G.Ü. İ.İ.B.F. Dergisi, 3/2000, 53-62

A Mathematical Programming Model for the Synchronized Development of Dynamic Inventory and Distribution Schedules

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This study presents the extensions of a comprehensive project completed for Petrol Ofisi, the major petroleum products distributor in Turkey. In the completed project a decision support system was developed to minimise the total transportation costs of petroleum products under the static environment assumption. The aim of the current extensions, presented in this paper, is to organise the transportation activities, to be carried out throughout the planning year between the existing refineries and the coastal depots in a manner to meet the forecasted demand at the coastal depots, with minimum total inventory and transportation costs. An integrated freight transport planning model, incorporating inventory holding costs, is introduced under the dynamic environment assumption and comments are made on further extensions of the model.

1. Introduction

The increasing relevance of transportation costs and the emphasis put on inventory reduction has recently stressed the importance of synchronizing the logistic decision process. This study presents such an effort to integrate inventory and transportation planning as an extension of a comprehensive project completed for Petrol Ofisi (POAŞ), the major petroleum products distributor in Turkey. The results of the aforementioned project and relevant literature were already reported by Ulucan and Tarım[1]. See [2]-[15] for the recent research on the subject.

The transportation model is given in details in §3 for the sake of completeness and future reference. The issues, such as forecasting the regional demand quantities for different types of petroleum products, which were individually covered in the scope of the project, are not discussed. Following the single period transportation planning model, the multi period integrated inventory-transportation planning model

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is introduced in §4. The inventory-transportation planning model aims at performing the transportation activities to be carried out throughout the planning year between the existing refineries and the coastal depots, in a manner to meet the forecasted demand at the coastal depots with minimum total inventory and transportation costs. The computation of model parameters are discussed in §5. §6, where the concluding remarks are given, finalises the paper.

2. Petrol Ofisi and Its Surroundings

Due to the difficulties that Turkey faced during the second world war in the supply and distribution of crude oil and petroleum products, the Turkish Government established Petrol Ofisi (POAŞ) in 1941. The basic duties of POAŞ are defined as to purchase and import petroleum products that are necessary for public and national defense requirements, to keep fuel inventories at various places in the country, to arrange distribution of products, to build, purchase and sell tanks and all types of transportation vehicles peculiar to transportation of petroleum products. POAŞ also carries out the tasks of procuring the fuel requirements of Turkish Armed Forces and transporting fuels to pipelines and units due to the protocols signed at various dates.

POAŞ is the largest petroleum distribution company in the Turkish market, by marketing 53.2% (in 1997) of the total fuels marketed by petroleum distribution companies in Turkey. In parallel to the magnitude of the distribution made, the planning of the petroleum distribution is one of the most significant problems of POAŞ. The importance of the savings to be achieved in the distribution and inventory expenses, which have a relatively huge portion in the overall expenses of the enterprise, is obvious.

The subject of the study is the planning, by means of a mathematical programming model, of the distribution to the depots and of storage of the petroleum products produced at the refineries. It is known that the crude oil is marketed several times, after it is refined as petroleum products, until delivered to the consumer. The transportation activities during these marketing processes can be examined under the following three major stages: (i) transportation of the crude oil to the refineries, (ii) transportation of the petroleum products produced at the refineries to the depots, (iii) transportation of the petroleum products from the depots to the retail service stations.

Among these, the transportation of crude oil to the refineries was not covered in the scope of the study due to the following reasons. First of all, in Turkey, the crude oil is produced principally at Southeast Anatolia and is forwarded to the refineries from this location. Since there is no other petroleum fields within the country, the optimization of the crude oil distribution within the country is senseless due to the absence of alternative sources. Similarly, since the factors determining the country of origin and quantity of the importation made to meet the petroleum demand depends on the international commercial and political relations rather than the transportation costs, the transportation of the crude oil to the refineries was not included in the transportation model.

The petroleum products transported from the refineries to the depots of POAŞ are forwarded from the depots to the retail service stations in order to be delivered to the consumers. POAŞ have no control on the purchasing decisions of the vendors. Therefore, the transportation of the petroleum products from the depots to the retail service stations is a problem of minimization of the overall transportation costs concerning only the vendors, and due to this reason, it is excluded from the scope of the study.

Almost all of the terminals of POAŞ are at the seashore, and its transport strategy is based on sea transportation. POAŞ utilizes the coastal depots for forwarding the petroleum products from the refineries to the consumers and the petroleum products are unloaded to these depots by means of tankers. As indicated above, the aim of the study is to find the policies that will provide the transportation of the petroleum products from the refineries to only the coastal depots by means of tankers with minimum transportation and inventory costs.

3. Single Period Transportation Model Formulation

Previous work of Ulucan and Tarım[1] covers the single period transportation model formulation and discusses the magnitude of savings achieved in POAŞ with the use of this model.

The Logistics department are responsible for providing transportation plans and used to perform their duty manually which is a difficult and time consuming task. A software system was developed to be used by the Logistics Department of POAŞ. A model generator that uses the relevant data and generates the associated mathematical model, which is a mixed integer programming model, was developed as part of the transportation software system. The generated mathematical programming model is, then, solved by means of the state-of-the-art solvers and, finally, the reports are generated. However, this software system, designed for the end user, is not discussed here.

In the mathematical model developed, the tankers owned by POAŞ as well as the time-chartered vessels available on the market are considered simultaneously. Not only the tankers to be employed in the transportation activity are determined, but also the refinery-depot route to be followed, the timing of the dispatches, the type of petroleum product to be transported during the year, the number of trips to be made by these tankers, included in the transportation plan, throughout the planning period can be specified.

This mathematical model and the notation are given below for future reference:

i	tanker index, $i=1,\dots,m$,
j	depot index, $j=1,\dots,n$,
k	refinery index, $k=1,\dots,p$,
A_k	the set of depots supplied by refinery k,
K_i	The {0,1} variable, equals to 1 if tanker i is in the transportation plan
$K_{ik}^B\,(K_{ik}^W)$	The {0,1} variable, equals to 1 if tanker i is used in transportation of black (white) products from refinery k, and equals to 0 otherwise.
$h_i^B(h_i^W)$	The parameter equals to 1 if tanker i is capable of carrying black (white) products and equals to 0 otherwise,
$X_{ijk}^B\,(X_{ijk}^W)$	Percentage of the service time of tanker i spent for carrying black (white) products from refinery k to depot j in one year,
$G_{ijk}^B\,(G_{ijk}^W)$	Maximum total amount of black (white) products could be carried by tanker i from refinery k to depot j in one year.
$d_i^B(d_i^W)$	Total black (white) product demand of depot j in one year.
F_i	Annual fixed contract cost for tanker i.
C_{ijk}	Variable freight cost of assigning tanker i to refinery k-depot j route.

Single Period Transportation Model:

$$\min Z = \sum_{i=1}^{m} F_i K_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} C_{ijk}\,(X_{ijk}^B + W_{ijk}^W) \tag{1}$$

subject to

$$K_i - \sum_{k=1}^{p} K_{ik}^B - \sum_{k=1}^{p} K_{ik}^W \geq 0 \qquad i=1,\dots,m, \tag{2}$$

$$h_i^B K_{ik}^B - \sum_{l\in A_k} X_{ilk}^B \geq 0 \qquad i=1,\dots,m, \quad k=1,\dots,p, \tag{3}$$

$$h_i^W K_{ik}^W - \sum_{l\in A_k} X_{ilk}^W \geq 0 \qquad i=1,\dots,m, \quad k=1,\dots,p, \tag{4}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{p} G_{ijk}^B X_{ijk}^B \geq d_j^B \qquad j=1,\dots,n, \tag{5}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{p} G_{ijk}^W X_{ijk}^W \geq d_j^W \qquad j=1,\dots,n, \tag{6}$$

$$X_{ijk}^B\,,\, X_{ijk}^W \geq 0,\;\; K_i,\, K_{ik}^B,\, K_{ik}^W \in \{0,\,1\} \tag{7}$$

The main assumptions of the above model are as follows. First, the dynamic nature of or seasonality in demand is ignored and it is assumed that demand is constant, which is not the case. Second, it is assumed that if a vessel is assigned to a certain refinery-depot route then it serves in this route for the rest of the planning horizon. The second assumption is actually a result of the static nature.

The petroleum products carried by tankers are classifed into two groups -black (fuel-oils) and white (jet fuels, gasoline types, kerosene, diesel fuel). Some tankers are equipped to carry just one type of petroleum products –black or white-, while the others can carry both one at a time. In the latter category, since it is rather a time consuming and expensive process to convert the tankers into the ones to carry white products instead of black, or visa versa, it is not considered as a standard practice by POAŞ. Therefore, colour switching is not permitted in the model. Tankers usually have number of holds and a tanker can carry several products, but only one product per hold. However, to satisfy the confines of POAŞ, in developing the model to assign the tankers, being equipped to carry products in both colours, to the depots, the tankers are constrained to carry only the same coloured products during the whole planning horizon.

Although the above assumptions are rather severe, considerable cost savings are achieved. In the following section, the above assumptions are relaxed and applicability of the modelling approach is extended.

4. Multi Period Inventory-Transportation Model Formulation

The integrated inventory-transportation model defined with its additional notation is given below.

t	time period index, t=1,...,r,
q	product index, q=1,...,s; the set of black (white) products is S_B (S_w),
$\overline{K}_i$	=1 if tanker i is in the plan, =0 otherwise,
P_{it}	the cost of assigning tanker i to a refinery in period t that is different from the one in period (t-1),
$J^B_{it} (J^W_{it})$	=1 if tanker i is assigned to a different refinery in period t, =0 otherwise,
$G^B_{ijk} (G^B_{ijk})$	maximum total amount of black (white) products could be carried by tanker i from refinery k to depot j in one complete period,
$I^B_{it} (I^W_{it})$	the period closing inventory level of depot j for black (white) products in period t,
$X^B_{ijkt} (X^B_{ijkt})$	percentage of the period service time of tanker i spent for carrying black (white) products from refinery k to depot j in period t,

$U^B_{jt}\,(U^W_{jt})$ predetermined black (white) buffer stock levels for depot j in period t,

Q_i the fixed cost of switching the colour of tanker i from black to white, or vice versa,

R_{it} =1 if the colour of tanker i is switched in period t, =0 otherwise,

$c^B_h\,(c^W_h)$ a linear holding cost is incurred on any black (white) product carried in inventory over from period t to period t+1. The holding costs are modelled by multiplying the weighted average price of black (white) products by the nominal interest rate.

Multi Period Inventory-Transportation Model :

min

$$\sum_{i=1}^{m} F_i\overline{K}_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} C_{ijk}\sum_{t=1}^{r}(X^B_{ijkt} + X^W_{ijkt}) + \sum_{j=1}^{n}\sum_{t=1}^{r}(c^B_h I^B_{jt} + c^W_h I^W_{jt}) + \sum_{i=1}^{m}\sum_{t=2}^{r} P_{it}(J^B_{it} + J^W_{it}) + \sum_{i=1}^{m} Q_i\sum_{t=2}^{r} R_{it} \tag{8}$$

subject to,

$$-\overline{K}_i + \sum_{i=1}^{m}(K^B_{ikt} + K^W_{ikt}) \le 0 \qquad i=1,\dots,m \tag{9}$$

$$h_i^B K^B_{ikt} - \sum_{j=1}^{n} X^B_{ijkt} \ge 0 \qquad i=1,\dots,m \quad t=1,\dots,r \quad k=1,\dots,p \tag{10}$$

$$h_i^W K^W_{ikt} - \sum_{j=1}^{n} X^W_{ijkt} \ge 0 \qquad i=1,\dots,m \quad t=1,\dots,r \quad k=1,\dots,p \tag{11}$$

$$I^B_{jt} = I^B_{j,t-1} + \sum_{i=1}^{m}\sum_{k=1}^{p} G^B_{ijk}X^B_{ijkt} - \sum_{q\in S_B} d^q_{jt} \qquad j=1,\dots,n \quad t=1,\dots,r \tag{12}$$

$$I^W_{jt} = I^W_{j,t-1} + \sum_{i=1}^{m}\sum_{k=1}^{p} G^W_{ijk}X^W_{ijkt} - \sum_{q\in S_W} d^q_{jt} \qquad j=1,\dots,n \quad t=1,\dots,r \tag{13}$$

$$\sum_{k=1}^{p} K^B_{ikt} + 2\sum_{k=1}^{p} K^W_{ikt} - \sum_{k=1}^{p} K^B_{ik(t+1)} - 2\sum_{k=1}^{p} K^W_{ik(t+1)} \le R_{i(t+1)} \qquad i=1,\dots,m \quad t=1,\dots,r\text{-}1 \tag{14}$$

$$-\sum_{k=1}^{p} K^B_{ikt} - 2\sum_{k=1}^{p} K^W_{ikt} + \sum_{k=1}^{p} K^B_{ik(t+1)} + 2\sum_{k=1}^{p} K^W_{ik(t+1)} \le R_{i(t+1)} \qquad i=1,\dots,m \quad t=1,\dots,r\text{-}1 \tag{15}$$

$$I^B_{jt} \ge U^B_{jt} \qquad j=1,\dots,n \quad t=1,\dots,r \tag{16}$$

$$I^W_{jt} \ge U^W_{jt} \qquad j=1,\dots,n \quad t=1,\dots,r \tag{17}$$

$$\sum_{k=1}^{p} 2^{k-1} K_{ikt}^{B} - \sum_{k=1}^{p} 2^{k-1} K_{ik(t+1)}^{B} \leq 2^{p-1} J_{i(t+1)}^{B} \qquad i=1,\dots,n \quad t=1,\dots,r\text{-}1 \quad (18)$$

$$-\sum_{k=1}^{p} 2^{k-1} K_{ikt}^{B} + \sum_{k=1}^{p} 2^{k-1} K_{ik(t+1)}^{B} \leq 2^{p-1} J_{i(t+1)}^{B} \qquad i=1,\dots,n \quad t=1,\dots,r\text{-}1 \quad (19)$$

$$\sum_{k=1}^{p} 2^{k-1} K_{ikt}^{W} + \sum_{k=1}^{p} 2^{k-1} K_{ik(t+1)}^{W} \leq 2^{p-1} J_{i(t+1)}^{W} \qquad i=1,\dots,n \quad t=1,\dots,r\text{-}1 \quad (20)$$

$$X_{ijkt}^{B},\ X_{ijkt}^{W},\ I_{jt}^{B},\ I_{jt}^{W} \geq 0 \qquad i=1,\dots,n \quad t=1,\dots,r\text{-}1 \quad (21)$$

$$\overline{K}_i, K_{ikt}^{B},\ J_{ikt}^{W},\ J_{it}^{B},\ J_{it}^{W},\ R_{it} \in \{0,1\}$$

Certain assumptions of the single period transportation model given in §3 are relaxed. To be more specific, the colour switching is incorporated in the multi period inventory-transportation model by means of incurring switching cost. In a similar way, assignment of the tankers to different refineries in different time periods is also allowed.

Objective function at (8) is equal to the minimisation of (i) the total fixed costs for the POAŞ owned tankers and the hiring costs for the time-chartered vessels, (ii) the total variable freight costs for all tankers, (iii) the total inventory holding costs, (iv) the total transfer cost for all tankers, and (v) the total colour switching costs. The model developed for this aim covers a planning period of one year, which is dictated by the rigid regulations (for example, the rental period for the tankers is strictly one year).

(9) assures that each tanker in the fleet will carry but one colour of products and be assigned to a single refinery. (10) and (11) provide that the tanker i should carry black or white products, respectively, during maximum 100 percent of the total period service time. (12) and (13) are the black and white inventory transition equations, respectively, and assure that the demand is met. (14) and (15) introduce the possibility of transporting different colour petroleum products in consequtive periods. The predetermined buffer stock levels are imposed by (16) and (17). The assignment of tankers to different refinery-depot routes in different time periods is allowed by the introduction of the constraint sets (18)-(21).

5. Computation of Model Parameters

The procedure followed in the computation of parameters of the mixed integer programming model is given below.

First, the total variable costs of tankers considered in the mathematical programming model were determined on the basis of the data from the previous planning years. The total variable costs of POAŞ tankers were found by subtracting the costs of personnel, materials and amortisations from the total costs. The hiring costs of the time-chartered vessels include personnel, goods and amortisations; therefore, the total variable costs of the time-chartered vessels were computed by subtracting the hiring costs from the total costs. In this study, a constant called the tanker variable cost coefficient in \$/(mile*ton) is computed for each tanker separately. The variable cost of each tanker is divided by the multiplication of total mileage (in miles) and total cargo (in tons) of the previous year. To test the validity of the tanker variable cost coefficients, the calculations are repeated, for all tankers, using the previous years' data and it is observed that the results are very consistent.

Second, the average time (in hours) spent in the refineries and depots (queueing, shipment, discharge, etc.) by each tanker with respect to their tonnage and the distance between the refineries and depots (in miles) are tabulated.

Third, all possible tanker-depot-refinery combinations are generated and for each combination the time each delivery takes (i.e., the time spent in the refinery, the return trip, the depot) is calculated. The data on average speed of empty and loaded tankers are provided by the experts of the Logistics Department. Hence, the maximum number of possible deliveries to be made in each period for each combination is determined.

Fourth, the total amount of petroleum products of black (or white) type to be carried by tanker i from refinery k to depot j, if the tanker is assigned to route for the whole period, is computed in tons (G^B_{ijk} and G^W_{ijk}) while the total distance covered is in miles. Finally, in the light of this information it is now possible to compute the variable costs, C_{ijk}.

6. Conclusion

In this paper, the extensions of a comprehensive project completed for Petrol Ofisi are presented. In the completed project a decision support system was developed to minimise the total transportation costs of petroleum products under the constant demand assumption. The aim of this paper is to develop a model to organise the transportation activities, to be carried out throughout the planning year between the existing refineries and the coastal depots in a manner to meet the forecasted demand at the coastal depots, with minimum total inventory and transportation costs. An integrated freight transport planning model, incorporating inventory holding costs, is introduced under the dynamic demand assumption. Previous constraints on colour switching and the assignment of a tanker to only one single refinery-depot route are relaxed. To have a modicum of resemblance to reality such additions to the model are crucial. Inevitably, the size of the model becomes enormous from a practical point of view and it is observed that the resulting model cannot be solved in feasible time using the standard mathematical programming softwares. Computationally feasible solution approaches, like Lagrangean relaxation technique, are currently being developed by the authors.

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