Cumhuriyet Üniversitesi Fen Fakültesi
Fen Bilimleri Dergisi (CFD), Cilt 38, No. 2 (2017)
ISSN: 1300-1949

Cumhuriyet University Faculty of Science
Science Journal (CSJ), Vol. 38, No. 2 (2017)
ISSN: 1300-1949

http://dx.doi.org/10.17776/cumuscij.301954

Investigation of Neutrino Magnetic Moment in the Process $e^+e^- \to \nu\bar{\nu}\gamma$

Coşkun AYDIN

Karadeniz Technical University, Department of Physics, 61080, Trabzon, TURKEY

Received: 28.03.2017; Accepted: 14.04.2017

Abstract: We calculate the differential and total cross section for the process $e^+e^- \to \nu\bar{\nu}\gamma$ assuming that the neutrino has an anomalous magnetic moment in the extended Standard Model. Using the LEP results on the total cross-section, we obtain a limit of $\mu \approx 8.4 \times 10^{-8}\,\mu_B$ for the neutrino magnetic moment.

Keywords: Neutrino, Magnetic Moment, Standard Model.

$e^+e^- \to \nu\bar{\nu}\gamma$ Sürecinde Nötrino Magnetik Momentinin Araştırılması

Özet: Bu çalışmada, $e^+e^- \to \nu\bar{\nu}\gamma$ sürecinin diferansiyel ve toplam tesir kesiti nötrinonun anomal çiftlenimleri göz önüne alınarak genişletilmiş Standart Modelde hesaplanıp, LEP deneysel sonuçlarını kullanarak, nötrino magnetic momenti için $\mu \approx 8.4 \times 10^{-8}\,\mu_B$ sınır değeri elde edildi.

Anahtar Kelimeler: Nötrino, Magnetik Moment, Standart Model.

1. INTRODUCTION

The question of whether the neutrinos are Dirac or Majorana particles is one of the most important issues in particle physics, astrophysics and cosmology. The properties of neutrinos have become the research area increasing interest in recent years. The searches for the neutrino mass, magnetic moment, dipole moment and anapole moment are of utmost important for the theory of elementary particles and for understanding the phenomena such as supernova dynamics, stellar evolution and the production of neutrino by the sun [1].

The Standard Model (SM) describes many processes in the range of energies which has been reached up to now. Neutrinos are the least known particles of the SM and they are treated as massless in the SM. Neutrinos seem to be one of the probable candidates of carrying on the future of physics beyond the SM. The purpose of some extended theories is to explain some fundamental aspects; for example, the neutrino mass, neutrino oscillations, neutrino magnetic moment, etc which are not clarified within the frame of SM. In many extensions of the Standard Model neutrino acquires nonzero mass and they are named as Dirac or Majorana neutrinos. These neutrinos have different electromagnetic properties. Dirac neutrino has three form factors which are the charge, magnetic moment and anapole moment since the electric dipole moment is zero for CP conserving theory. If there is no neutrino mixing, Majorana neutrino has only one form factor which is the anapole moment. If neutrino mixing is taken into account, then there are magnetic and electric transition moments as well. In this respect, as aforementioned, the neutrinos seem to be likely candidates of extending the physics beyond the

* Corresponding author. *Email address:* coskun@ktu.edu.tr

Standard Model. The electromagnetic properties of the neutrinos were calculated by many authors using various models [2,3].

The magnetic moment of the neutrino was also discussed intensively in the framework of SM and its various extensions. For example, SM predicts that if the neutrino has mass of $\sim 30\,\mathrm{eV}$ then $\mu \approx 10^{-19}\mu_B$, but the left-right symmetric model predicts $\mu \approx 10^{-13}\mu_B$ [2]. Modern Laboratories announced [4] that the neutrino magnetic moment are close to $10^{-10}\mu_B$; in fact, some published values are:

$$\begin{aligned} &\mu(\nu_e) < 4 \times 10^{-12}\mu_B \\ &\mu(\nu_\mu) < 1 \times 10^{-9}\mu_B \\ &\mu(\nu_\tau) < 4 \times 10^{-6}\mu_B\,. \end{aligned} \tag{1}$$

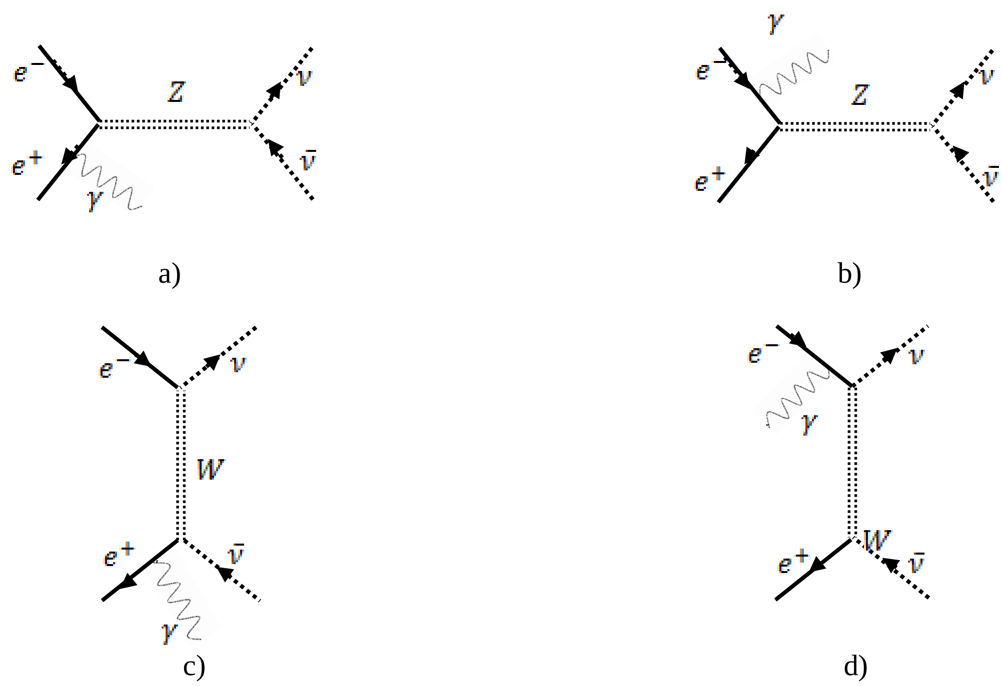
The upper bound for light neutrinos deduced from cosmological and astrophysical researches is given as [5]:

$$\mu \lesssim 4 \times 10^{-12}\mu_B \quad . \tag{2}$$

In the present research, we study the effects of the neutrino magnetic moment in a model dependent approach. For this purpose, we consider the process $e^+e^- \to \nu\bar{\nu}\gamma$. Note that, this process has been already discussed in various models by many researchers (see [6, 7, 8] and reference therein). The relevant Feymann diagrams are shown in Fig. 1. Assuming that the $(\gamma\nu\bar{\nu})$ vertex has only the anomalous magnetic moment (see [9] and reference therein), Forgion et al [10] calculated the differential cross sections of this process. But in their work the $Z\nu\bar{\nu}$ vertex anomalous coupling is not investigated. In this work, we assume that the $Z\nu\bar{\nu}$ vertex has anomalous coupling $\kappa\sigma_{\mu\nu}(1-\gamma_5)q^\nu$ [11].

2. CALCULATION

The Feymann diagrams considered in this study are displayed in Figure 1.



e) f)

Figure 1. Feynman diagrams for the process $e^- e^+ \rightarrow \nu\bar{\nu}\gamma$.

The expressions for the Feynman amplitude of this process are given in the following equations as;

$$M_{\mathrm{a}} = iee'\frac{1}{(q^2 - M^2 + iM_z\Gamma_z)}\cdot\frac{1}{(p_1-k)^2 - m^2}\bar{u}(q_1)\left[\frac{g}{4c_\theta}\gamma^\mu(1-\gamma_5) + \kappa\sigma^{\mu\upsilon}(1-\gamma_5)q_\nu\right]\upsilon(q_2)$$
$$\times\,\bar{\upsilon}(p_2)\gamma_\mu(\mathrm{a}+b\gamma_5)(\not{p}_1 - \not{k} + m)\not{\varepsilon}\,u(p_1)$$
(3)

$$M_{\mathrm{b}} = iee'\frac{1}{(q^2 - M_z{}^2 + iM_z\Gamma_z)}\cdot\frac{1}{(p_2-k)^2 - m^2}\bar{u}(q_1)\left[\frac{g}{4c_\theta}\gamma^\mu(1-\gamma_5) + \kappa\sigma^{\mu\upsilon}(1-\gamma_5)q_\nu\right]\upsilon(q_2)$$
$$\times\,\bar{\upsilon}(p_2)\not{\varepsilon}(-\not{p}_2 + \not{k} + m)\gamma_\mu(\mathrm{a}+b\gamma_5)u(p_1)$$
(4)

$$M_{\mathrm{c}} = ie\frac{g^2}{8}\frac{1}{(q^2 - M_w{}^2)}\cdot\frac{1}{(p_1-k)^2 - m^2}\bar{u}(q_1)\gamma^\mu(1-\gamma_5)(\not{p}_1 - \not{k} + m)\not{\varepsilon}\,u(p_1)$$
$$\times\,\bar{\upsilon}(p_2)\gamma_\mu(1-\gamma_5)\upsilon(q_2) \tag{5}$$

$$M_{\mathrm{d}} = ie\frac{g^2}{8}\frac{1}{(q^2 - M_w{}^2)}\cdot\frac{1}{(p_2-k)^2 - m^2}\bar{u}(q_1)\gamma^\mu(1-\gamma_5)u(p_1) \tag{6}$$
$$\times\,\bar{\upsilon}(p_2)\not{\varepsilon}(\not{p}_2 + \not{k} + m)\gamma_\mu(1-\gamma_5)\upsilon(q_2)$$

$$M_{\mathrm{e}} = ie^2\kappa'\frac{1}{q^2}\cdot\frac{1}{(p_1-k)^2 - m^2}\bar{u}(q_1)\sigma^{\mu\upsilon}q_\nu\upsilon(q_2)\bar{\upsilon}(p_2)\gamma_\mu(\not{p}_1 - \not{k} + m)\not{\varepsilon}\,u(p_1) \tag{7}$$

$$M_{\mathrm{f}} = ie^2\kappa'\frac{1}{q^2}\cdot\frac{1}{(p_2-k)^2 - m^2}\bar{u}(q_1)\sigma^{\mu\upsilon}q_\nu\upsilon(q_2)\bar{\upsilon}(p_2)(-\not{p}_1 + \not{k} + m)\gamma_\mu\not{\varepsilon}\,u(p_1) \tag{8}$$

Where

$$\frac{g^2}{(4c_g)^2} = \frac{G_F M_z^2}{2\sqrt{2}} \quad , \quad \frac{g^2}{8} = \frac{G_F M_w^2}{\sqrt{2}}$$

$$a = \sin^2\theta_w - \frac{1}{4}\ , \ b = \frac{1}{4}\ e' = \frac{e}{\sin\theta_w \cos\theta_w}\ ,$$

$$M = M_a + M_b + M_c + M_d + M_e + M_f \tag{9}$$

and, $\in_\mu$ is the photon four-polarization vector, k is photon momenta, p_2, p_1, q_1 and q_2 are the 4-momenta of e^+, e^-, υ and $\bar{\upsilon}$ describe by spinors $\upsilon(p_2), u(p_1), u(q_1)$, $\upsilon(q_2)$, respectively and $q = q_1 + q_2$. In order to obtain the unpolarized differential cross section, we take an average over the initial electron and the positron spins and a sum over the spins of final neutrinos and photon polarization.

The terms which are linear with κ are negligible since they are proportional to m_υ (the neutrino mass). Also we have assumed that $\kappa' = \kappa / (M_z \Gamma_z)$.

The squared matrix elements with κ^2 is written (by omitting the other contributions) as

$$|M(\kappa)|^2 = e^2\kappa^2 \left\{ \left[\frac{e^2}{q^4} + \frac{e'^2(a^2+b^2)}{(q^2-M_z^2)^2 + M_z^2\Gamma_z^2} + \frac{e'ea}{q^2(q^2-M_z^2+M_z\Gamma_z)} \right] \times \left[\frac{1}{(2p_1k)^2}\sum AA^+ + \frac{1}{(2p_2k)^2}\sum CC^+ + \frac{1}{(2p_1k)(2p_2k)}\sum (CA^+ + AC^+) \right] \right\}$$

(10)

where

$$A = \bar{\upsilon}(p_2)\gamma_\mu(\not{p}_1 - \not{k} + m)\not\in u(p_1)\bar{u}(q_1)\sigma^{\mu\upsilon}q_\nu \upsilon(q_2) \tag{11}$$

$$C = \bar{\upsilon}(p_2)\not\in(-\not{p}_2 + \not{k} + m)\gamma_\mu \not\in u(p_1)\bar{u}(q_1)\sigma^{\mu\upsilon}q_\nu \upsilon(q_2)\,. \tag{12}$$

The differential cross section $e^+e^- \to \upsilon\bar{\upsilon}\gamma$ is written as

$$d\sigma = \frac{m_e^2 m_\nu^2}{4p_1p_2}\frac{1}{(2\pi)^5}\frac{d^3q_1}{E_\nu}\frac{d^3q_2}{E_{\bar{\nu}}}\frac{d^3k}{E_\gamma}\delta^4(q-q_1-q_2)\frac{1}{4}\sum |M|^2\,.$$

(13)

Using the same notation as in [13, 14] and also neglecting the electron mass m, after a long and straightforward calculations, we can obtain the result for the differential cross section κ^2 as

$$\frac{d\sigma}{dxdy}=\frac{e^2\kappa^2}{32(2\pi)^3M_z^2\Gamma_z^2}\left\{\frac{1}{1-y}\left[sx(1-x)(1+y)-\frac{1}{3}\left[4s^2x\left[1-\frac{1}{2}x(1+y)\right]-4s^2x(1-x)(1+y)\right.\right.\right.$$
$$\left.\left.2s^2\left[1-\frac{1}{2}x(1+y)\right]^2-2s^2(1-x)-2s^2\right]\right]$$
$$+\frac{1}{1+y}\left[sx(1-x)(1-y)-\frac{1}{3}\left[2s^2x\left[1-\frac{1}{2}x(1-y)\right]-2s^2x(1-x)(1-y)\right.\right.$$
$$\left.+2s^2\left[1-\frac{1}{2}x(1+y)\right]^2+2s^2(1-x)(1+y)-2s^2(1-x)+s^2x(1+y)-2s^2\right]$$
$$\left.+\frac{s}{x(1-y^2)}-8(1-x)^2+\frac{1}{3}\left[4x\left[1-\frac{1}{2}x(1-y)\right]-4(1-x)+8(1-x)^2+8(1-x)-4\right]\right\}$$

(14)

where $x=2E_\gamma/\sqrt{s}$, E_γ is the photon energy, $y=\cos\theta_\gamma$, θ_γ is the angle between $\overrightarrow{p_1}$ and $\vec{k}$ and $\sqrt{s}$ is the center of mass energy.

In order to obtain the total cross-section, equation (14) should be integrated for the variables x and y (photon energy and scattering angle). However, in the general form, these integrals cannot be calculated analytically due to the term κ which depends on the transfer momentum square (the photon energy). Therefore, the integrations are calculated numerically.

In the calculations, the parameters are chosen as $E=\sqrt{s}/2, \sqrt{s}=90\ GeV$, photon energy is considered to be in the region $15\ GeV \leq E_\gamma \leq 44\ GeV$, and $-0.9 \leq y \leq 0.9$, which correspond to LEP experiments condition [12]. Previously single photon production at LEP was also performed by L3 collaborations experimentally. Under these conditions (i.e. $\sqrt{s} \gg m$ and in the integration we neglect the mass of the electron), for the total cross-section from (7) we obtained the result:

$$\sigma \approx 8.2x10^{-10}\mu^2. \tag{15}$$

If one expresses the anomalous magnetic moment via Bohr magneton, i.e. $\kappa=\mu=\beta\mu_B$, where $\mu_B=\frac{e}{2m}$, Using the experimental results as $\sqrt{s}=90\ GeV\ (L=48, N=14\ )$ from [4,6,12] we then obtained the result:

$$\beta \approx 8.4\times10^{-8}$$

(16)

Finally, the neutrino magnetic moment was obtained as:

$$\mu=8.4\times10^{-8}\mu_B$$

(17)

3. CONCLUSION

Comparing the obtained result with the minimal Standard Model (SM) prediction $\mu_\upsilon \approx 3 \times 10^{-19}\mu_B$ [2], it can be seen that the obtained value for the neutrino magnetic moment has an 10^{11} times order difference of the minimal SM value.

From the equation (1), it is already known that $10^{-10}\mu_B \leq \mu_\upsilon \leq 10^{-6}\mu_B$. If this value is compared with the obtained result, it is seen that, the muon neutrino magnetic moment is in a very good agreement with the values given in [4]. For the case Fig.1 e) and f) (this models corresponds to the magnetic model), we obtain $\mu = 4\times10^{-6}\mu_B$. This upper bound can be used in analysis of data of reactor neutrino experiments as well as stellar physics.

REFERENCES

[1]. Giunti, C., Studenikin, A., Neutrino electromagnetic interactions: A window to new physics, Rev. Mod. Phys. 2015; 87: 531-585.
[2]. Fujikawa K., Shrock E. E.Magnetic Moment of a Massive Neutrino and Neutrino-Spin Rotation, Phys. Rev. Lett. 1980; 45 :963-965.
[3]. Abak M and Aydın C.,Calculation of the Anapole Moment of The Neutrino, Europhys. Lett. 1987; 4: 881-885.
[4]. Olive K.A. et al., Partical Data Group, Chin. Phys. C 2014; 38: 14-15.
[5]. Lattimer J. M. And Goperstein J., Limits on the Neutrino Magnetic Moment from SN1987A, Phys. Rev. Lett. 1988; 61: 23-26.
[6]. Aydın C., Bayar M, Kılıç N., Bounds on The magnetic moment of the Tau-neutrino via the process $e^+e^- \to \nu\bar{\nu}\gamma$ Chinese Physics C 2008; 32 608-611.
[7]. Barut A. O., Aydın Z. Z., Duru I. H., Upper Limit on The Magnetic Moment of the Neutrino, Phys. Rev. D 1982; 26: 1794-1797;
[8]. Aydın C., Neutrino-Electron Elastic Scattering and Stellar Energy Loss through Electron-Positron Pair Annihilation in the Magnetic Model, Nuovo Cimento A 1992; 105: 843-849.
[9]. Abak M., Aydın C.,The Process $e^+e^- \to \mu^+\mu^-$, $\mu^- \to e^-\bar{\nu}_e\nu_\mu$, $\mu^- \to e^-\gamma$ and Heavy Neutrino Decay in the Magnetic Model, Int. Jour. of Modern Phys. A 1996; 11: 1329-1336.
[10].Fargion D., Konoplich R. V. and Mignani R., Constraints on the Two-Higgs Doublet Model and the Tau Neutrino Magnetic Moment from Recent LEP Data, Phys. Rev. D 1993; 47: 751-753
[11].Rizzo T., Investigation of Neutrino Properties in the Process $e^+e^- \to \nu\bar{\nu}\gamma$, Phys. Lett. 1990; 237: 88-90.
[12].Acciarri M. et al., Measurement of energetic single-photon production at LEP, Phys. Lett. B 1997; 346: 190-198.
[13].Ma E., Okada J., How Many Neutrinos?, Phys. Rev. Lett.1978; 41: 287-289.
[14].Tanimoto N., Nakano I., Sakuda M., Bound on the tau neutrino magnetic moment from the TRISTAN experiments, Phys. Lett. B. 2000; 478:1-4.