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Research Article

Hybrid bi-ideals in near-subtraction semigroups

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Abstract

The fuzzy set is an excellent solution for dealing with ambiguity and for expressing people's hesitation in regular life. Soft set theory is an innovative method for solving practical issues. This is useful in resolving a number of problems, and a lot of progress is being made at the moment. In order to develop hybrid structures, Jun et al. fused the fuzzy and soft sets. In this paper, the notion of hybrid bi-ideals in near-subtraction semigroups is proposed and their associated results are discussed. The notion of hybrid intersections is examined. Furthermore, we establish some results related to the homomorphic preimage of a hybrid bi-ideal in near-subtraction semigroups.

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1. Introduction

Schein [29] proposed the systems of the form $(\emptyset, \circ, \backslash)$ in which $\emptyset$ is a gathering of closed functions under the composition $\circ$ of functions (and thus $(\emptyset, \circ)$ is a function semigroup) and the set theoretic subtraction $\backslash$ (and thus $(\emptyset, \backslash)$ is a subtraction algebra). He also showed that every subtraction semigroup is isomorphic to an invertible difference semigroup function. Zelinka [32] discussed the structure of multiplication in a subtraction semigroup proposed by Schein. He discovered how to solve problems in atomic subtraction algebras, which are a type of subtraction algebra. Jun et al. [9] developed the concept of ideals by studying the characterization of ideals in subtraction algebras. Jun et al. looked at the properties of ideals obtained by sets as well as their associated outcomes in [10]. One of the generalised structures of rings is near-rings. Near-rings were studied by Zassenhaus and Wielandt in 1930 in relation to ring theory and group theory. Near-ring research was redeveloped in 1950 by Wielandt, Frohlich, and Blackett. Since then, the field has grown to include non-linear interpolation theory, automata theory, optimization theory, and formal language theory, among others (see [3,4,16,19,23]). Strongly regular near-subtraction

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semigroups as well as near-subtraction semigroups were presented by Dheena et al. [5]. In [14], the idea of bi-ideals of near-subtraction semigroups was explored by Mahalakshmi et al.

Zadeh [31] pioneered fuzzy set theory, which has since been used successfully in a wide range of fields including industrial automation, image processing, control systems engineering, optimization, and robotics. In [28], the idea of a fuzzy subgroup of a group was established by Rosenfeld in 1971. In gamma near-ring, Jun et al. [11] developed a definition for fuzzy ideals. In subtraction algebras, Lee et al. [13] defined a fuzzy ideal and outlined numerous assertions for a fuzzy set to be a fuzzy ideal. Prince Williams [30] defined the idea of fuzzy ideals in near-subtraction semigroups. Manikandan [15] discussed the idea of fuzzy bi-ideal of near-rings. In [2], the idea of fuzzy bi-ideals of near-subtraction semigroups was presented by Chinnadurai et al. and some of their characterizations were also discussed.

Molodtsov [20] founded soft set theory in 1999 as a generalisation of fuzzy set theory, and he has applied it successfully to a broad variety of fields. Soft sets were also used to solve decision-making problems by Maji et al. [17]. According to a set of parameters over an initial universe, Jun et al. [12] looked into a number of properties of hybrid structures. Based on this idea, they generated the concepts of a hybrid field, a hybrid linear space, and a hybrid subalgebra. Many algebraic issues have been resolved using hybrid structures, producing a wide range of outcomes (See [1, 6–8, 21, 22, 24–27]).

In [18], Meenakshi et al. established the idea of hybrid ideals in near-subtraction semigroups and some of their results were obtained. They also studied the notion of homomorphism of a hybrid structure in near-subtraction semigroups and hybrid intersection. In this paper, we extend these ideas by introducing the concept of hybrid bi-ideals in near-subtraction semigroups and their related assertions. We build an example that shows every hybrid bi-ideal need not be a hybrid ideal. The hybrid intersection of the near-subtraction semigroup is also described. In addition, we define the preimage of a hybrid bi-ideal in near-subtraction semigroups.

2. Preliminaries

We will provide some fundamental definitions of near-subtraction semigroups and hybrid structures. The power set of a set J is represented by $\mathfrak{P}(J)$.

Definition 2.1. [29] A set $\mathbb{B}(\neq \emptyset)$ with a binary operation "$-$" is said to be subtraction algebra that fulfils the following criteria:

(i) $h_1 - (w_1 - h_1) = h_1$,
(ii) $h_1 - (h_1 - w_1) = w_1 - (w_1 - h_1)$,
(iii) $(h_1 - w_1) - s_1 = (h_1 - s_1) - w_1 \quad \forall h_1, w_1, s_1 \in \mathbb{B}$.

Definition 2.2. [5] A set $\mathbb{B}(\neq \emptyset)$ with the binary operations "$-$" and "$\cdot$" is called as right near-subtraction semigroup (resp., left) that fulfils the assertions listed below:

(i) $(\mathbb{B}, -)$ is a subtraction algebra.
(ii) $(\mathbb{B}, \cdot)$ is a semigroup.
(iii) $(j_0 - j_1)j_2 = j_0j_2 - j_1j_2$ (resp., $j_0(j_1 - j_2) = j_0j_1 - j_0j_2$) $\forall j_0, j_1, j_2 \in \mathbb{B}$.

It is obvious that $0l_0 = 0 \quad \forall l_0 \in \mathbb{B}$.

Throughout the paper, $\mathbb{B}$ represents a near- subtraction semigroup (briefly, NSS) means only a right near-subtraction semigroup, unless otherwise stated.

Definition 2.3. [5] A near-subtraction semigroup $\mathbb{B}$ is called a zero-symmetric if $k_1 0 = 0$ for every $k_1 \in \mathbb{B}$.

Definition 2.4. [5] A subset $J(\neq \emptyset)$ of a subtraction algebra $\mathbb{B}$ is defined as a subalgebra of $\mathbb{B}$ if $h_0 - h_1 \in J$ whenever $h_0, h_1 \in J$.

Definition 2.5. [5] A subset $J(\neq \emptyset)$ of $\mathbb{B}$ is defined as a near-subtraction subsemigroup of $\mathbb{B}$ if $y_0 - y_1, y_0y_1 \in J$ whenever $y_0, y_1 \in J$.

Definition 2.6. [5] Let $(\mathbb{B}, -, \cdot)$ be a NSS. A subset $F(\neq \emptyset)$ of $\mathbb{B}$ is described as
(i) a left ideal if F is a subalgebra of $(\mathbb{B}, -)$ and $fs_0 - f(v - s_0) \in F \ \ \forall f, v \in \mathbb{B}; s_0 \in F$.
(ii) a right ideal if F is a subalgebra of $(\mathbb{B}, -)$ and $F\mathbb{B} \subseteq F$.
(iii) an ideal if F is both a left and right ideal.

Definition 2.7. [14] Let $C, D \in \mathfrak{P}(\mathbb{B})$. Then the product and $*$ product are described as below:

$$CD = \{c_1d_1 \mid c_1 \in C \ and \ d_1 \in D\}.$$
$$C * D = \{c_1d_1 - c_1(c_1' - d_1) \mid c_1, c_1' \in C \ and \ d_1 \in D\}.$$

Definition 2.8. [14] An subalgebra $\mathbb{W}$ of $\mathbb{B}$ is called as a bi-ideal if $\mathbb{W}\mathbb{B}\mathbb{W} \cap \mathbb{W}\mathbb{B} * \mathbb{W} \subseteq \mathbb{W}$.

Definition 2.9. [12] Let $\mathbb{T}$ be a universal set. A hybrid structure in $\mathbb{B}$ over $\mathbb{T}$ is defined to be a mapping

$$\tilde{l}_\varsigma := (\tilde{l}, \varsigma) : \mathbb{B} \to \mathfrak{P}(\mathbb{T}) \times [0, 1], \ u \mapsto (\tilde{l}(u), \varsigma(u))$$

where $\tilde{l} : \mathbb{B} \to \mathfrak{P}(\mathbb{T})$ and $\varsigma : \mathbb{B} \to [0, 1]$ are mappings.

Define a relation $\ll$ on the collection of all hybrid structures, represented by $\mathfrak{H}(\mathbb{B})$, in $\mathbb{B}$ over $\mathbb{T}$ as follows:

$$\left(\forall \tilde{l}_\varsigma, \tilde{b}_\gamma \in \mathfrak{H}(\mathbb{B})\right) \left(\tilde{l}_\varsigma \ll \tilde{b}_\gamma \iff \tilde{l} \,\tilde{\subseteq}\, \tilde{b}, \varsigma \succeq \gamma\right)$$

where $\tilde{l} \,\tilde{\subseteq}\, \tilde{b}$ stands for $\tilde{l}(q_1) \subseteq \tilde{b}(q_1)$ and $\varsigma \succeq \gamma$ stands for $\varsigma(q_1) \geq \gamma(q_1) \ \forall q_1 \in \mathbb{B}$. Then $(\mathfrak{H}(\mathbb{B}), \ll)$ is a poset.

Definition 2.10. [12] For $\tilde{l}_\tau \in \mathfrak{H}(\mathbb{B})$ and $V \in \mathfrak{P}(\mathbb{B}) \backslash \{\emptyset\}$, the characteristic hybrid structure in $\mathbb{B}$ over $\mathbb{T}$ is represented by $\chi_V(\tilde{l}_\tau)$ and it is defined as,

$$\chi_V(\tilde{l}_\tau) = (\chi_V(\tilde{l}), \chi_V(\tau)) : \mathbb{B} \longrightarrow \mathfrak{P}(\mathbb{T}) \times [0, 1],$$
$$w_0 \mapsto \left(\chi_V(\tilde{l})(w_0), \chi_V(\tau)(w_0)\right),$$

where

$$\chi_V(\tilde{l}) : \mathbb{B} \to \mathfrak{P}(\mathbb{T}), w_0 \mapsto \begin{cases} \mathbb{T} & if \ \ w_0 \in V \\ \emptyset & otherwise, \end{cases}$$
$$\chi_V(\tau) : \mathbb{B} \to [0, 1], w_0 \mapsto \begin{cases} 0 & if \ w_0 \in V \\ 1 & otherwise \end{cases}$$

for any $w_0 \in \mathbb{B}$.
If $V = \mathbb{B}$, then we use that $\chi_V(\tilde{l}_\tau) = \mathfrak{B}_{\mathfrak{b}}$.

Definition 2.11. [12] Let $\tilde{l}_\varsigma \in \mathfrak{H}(\mathbb{B})$. Then we define

$$\mathbb{B}_{\tilde{l}}^{\Upsilon} := \{s_0 \in \mathbb{B} \ : \ \tilde{l}(s_0) \supseteq \Upsilon\} \text{ and } \mathbb{B}_{\varsigma}^{\omega} := \{s_0 \in \mathbb{B} \ : \ \varsigma(s_0) \leq \omega\}.$$

for any $(\Upsilon, \omega) \in \mathfrak{P}(\mathbb{T}) \times [0, 1]$.

Definition 2.12. [12] For $\tilde{l}_\varsigma \in \mathfrak{H}(\mathbb{B})$, the set

$$\tilde{l}_\varsigma[Z, d] := \{s_0 \in \mathbb{B} \ : \ \tilde{l}(s_0) \supseteq Z \ \ and \ \ \varsigma(s_0) \leq d\}$$

is called as $[Z, d]-$hybrid cut of $\tilde{l}_\varsigma$, where $Z \in \mathfrak{P}(\mathbb{T})$ and $d \in [0, 1]$.
It is noted that $\mathbb{B}_{\tilde{l}}^{\Upsilon} \cap \mathbb{B}_{\varsigma}^{\omega} = \{s_0 \in \mathbb{B} \ : \ \tilde{l}(s_0) \supseteq \Upsilon \text{ and } \varsigma(s_0) \leq \omega\} = \tilde{l}_\varsigma[\Upsilon, \omega]$.

Definition 2.13. [18] A hybrid structure $\tilde{l}_\varsigma$ of $\mathbb{B}$ is known as a hybrid subalgebra of $\mathbb{B}$, if

$$(\forall \ a_1, j_1 \in \mathbb{B}) \begin{pmatrix} \tilde{l}(a_1 - j_1) \supseteq \tilde{l}(a_1) \cap \tilde{l}(j_1) \\ \varsigma(a_1 - j_1) \leq \varsigma(a_1) \vee \varsigma(j_1) \end{pmatrix}.$$

Definition 2.14. [18] A hybrid subalgebra $\tilde{l}_\varsigma$ of $\mathbb{B}$ is defined as a hybrid ideal if it satisfies the assertions listed below:

(i) $(\forall r_0, j_0, p_0 \in \mathbb{B}) \left(\begin{array}{c} \tilde{l}(r_0p_0 - r_0(j_0 - p_0)) \supseteq \tilde{l}(p_0) \\ \varsigma(r_0p_0 - r_0(j_0 - p_0)) \leq \varsigma(p_0) \end{array} \right)$.

(ii) $(\forall p_0, d_0 \in \mathbb{B}) \left(\begin{array}{c} \tilde{l}(p_0d_0) \supseteq \tilde{l}(p_0) \\ \varsigma(p_0d_0) \leq \varsigma(p_0) \end{array} \right)$.

Note that $\tilde{l}_\varsigma$ is a left hybrid ideal of $\mathbb{B}$ if it satisfies (i), and $\tilde{l}_\varsigma$ is a right hybrid ideal of $\mathbb{B}$ if it satisfies (ii).

Definition 2.15. Let $\tilde{l}_\varsigma, \tilde{k}_\vartheta \in \mathfrak{H}(\mathbb{B})$. Then $\tilde{l}_\varsigma \Cap \tilde{k}_\vartheta$, $\tilde{l}_\varsigma \Cup \tilde{k}_\vartheta$, $\tilde{l}_\varsigma - \tilde{k}_\vartheta$, $\tilde{l}_\varsigma\tilde{k}_\vartheta$ and $\tilde{l}_\varsigma * \tilde{k}_\vartheta$ are hybrid structures of $\mathbb{B}$ defined by

(i) $(\tilde{l}_\varsigma \Cap \tilde{k}_\vartheta) := (\tilde{l} \cap \tilde{k}, \varsigma \vee \vartheta)$, where $(\forall j \in \mathbb{B}) \left(\begin{array}{c} (\tilde{l} \cap \tilde{k})(j) = \tilde{l}(j) \cap \tilde{k}(j) \\ (\varsigma \vee \vartheta)(j) = \varsigma(j) \vee \vartheta(j) \end{array} \right)$.

(ii) $(\tilde{l}_\varsigma \Cup \tilde{k}_\vartheta) := (\tilde{l} \cup \tilde{k}, \varsigma \wedge \vartheta)$, where $(\forall j \in \mathbb{B}) \left(\begin{array}{c} (\tilde{l} \cup \tilde{k})(j) = \tilde{l}(j) \cup \tilde{k}(j) \\ (\varsigma \wedge \vartheta)(j) = \varsigma(j) \wedge \vartheta(j) \end{array} \right)$.

(iii) $(\tilde{l}_\varsigma - \tilde{k}_\vartheta) := (\tilde{l} - \tilde{k}, \varsigma - \vartheta)$, where

$$(\tilde{l} - \tilde{k})(s) = \begin{cases} \bigcup\limits_{s=f-c} \{\tilde{l}(f) \cap \tilde{k}(c)\} & if \quad s = f - c \\ \emptyset & otherwise, \end{cases}$$

$$(\varsigma - \vartheta)(s) = \begin{cases} \bigwedge\limits_{s=f-c} \{\varsigma(f) \vee \vartheta(c)\} & if \quad s = f - c \\ 1 & otherwise \end{cases}$$

for any $s \in \mathbb{B}$.

(iv) $(\tilde{l}_\varsigma\tilde{k}_\vartheta) := (\tilde{l}\tilde{k}, \varsigma\vartheta)$, where

$$(\tilde{l}\tilde{k})(j) = \begin{cases} \bigcup\limits_{j=yz} \{\tilde{l}(y) \cap \tilde{k}(z)\} & if \quad j = yz \\ \emptyset & otherwise, \end{cases}$$

$$(\varsigma\vartheta)(j) = \begin{cases} \bigwedge\limits_{j=yz} \{\varsigma(y) \vee \vartheta(z)\} & if \quad j = yz \\ 1 & otherwise \end{cases}$$

for any $j \in \mathbb{B}$.

(v) $(\tilde{l}_\varsigma * \tilde{k}_\vartheta) := (\tilde{l} * \tilde{k}, \varsigma * \vartheta)$, where

$$(\tilde{l} * \tilde{k})(j) = \begin{cases} \bigcup\limits_{j=zc-z(b-c)} \{\tilde{l}(z) \cap \tilde{k}(c)\} & if \quad j = zc - z(b - c) \\ \emptyset & otherwise, \end{cases}$$

$$(\varsigma * \vartheta)(j) = \begin{cases} \bigwedge\limits_{j=zc-z(b-c)} \{\varsigma(z) \vee \vartheta(c)\} & if \quad j = zc - z(b - c) \\ 1 & otherwise \end{cases}$$

for any $j \in \mathbb{B}$.

3. Properties of hybrid bi-ideals

In this section, we define the idea of hybrid bi-ideals of near-subtraction semigroups and build an example to show that every hybrid bi-ideal need not be a hybrid ideal of near-subtraction semigroups. Within near-subtraction semigroups, we define hybrid intersection and provide some results about hybrid bi-ideals.

Definition 3.1. A hybrid subalgebra $\tilde{l}_\varsigma$ of $\mathbb{B}$ is defined as a hybrid bi-ideal of $\mathbb{B}$, if $((\tilde{l}_\varsigma\mathfrak{B}_\mathfrak{b}\tilde{l}_\varsigma) \Cap (\tilde{l}_\varsigma\mathfrak{B}_\mathfrak{b} * \tilde{l}_\varsigma) \ll \tilde{l}_\varsigma) := ((\tilde{l}\mathfrak{B}\tilde{l}) \cap (\tilde{l}\mathfrak{B} * \tilde{l}) \subseteq \tilde{l}, (\varsigma\mathfrak{b}\varsigma) \vee (\varsigma\mathfrak{b} * \varsigma) \geq \varsigma)$.

Example 3.2. Let $X = \{0, p, w, z\}$ in which "$-$" and "$\cdot$" are defined by:

-	0	p	w	z
0	0	0	0	0
p	p	0	p	p
w	w	w	0	w
z	z	z	z	0

$\cdot$	0	p	w	z
0	0	0	0	0
p	p	p	p	p
w	0	0	0	w
z	0	0	0	z

Then $(\mathbb{B}, -, \cdot)$ is a NSS. For $Y, A, V, N \in \mathfrak{P}(\mathbb{T})$ and $y, a, v, n \in [0,1]$, define $\tilde{l}_\varsigma \in \mathfrak{H}(\mathbb{B})$ by $\tilde{l}(0) = Y$, $\tilde{l}(p) = A$, $\tilde{l}(w) = V$, $\tilde{l}(z) = N$ and $\varsigma(0) = y$, $\varsigma(p) = a$, $\varsigma(w) = v$, $\varsigma(z) = n$. If $Y \supset A \supset V \supset N$ and with $y < a < v < n$, then $\tilde{l}_\varsigma$ is a hybrid bi-ideal of $\mathbb{B}$.

It is clear that every hybrid ideal of $\mathbb{B}$ is a hybrid bi-ideal of $\mathbb{B}$ (see Lemma 3.4). The following example shows that the converse is generally not true.

Example 3.3. Let $X = \{0, p, w, z\}$ in which "$-$" and "$\cdot$" are defined as in Example 3.2. Let $\tilde{l}_\varsigma : \mathbb{B} \to \mathfrak{P}(\mathbb{T}) \times [0,1]$ be a hybrid structure of $\mathbb{B}$ defined by $\tilde{l}(0) = A$, $\tilde{l}(p) = U = \tilde{l}(z)$, $\tilde{l}(w) = F$ and $\varsigma(0) = a$, $\varsigma(p) = u = \varsigma(z)$, $\varsigma(w) = f$ for some $A, U, F \in \mathfrak{P}(\mathbb{T})$ and $f, u, a \in [0,1]$. If $F \subset U \subset A$ and $f > u > a$, then $\tilde{l}_\varsigma$ of $\mathbb{B}$ is a hybrid bi-ideal, but $\tilde{l}_\varsigma$ of $\mathbb{B}$ is not a left hybrid ideal, since $\tilde{l}(wz - w(0 - z)) = \tilde{l}(w) = F \not\supseteq U = \tilde{l}(z)$ and $\varsigma(wz - w(0-z)) = \varsigma(w) = f \not\leq u = \varsigma(z)$.

Lemma 3.4. *For $\tilde{l}_\varsigma \in \mathfrak{H}(\mathbb{B})$. If $\tilde{l}_\varsigma$ of $\mathbb{B}$ is a hybrid left ideal, then $\tilde{l}_\varsigma$ of $\mathbb{B}$ is a hybrid bi-ideal.*

Proof. Let $a' \in \mathbb{B}$ be such that $a' = vys = zd - z(q-d)$, where $v, y, s, z, q, d \in \mathbb{B}$. Then

$$\begin{aligned}
((\tilde{l}\mathfrak{B}\tilde{l}) \cap (\tilde{l}\mathfrak{B} * \tilde{l}))(a') &= (\tilde{l}\mathfrak{B}\tilde{l})(a') \cap (\tilde{l}\mathfrak{B} * \tilde{l})(a') \\
&= \left(\bigcup_{a'=vys} \tilde{l}(v) \cap \mathfrak{B}(y) \cap \tilde{l}(s) \right) \cap \left(\bigcup_{a'=zd-z(q-d)} (\tilde{l}\mathfrak{B})(z) \cap \tilde{l}(d) \right) \\
&\subseteq \bigcup_{a'=zd-z(q-d)} (\tilde{l}\mathfrak{B})(z) \cap \tilde{l}(d) \\
&\subseteq \bigcup_{a'=zd-z(q-d)} \{\mathfrak{B}(z) \cap \tilde{l}(zd - z(q-d))\} \\
&= \bigcup_{a'=zd-z(q-d)} \tilde{l}(zd - z(q-d)) = \tilde{l}(a'),
\end{aligned}$$

$$\begin{aligned}
((\varsigma\mathfrak{b}\varsigma) \vee (\varsigma\mathfrak{b} * \varsigma))(a') &= (\varsigma\mathfrak{b}\varsigma)(a') \vee (\varsigma\mathfrak{b} * \varsigma)(a') \\
&= \left(\bigwedge_{a'=vys} \varsigma(v) \vee \mathfrak{b}(y) \vee \varsigma(s) \right) \vee \left(\bigwedge_{a'=zd-z(q-d)} (\varsigma\mathfrak{b})(z) \vee \varsigma(d) \right) \\
&\geq \bigwedge_{a'=zd-z(q-d)} (\varsigma\mathfrak{b})(z) \vee \varsigma(d) \\
&\geq \bigwedge_{a'=zd-z(q-d)} \{\mathfrak{b}(z) \vee \varsigma(zd - z(q-d))\} \\
&= \bigwedge_{a'=zd-z(q-d)} \varsigma(zd - z(q-d)) = \varsigma(a').
\end{aligned}$$

If a' cannot be expressed as $a' = vys = zd - z(q-d)$, then $((\tilde{l}\mathfrak{B}\tilde{l}) \cap (\tilde{l}\mathfrak{B} * \tilde{l}))(a') = \emptyset \subseteq \tilde{l}(a')$ and $((\varsigma\mathfrak{b}\varsigma) \vee (\varsigma\mathfrak{b} * \varsigma))(a') = 1 \geq \varsigma(a')$. Thus $\tilde{l}_\varsigma \mathfrak{B}_\mathfrak{b} \tilde{l}_\varsigma \Cap \tilde{l}_\varsigma \mathfrak{B}_\mathfrak{b} * \tilde{l}_\varsigma \ll \tilde{l}_\varsigma$ and hence $\tilde{l}_\varsigma$ of $\mathbb{B}$ is a hybrid bi-ideal. $\square$

The proof of the below lemma is similar to the proof of Lemma 3.4, we provide the proof for readers' convenience.

Lemma 3.5. *If $\tilde{l}_\varsigma \in \mathfrak{H}(\mathbb{B})$ is a hybrid right ideal, then $\tilde{l}_\varsigma$ of $\mathbb{B}$ is a hybrid bi-ideal.*

Proof. Let $x' \in \mathbb{B}$ be such that $x' = vy = xj - x(q-j)$, $v = v_1v_2$, where $v, v_1, v_2, y, x, q, j \in \mathbb{B}$. Then

$$\begin{aligned}
((\tilde{l}\mathfrak{B}\tilde{l}) \cap (\tilde{l}\mathfrak{B} * \tilde{l}))(x') &= (\tilde{l}\mathfrak{B}\tilde{l})(x') \cap (\tilde{l}\mathfrak{B} * \tilde{l})(x') \\
&\subseteq \bigcup_{x'=vy} (\tilde{l}\mathfrak{B})(v) \cap \tilde{l}(y) \\
&= \left\{ \bigcup_{x'=vy} \left\{ \bigcup_{v=v_1v_2} \tilde{l}(v_1) \cap \mathfrak{B}(v_2) \right\} \cap \tilde{l}(y) \right\} \\
&= \left\{ \bigcup_{x'=vy} \left\{ \bigcup_{v=v_1v_2} \tilde{l}(v_1) \right\} \cap \tilde{l}(y) \right\} \\
&= \tilde{l}(v_1) \cap \tilde{l}(y) \\
&\quad (\text{since } \ \tilde{l}(vy) = \tilde{l}(v_1v_2y) \supseteq \tilde{l}(v_1)) \\
&\subseteq \tilde{l}(vy) = \tilde{l}(x'), \\
((\varsigma\mathfrak{b}\varsigma) \vee (\varsigma\mathfrak{b} * \varsigma))(x') &= (\varsigma\mathfrak{b}\varsigma)(x') \vee (\varsigma\mathfrak{b} * \varsigma)(x') \\
&\geq \bigwedge_{x'=vy} (\varsigma\mathfrak{b})(v) \vee \varsigma(y) \\
&= \left\{ \bigwedge_{x'=vy} \left\{ \bigwedge_{v=v_1v_2} \varsigma(v_1) \vee \mathfrak{b}(v_2) \right\} \vee \varsigma(y) \right\} \\
&= \left\{ \bigwedge_{x'=vy} \left\{ \bigwedge_{v=v_1v_2} \varsigma(v_1) \right\} \vee \varsigma(y) \right\} \\
&= \varsigma(v_1) \vee \varsigma(y) \\
&\quad (\text{since } \ \varsigma(vy) = \varsigma(v_1v_2y) \leq \varsigma(v_1)) \\
&\geq \varsigma(vy) = \varsigma(x').
\end{aligned}$$

If x' cannot be expressed as $x' = vy = xj - x(q-j)$, then $((\tilde{l}\mathfrak{B}\tilde{l}) \cap (\tilde{l}\mathfrak{B} * \tilde{l}))(x') = \emptyset \subseteq \tilde{l}(x')$ and $((\varsigma\mathfrak{b}\varsigma) \vee (\varsigma\mathfrak{b} * \varsigma))(x') = 1 \geq \varsigma(x')$. Thus $\tilde{l}_\varsigma \mathfrak{B}_\mathfrak{b} \tilde{l}_\varsigma \Cap \tilde{l}_\varsigma \mathfrak{B}_\mathfrak{b} * \tilde{l}_\varsigma \ll \tilde{l}_\varsigma$ and hence $\tilde{l}_\varsigma$ is a hybrid bi-ideal of $\mathbb{B}$. □

Theorem 3.6. *Let $\tilde{k}_\varrho$ and $\tilde{l}_\varsigma$ be any two hybrid bi-ideals of $\mathbb{B}$. Then $\tilde{k}_\varrho \Cap \tilde{l}_\varsigma$ is also a hybrid bi-ideal of $\mathbb{B}$.*

Proof. Let $\tilde{k}_\varrho$ and $\tilde{l}_\varsigma$ be any two hybrid bi-ideals of $\mathbb{B}$, and let $z_0, t_0 \in \mathbb{B}$. Then

$$\begin{aligned}
(\tilde{k} \cap \tilde{l})(z_0 - t_0) &= \tilde{k}(z_0 - t_0) \cap \tilde{l}(z_0 - t_0) \\
&\supseteq (\tilde{k}(z_0) \cap \tilde{k}(t_0)) \cap (\tilde{l}(z_0) \cap \tilde{l}(t_0)) \\
&= (\tilde{k}(z_0) \cap \tilde{l}(z_0)) \cap (\tilde{k}(t_0) \cap \tilde{l}(t_0)) \\
&= (\tilde{k} \cap \tilde{l})(z_0) \cap (\tilde{k} \cap \tilde{l})(t_0), \\
(\varrho \vee \varsigma)(z_0 - t_0) &= \varrho(z_0 - t_0) \vee \varsigma(z_0 - t_0) \\
&\leq (\varrho(z_0) \vee \varrho(t_0)) \vee (\varsigma(z_0) \vee \varsigma(t_0)) \\
&= (\varrho(z_0) \vee \varsigma(z_0)) \vee (\varrho(t_0) \vee \varsigma(t_0)) \\
&= (\varrho \vee \varsigma)(z_0) \vee (\varrho \vee \varsigma)(t_0).
\end{aligned}$$

Let $j' \in \mathbb{B}$. Choose $h, w, s, j, t, a \in \mathbb{B}$ such that $j' = hws = ja - j(t-a)$. Since $\tilde{k}_\varrho$ and $\tilde{l}_\varsigma$ are hybrid bi-ideals of $\mathbb{B}$, we get

$$\left\{\bigcup_{j'=hws}(\tilde{k}(h)\cap\tilde{k}(s))\right\}\cap\left\{\bigcup_{j'=ja-j(t-a)}(\tilde{k}(j)\cap\tilde{k}(a))\right\}\subseteq\tilde{k}(j'),$$

$$\left\{\bigwedge_{j'=hws}(\varrho(h)\vee\varrho(s))\right\}\vee\left\{\bigwedge_{j'=ja-j(t-a)}(\varrho(j)\vee\varrho(a))\right\}\geq\varrho(j')$$

and

$$\left\{\bigcup_{j'=hws}(\tilde{l}(h)\cap\tilde{l}(s))\right\}\cap\left\{\bigcup_{j'=ja-j(t-a)}(\tilde{l}(j)\cap\tilde{l}(a))\right\}\subseteq\tilde{l}(j'),$$

$$\left\{\bigwedge_{j'=hws}(\varsigma(h)\vee\varsigma(s))\right\}\vee\left\{\bigwedge_{j'=ja-j(t-a)}(\varsigma(j)\vee\varsigma(a))\right\}\geq\varsigma(j').$$

Now

$$\begin{aligned}
&((\tilde{k}\cap\tilde{l})\mathfrak{B}(\tilde{k}\cap\tilde{l}))(j')\cap((\tilde{k}\cap\tilde{l})\mathfrak{B}*(\tilde{k}\cap\tilde{l}))(j')\\
&=\left\{\bigcup_{j'=hws}((\tilde{k}\cap\tilde{l})(h)\cap(\tilde{k}\cap\tilde{l})(s))\right\}\cap\left\{\bigcup_{j'=ja-j(t-a)}((\tilde{k}\cap\tilde{l})(j)\cap(\tilde{k}\cap\tilde{l})(a))\right\}\\
&=\left\{\bigcup_{j'=hws}(\tilde{k}(h)\cap\tilde{l}(h))\cap(\tilde{k}(s)\cap\tilde{l}(s))\right\}\cap\left\{\bigcup_{j'=ja-j(t-a)}(\tilde{k}(j)\cap\tilde{l}(j))\cap(\tilde{k}(a)\cap\tilde{l}(a))\right\}\\
&=\left\{\bigcup_{j'=hws}(\tilde{k}(h)\cap\tilde{k}(s))\cap\bigcup_{j'=ja-j(t-a)}(\tilde{k}(j)\cap\tilde{k}(a))\right\}\\
&\qquad\qquad\cap\left\{\bigcup_{j'=hws}(\tilde{l}(h)\cap\tilde{l}(s))\cap\bigcup_{j'=ja-j(t-a)}(\tilde{l}(j)\cap\tilde{l}(a))\right\}\\
&\subseteq\tilde{k}(j')\cap\tilde{l}(j')=(\tilde{k}\cap\tilde{l})(j'),\\
&((\varrho\vee\varsigma)\mathfrak{b}(\varrho\vee\varsigma))(j')\vee((\varrho\vee\varsigma)\mathfrak{b}*(\varrho\vee\varsigma))(j')\\
&=\left\{\bigwedge_{j'=hws}((\varrho\vee\varsigma)(h)\vee(\varrho\vee\varsigma)(s))\right\}\vee\left\{\bigwedge_{j'=ja-j(t-a)}((\varrho\vee\varsigma)(j)\vee(\varrho\vee\varsigma)(a))\right\}\\
&=\left\{\bigwedge_{j'=hws}(\varrho(h)\vee\varsigma(h))\vee(\varrho(s)\vee\varsigma(s))\right\}\vee\left\{\bigwedge_{j'=ja-j(t-a)}(\varrho(j)\vee\varsigma(j))\vee(\varrho(a)\vee\varsigma(a))\right\}\\
&=\left\{\bigwedge_{j'=hws}(\varrho(h)\vee\varrho(s))\vee\bigwedge_{j'=ja-j(t-a)}(\varrho(j)\vee\varrho(a))\right\}\\
&\qquad\qquad\vee\left\{\bigwedge_{j'=hws}(\varsigma(h)\vee\varsigma(s))\vee\bigwedge_{j'=ja-j(t-a)}(\varsigma(j)\vee\varsigma(a))\right\}\\
&\geq\varrho(j')\vee\varsigma(j')=(\varrho\vee\varsigma)(j').
\end{aligned}$$

Hence $\tilde{k}_\varrho \Cap \tilde{l}_\varsigma := (\tilde{k}\cap\tilde{l};\varrho\vee\varsigma)$ is a hybrid bi-ideal of $\mathbb{B}$. □

Theorem 3.7. *Let $\tilde{l}_\varsigma\in\mathfrak{H}(\mathbb{B})$. If $\tilde{l}_\varsigma$ is a hybrid bi-ideal of $\mathbb{B}$, then the hybrid cut $\tilde{l}_\varsigma[\Gamma,\alpha]$ is a bi-ideal of $\mathbb{B}$, $\forall\ \Gamma\in\mathfrak{P}(\mathbb{T});\alpha\in[0,1]$.*

Proof. For $\Gamma\in\mathfrak{P}(\mathbb{T})$ and $\alpha\in[0,1]$, let $u,y\in\tilde{l}_\varsigma[\Gamma,\alpha]$. Then $\tilde{l}(u-y)\supseteq\tilde{l}(u)\cap\tilde{l}(y)\supseteq\Gamma$ and $\varsigma(u-y)\leq\varsigma(u)\vee\varsigma(y)\leq\alpha$. It follows that $u-y\in\tilde{l}_\varsigma[\Gamma,\alpha]$.

Let $z' \in \mathbb{B}$ and $z' \in \tilde{l}_\varsigma[\Gamma,\alpha]\mathfrak{B}_\mathfrak{b}\tilde{l}_\varsigma[\Gamma,\alpha] \Cap \tilde{l}_\varsigma[\Gamma,\alpha]\mathfrak{B}_\mathfrak{b} * \tilde{l}_\varsigma[\Gamma,\alpha]$. If there exist $f_1, q, u_1, c \in \tilde{l}_\varsigma[\Gamma,\alpha]$ and $f_2, f, u, u_2, y \in \mathbb{B}$ such that $z' = fq = uc - u(y-c)$, $f = f_1 f_2$ and $u = u_1 u_2$, then

$$\begin{aligned}
\tilde{l}(z') \supseteq& ((\tilde{l}\mathfrak{B}\tilde{l}) \cap (\tilde{l}\mathfrak{B} * \tilde{l}))(z') \\
=& (\tilde{l}\mathfrak{B}\tilde{l})(z') \cap (\tilde{l}\mathfrak{B} * \tilde{l})(z') \\
=& \left\{ \bigcup_{z'=fq} \{(\tilde{l}\mathfrak{B})(f) \cap \tilde{l}(q)\} \right\} \cap \left\{ \bigcup_{z'=uc-u(y-c)} \{(\tilde{l}\mathfrak{B})(u) \cap \tilde{l}(c)\} \right\} \\
=& \left\{ \bigcup_{z'=fq} \left(\bigcup_{f=f_1f_2} (\tilde{l}(f_1) \cap \mathfrak{B}(f_2)) \right) \cap \tilde{l}(q) \right\} \\
& \cap \left\{ \bigcup_{z'=uc-u(y-c)} \left(\bigcup_{u=u_1u_2} (\tilde{l}(u_1) \cap \mathfrak{B}(u_2)) \right) \cap \tilde{l}(c) \right\} \\
\supseteq& (\tilde{l}(f_1) \cap \tilde{l}(q) \cap \tilde{l}(u_1) \cap \tilde{l}(c)) \supseteq \Gamma,
\end{aligned}$$

$$\begin{aligned}
\varsigma(z') \leq& ((\varsigma\mathfrak{b}\varsigma) \vee (\varsigma\mathfrak{b} * \varsigma))(z') \\
=& (\varsigma\mathfrak{b}\varsigma)(z') \vee (\varsigma\mathfrak{b} * \varsigma)(z') \\
=& \left\{ \bigwedge_{z'=fq} \{(\varsigma\mathfrak{b})(f) \vee \varsigma(q)\} \right\} \vee \left\{ \bigwedge_{z'=uc-u(y-c)} \{(\varsigma\mathfrak{b})(u) \vee \varsigma(c)\} \right\} \\
=& \left\{ \bigwedge_{z'=fq} \left(\bigwedge_{f=f_1f_2} (\varsigma(f_1) \vee \mathfrak{b}(f_2)) \right) \vee \varsigma(q) \right\} \\
& \vee \left\{ \bigwedge_{z'=uc-u(y-c)} \left(\bigwedge_{u=u_1u_2} (\varsigma(u_1) \vee \mathfrak{b}(u_2)) \right) \vee \varsigma(c) \right\} \\
\leq& (\varsigma(f_1) \vee \varsigma(q) \vee \varsigma(u_1) \vee \varsigma(c)) \leq \alpha.
\end{aligned}$$

This implies that $z' \in \tilde{l}_\varsigma[\Gamma,\alpha]$. Thus $\tilde{l}_\varsigma[\Gamma,\alpha]\mathfrak{B}_\mathfrak{b}\tilde{l}_\varsigma[\Gamma,\alpha] \Cap \tilde{l}_\varsigma[\Gamma,\alpha]\mathfrak{B}_\mathfrak{b} * \tilde{l}_\varsigma[\Gamma,\alpha] \ll \tilde{l}_\varsigma[\Gamma,\alpha]$ and hence $\tilde{l}_\varsigma[\Gamma,\alpha]$ is a bi-ideal of $\mathbb{B}$. □

Lemma 3.8. *For $K, V \in \mathfrak{P}(\mathbb{B}) \setminus \{\emptyset\}$ and $\tilde{l}_\varrho \in \mathfrak{H}(\mathbb{B})$, the following statements are true:*
(i) $\chi_K(\tilde{l}_\varrho) \Cap \chi_V(\tilde{l}_\varrho) = \chi_{K\cap V}(\tilde{l}_\varrho)$.
(ii) $\chi_K(\tilde{l}_\varrho) \Cup \chi_V(\tilde{l}_\varrho) = \chi_{K\cup V}(\tilde{l}_\varrho)$.
(iii) $\chi_K(\tilde{l}_\varrho)\chi_V(\tilde{l}_\varrho) = \chi_{KV}(\tilde{l}_\varrho)$.
(iv) $\chi_K(\tilde{l}_\varrho) * \chi_V(\tilde{l}_\varrho) = \chi_{K*V}(\tilde{l}_\varrho)$.
(v) If $K \subseteq V$, then $\chi_K(\tilde{l}_\varrho) \ll \chi_V(\tilde{l}_\varrho)$.

Proof. The proofs are obvious. □

Lemma 3.9. *For $K \in \mathfrak{P}(\mathbb{B})\setminus\{\emptyset\}$ and $\tilde{l}_\varsigma \in \mathfrak{H}(\mathbb{B})$, the assertions listed below are equivalent:*
(i) K is a bi-ideal of $\mathbb{B}$,
(ii) $\chi_K(\tilde{l}_\varsigma)$ is a hybrid bi-ideal of $\mathbb{B}$.

Proof. $(i) \Rightarrow (ii)$ Let $u, z \in \mathbb{B}$. If $u, z \in K$, then $u - z \in K$ which implies $\chi_K(\tilde{l})(u-z) = \mathbb{T} = \chi_K(\tilde{l})(u) \cap \chi_K(\tilde{l})(z)$ and $\chi_K(\varsigma)(u-z) = 0 = \chi_K(\varsigma)(u) \vee \chi_K(\varsigma)(z)$. Otherwise $u \notin K$ or $z \notin K$. Then $\chi_K(\tilde{l})(u-z) \supseteq \emptyset = \chi_K(\tilde{l})(u) \cap \chi_K(\tilde{l})(z)$ and $\chi_K(\varsigma)(u-z) \leq 1 = \chi_K(\varsigma)(u) \vee \chi_K(\varsigma)(z)$. So $\chi_K(\tilde{l}_\varsigma)$ is a hybrid subalgebra of $\mathbb{B}$. By Lemma 3.8, we have

$$\chi_K(\tilde{l}_\varsigma)\mathfrak{B}_\mathfrak{b}\chi_K(\tilde{l}_\varsigma) \Cap \chi_K(\tilde{l}_\varsigma)\mathfrak{B}_\mathfrak{b} * \chi_K(\tilde{l}_\varsigma) = \chi_{K\mathbb{B}K}(\tilde{l}_\varsigma) \Cap \chi_{K\mathbb{B}*K}(\tilde{l}_\varsigma) = \chi_{(K\mathbb{B}K\cap(K\mathbb{B}*K))}(\tilde{l}_\varsigma) \ll \chi_K(\tilde{l}_\varsigma).$$

So, $\chi_K(\tilde{l}_\varsigma)$ of $\mathbb{B}$ is a hybrid bi-ideal.

$(ii) \Rightarrow (i)$ Let $u' \in K\mathbb{B}K \cap K\mathbb{B} * K$. Then $u' = hv = uq - u(z-q)$ and $h = h_1h_2$; $u = u_1u_2$ for some $h_1, v, q, u_1 \in K$ and $h_2, u, z, u_2 \in \mathbb{B}$. Now,

$$\begin{aligned}
\chi_K(\tilde{l})(u') \supseteq & (\chi_K\mathfrak{B}\chi_K \cap \chi_K\mathfrak{B} * \chi_K)(\tilde{l})(u') \\
= & (\chi_K\mathfrak{B}\chi_K)(\tilde{l})(u') \cap (\chi_K\mathfrak{B} * \chi_K)(\tilde{l})(u') \\
= & \bigcup_{u'=hv} \{(\chi_K\mathfrak{B})(\tilde{l})(h) \cap \chi_K(\tilde{l})(v)\} \cap \bigcup_{u'=uq-u(z-q)} \{(\chi_K\mathfrak{B})(\tilde{l})(u) \cap \chi_K(\tilde{l})(q)\} \\
= & \left\{ \bigcup_{u'=hv} \left(\bigcup_{h=h_1h_2} \chi_K(\tilde{l})(h_1) \cap \mathfrak{B}(h_2) \right) \cap \chi_K(\tilde{l})(v) \right\} \\
& \cap \left\{ \bigcup_{u'=uq-u(z-q)} \left(\bigcup_{u=u_1u_2} \chi_K(\tilde{l})(u_1) \cap \mathfrak{B}(u_2) \right) \cap \chi_K(\tilde{l})(q) \right\} \\
= & \chi_K(\tilde{l})(h_1) \cap \chi_K(\tilde{l})(v) \cap \chi_K(\tilde{l})(u_1) \cap \chi_K(\tilde{l})(q) = \mathbb{T},
\end{aligned}$$

$$\begin{aligned}
\chi_K(\varsigma)(u') \leq & (\chi_K\mathfrak{b}\chi_K \vee \chi_K\mathfrak{b} * \chi_K)(\varsigma)(u') \\
= & (\chi_K\mathfrak{b}\chi_K)(\varsigma)(u') \vee (\chi_K\mathfrak{b} * \chi_K)(\varsigma)(u') \\
= & \bigwedge_{u'=hv} \{(\chi_K\mathfrak{b})(\varsigma)(h) \vee \chi_K(\varsigma)(v)\} \vee \bigwedge_{u'=uq-u(z-q)} \{(\chi_K\mathfrak{b})(\varsigma)(u) \vee \chi_K(\varsigma)(q)\} \\
= & \left\{ \bigwedge_{u'=hv} \left(\bigwedge_{h=h_1h_2} \chi_K(\varsigma)(h_1) \vee \mathfrak{b}(h_2) \right) \vee \chi_K(\varsigma)(v) \right\} \\
& \vee \left\{ \bigwedge_{u'=uq-u(z-q)} \left(\bigwedge_{u=u_1u_2} \chi_K(\varsigma)(u_1) \vee \mathfrak{b}(u_2) \right) \vee \chi_K(\varsigma)(q) \right\} \\
= & \chi_K(\varsigma)(h_1) \vee \chi_K(\varsigma)(v) \vee \chi_K(\varsigma)(u_1) \vee \chi_K(\varsigma)(q) = 0.
\end{aligned}$$

This implies that $u' \in K$. Thus $K\mathbb{B}K \cap K\mathbb{B} * K \subseteq K$ and hence K of $\mathbb{B}$ is a bi-ideal. □

Theorem 3.10. *Let $\tilde{l}_\varsigma$ be a hybrid subalgebra of $\mathbb{B}$. If $\tilde{l}_\varsigma\mathfrak{B}_\mathfrak{b}\tilde{l}_\varsigma \ll \tilde{l}_\varsigma$, then $\tilde{l}_\varsigma$ of $\mathbb{B}$ is a hybrid bi-ideal.*

Proof. Consider $\tilde{l}\mathfrak{B}\tilde{l} \subseteq \tilde{l}$ and $\varsigma\mathfrak{b}\varsigma \geq \varsigma$. Let $y_1 \in \mathbb{B}$. Then

$$((\tilde{l}\mathfrak{B}\tilde{l}) \cap (\tilde{l}\mathfrak{B} * \tilde{l}))(y_1) = (\tilde{l}\mathfrak{B}\tilde{l})(y_1) \cap (\tilde{l}\mathfrak{B} * \tilde{l})(y_1) \subseteq (\tilde{l}\mathfrak{B}\tilde{l})(y_1) \subseteq \tilde{l}(y_1),$$
$$((\varsigma\mathfrak{b}\varsigma) \vee (\varsigma\mathfrak{b} * \varsigma))(y_1) = (\varsigma\mathfrak{b}\varsigma)(y_1) \vee (\varsigma\mathfrak{b} * \varsigma)(y_1) \geq (\varsigma\mathfrak{b}\varsigma)(y_1) \geq \varsigma(y_1).$$

Thus $\tilde{l}_\varsigma\mathfrak{B}_\mathfrak{b}\tilde{l}_\varsigma \Cap \tilde{l}_\varsigma\mathfrak{B}_\mathfrak{b} * \tilde{l}_\varsigma \ll \tilde{l}_\varsigma$ and so $\tilde{l}_\varsigma$ of $\mathbb{B}$ is a hybrid bi-ideal. □

Theorem 3.11. *If $\mathbb{B}$ is a zero-symmetric NSS and $\tilde{l}_\varsigma$ is a hybrid bi-ideal of $\mathbb{B}$, then $\tilde{l}_\varsigma\mathfrak{B}_\mathfrak{b}\tilde{l}_\varsigma \ll \tilde{l}_\varsigma$.*

Proof. Let $\tilde{l}_\varsigma$ of $\mathbb{B}$ be a hybrid bi-ideal. Then $\tilde{l}_\varsigma\mathfrak{B}_\mathfrak{b}\tilde{l}_\varsigma \Cap \tilde{l}_\varsigma\mathfrak{B}_\mathfrak{b} * \tilde{l}_\varsigma \ll \tilde{l}_\varsigma$. Clearly, $\tilde{l}(0) \supseteq \tilde{l}(d)$ and $\varsigma(1) \leq \varsigma(d)$. Thus $(\tilde{l}\mathfrak{B})(0) \supseteq (\tilde{l}\mathfrak{B})(d)$ and $(\varsigma\mathfrak{b})(1) \leq (\varsigma\mathfrak{b})(d)$ for all $d \in \mathbb{B}$. Since $\mathbb{B}$ is a zero-symmetric, $\tilde{l}\mathfrak{B}\tilde{l} \subseteq \tilde{l}\mathfrak{B} * \tilde{l}$ and $\varsigma\mathfrak{b}\varsigma \geq \varsigma\mathfrak{b} * \varsigma$. So $\tilde{l}\mathfrak{B}\tilde{l} \cap \tilde{l}\mathfrak{B} * \tilde{l} = \tilde{l}\mathfrak{B}\tilde{l} \subseteq \tilde{l}$ and $\varsigma\mathfrak{b}\varsigma \vee \varsigma\mathfrak{b} * \varsigma = \varsigma\mathfrak{b}\varsigma \geq \varsigma$. Hence $\tilde{l}_\varsigma\mathfrak{B}_\mathfrak{b}\tilde{l}_\varsigma \ll \tilde{l}_\varsigma$. □

Theorem 3.12. *Let $\mathbb{B}$ be a zero-symmetric NSS and $\tilde{l}_\varsigma$ be a hybrid subalgebra of $\mathbb{B}$. Then the following assertions are equivalent:*
(i) $\tilde{l}_\varsigma$ be a hybrid bi-ideal of $\mathbb{B}$,
(ii) $\tilde{l}_\varsigma\mathfrak{B}_\mathfrak{b}\tilde{l}_\varsigma \ll \tilde{l}_\varsigma$.

Proof. By Theorem 3.10 and Theorem 3.11, the proof is obvious. □

Theorem 3.13. *Let $\tilde{l}_\varsigma$ of $\mathbb{B}$ be a hybrid bi-ideal of a zero-symmetric NSS. Then* $(\forall q, j, c \in \mathbb{B}) \begin{pmatrix} \tilde{l}(qjc) \supseteq \tilde{l}(q) \cap \tilde{l}(c) \\ \varsigma(qjc) \leq \varsigma(q) \vee \varsigma(c) \end{pmatrix}$.

Proof. Suppose $\tilde{l}_\varsigma$ of $\mathbb{B}$ be a hybrid bi-ideal of a zero-symmetric NSS. By Theorem 3.11, $\tilde{l}_\varsigma \mathfrak{B}_\mathfrak{b} \tilde{l}_\varsigma \ll \tilde{l}_\varsigma$. Let $q, j, c \in \mathbb{B}$. Then $\tilde{l}(qjc) \supseteq (\tilde{l}\mathfrak{B}\tilde{l})(qjc) = \bigcup_{qjc=dm} (\tilde{l}\mathfrak{B})(d) \cap \tilde{l}(m) \supseteq (\tilde{l}\mathfrak{B})(qj) \cap \tilde{l}(c) \supseteq \tilde{l}(q) \cap \mathfrak{B}(j) \cap \tilde{l}(c) = \tilde{l}(q) \cap \mathbb{T} \cap \tilde{l}(c) = \tilde{l}(q) \cap \tilde{l}(c)$, $\varsigma(qjc) \leq (\varsigma\mathfrak{b}\varsigma)(qjc) = \bigwedge_{qjc=dm} (\varsigma\mathfrak{b})(d) \vee \varsigma(m) \leq (\varsigma\mathfrak{b})(qj) \vee \varsigma(c) \leq \varsigma(q) \vee \mathfrak{b}(j) \vee \varsigma(c) = \varsigma(q) \vee 0 \vee \varsigma(c) = \varsigma(q) \vee \varsigma(c)$.
Hence $\tilde{l}(qjc) \supseteq \tilde{l}(q) \cap \tilde{l}(c)$ and $\varsigma(qjc) \leq \varsigma(q) \vee \varsigma(c)$. □

Theorem 3.14. *Let $\tilde{l}_\varsigma$ of $\mathbb{B}$ be a hybrid bi-ideal of a zero-symmetric NSS. Then the below mentioned assertions are equivalent:*

(i) $(\forall d, m, g \in \mathbb{B}) \begin{pmatrix} \tilde{l}(dmg) \supseteq \tilde{l}(d) \cap \tilde{l}(g) \\ \varsigma(dmg) \leq \varsigma(d) \vee \varsigma(g) \end{pmatrix}$,

(ii) $\tilde{l}_\varsigma \mathfrak{B}_\mathfrak{b} \tilde{l}_\varsigma \ll \tilde{l}_\varsigma$.

Proof. $(i) \Rightarrow (ii)$ Let $v' \in \mathbb{B}$. If there exist $d, m, d_1, d_2 \in \mathbb{B}$ such that $v' = dm$ and $d = d_1 d_2$. Then by hypothesis, $\tilde{l}(d_1 d_2 m) \supseteq \tilde{l}(d_1) \cap \tilde{l}(m)$ and $\varsigma(d_1 d_2 m) \leq \varsigma(d_1) \vee \varsigma(m)$. Now,

$$\begin{aligned}
(\tilde{l}\mathfrak{B}\tilde{l})(v') &= \bigcup_{v'=dm} (\tilde{l}\mathfrak{B})(d) \cap \tilde{l}(m) \\
&= \bigcup_{v'=dm} \left(\bigcup_{d=d_1 d_2} \tilde{l}(d_1) \cap \mathfrak{B}(d_2) \right) \cap \tilde{l}(m) \\
&= \bigcup_{v'=dm} \left(\bigcup_{d=d_1 d_2} \tilde{l}(d_1) \cap \mathbb{T} \right) \cap \tilde{l}(m) \\
&= \bigcup_{v'=d_1 d_2 m} (\tilde{l}(d_1) \cap \tilde{l}(m)) \subseteq \bigcup_{v'=d_1 d_2 m} \tilde{l}(d_1 d_2 m) = \tilde{l}(v'), \\
(\varsigma\mathfrak{b}\varsigma)(v') &= \bigwedge_{v'=dm} (\varsigma\mathfrak{b})(d) \vee \varsigma(m) \\
&= \bigwedge_{v'=dm} \left(\bigwedge_{d=d_1 d_2} \varsigma(d_1) \vee \mathfrak{b}(d_2) \right) \vee \varsigma(m) \\
&= \bigwedge_{v'=dm} \left(\bigwedge_{d=d_1 d_2} \varsigma(d_1) \vee 0 \right) \vee \varsigma(m) \\
&= \bigwedge_{v'=d_1 d_2 m} (\varsigma(d_1) \vee \varsigma(m)) \geq \bigwedge_{v'=d_1 d_2 m} \varsigma(d_1 d_2 m) = \varsigma(v').
\end{aligned}$$

So $\tilde{l}_\varsigma \mathfrak{B}_\mathfrak{b} \tilde{l}_\varsigma \ll \tilde{l}_\varsigma$.

$(ii) \Rightarrow (i)$ Suppose that $\tilde{l}_\varsigma \mathfrak{B}_\mathfrak{b} \tilde{l}_\varsigma \ll \tilde{l}_\varsigma$ and let $d, m, g, v' \in \mathbb{B}$ be such that $v' = dmg$. Then

$$\begin{aligned}
\tilde{l}(dmg) = \tilde{l}(v') \supseteq (\tilde{l}\mathfrak{B}\tilde{l})(v') &= \bigcup_{v'=fy} (\tilde{l}\mathfrak{B})(f) \cap \tilde{l}(y) \\
&= \bigcup_{v'=fy} \left(\bigcup_{f=f_1 f_2} \tilde{l}(f_1) \cap \mathfrak{B}(f_2) \right) \cap \tilde{l}(y) \\
&\supseteq \tilde{l}(d) \cap \mathfrak{B}(m) \cap \tilde{l}(g) = (\tilde{l}(d) \cap \mathbb{T} \cap \tilde{l}(g)) = \tilde{l}(d) \cap \tilde{l}(g),
\end{aligned}$$

$$\begin{aligned}\varsigma(dmg) = \varsigma(v') \leq (\varsigma\mathfrak{b}\varsigma)(v') &= \bigwedge_{v'=fy} (\varsigma\mathfrak{b})(f) \vee \varsigma(y) \\ &= \bigwedge_{v'=fy} \left(\bigwedge_{f=f_1f_2} \varsigma(f_1) \vee \mathfrak{b}(f_2) \right) \vee \varsigma(y) \\ &\leq \varsigma(d) \vee \mathfrak{b}(m) \vee \varsigma(g) = (\varsigma(d) \vee 0 \vee \varsigma(g)) = \varsigma(d) \vee \varsigma(g).\end{aligned}$$

Hence $\tilde{l}(dmg) \supseteq \tilde{l}(d) \cap \tilde{l}(g)$ and $\varsigma(dmg) \leq \varsigma(d) \vee \varsigma(g)$. □

Theorem 3.15. *Let $\tilde{l}_\varsigma$ of $\mathbb{B}$ be a hybrid subalgebra of a zero-symmetric NSS. Then the below mentioned assertions are equivalent:*

(i) $\tilde{l}_\varsigma$ is a hybrid bi-ideal of $\mathbb{B}$,

(ii) $(\forall q, j, c \in \mathbb{B}) \begin{pmatrix} \tilde{l}(qjc) \supseteq \tilde{l}(q) \cap \tilde{l}(c) \\ \varsigma(qjc) \leq \varsigma(q) \vee \varsigma(c) \end{pmatrix}$.

(iii) $\tilde{l}_\varsigma \mathfrak{B}_\mathfrak{b} \tilde{l}_\varsigma \ll \tilde{l}_\varsigma$.

Proof. It follows from Theorem 3.13 and Theorem 3.14. □

4. Homomorphism of a hybrid structure

In this portion, we explore some properties of hybrid structures that are homomorphic to near-subtraction semigroups. Hereafter, $\mathbb{B}$ and $\mathbb{B}'$ denote the zero-symmetric near-subtraction semigroups.

Definition 4.1. [18] A homomorphism of $\mathbb{B}$ into $\mathbb{B}'$ such that $\psi(m_1 - v_1) = \psi(m_1) - \psi(v_1)$ and $\psi(m_1 v_1) = \psi(m_1)\psi(v_1)$ $\forall m_1, v_1 \in \mathbb{B}$ is defined.

Definition 4.2. [18] Let $\psi : \mathbb{B} \to \mathbb{B}'$ be a mapping and $\tilde{l}_\varsigma \in \mathfrak{H}(\mathbb{B}')$. The preimage of $\tilde{l}_\varsigma$ under ψ, represented as $\psi^{-1}(\tilde{l}_\varsigma)$, is a hybrid structure of $\mathbb{B}$ defined by $\psi^{-1}(\tilde{l}_\varsigma) := (\psi^{-1}(\tilde{l}), \psi^{-1}(\varsigma))$ where $\psi^{-1}(\tilde{l})(r_1) = \tilde{l}(\psi(r_1))$ and $\psi^{-1}(\varsigma)(r_1) = \varsigma(\psi(r_1))$ $\forall r_1 \in \mathbb{B}$.

Theorem 4.3. *Let $\psi : \mathbb{B} \to \mathbb{B}'$ be a homomorphism of a zero-symmetric NSS. If $\tilde{l}_\varsigma$ of $\mathbb{B}'$ is a hybrid bi-ideal, then $\psi^{-1}(\tilde{l}_\varsigma)$ of $\mathbb{B}$ is a hybrid bi-ideal.*

Proof. Suppose $\tilde{l}_\varsigma$ of $\mathbb{B}'$ is a hybrid bi-ideal. Let $w_0, d_0 \in \mathbb{B}$. Then $\psi^{-1}(\tilde{l})(w_0 - d_0) = \tilde{l}(\psi(w_0 - d_0)) = \tilde{l}(\psi(w_0) - \psi(d_0)) \supseteq \tilde{l}(\psi(w_0)) \cap \tilde{l}(\psi(d_0)) = \psi^{-1}(\tilde{l})(w_0) \cap \psi^{-1}(\tilde{l})(d_0), \psi^{-1}(\varsigma)(w_0 - d_0) = \varsigma(\psi(w_0 - d_0)) = \varsigma(\psi(w_0) - \psi(d_0)) \leq \varsigma(\psi(w_0)) \vee \varsigma(\psi(d_0)) = \psi^{-1}(\varsigma)(w_0) \vee \psi^{-1}(\varsigma)(d_0)$.

By Theorem 3.15, assume that $\tilde{l}_\varsigma \mathfrak{B}_\mathfrak{b} \tilde{l}_\varsigma \ll \tilde{l}_\varsigma$. Let $w, g, m, w' \in \mathbb{B}$ be such that $w' = wgm$. Then

$$\begin{aligned}\psi^{-1}(\tilde{l})(wgm) = \tilde{l}(\psi(w')) &\supseteq (\tilde{l}\mathfrak{B}\tilde{l})(\psi(w')) \\ &= \bigcup_{w'=fy} (\tilde{l}\mathfrak{B})(\psi(f)) \cap \tilde{l}(\psi(y)) \\ &= \bigcup_{w'=fy} \left(\bigcup_{f=f_1f_2} \tilde{l}(\psi(f_1)) \cap \mathfrak{B}(f_2) \right) \cap \tilde{l}(\psi(y)) \\ &\supseteq \tilde{l}(\psi(w)) \cap \mathfrak{B}(g) \cap \tilde{l}(\psi(m)) \\ &= \tilde{l}(\psi(w)) \cap \mathbb{T} \cap \tilde{l}(\psi(m)) \\ &= \tilde{l}(\psi(w)) \cap \tilde{l}(\psi(m)) = \psi^{-1}(\tilde{l})(w) \cap \psi^{-1}(\tilde{l})(m),\end{aligned}$$

$$\begin{aligned}\psi^{-1}(\varsigma)(wgm) = \varsigma(\psi(w')) &\leq (\varsigma\mathfrak{b}\varsigma)(\psi(w'))\\ &= \bigwedge_{w'=fy} (\varsigma\mathfrak{b})(\psi(f)) \vee \varsigma(\psi(y))\\ &= \bigwedge_{w'=fy}\left(\bigwedge_{f=f_1f_2} \varsigma(\psi(f_1)) \vee \mathfrak{b}(f_2)\right) \vee \varsigma(\psi(y))\\ &\leq \varsigma(\psi(w)) \vee \mathfrak{b}(g) \vee \varsigma(\psi(m))\\ &= \varsigma(\psi(w)) \vee 0 \vee \varsigma(\psi(m))\\ &= \varsigma(\psi(w)) \vee \varsigma(\psi(m)) = \psi^{-1}(\varsigma)(w) \vee \psi^{-1}(\varsigma)(m).\end{aligned}$$

So $\psi^{-1}(\tilde{l}_\varsigma)$ of $\mathbb{B}$ is a hybrid bi-ideal. □

Theorem 4.4. *Let $\psi : \mathbb{B} \to \mathbb{B}^{'}$ be an onto homomorphism of a zero-symmetric NSS and $\tilde{l}_\varsigma \in \mathfrak{H}(\mathbb{B}^{'})$. If $\psi^{-1}(\tilde{l}_\varsigma)$ of $\mathbb{B}$ is a hybrid bi-ideal, then $\tilde{l}_\varsigma$ of $\mathbb{B}^{'}$ is a hybrid bi-ideal.*

Proof. Let $\psi^{-1}(\tilde{l}_\varsigma)$ in $\mathbb{B}$ be a hybrid bi-ideal and $k', z' \in \mathbb{B}^{'}$. Then $\exists\ k, z \in \mathbb{B}$ such that $\psi(k) = k'$ and $\psi(z) = z'$. Now, $\tilde{l}(k' - z') = \tilde{l}(\psi(k) - \psi(z)) = \tilde{l}(\psi(k - z)) = \psi^{-1}(\tilde{l})(k - z) \supseteq \psi^{-1}(\tilde{l})(k) \cap \psi^{-1}(\tilde{l})(z) = \tilde{l}(\psi(k)) \cap \tilde{l}(\psi(z)) = \tilde{l}(k') \cap \tilde{l}(z'), \varsigma(k' - z') = \varsigma(\psi(k) - \psi(z)) = \varsigma(\psi(k - z)) = \psi^{-1}(\varsigma)(k - z) \leq \psi^{-1}(\varsigma)(k) \vee \psi^{-1}(\varsigma)(z) = \varsigma(\psi(k)) \vee \varsigma(\psi(z)) = \varsigma(k') \vee \varsigma(z')$.

By Theorem 3.15, assume that $\tilde{l}_\varsigma \mathfrak{B}_\mathfrak{b} \tilde{l}_\varsigma \ll \tilde{l}_\varsigma$. Let $k', r', m', g' \in \mathbb{B}^{'}$. Then $\exists\ k, r, m \in \mathbb{B}$ such that $\psi(k) = k'$, $\psi(r) = r'$, $\psi(m) = m'$ and $g' = k'r'm'$. Then

$$\begin{aligned}\tilde{l}(k'r'm') = \tilde{l}(\psi(g')) &\supseteq (\tilde{l}\mathfrak{B}\tilde{l})(\psi(g')) = \psi^{-1}(\tilde{l}\mathfrak{B}\tilde{l})(g')\\ &= \bigcup_{g'=fy} \psi^{-1}(\tilde{l}\mathfrak{B})(f) \cap \psi^{-1}(\tilde{l})(y)\\ &= \bigcup_{g'=fy}\left(\bigcup_{f=f_1f_2} \psi^{-1}(\tilde{l}(f_1) \cap \mathfrak{B}(f_2))\right) \cap \psi^{-1}(\tilde{l})(y)\\ &\supseteq \psi^{-1}(\tilde{l})(k) \cap \mathfrak{B}(r) \cap \psi^{-1}(\tilde{l})(m)\\ &= \psi^{-1}(\tilde{l})(k) \cap \mathbb{T} \cap \psi^{-1}(\tilde{l})(m)\\ &= \psi^{-1}(\tilde{l})(k) \cap \psi^{-1}(\tilde{l})(m) = \tilde{l}(\psi(k)) \cap \tilde{l}(\psi(m)) = \tilde{l}(k') \cap \tilde{l}(m'),\\ \varsigma(k'r'm') = \varsigma(\psi(g')) &\leq (\varsigma\mathfrak{b}\varsigma)(\psi(g')) = \psi^{-1}(\varsigma\mathfrak{b}\varsigma)(g')\\ &= \bigwedge_{g'=fy} \psi^{-1}(\varsigma\mathfrak{b})(f) \vee \psi^{-1}(\varsigma)(y)\\ &= \bigwedge_{g'=fy}\left(\bigwedge_{f=f_1f_2} \psi^{-1}(\varsigma(f_1) \vee \mathfrak{b}(f_2))\right) \vee \psi^{-1}(\varsigma)(y)\\ &\leq \psi^{-1}(\varsigma)(k) \vee \mathfrak{b}(r) \vee \psi^{-1}(\varsigma)(m)\\ &= \psi^{-1}(\varsigma)(k) \vee 0 \vee \psi^{-1}(\varsigma)(m)\\ &= \psi^{-1}(\varsigma)(k) \vee \psi^{-1}(\varsigma)(m) = \varsigma(\psi(k)) \vee \varsigma(\psi(m)) = \varsigma(k') \vee \varsigma(m').\end{aligned}$$

So $\tilde{l}_\varsigma$ in $\mathbb{B}^{'}$ is a hybrid bi-ideal. □

5. Conclusions

In this paper, we investigated the properties of hybrid bi-ideals and generated bi-ideals for hybrid bi-ideals in near-subtraction semigroups. Under homomorphism mapping, various properties of the hybrid preimage of the hybrid bi-ideal of a near-subtraction semigroup had also been discussed. Using the concepts and results discussed in this work, it is hoped to introduce the idea of a hybrid prime bi-ideal and its associated properties in near-subtraction semigroups.

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Conflict of interest

The authors declare that they have no conflict of interest.

Data Availability

No data were used to support the study.

References

[1] S. Anis, M. Khan and Y. B. Jun, *Hybrid ideals in semigroups*, Cogent Math. **4**, 1352117, 2017.

[2] V. Chinnadurai and S. Kadalarasi, *Fuzzy bi-ideals of near-subtraction semigroups*, Annals of Fuzzy Mathematics and Informatics **12**(6), 781–790, 2016.

[3] J. R. Clay, *The near-rings on groups of low order*, Math. Z. **104** (5), 364–371, 1968.

[4] P. Dheena and B. Elavarasan, *An ideal-based zero-divisor graph of 2-primal near-rings*, Bull. Korean Math. Soc. **46**(6), 1051–1060, 2009.

[5] P. Dheena and G. Satheesh kumar, *On storngly regular near-subtraction semigroups*, Commun. Korean Math. Soc. **22**(3), 323–330, 2007.

[6] B. Elavarasan and Y. B. Jun, *Regularity of semigroups in terms of hybrid ideals and hybrid bi-ideals*, Kragujev. J. Math. **46**(6), 857–864, 2022.

[7] B. Elavarasan, G. Muhiuddin, K. Porselvi and Y. B. Jun, *Hybrid structures applied to ideals in near-rings*, Complex Intell. Syst. **7**(3), 1489–1498, 2021.

[8] B. Elavarasan, K. Porselvi and Y. B. Jun, *Hybrid generalized bi-ideals in semigroups*, Int. J. Math. Comput. Sci. **14**(3), 601–612, 2019.

[9] Y. B. Jun, H. S. Kim and E. H. Roh, *Ideal theory of subtraction algebras*, Sci. Math. Jpn. **61**(3), 459–464, 2005.

[10] Y. B. Jun and H. S. Kim, *On ideals in subtraction algebras*, Sci. Math. Jpn. **65**(1), 129–134, 2007.

[11] Y. B. Jun, M. Sapanci and M. A. Ozturk, *Fuzzy ideals in Gamma near-rings*, Tr. J. Math. **22**, 449–459, 1998.

[12] Y. B. Jun, S. Z. Song and G. Muhiuddin, *Hybrid structures and applications*, Annals of communications in Mathematics **1**(1), 11–25, 2018.

[13] K. J. Lee and C. H. Park, *Some questions on fuzzifications of ideals in subtraction algebras*, Commun. Korean Math. Soc. **22**(3), 359–363, 2007.

[14] V. Mahalakshmi, S. Maharasi and S. Jayalakshmi, *Bi-ideals in near- subtraction semigroup*, Indian Advances in Algebra **6**(1), 35–48, 2013.

[15] T. Manikandan, *Fuzzy bi-ideals of near-rings*, J. Fuzzy Math. **17**(3), 659–671, 2009.

[16] G. Mason, *Strongly regular near-rings*, Proc. Edinb. Math. Soc. **23**(1), 27–35, 1980.

[17] P. K. Maji, A. R. Roy and R. Biswas, *An application of soft sets in a decision making problem*, Comput. Math. Appl. **44**, 1077–1083, 2002.

[18] S. Meenakshi, G. Muhiuddin, B. Elavarasan and D. Al-Kadi, *Hybrid ideals in near-subtraction semigroups*, AIMS Mathematics **7**(7), 13493–13507, 2022.

[19] J. D. P. Meldrum, *Varieties and d.g. near-rings*, Proc. Edinb. Math. Soc. **17**(3), 271–274, 1971.

[20] D. Molodtsov, *Soft set theory–first results*, Comput. Math. Appl. **37**, 19–31, 1999.
[21] G. Muhiuddin, J. Catherine Grace John, B. Elavarasan, Y. B. Jun and K. Porselvi, *Hybrid structures applied to modules over semirings*, J. Intell. Fuzzy Syst. **42**(3), 2521–2531, 2022.
[22] G. Muhiuddin, J. Catherine Grace John, B. Elavarasan, K. Porselvi and D. Al-Kadi, *Properties of k-hybrid ideals in ternary semiring*, J. Intell. Fuzzy Syst. **42**(6), 5799–5807, 2022.
[23] G. Pilz, *Near-rings*, North-Holland, Amsterdam, 1983.
[24] K. Porselvi and B. Elavarasan, *On hybrid interior ideals in semigroups*, Probl. Anal. Issues. Anal. **8**(26)(3), 137–146, 2019.
[25] K. Porselvi, B. Elavarasan and Y. B. Jun, *Hybrid interior ideals in ordered semigroups*, New Math. Nat. Comput. **18**(1), 1–8, 2022.
[26] K. Porselvi, G. Muhiuddin, B. Elavarasan and A. Assiry, *Hybrid nil radical of a ring*, Symmetry **14**, 1367, 2022.
[27] K. Porselvi, G. Muhiuddin, B. Elavarasan, Y. B. Jun and J. Catherine Grace John, *Hybrid ideals in an AG-groupoid*, New Math. Nat. Comput. **19**(1), 289–305, 2023.
[28] A. Rosenfeld, *Fuzzy groups*, J. Math. Anal. Appl. **35**, 512–517, 1971.
[29] B. M. Schein, *Difference semigroups*, Commun. Algebra. **20**(8), 2153–2169, 1992.
[30] D. R. P. Williams, *Fuzzy ideals in near-subtraction semigroups*, International scholarly and scientific research and innovation **2**(7), 625–632, 2008.
[31] L. A. Zadeh, *Fuzzy sets*, Inf. Control. **8**, 338–353, 1965.
[32] B. Zelinka, *Subtraction semigroups*, Math. Bohem. **120**(4), 445–447, 1995.