MANAS Journal of Engineering
ISSN 1694-7398 — e-ISSN 1694-7398

*Volume 11 (Issue 1) (2023) Pages 158-165*
*https://doi.org/10.51354/mjen.1233063*

# The Solution and Dynamic Behaviour of Difference Equations of Twenty-First Order

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**ABSTRACT**

We explore the dynamics of adhering to rational difference formula

$$\psi_{m+1} = \frac{\psi_{m-20}}{\pm 1 \pm \psi_{m-2}\psi_{m-5}\psi_{m-8}\psi_{m-11}\psi_{m-14}\psi_{m-17}\psi_{m-20}}, \quad m \in \mathbb{N}_0$$

where the initials are arbitrary nonzero real numbers. Specifically, we examine global asymptotically stability. Additionally, we provide examples and solutions graphs of some special cases.

**ARTICLE INFO**

***Research article***

*Received*: 12.01.2023
*Accepted*: 30.03.2023

***Keywords***: Equilibrium point, solution of difference equation, stability, boundedness, global asymptotic stability

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## 1. Introduction

Our objective in this paper is to check out the actions of the solution of the adhering to nonlinear difference formula

$$\psi_{m+1} = \frac{\psi_{m-20}}{\pm 1 \pm \psi_{m-2}\psi_{m-5}\psi_{m-8}\psi_{m-11}\psi_{m-14}\psi_{m-17}\psi_{m-20}}, \quad (1)$$

$m \in \mathbb{N}_0$, where the initials are real numbers. Additionally, we obtain these types of solutions.

Just recently there has been an expanding rate of interest in researching rational difference equations and also their qualitative properties.

Ahmed et al., in [2] discovered a collection of first order sine-type difference equations that are constructively solvable in closed form, and they provided a general solution to each of the equations.

Aloqeili, [6] has actually gotten the solutions to

$$\psi_{m+1} = \frac{\psi_{m-1}}{a - \psi_m\psi_{m-1}}.$$

Elsayed [14] got the solution of

$$\psi_{m+1} = \frac{\psi_{m-5}}{-1 + \psi_{m-2}\psi_{m-5}}.$$

In [17] Ibrahim et al., explored the boundedness and global stability of a nonlinear generalized high-order difference equation with delay.

Khan et. al. in [18], investigated the second-order nonlinear difference equation's local stability, attractor, periodicity feature, and boundedness solutions. Finally, the resulting findings are quantitatively validated.

Rahaman et. al., In light of Zadeh's expansion principle, they gave a new perspective on the fuzzy difference equation in [22].

Stevic et. al., [26] investigated a nonlinear second-order difference equation that significantly extends several previous equations. Their primary finding is that the difference equation may be solved in closed form. There are also several applications of the main result provided.

Yalcinkaya, [29] studied the existence, boundedness, and asymptotic behavior of positive solutions to the fuzzy difference equation,

$$\psi_{m+1} = \frac{A\psi_{m-1}}{1+\psi_{m-2}^{p}}, m \in \mathbf{N_0}$$

where $(\psi_m)$ is a sequence of positive fuzzy numbers, A and the initial conditions $\psi_{-j}$ are positive fuzzy numbers and $p$ is a positive integer.

Soykan, et al., the binomial transform of the generalized third-order Jacobsthal sequence is established in [27]. They also describe the binomial transform of four special cases of third-order Jacobsthal sequences: the binomial transform of the third-order Jacobsthal, the binomial transform of the third-order Jacobsthal-Lucas, the modified third-order Jacobsthal-Lucas, and the binomial transform of the third-order Jacobsthal-Perrin. Furthermore, they investigated their characteristics in greater depth.

## 2. First Equation

In this part we give the solutions of

$$\psi_{m+1} = \frac{\psi_{m-20}}{1+\psi_{m-2}\psi_{m-5}\psi_{m-8}\psi_{m-11}\psi_{m-14}\psi_{m-17}\psi_{m-20}}, \quad m \in \mathbb{N}_0, \qquad (2)$$

where the initials are arbitrary real numbers.

**Theorem 1.** *Let $\{x_m\}_{m=-20}^{\infty}$ be a solution of Eq. 2. Then for $m \in \mathbb{N}_0$*

$$\psi_{21m+1} = \frac{A_{20}\prod_{i=0}^{m-1}(1+7iA_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m}(1+(7i+1)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$

$$\psi_{21m+2} = \frac{A_{19}\prod_{i=0}^{m-1}(1+7iA_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m}(1+(7i+1)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$

$$\psi_{21m+3} = \frac{A_{18}\prod_{i=0}^{m-1}(1+7iA_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m}(1+(7i+1)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$

$$\psi_{21m+4} = \frac{A_{17}\prod_{i=0}^{m}(1+(7i+1)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m}(1+(7i+2)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$

$$\psi_{21m+5} = \frac{A_{16}\prod_{i=0}^{m}(1+(7i+1)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m}(1+(7i+2)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$

$$x_{21n+6} = \frac{A_{15}\prod_{i=0}^{m}(1+(7i+1)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m}(1+(7i+2)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$

$$\psi_{21m+7} = \frac{A_{14}\prod_{i=0}^{m}(1+(7i+2)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m}(1+(7i+3)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$

$$\psi_{21m+8} = \frac{A_{13}\prod_{i=0}^{m}(1+(7i+2)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m}(1+(7i+3)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$

$$\psi_{21m+9} = \frac{A_{12}\prod_{i=0}^{m}(1+(7i+2)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m}(1+(7i+3)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$

$$\psi_{21m+10} = \frac{A_{11}\prod_{i=0}^{m}(1+(7i+3)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m}(1+(7i+4)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$

$$\psi_{21m+11} = \frac{A_{10}\prod_{i=0}^{m}(1+(7i+3)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m}(1+(7i+4)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$

$$\psi_{21n+12} = \frac{A_9\prod_{i=0}^{m}(1+(7i+3)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m}(1+(7i+4)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$

$$\psi_{21m+13} = \frac{A_8\prod_{i=0}^{m}(1+(7i+4)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m}(1+(7i+5)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$

$$\psi_{21m+14} = \frac{A_7\prod_{i=0}^{m}(1+(7i+4)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m}(1+(7i+5)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$

$$\psi_{21n+15} = \frac{A_6\prod_{i=0}^{m}(1+(7i+4)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m}(1+(7i+5)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$

$$\psi_{21m+16} = \frac{A_5\prod_{i=0}^{m}(1+(7i+5)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m}(1+(7i+6)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$

$$\psi_{21m+17} = \frac{A_4\prod_{i=0}^{m}(1+(7i+5)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m}(1+(7i+6)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$

$$\psi_{21m+18} = \frac{A_3\prod_{i=0}^{m}(1+(7i+5)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m}(1+(7i+6)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$

$$\psi_{21m+19} = \frac{A_2\prod_{i=0}^{m}(1+(7i+6)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m}(1+(7i+7)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$

$$\psi_{21m+20} = \frac{A_1\prod_{i=0}^{m}(1+(7i+6)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m}(1+(7i+7)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$

$$\psi_{21m+21} = \frac{A_0\prod_{i=0}^{m}(1+(7i+6)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m}(1+(7i+7)A_0A_3A_6A_9A_{12}A_{15}A_{18})}.$$

*where, $\psi_{-20} = A_{20}$, $\psi_{-19} = A_{19}$, ..., $\psi_{-1} = A_1$, $\psi_0 = A_0$.*

**Proof** Suppose that $m > 0$ and that our assumption holds for $m-1$. That is,

$$\psi_{21m-20} = \frac{A_{20}\prod_{i=0}^{m-2}(1+7iA_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m-1}(1+(7i+1)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$

$$\psi_{21m-19} = \frac{A_{19}\prod_{i=0}^{m-2}(1+7iA_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m-1}(1+(7i+1)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$

$$\psi_{21m-18} = \frac{A_{18}\prod_{i=0}^{m-2}(1+7iA_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m-1}(1+(7i+1)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$

$$\psi_{21m-17} = \frac{A_{17}\prod_{i=0}^{m-1}(1+(7i+1)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m-1}(1+(7i+2)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$

$$\psi_{21m-16} = \frac{A_{16}\prod_{i=0}^{m-1}(1+(7i+1)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m-1}(1+(7i+2)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$

$$\psi_{21m-15} = \frac{A_{15}\prod_{i=0}^{m-1}(1+(7i+1)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m-1}(1+(7i+2)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$

$$\psi_{21m-14} = \frac{A_{14}\prod_{i=0}^{m-1}(1+(7i+2)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m-1}(1+(7i+3)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$

$$\psi_{21m-13} = \frac{A_{13}\prod_{i=0}^{m-1}(1+(7i+2)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m-1}(1+(7i+3)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$

$$\psi_{21m-12} = \frac{A_{12}\prod_{i=0}^{m-1}(1+(7i+2)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m-1}(1+(7i+3)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$

$$\psi_{21m-11} = \frac{A_{11}\prod_{i=0}^{m-1}(1+(7i+3)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m-1}(1+(7i+4)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$

$$\psi_{21m-10} = \frac{A_{10}\prod_{i=0}^{m-1}(1+(7i+3)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m-1}(1+(7i+4)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$

$$\psi_{21m-9} = \frac{A_9\prod_{i=0}^{m-1}(1+(7i+3)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m-1}(1+(7i+4)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$

$$\psi_{21m-8} = \frac{A_8\prod_{i=0}^{m-1}(1+(7i+4)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m-1}(1+(7i+5)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$

$$\psi_{21m-7} = \frac{A_7\prod_{i=0}^{m-1}(1+(7i+4)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m-1}(1+(7i+5)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$

$$\psi_{21m-6} = \frac{A_6\prod_{i=0}^{m-1}(1+(7i+4)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m-1}(1+(7i+5)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$

$$\psi_{21m-5} = \frac{A_5\prod_{i=0}^{m-1}(1+(7i+5)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m-1}(1+(7i+6)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$

$$\psi_{21m-4} = \frac{A_4 \prod_{i=0}^{m-1}(1+(7i+5)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m-1}(1+(7i+6)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$

$$\psi_{21m-3} = \frac{A_3 \prod_{i=0}^{m-1}(1+(7i+5)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m-1}(1+(7i+6)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$

$$\psi_{21m-2} = \frac{A_2 \prod_{i=0}^{m-1}(1+(7i+6)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m-1}(1+(7i+7)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$

$$\psi_{21m-1} = \frac{A_1 \prod_{i=0}^{m-1}(1+(7i+6)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m-1}(1+(7i+7)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$

$$\psi_{21m} = \frac{A_0 \prod_{i=0}^{m-1}(1+(7i+6)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m-1}(1+(7i+7)A_0A_3A_6A_9A_{12}A_{15}A_{18})}.$$

Now, using the main 2, one has

$$\psi_{21m+1} = \frac{\psi_{21m-20}}{1+\psi_{21m-2}\psi_{21m-5}\psi_{21m-8}\psi_{21m-11}\psi_{21m-14}\psi_{21m-17}\psi_{21m-20}},$$

Hence, we have

$$\psi_{21m+1} = \frac{A_{20} \prod_{i=0}^{m-1}(1+7iA_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m}(1+(7i+1)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}.$$

Other relations can be proved similarly way.

**Theorem 2.** *Eq. 2 has unique equilibrium point which is the number zero and this equilibrium isn't locally asymptotically stable. Also $\overline{\psi}$ is non hyperbolic.*

**Proof** For the equilibrium points of Eq. 2, we can write

$$\overline{\psi} = \frac{\overline{\psi}}{1+\overline{\psi}^7}.$$

Then

$$\overline{\psi}+\overline{\psi}^8 = \overline{\psi}, \quad \overline{\psi}^8 = 0.$$

Thus the equilibrium point of Eq. 2 is $\overline{\psi} = 0$.
So,

$$f(l,o,t,w,\alpha,\beta,\gamma) = \frac{l}{1+lotw\alpha\beta\gamma}$$

$$f_l(l,o,t,w,\alpha,\beta,\gamma) = \frac{1}{(1+lotw\alpha\beta\gamma)^2}, \quad f_o(l,o,t,w,\alpha,\beta,\gamma) = \frac{-l^2tw\alpha\beta\gamma}{(1+lotw\alpha\beta\gamma)^2},$$

$$f_t(l,o,t,w,\alpha,\beta,\gamma) = \frac{-l^2ow\alpha\beta\gamma}{(1+lotw\alpha\beta\gamma)^2}, \quad f_w(l,o,t,w,\alpha,\beta,\gamma) = \frac{-l^2ot\alpha\beta\gamma}{(1+lotw\alpha\beta\gamma)^2}$$

$$f_\alpha(l,o,t,w,\alpha,\beta,\gamma) = \frac{-l^2owt\beta\gamma}{(1+lotw\alpha\beta\gamma)^2}, \quad f_\beta(l,o,t,w,\alpha,\beta,\gamma) = \frac{-l^2ot\alpha w\gamma}{(1+lotw\alpha\beta\gamma)^2},$$

$$f_\gamma(l,o,t,w,\alpha,\beta,\gamma) = \frac{-l^2ot\alpha w\beta}{(1+lotw\alpha\beta\gamma)^2}.$$

We have

$$f_l(\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi}) = 1, \quad f_o(\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi}) = -1,$$

$$f_t(\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi}) = -1, \quad f_w(\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi}) = -1,$$

$$f_\alpha(\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi}) = -1, \quad f_\beta(\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi},\overline{\psi}) = -1,$$

$$f_\gamma(\overline{\psi},\overline{\psi},\overline{x},\overline{x},\overline{\psi},\overline{\psi},\overline{\psi}) = -1.$$

We confirm our results with the following numerical examples.

**Example 1.** *Assume that*

$\psi_{-20} = 0.3$, $\psi_{-19} = 0.35$, $\psi_{-18} = 0.4$, $\psi_{-17} = 0.87$, $\psi_{-16} = 0.86$,
$\psi_{-15} = 0.85$, $\psi_{-14} = 1.84$, $\psi_{-13} = 0.96$, $\psi_{-12} = 0.82$, $\psi_{-11} = 0.81$,
$\psi_{-10} = 0.8$, $\psi_{-9} = 0.79$, $\psi_{-8} = 0.78$, $\psi_{-7} = 1.77$, $\psi_{-6} = 0.76$,
$\psi_{-5} = 0.75$, $\psi_{-4} = 0.9$, $\psi_{-3} = 0.73$, $\psi_{-2} = 0.72$, $\psi_{-1} = 0.91$,
$\psi_0 = 0.7$,

*See figure 1*

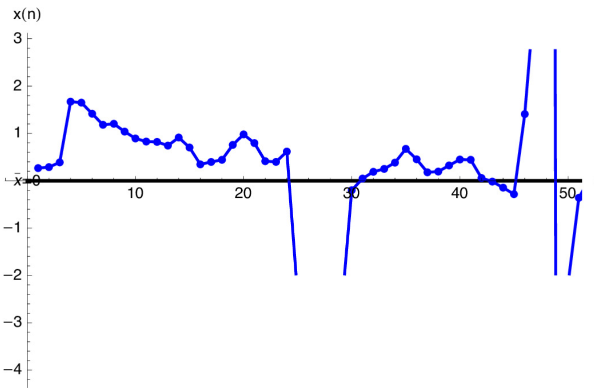


Figure 1

**Example 2.** *Assume that,*

$\psi_{-20} = 0.5$, $\psi_{-19} = 0.3$, $\psi_{-18} = 0.42$, $\psi_{-17} = 0.83$, $\psi_{-16} = 0.81$,
$\psi_{-15} = 0.85$, $\psi_{-14} = 1.84$, $\psi_{-13} = 0.91$, $\psi_{-12} = 0.81$, $\psi_{-11} = 0.84$,
$\psi_{-10} = 0.342$, $\psi_{-9} = 0.79$, $\psi_{-8} = 0.78$, $\psi_{-7} = 1.77$, $\psi_{-6} = 0.76$,
$\psi_{-5} = 0.75$, $\psi_{-4} = 0.9$, $\psi_{-3} = 0.73$, $\psi_{-2} = 0.72$, $\psi_{-1} = 0.91$,
$\psi_0 = 0.7$,

*See figure 2.*

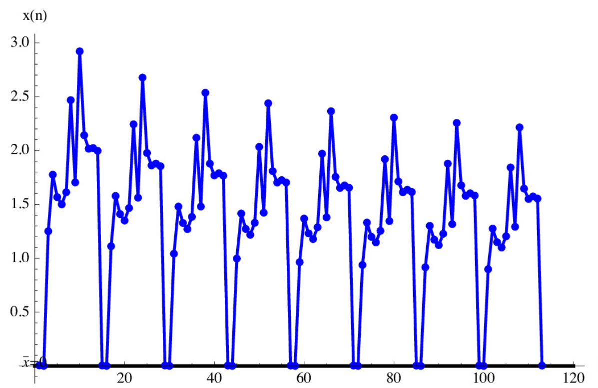


Figure 2

## 3. The Equation $\psi_{m+1} = \frac{\psi_{m-20}}{1-\psi_{m-2}\psi_{m-5}\psi_{m-8}\psi_{m-11}\psi_{m-14}\psi_{m-17}\psi_{m-20}}$

We deal with the difference equation

$$\psi_{m+1} = \frac{\psi_{m-20}}{1-\psi_{m-2}\psi_{m-5}\psi_{m-8}\psi_{m-11}\psi_{m-14}\psi_{m-17}\psi_{m-20}} \quad m \in \mathbb{N}_0. \tag{3}$$

**Theorem 3.** *Let $\{\psi_m\}_{m=-20}^{\infty}$ be a solution of Eq. 3 Then for $m \in \mathbb{N}_0$*

$$\psi_{21m+1} = \frac{A_{20} \prod_{i=0}^{m-1}(1-7iA_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m}(1-(7i+1)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$

$$\psi_{21m+2} = \frac{A_{19} \prod_{i=0}^{m-1}(1-7iA_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m}(1-(7i+1)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$

$$\psi_{21m+3} = \frac{A_{18} \prod_{i=0}^{m-1}(1-7iA_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m}(1-(7i+1)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$

$$\psi_{21m+4} = \frac{A_{17} \prod_{i=0}^{m}(1-(7i+1)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m}(1-(7i+2)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$

$$\psi_{21m+5} = \frac{A_{16}\prod_{i=0}^{m}(1-(7i+1)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m}(1-(7i+2)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$
$$\psi_{21m+6} = \frac{A_{15}\prod_{i=0}^{m}(1-(7i+1)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m}(1-(7i+2)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$
$$\psi_{21m+7} = \frac{A_{14}\prod_{i=0}^{m}(1-(7i+2)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m}(1-(7i+3)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$
$$\psi_{21m+8} = \frac{A_{13}\prod_{i=0}^{m}(1-(7i+2)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m}(1-(7i+3)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$
$$\psi_{21m+9} = \frac{A_{12}\prod_{i=0}^{m}(1-(7i+2)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m}(1-(7i+3)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$
$$\psi_{21m+10} = \frac{A_{11}\prod_{i=0}^{m}(1-(7i+3)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m}(1-(7i+4)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$
$$\psi_{21m+11} = \frac{A_{10}\prod_{i=0}^{m}(1-(7i+3)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m}(1-(7i+4)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$
$$\psi_{21m+12} = \frac{A_9\prod_{i=0}^{m}(1-(7i+3)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m}(1-(7i+4)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$
$$\psi_{21m+13} = \frac{A_8\prod_{i=0}^{m}(1-(7i+4)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m}(1-(7i+5)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$
$$\psi_{21m+14} = \frac{A_7\prod_{i=0}^{m}(1-(7i+4)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m}(1-(7i+5)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$
$$\psi_{21m+15} = \frac{A_6\prod_{i=0}^{m}(1-(7i+4)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m}(1-(7i+5)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$
$$\psi_{21m+16} = \frac{A_5\prod_{i=0}^{m}(1-(7i+5)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m}(1-(7i+6)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$
$$\psi_{21m+17} = \frac{A_4\prod_{i=0}^{m}(1-(7i+5)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m}(1-(7i+6)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$
$$\psi_{21m+18} = \frac{A_3\prod_{i=0}^{m}(1-(7i+5)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m}(1-(7i+6)A_0A_3A_6A_9A_{12}A_{15}A_{18})},$$
$$\psi_{21m+19} = \frac{A_2\prod_{i=0}^{m}(1-(7i+6)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})}{\prod_{i=0}^{m}(1-(7i+7)A_2A_5A_8A_{11}A_{14}A_{17}A_{20})},$$
$$\psi_{21m+20} = \frac{A_1\prod_{i=0}^{m}(1-(7i+6)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})}{\prod_{i=0}^{m}(1-(7i+7)A_1A_4A_7A_{10}A_{13}A_{16}A_{19})},$$
$$\psi_{21n+21} = \frac{A_0\prod_{i=0}^{m}(1-(7i+6)A_0A_3A_6A_9A_{12}A_{15}A_{18})}{\prod_{i=0}^{m}(1-(7i+7)A_0A_3A_6A_9A_{12}A_{15}A_{18})}.$$

*where,* $\psi_{-20} = A_{20}$, $\psi_{-19} = A_{19}$, $\dots$, $\psi_{-1} = A_1$, $\psi_0 = A_0$.

**Proof** Theorem 3 proof can be obtained similar way to Theroem 1.

**Theorem 4.** *Eq. 3 has a unique equilibrium point* $\overline{\psi} = 0$, *which isn't locally asymptotically stable.*

**Proof** Theorem 4 proof can be obtained similar way to Theroem 2.

For confirming the outcomes of this section, we take into consideration mathematical instances which stand for various kind of solutions to (3).

**Example 3.** *Figure 3 gives the behavior when,*

$\psi_{-20} = 0.4$, $\psi_{-19} = 0.42$, $\psi_{-18} = 0.43$, $\psi_{-17} = 0.44$, $\psi_{-16} = 0.45$,
$\psi_{-15} = 0.46$, $\psi_{-14} = 0.47$, $\psi_{-13} = 0.48$, $\psi_{-12} = 0.49$, $\psi_{-11} = 0.5$,
$\psi_{-10} = 0.51$, $\psi_{-9} = 0.52$, $\psi_{-8} = 0.53$, $\psi_{-7} = 0.54$, $\psi_{-6} = 0.55$,
$\psi_{-5} = 0.56$, $\psi_{-4} = 0.57$, $\psi_{-3} = 0.58$, $\psi_{-2} = 0.59$, $\psi_{-1} = 0.6$,
$\psi_0 = 0.61$

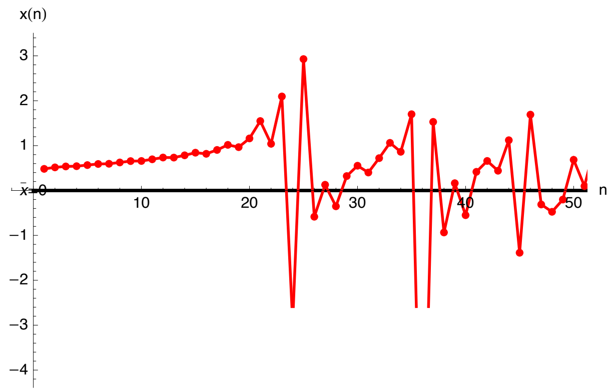


Figure 3

**Example 4.** *Figure 4 gives the behavior when,*

$\psi_{-20} = 0.8$, $\psi_{-19} = 0.65$, $\psi_{-18} = 0.43$, $\psi_{-17} = 0.44$, $\psi_{-16} = 0.45$,
$\psi_{-15} = 0.46$, $\psi_{-14} = 0.47$, $\psi_{-13} = 0.48$, $\psi_{-12} = 0.49$, $\psi_{-11} = 0.5$,
$\psi_{-10} = 0.51$, $\psi_{-9} = 0.52$, $\psi_{-8} = 0.53$, $\psi_{-7} = 0.54$, $\psi_{-6} = 0.55$,
$\psi_{-5} = 0.56$, $\psi_{-4} = 0.57$, $\psi_{-3} = 0.74$, $\psi_{-2} = 0.75$, $\psi_{-1} = 0.76$,
$\psi_0 = 0.77$

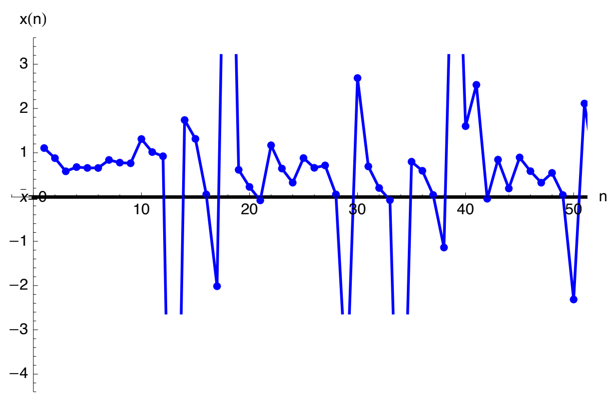


Figure 4

## 4. The Equation $\psi_{m+1} = \frac{\psi_{m-20}}{-1+\psi_{m-2}\psi_{m-5}\psi_{m-8}\psi_{m-11}\psi_{m-14}\psi_{m-17}\psi_{m-20}}$

In this part, we study

$$\psi_{m+1} = \frac{\psi_{m-20}}{-1+\psi_{m-2}\psi_{m-5}\psi_{m-8}\psi_{m-11}\psi_{m-14}\psi_{m-17}\psi_{m-20}}, \quad m \in \mathbb{N}_0, \qquad (4)$$

where the initial conditions are arbitrary nonzero real numbers with
$\psi_{m-2}\psi_{m-5}\psi_{m-8}\psi_{m-11}\psi_{m-14}\psi_{m-17}\psi_{m-20} \neq 1$.

**Theorem 5.** *Let* $\{\psi_m\}_{m=-20}^{\infty}$ *be a solution of difference equation 4. Then for,* $m = 0, 1, 2, \dots$

$$\psi_{42m+1} = \frac{A_{20}}{-1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}},$$
$$\psi_{42m+2} = \frac{A_{19}}{-1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}},$$
$$\psi_{42m+3} = \frac{A_{18}}{-1+A_0A_3A_6A_9A_{12}A_{15}A_{18}},$$
$$\psi_{42m+4} = A_{17}(-1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}),$$
$$\psi_{42m+5} = A_{16}(-1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}),$$
$$x_{42n+6} = A_{15}(-1+A_0A_3A_6A_9A_{12}A_{15}A_{18}),$$

$$\psi_{42m+7}=\frac{A_{14}}{-1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}},$$
$$\psi_{42m+8}=\frac{A_{13}}{-1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}},$$
$$\psi_{42m+9}=\frac{A_{12}}{-1+A_0A_3A_6A_9A_{12}A_{15}A_{18}},$$
$$\psi_{42m+10}=A_{11}(-1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}),$$
$$\psi_{42m+11}=A_{10}(-1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}),$$
$$x_{42n+12}=A_9(-1+A_0A_3A_6A_9A_{12}A_{15}A_{18}),$$
$$\psi_{42m+13}=\frac{A_8}{-1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}},$$
$$\psi_{42m+14}=\frac{A_7}{-1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}},$$
$$\psi_{42m+15}=\frac{A_6}{-1+A_0A_3A_6A_9A_{12}A_{15}A_{18}},$$
$$\psi_{42m+16}=A_5(-1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}),$$
$$\psi_{42m+17}=A_4(-1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}),$$
$$x_{42n+18}=A_3(-1+A_0A_3A_6A_9A_{12}A_{15}A_{18}),$$
$$\psi_{42m+19}=\frac{A_2}{-1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}},$$
$$\psi_{42m+20}=\frac{A_1}{-1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}},$$
$$\psi_{42m+21}=\frac{A_0}{-1+A_0A_3A_6A_9A_{12}A_{15}A_{18}},$$

$$\psi_{42m+22}=A_{20},\quad \psi_{42m+23}=A_{19},\quad \psi_{42m+24}=A_{18},\quad \ldots,\quad \psi_{42m+40}=A_2,$$
$$\psi_{42m+41}=A_1,\quad \psi_{42m+42}=A_0,$$

**Proof** Suppose

$$\psi_{42m-41}=\frac{A_{20}}{-1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}},$$
$$\psi_{42m-40}=\frac{A_{19}}{-1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}},$$
$$\psi_{42m-39}=\frac{A_{18}}{-1+A_0A_3A_6A_9A_{12}A_{15}A_{18}},$$
$$\psi_{42m-38}=A_{17}(-1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}),$$
$$\psi_{42m-37}=A_{16}(-1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}),$$
$$\psi_{42m-36}=A_{15}(-1+A_0A_3A_6A_9A_{12}A_{15}A_{18}),$$
$$\psi_{42m-35}=\frac{A_{14}}{-1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}},$$
$$\psi_{42m-34}=\frac{A_{13}}{-1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}},$$
$$\psi_{42m-33}=\frac{A_{12}}{-1+A_0A_3A_6A_9A_{12}A_{15}A_{18}},$$
$$\psi_{42m-32}=A_{11}(-1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}),$$
$$\psi_{42m-31}=A_{10}(-1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}),$$
$$x_{42n-30}=A_9(-1+A_0A_3A_6A_9A_{12}A_{15}A_{18}),$$
$$\psi_{42m-29}=\frac{A_8}{-1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}},$$
$$\psi_{42m-28}=\frac{A_7}{-1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}},$$
$$\psi_{42m-27}=\frac{A_6}{-1+A_0A_3A_6A_9A_{12}A_{15}A_{18}},$$
$$\psi_{42m-26}=A_5(-1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}),$$
$$\psi_{42m-25}=A_4(-1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}),$$
$$\psi_{42m-24}=A_3(-1+A_0A_3A_6A_9A_{12}A_{15}A_{18}),$$
$$\psi_{42m-23}=\frac{A_2}{-1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}},$$
$$\psi_{42m-22}=\frac{A_1}{-1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}},$$
$$\psi_{42m-21}=\frac{A_0}{-1+A_0A_3A_6A_9A_{12}A_{15}A_{18}},$$

$$\psi_{42m-20}=A_{20},$$
$$\psi_{42m-19}=A_{19},\quad \psi_{42m-18}=A_{18},$$
$$\psi_{42m-17}=A_{17},\quad \psi_{42m-16}=A_{16},$$
$$\psi_{42m-15}=A_{15},\quad \psi_{42m-14}=A_{14},$$
$$\vdots \qquad \vdots$$
$$\psi_{42m-5}=A_5,\quad \psi_{42m-4}=A_4,$$
$$\psi_{42m-3}=A_3,\quad \psi_{42m-2}=A_2,$$
$$\psi_{42m-1}=A_1,\quad \psi_{42m}=A_0,$$

Now, it follows from 4 that

$$\psi_{42m+1}=\frac{\psi_{42m-20}}{-1+\psi_{42m-2}\psi_{42m-5}\psi_{42m-8}\psi_{42m-11}\psi_{42m-14}\psi_{42m-17}\psi_{42m-20}}$$

Then, we have

$$\psi_{21m+1}=\frac{A_{20}}{-1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}}.$$

Other relations can be given by the same way.

**Theorem 6.** *Eq. 4 has three equilibrium points which are* $0,\pm\sqrt[7]{2}$*, and these equilibrium points aren't locally asymptotically stable.*

**Proof** Theorem 6 can be obtained similar way to proofs Theorem 2.

**Example 5.** *Figure 5 gives the behavior with*

$\psi_{-20}=0.8,\ \psi_{-19}=0.65,\ \psi_{-18}=0.43,\ \psi_{-17}=0.44,\ \psi_{-16}=0.45,$
$\psi_{-15}=0.46,\ \psi_{-14}=0.47,\ \psi_{-13}=0.48,\ \psi_{-12}=0.49,\ \psi_{-11}=0.5,$
$\psi_{-10}=0.51,\ \psi_{-9}=0.52,\ \psi_{-8}=0.53,\ \psi_{-7}=0.54,\ \psi_{-6}=0.55,$
$\psi_{-5}=0.56,\ \psi_{-4}=0.57,\ \psi_{-3}=0.74,\ \psi_{-2}=0.75,\ \psi_{-1}=0.76,$
$\psi_0=0.77$

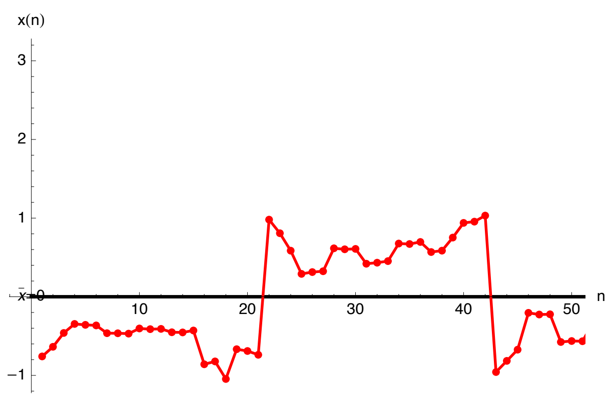


Figure 5

**Example 6.** *Figure 6 gives the behavior with*

$\psi_{-20}=0.79,\ \psi_{-19}=0.65,\ \psi_{-18}=0.41,\ \psi_{-17}=0.44,\ \psi_{-16}=0.45,$
$\psi_{-15}=0.465,\ \psi_{-14}=0.472,\ \psi_{-13}=0.48,\ \psi_{-12}=0.494,\ \psi_{-11}=0.5,$
$\psi_{-10}=0.513,\ \psi_{-9}=0.52,\ \psi_{-8}=0.53,\ \psi_{-7}=0.54,\ \psi_{-6}=0.55,$
$\psi_{-5}=0.953,\ \psi_{-4}=0.94,\ \psi_{-3}=0.74,\ \psi_{-2}=0.75,\ \psi_{-1}=0.76,$
$\psi_0=0.77$

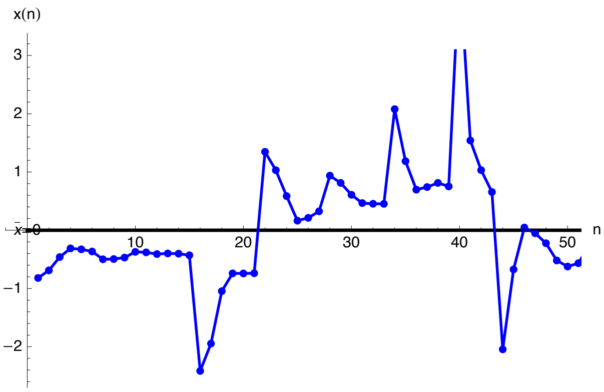


Figure 6

## 5. The Equation $\frac{\psi_{m-20}}{-1-\psi_{m-2}\psi_{m-5}\psi_{m-8}\psi_{m-11}\psi_{m-14}\psi_{m-17}\psi_{m-20}}$,

In this section, we find the solutions of

$$\frac{\psi_{m-20}}{-1-\psi_{m-2}\psi_{m-5}\psi_{m-8}\psi_{m-11}\psi_{m-14}\psi_{m-17}\psi_{m-20}}, \quad m \in \mathbb{N}_0, \tag{5}$$

where the initial conditions are arbitrary nonzero real numbers with $\psi_{m-2}\psi_{m-5}\psi_{m-8}\psi_{m-11}\psi_{m-14}\psi_{m-17}\psi_{m-20} \neq -1$.

**Theorem 7.** *Let $\{\psi_m\}_{m=-20}^{\infty}$ be a solution of 5.*

$$\psi_{42m+1} = \frac{-A_{20}}{1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}},$$
$$\psi_{42m+2} = \frac{-A_{19}}{1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}},$$
$$\psi_{42m+3} = \frac{-A_{18}}{1+A_0A_3A_6A_9A_{12}A_{15}A_{18}},$$
$$\psi_{42m+4} = -A_{17}(1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}),$$
$$\psi_{42m+5} = -A_{16}(1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}),$$
$$\psi_{42m+6} = -A_{15}(1+A_0A_3A_6A_9A_{12}A_{15}A_{18}),$$
$$\psi_{42m+7} = \frac{-A_{14}}{1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}},$$
$$\psi_{42m+8} = \frac{-A_{13}}{1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}},$$
$$\psi_{42m+9} = \frac{-A_{12}}{1+A_0A_3A_6A_9A_{12}A_{15}A_{18}},$$
$$\psi_{42m+10} = -A_{11}(1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}),$$
$$\psi_{42m+11} = -A_{10}(1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}),$$
$$x_{42n+12} = -A_9(1+A_0A_3A_6A_9A_{12}A_{15}A_{18}),$$
$$\psi_{42m+13} = \frac{-A_8}{1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}},$$
$$\psi_{42m+14} = \frac{-A_7}{1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}},$$
$$\psi_{42m+15} = \frac{-A_6}{1+A_0A_3A_6A_9A_{12}A_{15}A_{18}},$$
$$\psi_{42m+16} = -A_5(1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}),$$
$$\psi_{42m+17} = -A_4(1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}),$$
$$\psi_{42m+18} = -A_3(1+A_0A_3A_6A_9A_{12}A_{15}A_{18}),$$
$$\psi_{42m+19} = \frac{-A_2}{1+A_2A_5A_8A_{11}A_{14}A_{17}A_{20}},$$
$$\psi_{42m+20} = \frac{-A_1}{1+A_1A_4A_7A_{10}A_{13}A_{16}A_{19}},$$
$$\psi_{42m+21} = \frac{-A_0}{1+A_0A_3A_6A_9A_{12}A_{15}A_{18}}, \quad \psi_{42m+22} = A_{20},$$
$$\psi_{42m+23} = A_{19}, \quad \psi_{42m+24} = A_{18},$$
$$\psi_{42m+25} = A_{17}, \quad \psi_{42m+26} = A_{16},$$
$$\psi_{42m+27} = A_{15}, \quad \psi_{42m+28} = A_{14},$$
$$\vdots \qquad \vdots$$
$$\psi_{42m+36} = A_6, \quad \psi_{42m+37} = A_5,$$
$$\psi_{42m+38} = A_4, \quad \psi_{42m+39} = A_2,$$
$$\psi_{42m+41} = A_1, \quad \psi_{42m+42} = A_0,$$

*where the initial conditions are arbitrary nonzero real numbers with* $\psi_{m-2}\psi_{m-5}\psi_{m-8}\psi_{m-11}\psi_{m-14}\psi_{m-17}\psi_{m-20} \neq -1$.

**Proof** Theorem 7 can be obtained similar way to Theorem 5.

**Theorem 8.** *Eq. 5 has three equilibrium point which are $0, \pm\sqrt[5]{-2}$ and these equilbrium points aren't locally asymptotically stable.*

**Proof** Theorem 8 can be obtained similar way to Theorem 2.

**Example 7.** *See 7 for the initials*

$\psi_{-20} = 0.8$, $\psi_{-19} = 0.65$, $\psi_{-18} = 0.43$, $\psi_{-17} = 0.44$, $\psi_{-16} = 0.45$,
$\psi_{-15} = 0.46$, $\psi_{-14} = 0.9$, $\psi_{-13} = 0.91$, $\psi_{-12} = 0.92$, $\psi_{-11} = 0.5$,
$\psi_{-10} = 0.51$, $\psi_{-9} = 0.52$, $\psi_{-8} = 0.53$, $\psi_{-7} = 0.54$, $\psi_{-6} = 0.55$,
$\psi_{-5} = 0.95$, $\psi_{-4} = 0.94$, $\psi_{-3} = 0.74$, $\psi_{-2} = 0.75$, $\psi_{-1} = 0.76$,
$\psi_0 = 0.77$

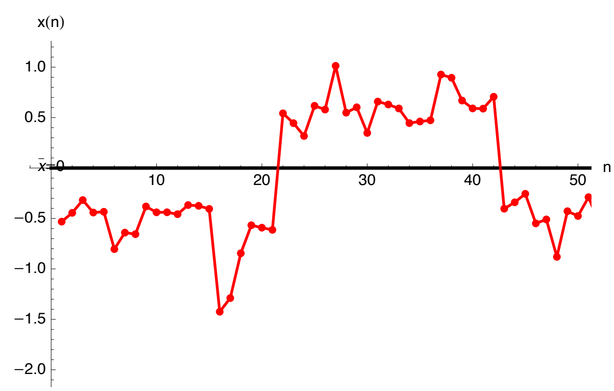


Figure 7

**Example 8.** *We consider*

$\psi_{-20} = 0.8$, $\psi_{-19} = 0.65$, $\psi_{-18} = 0.43$, $\psi_{-17} = 0.44$, $\psi_{-16} = 0.45$,
$\psi_{-15} = 0.46$, $\psi_{-14} = 0.9$, $\psi_{-13} = 0.91$, $\psi_{-12} = 0.92$, $\psi_{-11} = 0.5$,
$\psi_{-10} = 0.51$, $\psi_{-9} = 0.87$, $\psi_{-8} = 0.53$, $\psi_{-7} = 0.54$, $\psi_{-6} = 0.88$,
$\psi_{-5} = 0.95$, $\psi_{-4} = 0.94$, $\psi_{-3} = 0.74$, $\nu_{-2} = 0.96$, $\psi_{-1} = 0.76$,
$\psi_0 = 0.77$

*See figure 8*

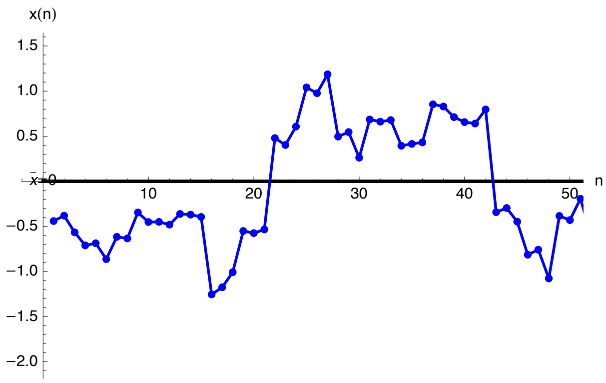


Figure 8

## 6. Conclusion

We study the behavior of the difference equation

$$\psi_{m+1} = \frac{\psi_{m-20}}{\pm 1 \pm \psi_{m-2}\psi_{m-5}\psi_{m-8}\psi_{m-11}\psi_{m-14}\psi_{m-17}\psi_{m-20}}$$

where the initials are positive real numbers. Local stability is discussed. More- over,we get the solution of some special cases.Finally, some numerical examples are given.

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