart control charts only monitor the large shift in process. In contrast, memory-type control charts are preferable for monitoring small and moderate shifts. The study aims to design new and efficient memory-type control charts based on MSS to monitor the processs location parameter. To overcome this deficiency, in this study, we propose memory-type MA and DMA control charts based on an improved and cost-efficient sampling scheme MSS to monitor the processs location. To monitor the performance of proposed control charts, run-length metrics are used. We also designed a comparative study between the proposed and Shewhart control charts. Also, a real-time dataset is used to demonstrate the application of proposed charts in real-life scenarios. The respite of the stated proposal is structured as follows; the introduction to MSS schemes and the construction of the proposed charts are presented in Section 2. In Section 3, the performance evaluation of the proposed charts is evaluated using the simulation. Comparative analysis is delivered in Section 4, in Section 5, a case study is displayed, and Section 6 shows the conclusions and future recommendations of the article.

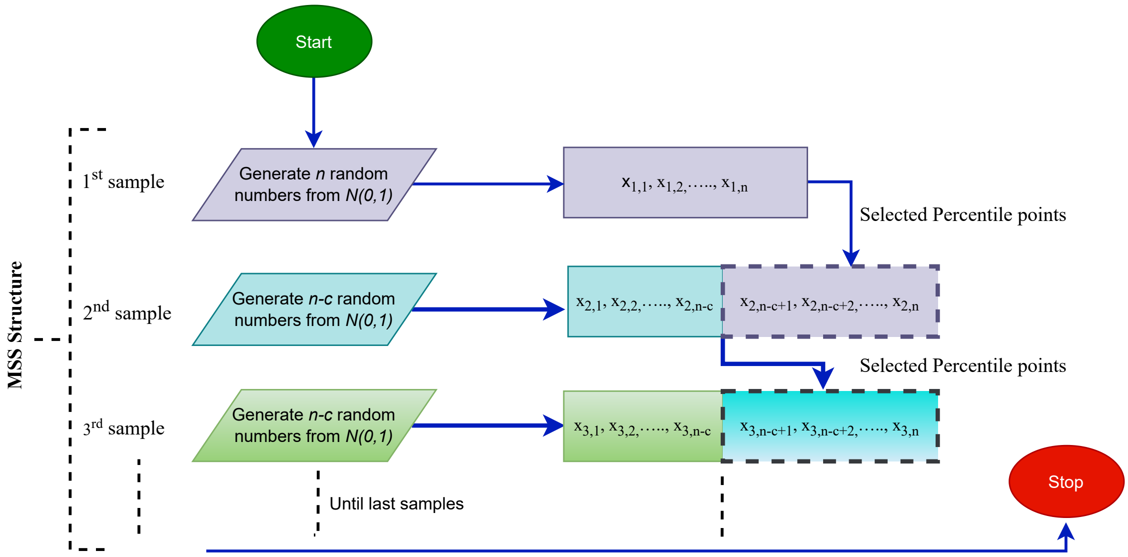

Figure 1. Structural diagram of the MSS scheme

2. Methodology

This section is intended to detect the processs location parameter effectively by developing proposed MA and DMA charts under modified successive sampling schemes. Further, the structures of the proposed MA and DMA charts are also presented in this segment.

2.1. Modified successive sampling

Nowadays, it is preferred to pattern the variation of a series on different occasions. For sorting out this problem, a successive sampling scheme is preferable for collecting information at regular intervals with minimum expenditure. Because SRS scheme provides reliable estimates for single occasion problems. Nevertheless, successive sampling scheme offers more reliable and efficient estimates for repetitive surveys. Jessen [20] first time introduced successive sampling. The procedure of successive sampling proceeds as follows: for the first occasion, the first sample is selected, and then a second sample is chosen for the next occasion, including some previous sample points. Hence, this sampling scheme delivers more efficient estimates and cost-effective procedures than a simple random sampling scheme.

Furthermore, Successive sampling has been modified as a new sampling technique named MSS, specially designed for SPC applications. Figure 1 presents the structure of the MSS scheme, which is further explained in the following steps:

i. The process begins with SRS, the subgroup of size n $(z_{1,1}, z_{1,2}, \ldots, z_{1,n})$ is selected.
ii. Again, by using SRS, draw new $(n-c)$ observations as a second sample and use some information such as quantiles (Q_1& Q_3), chosen from the previous sample as the remaining c observations. They are represented by $[(z_{2,1}, z_{2,2}, \ldots, z_{2,n-c}),$ $(z_{2,n-c+1}, z_{2,n-c+2}, \ldots, z_{2,n})]$, where $z_{2,n-c+1}{=}f_1\left(z_{1,1}, z_{1,2}, \ldots, z_{1,n}\right)$, $z_{2,n-c+2}{=}\ f_2\left(z_{1,1}, z_{1,2}, \ldots, z_{1,n}\right)$ and so on, up to $z_{2,n}{=}\ f_c\left(z_{1,1}, z_{1,2}, \ldots, z_{1,n}\right)$.
iii. For the selection of the third sample, the procedure mentioned above is repeated by selecting $(n-c)$ new observations, and c observations are chosen in the form of quantiles from the second sample. In a similar pattern, this sampling method can be repeated for the complete run of production.

Following Riaz et al. [31] and Hyder et al. [18], the four different sampling schemes are used and represented by $MSS_{(S)}$, where the subscript S has been used to identify the four different choices of modified successive sampling schemes. Where,

i. $S = 1$ indicates about first MSS scheme ($\mathbf{MSS_{n,2,min,max,}}$),
ii. $S = 2$ referred to the second MSS scheme ($\mathbf{MSS_{n,2,Q1,Q3}}$),
iii. $S = 3$ shows the third MSS scheme ($\mathbf{MSS_{n,3,min,Q2,max,}}$),
iv. $S = 4$ defines the fourth MSS scheme ($\mathbf{MSS_{n,3,Q1,Q2,Q3}}$).

The representation mentioned above is further followed throughout this study.

2.2. The $MA_{MSS(S)}$ control chart

The moving average (MA) control chart was proposed by [41]. The statistic of the proposed chart ($MA_{MSS(S)}$) based on the various sampling schemes of MSS is described as follows for detecting location parameter.

$$M_{i(S)} = \begin{cases} \frac{\sum_{j=1}^{i} \bar{X}_{j(S)}}{i}, & \text{for i < w,} \\ \frac{\sum_{j=i-w+1}^{i} \bar{X}_{j(S)}}{w}, & for\ i \geqslant w. \end{cases} \tag{2.1}$$

Where i is the sample number, w is the moving average span, $\bar{x}_{j(S)}$ is the mean of sample number j with the various sampling schemes shown by the subscript(S). The mean of $M_{i(S)}$ statistic is $\mu_0 = E(M_{i(S)})$, and the variance is determined by:

$$Var\left(M_{i(S)}\right) = \begin{cases} \frac{\sigma_0{}^2}{ni}, & \text{for i < w,} \\ \frac{\sigma_0{}^2}{nw}, & for\ i \geqslant w. \end{cases} \tag{2.2}$$

The control limits of $M_{i(S)}$ are described as:

$$\text{LCL}_{(S)} = \mu_0 - \text{L}_{(S)}\sqrt{Var\left(M_{i(S)}\right)} \tag{2.3}$$

$$CL_{(S)} = \mu_0 \tag{2.4}$$

$$\text{UCL}_{(S)} = \mu_0 + \text{L}_{(S)}\sqrt{Var\left(M_{i(S)}\right)} \tag{2.5}$$

Against the subgroup size n and given ARL_0, $\text{L}_{(S)}$ is a charting constant and it is determined using simulation algorithm provided in Section 3.1. Further, μ_0 and σ_0 are the mean and standard deviation of IC process, respectively.

2.3. The $DMA_{MSS(S)}$ control chart

For the efficient detection of location parameter, Alevizakos et al. [7] developed a memory-type DMA control chart. The statistic of $DMA_{MSS(S)}$ based on different MSS schemes is described as:

$$D_{i(S)} = \begin{cases} \frac{\sum_{j=1}^{i} M_{j(S)}}{i}, & \text{for i < w,} \\ \frac{\sum_{j=i-w+1}^{i} M_{j(S)}}{w}, & for\ i \geqslant w. \end{cases} \tag{2.6}$$

The expected value of $D_{(i(S))}$ statistic for any i is equivalent to $\mu_0 = E(D_{i(S)})$ and the variance is defined as:

For $i < w$,

$$Var\left(D_{i(S)}\right) = \frac{\sum_{l=1}^{i} {a_l}^2 {\sigma_0}^2}{ni^2} \tag{2.7}$$

For $w \leq i < 2w - 1$,

$$\begin{aligned} Var\left(D_{i(S)}\right) = \left(\frac{{\sigma_0}^2}{nw^2}\right)\Bigg[& \sum_{j_1=i-w+1}^{w-1} \frac{1}{j_1} + \sum_{i-w+1 \leqslant j_{11} < j_{12} \leqslant w-1} \frac{2}{j_{12}} \\ & + \sum_{j_1=i-w+1}^{w-1} \sum_{j_2=w}^{i} \frac{2\left(j_1 - j_2 + w\right)}{j_1 w} + \frac{i-w+1}{w} \\ & + \sum_{w \leqslant j_{21} < j_{22} \leqslant i} \frac{2\left(j_{21} - j_{22} + w\right)}{w^2} \Bigg], \end{aligned} \tag{2.8}$$

and for $i \geq 2w - 1$

$$Var\left(D_{i(S)}\right) = \frac{{\sigma_0}^2}{nw^2}\left[1 + \sum_{i-w+1 \leqslant j_1 < j_2 \leqslant i} \frac{2\left(j_1 - j_2 + w\right)}{w^2}\right]. \tag{2.9}$$

The derivation of the above-stated mean and variances is briefly described in Alevizakos et al. [7]. The $LCL_{(S)}$ and $UCL_{(S)}$ of $DMA_{MSS(S)}$ are calculated as:

$$\mathrm{LCL}_{(S)} = \mu_0 - \mathrm{K}_{(S)}\sqrt{Var\left(D_{i(S)}\right)} \tag{2.10}$$

$$CL_{(S)} = \mu_0 \tag{2.11}$$

$$\mathrm{UCL}_{(S)} = \mu_0 + \mathrm{K}_{(S)}\sqrt{Var\left(D_{i(S)}\right)} \tag{2.12}$$

The constant $K_{(S)}$ defines the width of control limits. By carrying a procedure of the simulation algorithm provided in Section 3.1, the value of $K_{(S)}$ is determined. If the statistic $D_{i(S)}$ lies across these limits, then it is said to be OOC process; otherwise, it is called IC process. Further, a general layout of the implementation of proposed $MA_{MSS(S)}$ and $DMA_{MSS(S)}$ charts is presented in Figure 2.

3. Performance evaluations and simulation procedure

The performance of the designed $MA_{MSS(S)}$ and $DMA_{MSS(S)}$ charts were evaluated by run-length metrics. This section highlights the proposed charts performance evaluations and comparative analysis with the existing Shewhart control charts designed by [42]. To monitor the performance of designed $MA_{MSS(S)}$ and $DMA_{MSS(S)}$ charts, the different performance measures (i.e., ARL, SDRL, and MDRL) have been used. The ARL (average run length) is the most frequently used performance evaluation measure [26]. It is attained by noticing the sample average values when a single OOC signal is observed. ARL_0 (for IC situation) and ARL_1 (for OOC situation) are two categories of ARL (cf. [19]). At a pre-specified value of ARL_0, a control chart is said to outperform with a low value of ARL_1. For more brevity of results, with ARL, the MDRL (median run length) is also calculated to define the robust average value, and the SDRL (standard deviation run length) is included in the discussion to describe the variability of the run length. Moreover, this study covers zero-state as well as steady-state run-length performances.

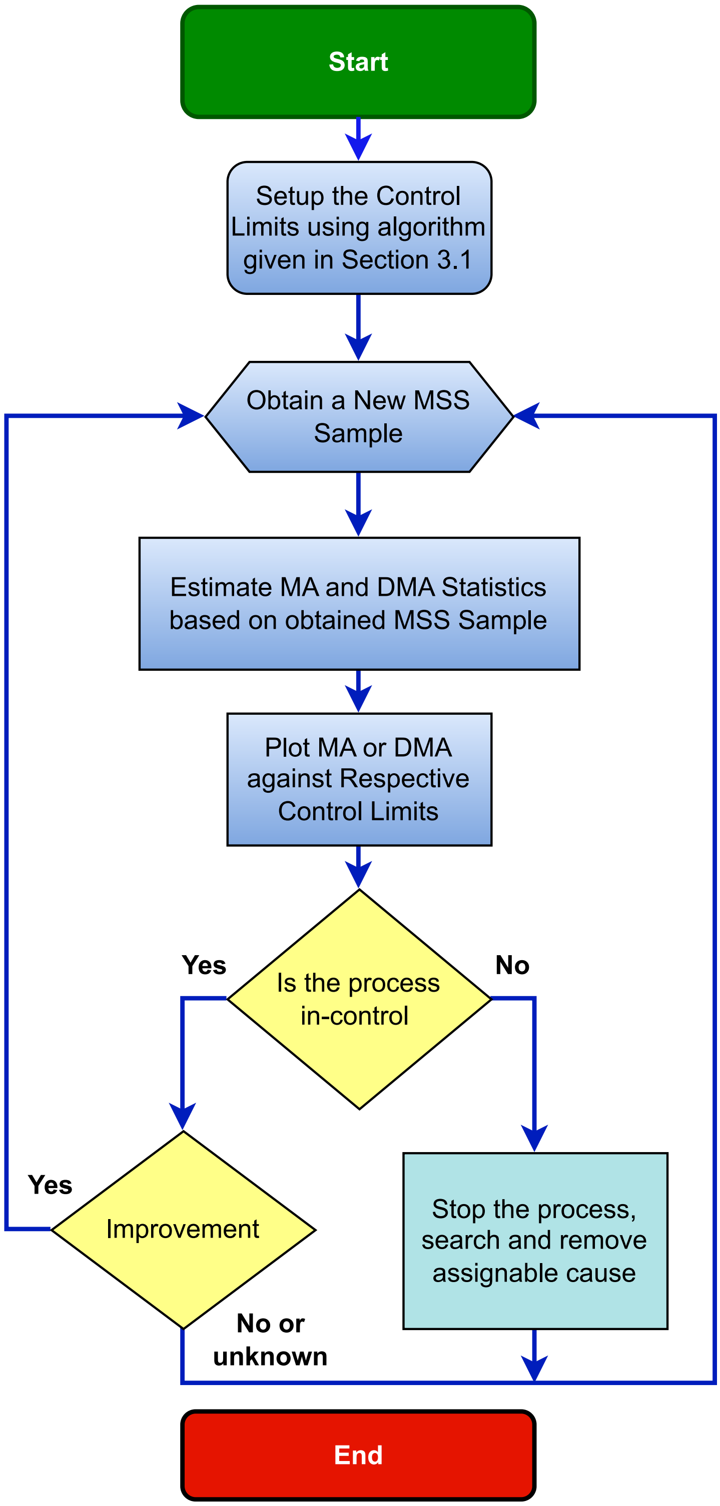

Figure 2. Paradigm of the proposed control chart structures

3.1. Algorithm for control charting coefficients

The control limits of the designed $MA_{MSS(S)}$ and $DMA_{MSS(S)}$ charts are defined by using control charting constant $L_{(S)}$ and $K_{(S)}$, respectively. By following the given steps, the values of these charting constants are determined at a fixed value of ARL_0.

i To build the structure of control limits, choose the values of ARL_0 and parameters $(\mu_0,\ {\sigma_0}^2,\ w,\ n)$ by setting the arbitrary values of $L_{(S)}$ and $K_{(S)}$.
ii By using a normal distribution with parameters μ_0 and ${\sigma_0}^2$, generate the random sample using MSS structure, and compute the statistics of the designed $MA_{MSS(S)}$ and $DMA_{MSS(S)}$ charts described in Eq's. (2.1 and 2.6).
iii Now, compare these computed statistics with their corresponding control limits.
iv To obtain a single run length, repeat steps (ii)-(iii).
v To attain the ARL_0, repeat the steps (ii)-(iv), 10,000 times.
vi If the desired ARL_0 is not attained, stop the procedure and readjust the values of $L_{(S)}$ and $K_{(S)}$. To obtain the desired ARL_0, repeat steps (i)(v), a large number of times.

Followed by [42], the choices of design parameters are considered as: (i) the mean $(\mu_0 = 0)$, (ii) the standard deviation $({\sigma_0}^2 = 1)$, (iii) subgroup size (n = 5&7), and (iv) moving average span $(w = 2)$. Table 1 presents the values of charting constants $L_{(S)}$ for $MA_{MSS(S)}$ and $K_{(S)}$ for $DMA_{MSS(S)}$ control charts against $ARL_0 = 200$, 370, and 500.

Table 1. Control charting constants $L_{(S)}$ and $K_{(S)}$ for the proposed control charts under different MSS schemes for $(w = 2)$

Chart	Scheme	$\mathbf{ARL_0 = 200}$		$\mathbf{ARL_0 = 370}$		$\mathbf{ARL_0 = 500}$	
		$n = 5$	$n = 7$	$n = 5$	$n = 7$	$n = 5$	$n = 7$
$\mathbf{MA_{MSS(S)}}$	$\mathbf{MSS_{n,2,min,max,}}$	2.31	2.45	2.46	2.62	2.52	2.7
	$\mathbf{MSS_{n,2,Q1,Q3}}$	2.71	2.75	2.92	2.96	3.01	3.07
	$\mathbf{MSS_{n,3,min,Q2,max,}}$	2.32	2.45	2.47	2.61	2.55	2.69
	$\mathbf{MSS_{n,3,Q1,Q2,Q3}}$	2.69	2.73	2.9	2.94	3	3.04
$\mathbf{DMA_{MSS(S)}}$	$\mathbf{MSS_{n,2,min,max,}}$	2.32	2.45	2.47	2.61	2.54	2.7
	$\mathbf{MSS_{n,2,Q1,Q3}}$	2.85	2.84	3.06	3.06	3.16	3.16
	$\mathbf{MSS_{n,3,min,Q2,max,}}$	2.41	2.51	2.59	2.69	2.66	2.77
	$\mathbf{MSS_{n,3,Q1,Q2,Q3}}$	2.88	2.87	3.14	3.1	3.25	3.2

4. Comparative analysis

When the shifts are introduced in location parameter (i.e., $\mu_1 = \mu_0 + \delta\sigma_0$), the process is declared as OOC; otherwise, a normal process with parameters $\mu_0 = 0$ and ${\sigma_0}^2 = 1$ is considered as IC process. At the fixed $ARL_0 = 200, 370$, and 500, the ARL_1, $SDRL_1$, and $MDRL_1$ are recorded by introducing both increasing and decreasing shifts $[(0 < \delta \leqslant 2)$ and $(-2 \leqslant \delta < 0)]$ to evaluate the performance of proposed charts (cf. Tables 2-5). A brief discussion on the zero-state performance of designed $MA_{MSS(S)}$ and $DMA_{MSS(S)}$ control charts and a comparative analysis between them and with exiting $Shewhart_{MSS(S)}$ are presented below:

- It is observed that the proposed $MA_{MSS(S)}$ and $DMA_{MSS(S)}$ charts have more detection capability against shifts in the μ as compared to the existing $Shewhart_{MSS(S)}$ chart. For illustration, on the fixed $ARL_0 = 200, n = 7$ and $\delta = 0.25$, ARL_1 for $Shewhart_{MSS(S)}$, $MA_{MSS(S)}$ and $DMA_{MSS(S)}$ are observed at 57.44, 36.82, and 32.40, respectively. When a shift in mean increases as $\delta = 0.50$, the ARL_1 decreases and recorded 11.82, 6.92, and 6.46 for $Shewhart_{MSS(S)}$, $MA_{MSS(S)}$ and $DMA_{MSS(S)}$, respectively (cf. Table 2). By increasing δ (0.25 to 0.50), it is captured that the performance of $Shewhart_{MSS(S)}$, $MA_{MSS(S)}$ and $DMA_{MSS(S)}$ increased as the ARL_1 values are decreased at the pre-specified $ARL_0 = 200$ and n = 7 (cf. Table 3). Tables 2-5 shows that the same performance outlines are also detected for all choices of n and ARL_0.

Table 2. Zero-state run length profile of the proposed charts under $MSS_{n,2,min,max}$

		$\mathbf{Shewhart_{MSS(1)}}$						$\mathbf{MA_{MSS(1)}}$						**DMA – MSS(1)**					
$\mathbf{ARL_0}$	δ	**n=5**			**n=7**			**n=5**			**n=7**			**n=5**			**n=7**		
		ARL	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**
	-2	1.01	1	0.13	1	1	0.05	1.01	1	0.12	1	1	0.05	1.02	1	0.17	1	1	0.07
	-1.75	1.06	1	0.28	1.02	1	0.14	1.06	1	0.25	1.02	1	0.13	1.08	1	0.35	1.02	1	0.14
	-1.5	1.2	1	0.6	1.07	1	0.29	1.17	1	0.45	1.07	1	0.26	1.25	1	0.62	1.08	1	0.33
	-1.25	1.59	1	1.21	1.27	1	0.66	1.43	1	0.8	1.22	1	0.49	1.6	1	0.96	1.31	1	0.68
	-1	2.58	2	2.5	1.9	1	1.5	2.09	2	1.63	1.63	1	0.95	2.27	2	1.57	1.83	1	1.07
	-0.75	5.59	3	6.52	3.78	2	3.78	3.87	2	4.07	2.73	2	2.26	3.82	3	3.46	2.86	3	1.93
	-0.5	17.2	10	22.18	11.87	7	13.71	10.53	6	14.08	7.06	4	7.8	9.68	6	12.36	6.39	4	6.14
	-0.25	75.21	41	100.62	57.34	34	68.37	53.59	26	80.83	36.1	20	45.72	47.09	22	72.68	31.69	17	40.9
200	**0**	**200.36**	**127**	**224.85**	**199.99**	**132**	**212.13**	**199.49**	**113**	**249.61**	**199.06**	**119**	**222.84**	**201.32**	**106**	**262.29**	**199.82**	**117**	**223.77**
	0.25	73.36	41	92.99	57.44	34	67.9	52.65	26	77.85	36.82	20	47.7	47.55	22	76.05	32.4	18	43.33
	0.5	17.49	10	22.54	11.82	7	13.57	10.66	6	13.44	6.92	4	7.3	9.52	6	11.49	6.46	4	6.26
	0.75	5.55	3	6.26	3.83	2	3.85	3.81	2	3.94	2.66	2	2.18	3.88	3	3.52	2.89	3	1.95
	1	2.62	2	2.61	1.9	1	1.54	2.08	2	1.66	1.61	1	0.93	2.28	2	1.6	1.81	1	1.07
	1.25	1.58	1	1.2	1.27	1	0.67	1.42	1	0.77	1.23	1	0.5	1.57	1	0.94	1.31	1	0.67
	1.5	1.2	1	0.58	1.07	1	0.29	1.17	1	0.44	1.07	1	0.27	1.24	1	0.61	1.08	1	0.35
	1.75	1.06	1	0.28	1.01	1	0.12	1.06	1	0.24	1.01	1	0.12	1.08	1	0.34	1.01	1	0.13
	2	1.01	1	0.12	1	1	0.04	1.02	1	0.13	1	1	0.05	1.02	1	0.18	1	1	0.06
	-2	1.02	1	0.16	1	1	0.06	1.02	1	0.15	1	1	0.06	1.03	1	0.21	1	1	0.08
	-1.75	1.08	1	0.34	1.03	1	0.18	1.08	1	0.28	1.02	1	0.15	1.11	1	0.41	1.03	1	0.19
	-1.5	1.26	1	0.69	1.1	1	0.37	1.21	1	0.51	1.1	1	0.31	1.3	1	0.68	1.12	1	0.43
	-1.25	1.76	1	1.48	1.38	1	0.81	1.53	1	0.91	1.28	1	0.55	1.72	1	1.05	1.41	1	0.77
	-1	3.07	2	3.16	2.19	1	1.86	2.31	2	1.88	1.74	1	1.07	2.49	3	1.71	1.96	1	1.14
	-0.75	7.17	4	8.59	4.8	3	4.86	4.48	3	5.12	3.08	2	2.7	4.37	3	4.02	3.22	3	2.22
	-0.5	24.83	14	34.14	16.82	10	19.74	13.74	7	19.38	8.96	6	9.9	11.84	7	14.95	7.78	5	8.43
	-0.25	125.74	69	158.63	94.6	56	114.06	82.48	39	125.33	55.11	30	74.31	69.37	31	120.52	47.5	25	64.13
370	**0**	**370.76**	**233**	**418.6**	**370.68**	**249.5**	**401.99**	**369.26**	**198**	**467.19**	**369.8**	**229**	**412.43**	**370.95**	**194**	**491.64**	**369.44**	**217**	**431.81**
	0.25	122.29	67	158.75	92.15	57	108.73	81.34	38	126.88	56.11	30	73.68	71.18	31	114.12	45.94	25	64.05
	0.5	25.32	14	32.81	16.47	10	19.06	13.31	7	17.92	8.8	6	9.73	11.96	6	15.79	7.74	5	7.76
	0.75	7.29	4	8.8	4.91	3	5.31	4.44	3	4.99	3.11	2	2.7	4.39	3	4.21	3.24	3	2.33
	1	3.07	2	3.22	2.18	1	1.86	2.3	2	1.88	1.76	1	1.08	2.45	3	1.69	1.98	1	1.16
	1.25	1.76	1	1.5	1.38	1	0.82	1.53	1	0.91	1.29	1	0.57	1.7	1	1.04	1.4	1	0.75
	1.5	1.26	1	0.68	1.1	1	0.35	1.21	1	0.5	1.09	1	0.29	1.31	1	0.69	1.12	1	0.43
	1.75	1.08	1	0.35	1.02	1	0.16	1.08	1	0.28	1.02	1	0.14	1.11	1	0.41	1.03	1	0.2
	2	1.02	1	0.15	1	1	0.06	1.02	1	0.15	1	1	0.06	1.03	1	0.2	1	1	0.07
	-2	1.03	1	0.18	1	1	0.06	1.03	1	0.17	1.01	1	0.07	1.04	1	0.24	1	1	0.07
	-1.75	1.09	1	0.37	1.03	1	0.19	1.08	1	0.29	1.02	1	0.16	1.13	1	0.44	1.03	1	0.21
	-1.5	1.29	1	0.74	1.12	1	0.4	1.24	1	0.53	1.11	1	0.33	1.34	1	0.72	1.14	1	0.45
	-1.25	1.87	1	1.64	1.43	1	0.89	1.59	1	0.97	1.32	1	0.59	1.78	1	1.08	1.46	1	0.81
	-1	3.34	2	3.46	2.36	2	2.08	2.39	2	2	1.83	2	1.15	2.59	3	1.81	2.09	2	1.22
	-0.75	8.12	5	9.86	5.41	3	5.82	4.84	3	5.32	3.42	2	3.03	4.55	3	4.21	3.46	3	2.47
	-0.5	29.93	16	42.45	19.91	12	24.13	14.87	8	20.39	9.97	6	11.16	13.36	7	18.01	8.57	5	8.76
	-0.25	158.45	87	208.6	119.04	72	139.07	100.14	45.5	162.25	68.77	38	93.77	89.63	39	152.24	56.11	30	77.55
500	**0**	**500.64**	**312**	**561.21**	**500.02**	**336**	**529.21**	**501.97**	**276**	**631.23**	**501.07**	**311**	**575.97**	**500.43**	**260**	**636.63**	**501.38**	**306**	**584.57**
	0.25	161.71	87	219.11	118.19	71	146.66	98.49	46	154.06	68.04	37	92.23	88.67	37	150.02	56.95	30	79.59
	0.5	29.54	16	39.72	19.77	12	23.19	15.65	8	23.42	10.14	6	11.69	13.24	7	17.72	8.66	6	8.99
	0.75	8.31	5	9.9	5.54	4	5.85	4.82	3	5.48	3.37	2	2.92	4.58	3	4.47	3.44	3	2.44
	1	3.42	2	3.69	2.34	2	2.08	2.41	2	2.01	1.83	2	1.15	2.58	3	1.78	2.07	2	1.19
	1.25	1.83	1	1.52	1.43	1	0.91	1.56	1	0.92	1.32	1	0.58	1.77	1	1.06	1.45	1	0.8
	1.5	1.3	1	0.75	1.11	1	0.39	1.24	1	0.52	1.1	1	0.31	1.34	1	0.72	1.15	1	0.47
	1.75	1.09	1	0.35	1.03	1	0.18	1.09	1	0.31	1.03	1	0.18	1.12	1	0.42	1.03	1	0.21
	2	1.02	1	0.17	1	1	0.06	1.02	1	0.16	1.01	1	0.07	1.04	1	0.23	1	1	0.07

- We have also seen that the designed $DMA_{MSS(S)}$ has the lowest value of ARL_1 as compared to $Shewhart_{MSS(S)}$, and $MA_{MSS(S)}$ observed as 62.89, 105.11, and 72.54, respectively, for the constant $ARL_0 = 500, \delta = 0.25$, and n = 7. Under the mentioned MSS schemes and for pre-fixed ARL_0 (i.e., 200, 370, 500), Tables 2-5 presented that the designed $DMA_{MSS(S)}$ control chart performs efficiently as compared to $Shewhart_{MSS(S)}$ and $MA_{MSS(S)}$ control charts in terms of ARL_1.

- For all mentioned MSS schemes, the values of ARL_1 of $DMA_{MSS(S)}$ control chart are recorded as 45.94, 55.86, 53.90, and 60.79, respectively, for the fixed values of $n = 7, \delta = 0.25$, and $ARL_0 = 370$. Similarly, the ARL_1 of $MA_{MSS(S)}$ control chart is detected as 56.11, 62.23, 58.54, and 66.12, respectively, for each MSS scheme. Table 2-5 shows that the MSS scheme $MSS_{n,2,min,max}$ (S=1) executes better than other MSS schemes, and similar results are also shown at fixed $ARL_0 = 200$ and 500.

- The monitoring ability of the designed charts increases with the n (subgroup size). From Table 3, at fixed value of n=5, $\delta = 0.5$ and $ARL_0 = 370$, ARL_1 for $Shewhart_{MSS(S)}$, $MA_{MSS(S)}$ and $DMA_{MSS(S)}$ is observed as 24.21, 13.31, and 11.96, respectively, while for $n = 7$, the decrease in ARL_1 are seen as 16.79, 8.80, and 7.74, respectively, for all charts. The same pattern of the performance is also revealed in all choices of ARL_0 and δ (cf. Tables 2-5).

Table 3. Zero-state run length profile of the proposed charts under $MSS_{n,2,Q_1,Q_3}$

		$\mathbf{Shewhart_{MSS(1)}}$						$\mathbf{MA_{MSS(1)}}$						**DMA − MSS(1)**					
$\mathbf{ARL_0}$	δ	**n=5**			**n=7**			**n=5**			**n=7**			**n=5**			**n=7**		
		ARL	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**
	-2	1.02	1	0.15	1	1	0.05	1.04	1	0.2	1.01	1	0.08	1.07	1	0.33	1.01	1	0.1
	-1.75	1.07	1	0.32	1.02	1	0.15	1.13	1	0.36	1.03	1	0.17	1.22	1	0.58	1.04	1	0.24
	-1.5	1.23	1	0.64	1.08	1	0.32	1.3	1	0.58	1.12	1	0.34	1.55	1	0.87	1.18	1	0.51
	-1.25	1.67	1	1.28	1.3	1	0.69	1.7	1	1.01	1.33	1	0.58	2.06	2	1.17	1.55	1	0.86
	-1	2.73	2	2.47	1.94	1	1.49	2.58	2	1.97	1.85	2	1.11	3.02	3	1.93	2.22	2	1.25
	-0.75	5.88	4	5.87	3.91	3	3.6	4.98	3	4.47	3.37	2	2.72	5.34	4	4.2	3.61	3	2.36
	-0.5	16.77	12	16.91	11.76	8	11.71	13.14	9	12.67	8.61	6	8.11	13.13	9	11.97	8.48	6	7.13
	-0.25	65.39	45	67.15	52.29	35	53.34	52.85	37	52.57	40.25	28	39.55	52.85	37	51.66	37.49	26	36.73
200	**0**	**199.67**	**138**	**201.83**	**199.9**	**137**	**200.09**	**199.88**	**133**	**202.01**	**199.73**	**133**	**198.15**	**201.98**	**143**	**204.55**	**201.9**	**143**	**204.85**
	0.25	66.6	46	68.53	52.31	35	53.49	53.47	37	53.36	39.89	28	39.01	52.51	37	51.46	37.48	26	36.21
	0.5	16.89	11	17.29	11.78	8	11.9	13.07	9	12.88	8.63	6	8.09	12.98	9	11.55	8.44	6	7.12
	0.75	5.92	4	5.92	3.99	3	3.71	4.97	3	4.49	3.34	2	2.63	5.25	4	4.09	3.6	3	2.37
	1	2.8	2	2.6	1.95	1	1.52	2.59	2	2	1.86	2	1.1	2.99	3	1.87	2.18	2	1.22
	1.25	1.67	1	1.27	1.29	1	0.66	1.7	1	1.01	1.34	1	0.6	2.06	2	1.19	1.56	1	0.86
	1.5	1.24	1	0.63	1.08	1	0.32	1.3	1	0.57	1.12	1	0.34	1.54	1	0.86	1.19	1	0.52
	1.75	1.08	1	0.32	1.02	1	0.15	1.12	1	0.35	1.03	1	0.17	1.22	1	0.58	1.05	1	0.26
	2	1.02	1	0.15	1	1	0.05	1.04	1	0.2	1.01	1	0.08	1.06	1	0.31	1.01	1	0.1
	-2	1.03	1	0.18	1	1	0.07	1.06	1	0.24	1.01	1	0.1	1.1	1	0.38	1.02	1	0.14
	-1.75	1.1	1	0.38	1.03	1	0.18	1.17	1	0.41	1.05	1	0.22	1.3	1	0.66	1.08	1	0.33
	-1.5	1.32	1	0.78	1.12	1	0.39	1.41	1	0.68	1.17	1	0.4	1.69	1	0.94	1.28	1	0.64
	-1.25	1.89	1	1.57	1.41	1	0.85	1.92	2	1.24	1.44	1	0.68	2.31	3	1.27	1.72	1	0.94
	-1	3.35	2	3.11	2.28	2	1.88	3.06	2	2.45	2.09	2	1.35	3.4	3	2.16	2.49	3	1.33
	-0.75	7.57	5	7.61	5.03	4	4.75	6.29	4	5.82	3.98	3	3.25	6.3	5	5.07	4.14	3	2.81
	-0.5	24.06	16	24.47	16.49	11	16.93	18.57	13	18.1	11.78	8	11.09	17.63	13	16.73	10.74	8	9.31
	-0.25	107.11	74	107.02	84.72	58	85.55	86.32	60	86.47	61.89	43	61.26	80.46	57	78.2	56.61	40	54.23
370	**0**	**369.12**	**253**	**363.78**	**370.07**	**264**	**373.88**	**371.92**	**257**	**374.52**	**369.13**	**256**	**361.31**	**369.04**	**253**	**365.74**	**370.24**	**262**	**377.19**
	0.25	108.06	75	109.35	85.22	60	85.01	88.56	62	87.43	62.23	43	61.7	80.41	56	79.93	55.86	39	55.63
	0.5	24.21	17	24.71	16.79	12	16.97	18.47	13	18.17	11.57	8	10.97	17.37	12	16.12	10.83	8	9.63
	0.75	7.86	5	8	5.15	3	5.07	6.31	4	5.79	4.03	3	3.31	6.33	5	5.13	4.25	3	2.87
	1	3.37	2	3.25	2.25	1	1.87	3.04	2	2.42	2.08	2	1.32	3.43	3	2.23	2.46	3	1.33
	1.25	1.87	1	1.49	1.41	1	0.84	1.88	2	1.19	1.44	1	0.68	2.32	3	1.28	1.71	1	0.94
	1.5	1.31	1	0.75	1.11	1	0.38	1.41	1	0.68	1.17	1	0.4	1.68	1	0.95	1.28	1	0.63
	1.75	1.1	1	0.37	1.03	1	0.18	1.17	1	0.42	1.05	1	0.21	1.32	1	0.68	1.08	1	0.33
	2	1.03	1	0.19	1	1	0.07	1.06	1	0.25	1.01	1	0.09	1.11	1	0.4	1.01	1	0.13
	-2	1.03	1	0.2	1	1	0.07	1.07	1	0.27	1.01	1	0.11	1.14	1	0.45	1.02	1	0.16
	-1.75	1.12	1	0.41	1.04	1	0.2	1.2	1	0.45	1.06	1	0.24	1.38	1	0.73	1.09	1	0.35
	-1.5	1.36	1	0.85	1.13	1	0.43	1.46	1	0.72	1.19	1	0.42	1.77	1	0.98	1.32	1	0.67
	-1.25	2.04	1	1.72	1.47	1	0.93	2.01	2	1.28	1.51	1	0.74	2.45	3	1.35	1.79	1	0.97
	-1	3.7	2	3.5	2.48	2	2.11	3.29	2	2.71	2.22	2	1.43	3.64	3	2.36	2.61	3	1.42
	-0.75	8.73	6	8.78	5.69	4	5.45	7.06	5	6.58	4.37	3	3.62	6.99	5	5.69	4.47	3	3.06
	-0.5	28.98	20	29.82	19.62	13	20.1	21.73	15	21.39	13.59	9	13.06	19.8	14	18.16	12.31	9	10.77
	-0.25	136.14	95	137	106.52	74	106.96	110.2	75	111.27	79.89	55	80.28	98.33	68	98.13	68.15	47	67.65
500	**0**	**499.26**	**349**	**500.03**	**500.02**	**349**	**513.52**	**500.6**	**348**	**501.89**	**499.91**	**346**	**497.79**	**499.93**	**341**	**498.76**	**499.79**	**348**	**495.13**
	0.25	137.07	93	139.33	106.52	73	107.83	110.16	77	109.67	78.42	55	77.33	98.46	69	98.16	68.59	47	68.17
	0.5	29.03	20	29.21	20	14	20.23	21.99	15	21.44	13.57	10	12.91	19.9	14	18.8	12.09	9	10.94
	0.75	8.87	6	9.05	5.72	4	5.57	7.06	5	6.58	4.4	3	3.72	6.93	5	5.7	4.47	3	3.04
	1	3.71	2	3.55	2.47	2	2.12	3.3	2	2.7	2.23	2	1.46	3.62	3	2.3	2.63	3	1.39
	1.25	2.02	1	1.69	1.48	1	0.93	2.02	2	1.3	1.5	1	0.73	2.41	3	1.32	1.82	1	0.98
	1.5	1.38	1	0.86	1.13	1	0.43	1.46	1	0.73	1.2	1	0.43	1.77	1	0.97	1.32	1	0.67
	1.75	1.12	1	0.42	1.03	1	0.18	1.2	1	0.45	1.06	1	0.24	1.37	1	0.72	1.1	1	0.36
	2	1.03	1	0.21	1.01	1	0.08	1.08	1	0.28	1.01	1	0.11	1.13	1	0.44	1.02	1	0.15

To evaluate the performance of the designed $MA_{MSS(S)}$ and $DMA_{MSS(S)}$ control charts, we observed different choices of moving average span, such as $w = 2, 3, 5, 10$, and 15 for both charts, respectively. From Figures 3 and 4, at a pre-specified $ARL_0 = 200$, the consequences of moving average span (w) on the performance of both proposed charts (i.e., $MA_{MSS(S)}$ and $DMA_{MSS(S)}$) can be seen under the different MSS schemes respectively. Figure 3 highlights the influence of the parameter (w) of $MA_{MSS(S)}$ control chart on various MSS schemes. It is examined that the higher value of (w) may lead to the high-detection ability of $MA_{MSS(S)}$ chart because $w = 15$ has the lowest ARL curves for all MSS schemes as compared to other choices of w (2, 3, 5 and 10).

Similarly, the value of the moving average span (w) of the $DMA_{MSS(S)}$ control chart has the lowest curves of ARL at $w = 15$ in all choices of (w) under different MSS schemes (cf. Figure 4). Hence, the large choice of (w) may lead to the high-monitoring ability of both $MA_{MSS(S)}$ and $DMA_{MSS(S)}$ control charts. Further, to show the steady-state performance of the designed $MA_{MSS(S)}$ and $DMA_{MSS(S)}$ control charts, we have added shifts after the 50 plotting points. Table 6 consists of the ARL profile of $MA_{MSS(S)}$ and $DMA_{MSS(S)}$ control charts at a pre-specified $ARL_0 = 200$. It is noted that $DMA_{MSS(S)}$ outperformed $MA_{MSS(S)}$ under the different MSS schemes. Moreover, both charts performance increased with the increase in subgroup size.

5. Real data application

The section is designed for real-life application of proposed $MA_{MSS(S)}$ and $DMA_{MSS(S)}$ control charts based on MSS for the location parameters monitoring using the chemical composition of the ceramic data set. He et al. [15] examined the sample of Longquan

Table 4. Zero-state run length profile of the proposed charts under $MSS_{n,3,min,Q_2,max}$

		$\text{Shewhart}_{\text{MSS(1)}}$						$\text{MA}_{\text{MSS(1)}}$						DMA – MSS(1)					
ARL_0	δ	n=5			n=7			n=5			n=7			n=5			n=7		
		ARL	MDRL	SDRL	ARL	MDRL	SDRL	ARL	MDRL	SDRL	ARL	MDRL	SDRL	ARL	MDRL	SDRL	ARL	MDRL	SDRL
	-2	1.01	1	0.1	1	1	0.03	1.02	1	0.13	1	1	0.05	1.03	1	0.21	1	1	0.06
	-1.75	1.03	1	0.23	1.01	1	0.1	1.06	1	0.26	1.02	1	0.12	1.11	1	0.42	1.02	1	0.17
	-1.5	1.14	1	0.54	1.05	1	0.26	1.18	1	0.48	1.06	1	0.25	1.3	1	0.71	1.1	1	0.39
	-1.25	1.46	1	1.16	1.22	1	0.62	1.5	1	0.98	1.23	1	0.53	1.71	1	1.14	1.36	1	0.73
	-1	2.37	1	2.51	1.72	1	1.35	2.32	2	2.22	1.66	1	1.08	2.55	3	2.02	1.94	1	1.2
	-0.75	5.06	3	6.43	3.44	2	3.6	4.41	2	5.11	2.86	2	2.5	4.6	3	4.54	3.12	3	2.31
	-0.5	14.97	8	20.72	10.55	6	12.71	12.37	6	17.18	7.7	5	8.94	11.93	6	15.75	7.36	5	7.72
	-0.25	69.7	35	100.19	52.82	31	65.5	60.93	28	90.18	40.75	22	53.8	56.4	26	87.26	36.4	20	47.46
200	**0**	**199.32**	**116**	**236.92**	**199.13**	**124**	**213.44**	**201.03**	**109**	**255.99**	**201.32**	**122**	**231.79**	**199.84**	**104.5**	**253.19**	**200.15**	**121.5**	**244.03**
	0.25	69.46	36	94.98	52.48	30.5	65.77	59.16	29	86.41	40.32	22	53.75	55.14	25	85.21	36.05	20	45.92
	0.5	15.37	8	20.72	10.31	6	11.81	12.73	7	17.34	7.88	5	8.76	12.11	7	15.87	7.47	5	7.88
	0.75	4.97	3	6.33	3.47	2	3.6	4.48	2	5.31	2.9	2	2.57	4.63	3	4.81	3.15	3	2.31
	1	2.37	1	2.58	1.74	1	1.4	2.25	2	2.1	1.67	1	1.07	2.54	3	1.93	1.92	1	1.2
	1.25	1.45	1	1.15	1.21	1	0.6	1.51	1	1.05	1.23	1	0.53	1.73	1	1.14	1.38	1	0.75
	1.5	1.14	1	0.57	1.05	1	0.24	1.18	1	0.51	1.07	1	0.26	1.3	1	0.72	1.11	1	0.41
	1.75	1.04	1	0.25	1.01	1	0.1	1.06	1	0.27	1.02	1	0.13	1.1	1	0.4	1.02	1	0.16
	2	1.01	1	0.09	1	1	0.04	1.02	1	0.13	1	1	0.05	1.03	1	0.2	1	1	0.07
	-2	1.01	1	0.12	1	1	0.04	1.02	1	0.16	1	1	0.07	1.04	1	0.25	1.01	1	0.08
	-1.75	1.05	1	0.3	1.02	1	0.14	1.08	1	0.31	1.02	1	0.15	1.14	1	0.48	1.03	1	0.22
	-1.5	1.18	1	0.64	1.07	1	0.3	1.23	1	0.56	1.09	1	0.3	1.38	1	0.79	1.15	1	0.47
	-1.25	1.59	1	1.39	1.29	1	0.73	1.62	1	1.18	1.29	1	0.6	1.9	1	1.27	1.47	1	0.83
	-1	2.71	1	3	1.96	1	1.68	2.51	2	2.37	1.81	1	1.22	2.9	3	2.39	2.1	2	1.28
	-0.75	6.26	3	8.18	4.24	3	4.71	5.32	3	6.45	3.33	2	3.17	5.36	3	5.8	3.52	3	2.77
	-0.5	21.23	11	31	14.35	8	17.31	16.7	8	23.74	9.57	6	11.3	16	8	23.45	8.91	6	9.64
	-0.25	111.27	54	161.45	84.74	49	105.67	96.58	43	149.54	58.08	31	77.13	84.84	39	130.41	51.85	28	71.46
370	**0**	**370.65**	**216**	**443.7**	**370.17**	**243**	**402.96**	**370.05**	**193**	**471.84**	**369.63**	**220**	**421.23**	**370.45**	**202**	**484.43**	**370.11**	**225**	**432.35**
	0.25	111.02	55	161.28	84.86	49	104.02	95.62	44	146.94	58.54	31	80.21	84.93	40	131.53	53.9	28	76.71
	0.5	21.33	11	30.27	14.7	9	17.75	17.05	9	24.34	9.52	6	11.34	15.42	8	22.03	8.89	6	9.63
	0.75	6.36	3	8.06	4.41	3	4.99	5.4	3	6.54	3.35	2	3.11	5.43	3	5.78	3.55	3	2.66
	1	2.67	1	2.95	1.96	1	1.7	2.55	2	2.51	1.81	1	1.23	2.86	3	2.3	2.11	2	1.28
	1.25	1.6	1	1.41	1.29	1	0.77	1.63	1	1.13	1.29	1	0.58	1.87	1	1.24	1.45	1	0.82
	1.5	1.18	1	0.64	1.06	1	0.3	1.24	1	0.59	1.1	1	0.31	1.41	1	0.83	1.15	1	0.48
	1.75	1.05	1	0.26	1.01	1	0.12	1.08	1	0.31	1.02	1	0.15	1.14	1	0.48	1.03	1	0.22
	2	1.01	1	0.1	1	1	0.05	1.02	1	0.16	1	1	0.05	1.04	1	0.26	1.01	1	0.08
	-2	1.01	1	0.13	1	1	0.04	1.03	1	0.17	1	1	0.07	1.06	1	0.29	1.01	1	0.11
	-1.75	1.06	1	0.31	1.02	1	0.15	1.1	1	0.33	1.03	1	0.16	1.17	1	0.52	1.04	1	0.26
	-1.5	1.21	1	0.71	1.08	1	0.35	1.27	1	0.61	1.11	1	0.34	1.43	1	0.83	1.17	1	0.5
	-1.25	1.67	1	1.54	1.32	1	0.78	1.69	1	1.21	1.34	1	0.65	1.98	1	1.33	1.53	1	0.87
	-1	2.93	1	3.31	2.1	1	1.89	2.73	2	2.7	1.88	2	1.29	2.99	3	2.34	2.18	2	1.31
	-0.75	6.96	4	9.09	4.66	3	5.05	5.72	3	6.8	3.58	2	3.39	5.84	3	6.27	3.76	3	2.93
	-0.5	24.89	13	36.56	16.74	10	20.41	18.54	10	26.68	10.67	6	12.35	17.01	9	23.55	9.98	6	11.15
	-0.25	141.9	68	207.87	104.65	60	130.24	114.4	53	180.64	71.2	38	99.4	103.16	46	160.01	64.59	34	91.01
500	**0**	**499.77**	**292**	**596.41**	**500.71**	**328**	**542.18**	**499.45**	**275**	**623.35**	**499.44**	**309**	**558.74**	**501.92**	**264**	**654.12**	**499.75**	**297**	**576.17**
	0.25	141.42	72	202.53	105.11	59	136.05	112.75	53	170.69	72.54	39	100.97	106.72	47	172.21	62.89	34	85.91
	0.5	24.45	12	34.51	16.31	10	20.1	19.21	10	28.55	10.78	7	12.69	17.55	9	26.26	9.95	6	10.89
	0.75	7.06	4	9.27	4.79	3	5.41	5.69	3	7.03	3.51	2	3.24	5.82	3	6.31	3.8	3	3.04
	1	2.94	1	3.4	2.1	1	1.9	2.73	2	2.9	1.89	2	1.3	3.03	3	2.45	2.19	2	1.31
	1.25	1.63	1	1.44	1.32	1	0.82	1.69	1	1.21	1.32	1	0.63	1.99	1	1.32	1.51	1	0.86
	1.5	1.22	1	0.69	1.08	1	0.36	1.27	1	0.63	1.1	1	0.33	1.44	1	0.84	1.18	1	0.52
	1.75	1.06	1	0.33	1.02	1	0.14	1.1	1	0.33	1.03	1	0.16	1.17	1	0.52	1.04	1	0.25
	2	1.01	1	0.12	1	1	0.05	1.03	1	0.17	1	1	0.06	1.05	1	0.28	1.01	1	0.1

celadon in Yuan and Ming, Southern Song, and Northern Song dynasties and also Longquan celadon in Jingdezhen in the Ming dynasty with respect to glaze and body by the energy-dispersive X-ray fluorescence (EDXRF) microprobe. In this research, using the data of glaze and body chemical elements compositions, they determined that the contents of $Na_2O, Fe_2O_3, TiO_2, SiO_2$ and CaO differs significantly in the celadon body for both kilns. The Longquan celadon in Jingdezhens body has low contents of titanium and iron and high contents of silicon. However, by comparing the chemical composition and firing technology in glaze, they have revealed that both kilns are very similar to each other. Jingdezhen craftsmen used similar raw materials of glaze in the local area and produced the imitated glazes; the firing temperature of 1140^oC was also employed. The melting temperature variation is lower in glaze and higher viscosity in celadon bodies in Na_2Ocontents. By comparing the local area raw materials, both kilns are not completely the same as each other because the contents of Na_2O in Jingdezhen differ by 1.01% from those in Longquan. Longquan celadon glaze had irregular cracks in glaze layers due to relatively high contents of Na_2O. Therefore, to demonstrate the performance of the proposed control charts, we have chosen the chemical composition of Na_2O as our variable of interest from the above-mentioned chemical elements.

The data set comprises 86 values allocated into two groups concerning the celadon body and glaze (i.e., 44 IC values and 42 OOC values). To check the normality of IC data, PP-plot is presented in Figure 5 (Left window). Andersons Darling test showed that IC data follows the normal distribution with mean equals 0.5459 and 0.4360 standard

Table 5. Zero-state run length profile of the proposed charts under $MSS_{n,3,Q_1,Q_2,Q_3}$

		Shewhart$_{\mathbf{MSS(1)}}$						**MA$_{\mathbf{MSS(1)}}$**						**DMA − MSS(1)**					
ARL$_0$	δ	**n=5**			**n=7**			**n=5**			**n=7**			**n=5**			**n=7**		
		ARL	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**
	-2	1.01	1	0.12	1	1	0.04	1.04	1	0.2	1.01	1	0.08	1.08	1	0.36	1.01	1	0.11
	-1.75	1.06	1	0.32	1.02	1	0.13	1.13	1	0.38	1.03	1	0.18	1.25	1	0.62	1.06	1	0.29
	-1.5	1.21	1	0.68	1.06	1	0.28	1.32	1	0.67	1.11	1	0.34	1.57	1	0.93	1.22	1	0.57
	-1.25	1.65	1	1.41	1.26	1	0.68	1.79	1	1.26	1.34	1	0.63	2.21	2	1.42	1.6	1	0.92
	-1	2.79	1	2.86	1.86	1	1.47	2.83	2	2.5	1.9	2	1.23	3.31	3	2.4	2.29	3	1.33
	-0.75	6.06	4	6.48	3.75	2	3.72	5.87	4	5.84	3.44	2	2.88	6.19	4	5.4	3.87	3	2.74
	-0.5	16.87	11	17.67	11.22	8	11.42	15.7	11	16.18	9.27	6	8.84	15.05	10	14.55	9.15	6	8.07
	-0.25	64.94	43	68.22	49.76	34	50	59.85	41	60.38	42.72	30	42.6	56.57	40	56.08	39.7	27	39.33
200	**0**	**199.27**	**136**	**206.82**	**199.14**	**139**	**198.69**	**201.7**	**138**	**209.81**	**199.09**	**136**	**202.06**	**199.61**	**134**	**192.87**	**200.95**	**141**	**204.8**
	0.25	65.62	45	69.24	50.06	35	51.04	60.95	42	61.68	42	28	42.34	56.73	39	57.65	40.11	28	38.98
	0.5	17.06	11	18.02	11.18	8	11.25	15.65	11	15.9	9.26	6	8.95	15.45	11	14.8	9.21	6	8.3
	0.75	6.1	4	6.69	3.84	2	3.75	5.84	4	5.81	3.49	2	2.93	6.1	4	5.32	3.87	3	2.68
	1	2.83	1	2.94	1.89	1	1.57	2.89	2	2.59	1.9	2	1.27	3.35	3	2.42	2.31	3	1.36
	1.25	1.64	1	1.39	1.26	1	0.71	1.79	1	1.26	1.34	1	0.65	2.16	2	1.35	1.6	1	0.9
	1.5	1.21	1	0.68	1.06	1	0.29	1.33	1	0.68	1.12	1	0.35	1.58	1	0.93	1.21	1	0.57
	1.75	1.05	1	0.29	1.01	1	0.11	1.12	1	0.38	1.03	1	0.18	1.25	1	0.63	1.05	1	0.28
	2	1.01	1	0.13	1	1	0.04	1.04	1	0.19	1	1	0.06	1.09	1	0.37	1.01	1	0.1
	-2	1.02	1	0.16	1	1	0.04	1.06	1	0.24	1.01	1	0.09	1.14	1	0.46	1.02	1	0.16
	-1.75	1.09	1	0.41	1.02	1	0.15	1.18	1	0.43	1.04	1	0.21	1.37	1	0.74	1.09	1	0.37
	-1.5	1.28	1	0.83	1.1	1	0.37	1.43	1	0.79	1.16	1	0.41	1.78	1	1.04	1.32	1	0.68
	-1.25	1.88	1	1.76	1.35	1	0.82	2.01	2	1.51	1.45	1	0.71	2.49	3	1.6	1.77	1	1
	-1	3.38	2	3.49	2.16	1	1.87	3.4	2	3.14	2.16	2	1.51	3.92	3	2.92	2.59	3	1.48
	-0.75	7.85	5	8.39	4.71	3	4.69	7.36	5	7.46	4.15	3	3.55	7.62	5	6.9	4.51	3	3.32
	-0.5	24.57	17	25.67	15.61	11	15.98	21.26	14	21.81	12.55	9	12.25	20.93	14	20.74	12	9	10.84
	-0.25	104.15	71	107.29	79.84	55	80.4	95.13	65	97.23	65.1	46	64.97	92.81	64	93.5	61.77	43	61.26
370	**0**	**369.19**	**253**	**366.21**	**370.82**	**254**	**378.31**	**369.64**	**251**	**376.7**	**370.38**	**258**	**374.88**	**370.09**	**259**	**376.8**	**370.87**	**256**	**378.34**
	0.25	106.46	73	110.37	81.26	57	81.31	93.65	65	96.17	66.12	47	65.25	92.49	64	92.86	60.79	43	58.73
	0.5	24.35	17	25.4	15.73	11	16.11	21.49	14	22.5	12.56	9	12.15	21.21	15	21.03	11.78	8	10.73
	0.75	7.86	5	8.41	4.85	3	4.83	7.29	5	7.33	4.2	3	3.66	7.62	5	6.72	4.53	3	3.24
	1	3.39	2	3.57	2.19	1	1.94	3.44	2	3.14	2.14	2	1.47	3.87	3	2.88	2.6	3	1.48
	1.25	1.86	1	1.68	1.37	1	0.87	2.01	2	1.5	1.45	1	0.75	2.52	3	1.54	1.77	1	0.99
	1.5	1.28	1	0.82	1.09	1	0.36	1.42	1	0.79	1.17	1	0.41	1.79	1	1.05	1.31	1	0.67
	1.75	1.09	1	0.39	1.02	1	0.15	1.18	1	0.45	1.05	1	0.22	1.37	1	0.74	1.09	1	0.36
	2	1.02	1	0.18	1	1	0.05	1.06	1	0.26	1.01	1	0.1	1.14	1	0.47	1.02	1	0.15
	-2	1.02	1	0.18	1	1	0.05	1.08	1	0.28	1.01	1	0.11	1.16	1	0.5	1.02	1	0.17
	-1.75	1.1	1	0.45	1.02	1	0.17	1.21	1	0.48	1.06	1	0.24	1.44	1	0.8	1.1	1	0.39
	-1.5	1.32	1	0.9	1.11	1	0.41	1.5	1	0.86	1.19	1	0.43	1.88	1	1.1	1.36	1	0.71
	-1.25	2.01	1	1.92	1.41	1	0.9	2.14	2	1.64	1.52	1	0.78	2.64	3	1.63	1.86	1	1.03
	-1	3.73	2	3.9	2.35	1	2.1	3.69	2	3.36	2.28	2	1.66	4.21	3	3.18	2.71	3	1.55
	-0.75	8.94	6	9.69	5.31	4	5.33	8.3	5	8.41	4.61	3	4.09	8.59	6	7.92	4.83	3	3.61
	-0.5	28.98	20	30.27	18.51	13	18.8	25.29	17	25.5	14.34	10	14.08	24.73	17	24.49	13.36	9	12.32
	-0.25	132.61	90	135.15	99.62	69	100.44	118.98	82	119.86	81.13	56	81.64	116.88	81.5	118.04	74.6	51	74.34
500	**0**	**499.55**	**340**	**499.45**	**500.25**	**340.5**	**515.82**	**499.29**	**342**	**498.12**	**499.19**	**338**	**493.97**	**500.29**	**348**	**512.35**	**499.65**	**341**	**494.5**
	0.25	133.66	90	139.55	100.71	70	101.05	117.79	80	120.97	79.13	55	79.07	113.99	78.5	114.37	72.53	50	71.2
	0.5	29.1	20	30.58	18.11	12	18.5	25.52	18	25.74	14.24	10	14.13	24.77	17	24.8	13.48	10	12.46
	0.75	9	6	9.74	5.48	4	5.55	8.33	6	8.39	4.69	3	4.19	8.5	6	7.72	4.85	3	3.62
	1	3.79	2	3.98	2.35	1	2.09	3.7	2	3.41	2.29	2	1.62	4.2	3	3.24	2.72	3	1.56
	1.25	1.98	1	1.84	1.41	1	0.9	2.14	2	1.61	1.51	1	0.78	2.67	3	1.7	1.86	1	1.02
	1.5	1.33	1	0.91	1.11	1	0.4	1.49	1	0.84	1.19	1	0.44	1.9	1	1.09	1.36	1	0.71
	1.75	1.1	1	0.42	1.02	1	0.17	1.2	1	0.49	1.06	1	0.24	1.45	1	0.81	1.1	1	0.39
	2	1.03	1	0.2	1	1	0.06	1.08	1	0.28	1.01	1	0.11	1.17	1	0.51	1.02	1	0.18

deviation. Similarly, for the OOC data, normality is assessed using PP-plot given in Figure 5 (Right Window) and Andersons Darling test. It is found that OOC data is not normally distributed and has a mean equal 0.3975 and 0.2115 standard deviation. Furthermore, we have distributed this dataset into 28 subgroups of size n = 5, comprised of 14 IC and 14 OOC subgroups by using the MSS scheme ($MSS_{n,2,min,max}$). In this case study, The primary purpose of choosing MSS as a sampling scheme is that it is a time and cost-effective scheme; through it, we can examine the variation in the processs mean over regular time intervals. To develop the control limits of proposed $MA_{MSS(1)}$ and $DMA_{MSS(1)}$ control charts, at pre-specified $ARL_0 = 200$, IC subgroups were used, and after that, the statistics of both proposed $MA_{MSS(1)}$ and $DMA_{MSS(1)}$ charts are computed. The calculated $MA_{MSS(1)}$ and $DMA_{MSS(1)}$ charting statistics are plotted on the y-axis against their corresponding control limits and subgroups (sample numbers on the x-axis). The graphical representation shows that the $MA_{MSS(1)}$ control chart monitors the OOC signal after 14 samples while $DMA_{MSS(1)}$ control chart detects the OOC signal after 15 samples (cf. Figure 6). Figure 6(a) presents that $MA_{MSS(1)}$ control chart has some IC signals along with OOC signals in the OOC region, and $DMA_{MSS(1)}$ control chart (cf. Figure 6(b)) does not have such a pattern. Hence, the $DMA_{MSS(1)}$ control chart has more monitoring capability than the $MA_{MSS(1)}$ control chart for this dataset.

Table 6. Steady-state run length profile of the proposed charts at fixed $ARL_0 = 200$

		$\mathbf{MA_{MSS(1)}}$						$\mathbf{DMA_{MSS(1)}}$					
ARL_0	δ	**n=5**			**n=7**			**n=5**			**n=7**		
		ARL	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**	**ARL**	**MDRL**	**SDRL**
	-2	40.73	51	17.81	42.63	51	15.92	40.4	52	18.16	42.78	52	16.21
	-1.75	40.92	52	17.83	42.72	51	16.06	40.54	52	18.15	42.77	52	16.26
	-1.5	40.77	52	18.21	42.97	51	16.08	41.14	52	18.07	42.93	52	16.24
	-1.25	41.89	52	18.36	42.99	52	16.44	41.45	52	18.52	43.35	52	16.32
	-1	42.99	52	19.58	43.68	52	16.52	42.29	53	19.52	43.33	52	16.9
	-0.75	46.87	54	23.11	45.01	52	18.1	45.25	53	22.11	44.71	53	17.9
	-0.5	56.84	57	36.72	51.23	55	24.61	54.6	57	34.58	50	55	22.76
	-0.25	100.88	75	103.18	82.71	69	67.13	94.88	70	100.66	77.08	66	61.17
$\mathbf{MSS_{n,2,min,max}}$	**0**	**200.22**	**108.5**	**253.48**	**199.29**	**117**	**220.88**	**201.7**	**108**	**262.84**	**199.29**	**118**	**231.23**
	0.25	97.38	73	100.74	81.65	69	67.34	94.66	69	98.88	78.39	67	63.75
	0.5	56.48	57	36.29	51.48	55	24.62	54.78	57	35.17	49.62	55	22.71
	0.75	46.52	54	22.62	45.42	52	17.86	45.27	53	22.26	44.6	53	18.09
	1	43	52	19.48	43.92	52	16.31	42.66	53	19.43	43.77	52	16.54
	1.25	41.86	52	18.4	42.77	52	16.51	41.5	52	18.57	43.15	52	16.43
	1.5	41.03	52	18.04	42.83	51	16.17	41.09	52	18.06	42.94	52	16.25
	1.75	40.72	52	17.97	42.66	51	16.13	40.81	52	18.08	42.91	52	16.13
	2	40.6	51	17.83	42.51	51	16.12	40.76	52	17.85	42.75	52	16.25
	-2	45.03	52	14.22	45.07	51	13.66	46.12	52	13.31	45.97	52	13.1
	-1.75	45.54	52	13.73	45.21	52	13.66	46.27	52	13.31	46	52	13.06
	-1.5	45.45	52	14.2	45.55	52	13.41	46.51	52	13.38	46.01	52	13.36
	-1.25	46.08	52	14.18	45.7	52	13.58	46.94	53	13.54	46.09	52	13.42
	-1	46.83	53	14.68	45.96	52	14	47.66	53	13.93	46.87	52	13.31
	-0.75	48.92	54	16.07	47.13	53	14.82	49.21	54	15.62	47.72	53	14.18
	-0.5	55.51	58	22.1	51.64	55	17.92	55.76	58	20.85	51.66	55	16.6
	-0.25	87.45	75	59.68	75.53	68	45.46	87.93	75	58.65	73.94	68	41.26
$\mathbf{MSS_{n,2,Q_1,Q_3}}$	**0**	**199.02**	**138**	**199.6**	**199.57**	**132.5**	**195.07**	**201.14**	**140**	**204.44**	**200.4**	**141**	**203.76**
	0.25	87.85	75	60.85	75.75	69	45.59	85.83	75	56.86	73.57	68	40.92
	0.5	55.24	57	22.04	51.27	55	18.17	55.61	58	20.5	51.59	55	16.83
	0.75	48.49	54	16.32	47.16	53	14.85	49.44	54	15.42	47.72	53	14.09
	1	46.72	53	14.7	45.99	52	13.97	47.65	53	14.02	46.44	52	13.72
	1.25	45.98	52	14.22	45.48	52	13.78	46.79	53	13.69	46.42	52	13.1
	1.5	45.4	52	14.23	45.44	52	13.65	46.7	52	13.12	46.07	52	13.3
	1.75	45.36	52	14.08	45.41	52	13.6	46.27	52	13.36	45.82	52	13.3
	2	45.51	52	13.59	45.02	51	13.76	46.1	52	13.33	45.8	52	13.3
	-2	40.74	52	18.28	42.57	51	16.33	40.86	52	18.18	42.66	52	16.49
	-1.75	41.01	52	18.39	42.58	52	16.48	40.53	52	18.77	43.17	52	16.16
	-1.5	41.44	52	18.72	42.64	52	16.62	41.61	52	18.48	43.3	52	16.33
	-1.25	42.17	52	19.33	42.76	52	16.95	41.85	53	19.27	43.42	52	16.66
	-1	43.76	53	20.95	44.26	52	16.84	42.97	53	20.67	43.79	53	17.17
	-0.75	48.28	55	24.96	45.56	53	18.71	46.96	54	24.43	45.63	53	18.38
	-0.5	60.27	59	42.27	52.65	56	26.04	57.81	58	38.95	51.6	56	24.29
	-0.25	106.85	76	117.35	83.75	70	70.89	100.26	73	108.83	81.06	68	65.7
$\mathbf{MSS_{n,3,min,Q2,max}}$	**0**	**201.2**	**112**	**263.51**	**199.44**	**119**	**231.19**	**199.19**	**105**	**244.81**	**201.07**	**124**	**247.67**
	0.25	106.35	75.5	116.55	85.86	71	72.8	98.66	72	105.25	81.14	69	65.95
	0.5	60.63	59	41.82	52.54	56	25.58	57.37	59	38.38	51.43	56	24.18
	0.75	47.92	55	25.36	46.03	53	18.58	46.55	54	24.42	45.53	53	18.32
	1	43.5	53	20.87	43.87	52	17.08	43.36	53	20.54	43.85	52	16.96
	1.25	42.5	52	18.97	42.99	52	16.76	41.77	53	19.31	43.37	52	16.66
	1.5	41.36	52	18.68	43.03	52	16.34	41.29	52	18.71	42.9	52	16.62
	1.75	40.77	52	18.64	42.64	52	16.49	40.92	52	18.43	43.01	52	16.34
	2	40.88	52	18.19	42.53	51	16.48	41.09	52	18.23	42.97	52	16.25
	-2	45.42	52	14.42	45.52	52	13.62	46.07	52	13.94	45.78	52	13.53
	-1.75	45.57	52	14.6	45.39	52	13.79	46.01	53	14.39	45.98	52	13.41
	-1.5	45.93	52	14.72	45.47	52	14.02	46.24	53	14.59	46.11	52	13.59
	-1.25	46.29	53	15.21	45.6	52	14.17	46.72	53	15.01	46.33	52	13.69
	-1	47.43	53	15.86	46.33	52	14.34	47.75	54	15.41	46.86	53	13.89
	-0.75	50.05	55	17.54	47.64	53	15.25	50.59	55	16.76	48.06	53	14.8
	-0.5	57.52	59	25.49	52.57	56	19.18	57.01	59	24.12	52.48	56	18.12
	-0.25	91.63	78	66.83	77.82	70	47.19	89.08	77	61.91	76.18	70	44.24
$\mathbf{MSS_{n,3,Q_1,Q_2,Q_3}}$	**0**	**199.41**	**136**	**201.89**	**200.74**	**141**	**202.15**	**199.41**	**133**	**192.31**	**200.45**	**141**	**196.75**
	0.25	91.63	78	66.24	76.79	70	47.25	89.07	76	61.4	76.44	70	44.98
	0.5	57.68	59	25.31	52.33	56	18.92	57.17	59	23.81	52.26	56	18.06
	0.75	49.91	55	17.91	47.74	53	15.24	50.01	55	17.41	48.13	53	14.64
	1	47.4	54	15.93	46.18	52	14.44	47.71	54	15.55	46.84	53	14.02
	1.25	46.19	53	15.16	45.84	52	13.95	46.87	53	14.69	46.48	52	13.61
	1.5	45.78	52	14.88	45.47	52	13.92	45.91	53	14.9	46.2	52	13.46
	1.75	45.41	52	14.85	45.49	52	13.75	45.77	53	14.56	45.92	52	13.62
	2	45.21	52	14.62	45.14	52	13.97	46.06	52	13.95	45.86	52	13.47

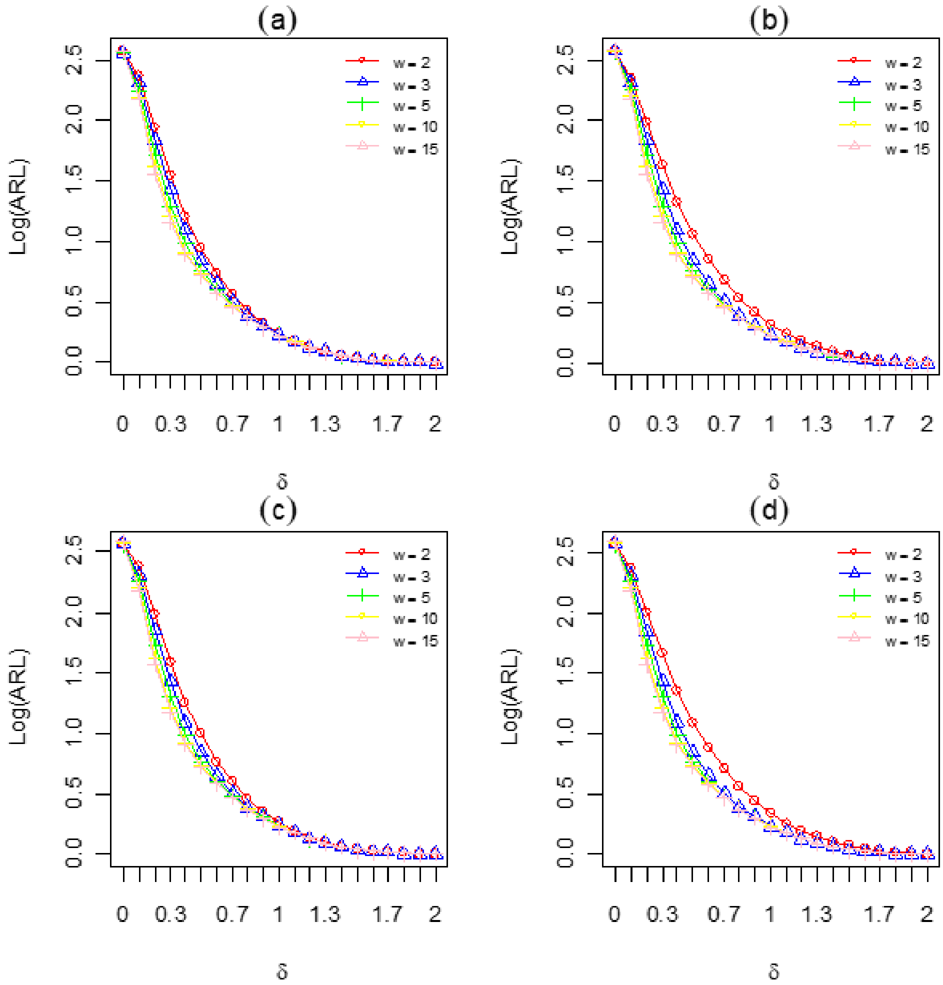

Figure 3. The effect of moving average span parameter (w) on the performance of $MA_{MSS(S)}$ control chart under MSS schemes; (a) $\mathbf{MSS_{n,2,min,max}}$,, (b) $\mathbf{MSS_{n,2,Q1,Q3}}$, (c) $\mathbf{MSS_{n,3,min,Q2,max}}$,, (d) $\mathbf{MSS_{n,3,Q1,Q2,Q3}}$.

6. Some concluding remarks

For observing the performance of a process, control chart is a popular tool in the SPC kit. This study aims to develop useful memory-type location control charts using moving and double moving averages under modified successive sampling. The performance of the proposed control charts (i.e., $MA_{MSS(S)}$ and $DMA_{MSS(S)}$) is compared with existing control charts using run-length matrices (ARL, MDRL, and SDRL). For all the mentioned MSS schemes, the results depict that the designed $DMA_{MSS(S)}$ control chart outperforms $MA_{MSS(S)}$ and $Shewhart_{MSS(S)}$ control charts at fixed ARL_0 choices (e.g., 200, 370, and 500). We have concluded it through different aspects by using various choices of MSS scheme, subgroups (n), shift in mean (δ), and moving average span (w). In various choices of MSS schemes, It is revealed that ($MSS_{n,2,min,max}$) ($S = 1$) has shown dominance over other MSS schemes (cf. Table 2). The values of ARL_1, $MDRL_1$, and $SDRL_1$ of the proposed charts depict a decreasing pattern, with an increase in δ and n for all the schemes of MSS (cf. Tables 2-4). Moreover, as the moving average span (w) increases, the values of ARL_1, $MDRL_1$, and $SDRL_1$ of $MA_{MSS(S)}$ and $SMA_{MSS(S)}$ control charts decrease for all sampling schemes (cf. Figures 2-3). It is noticed that in all mentioned MSS schemes, the proposed $DMA_{MSS(1)}$ control chart based on ($MSS_{n,2,min,max}$) ($S = 1$) scheme performs better as compared to $Shewhart_{MSS(S)}$ and $MA_{MSS(1)}$ control charts. We have

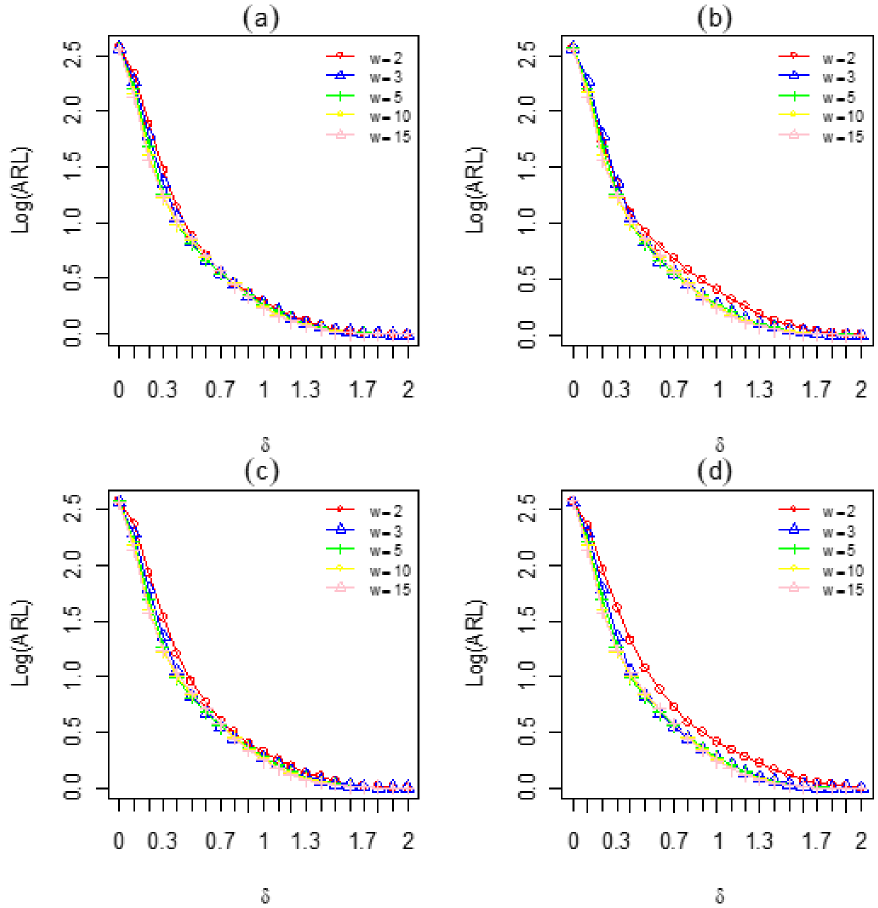

Figure 4. The effect of moving average span parameter (w) on the performance of $DMA_{MSS(S)}$ control chart under MSS schemes; (a) $\mathbf{MSS_{n,2,min,max}}$,, (b) $\mathbf{MSS_{n,2,Q1,Q3}}$, (c) $\mathbf{MSS_{n,3,min,Q2,max}}$,, (d) $\mathbf{MSS_{n,3,Q1,Q2,Q3}}$.

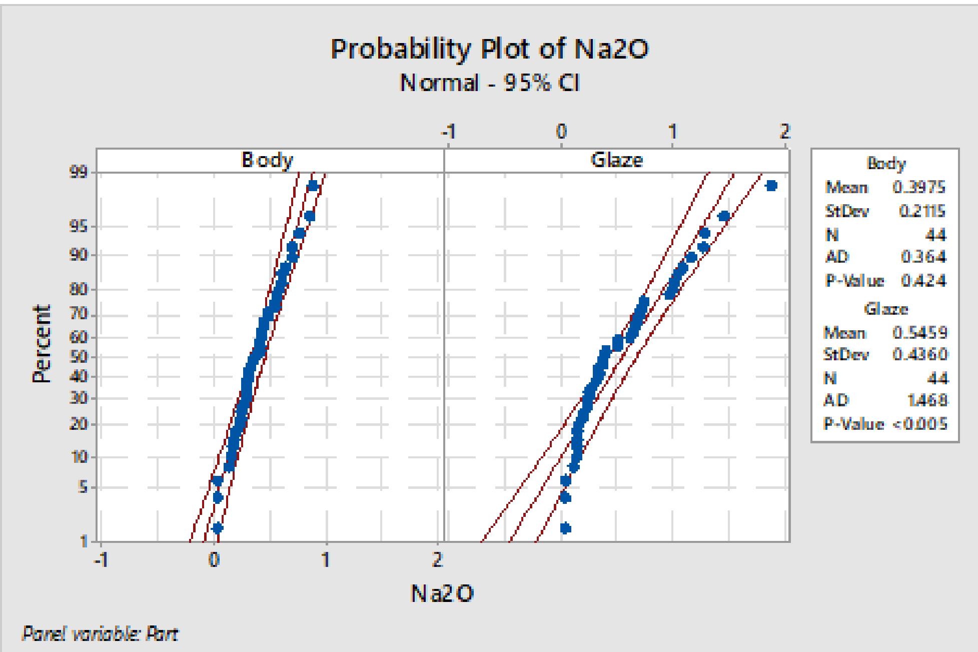

Figure 5. Probability plot of Na_2O with respect to celadon body and glaze.

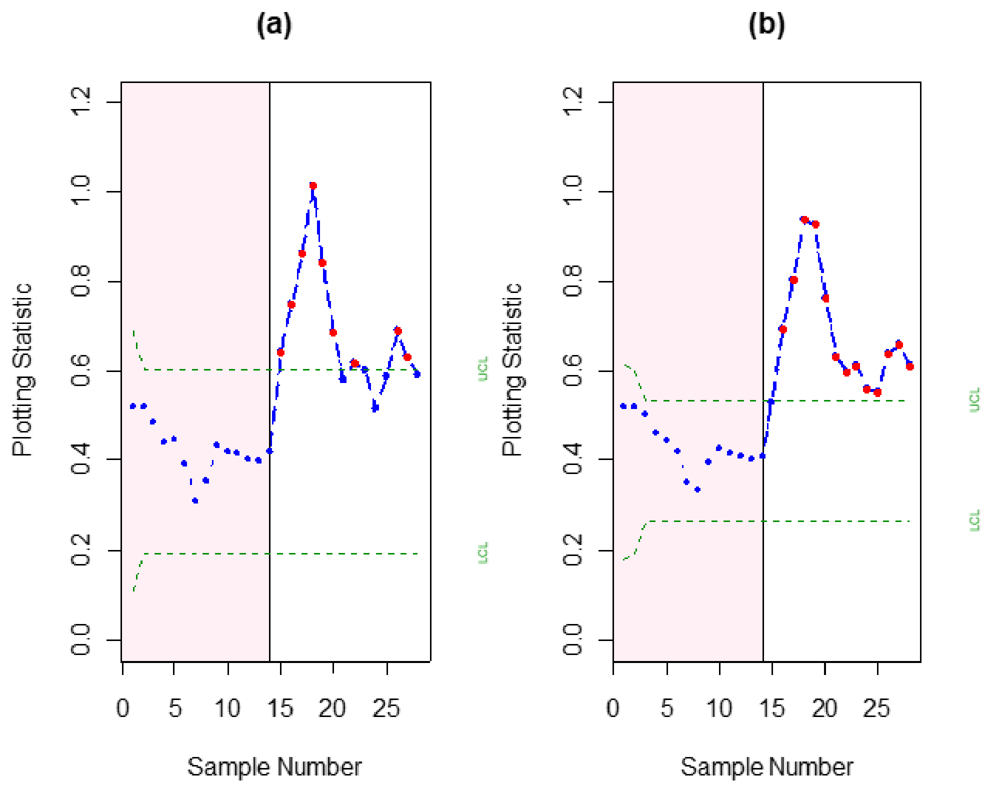

Figure 6. Illustration for (a) $MA_{MSS(1)}$ and (b) $DMA_{MSS(1)}$ control charts for Chemical Composition of Na_2O dataset.

observed a similar performance under steady-state. This study is limited to the four above-mentioned MSS schemes at particular choices of subgroup size, which can be extended to other similar schemes. This study is designed based on the known in-control parameters and for the normally distributed data; one may explore the estimation effects and effect of non-normal distributions in future studies. In the current study, charts are proposed based on a simple moving average, while the usage of the weighted moving average may be a future addition to this study. Moreover, proposing hybrid control chart proposed by [12] under MSS schemes and considering the mixed MA-DMA or mixed DMA-MA location charts under MSS will be interesting future research works.

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