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RENEWED STRUCTURE OF NEUTROSOPHIC SOFT GRAPHS AND ITS APPLICATION IN DECISION-MAKING PROBLEM

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Abstract. This study is designed with the renewed concept of neutrosophic soft graph (briefly ns-graph) which is a combination of graphs and neutrosophic soft sets. We re-define notions of ns-graphs and ns-subgraphs with the different perspective from the study in [4]. Also, we introduce some new operations on ns-graphs and detailed them with convenient examples. Moreover, we present an application of ns-graphs to determine of optimal object by using given data with the help of an algorithm. This algorithm we developed is new inventiveness domain for problems which are involving uncertainty, and effectively finds the optimal result between the states where vagueness exists. We also provide a comparative analysis with the existing method given in [4].

1. Introduction

It is clear that the uncertainty arising from different areas cannot be explained and expressed with precise definitions. Different types of uncertainty are common in many fields such as economics, biology, physics, engineering, medicine and social sciences. Since uncertainty is a complex and broad concept that is not clearly defined, many fields dealing with uncertainty have failed to model this situation successfully with classical mathematical methods. In valuing a phenomenon in real life, we use intermediate values, that is, fuzzy values. For example, when evaluating the temperature of the air, we make ratings such as cold, slightly cold, warm, slightly hot and hot. Therefore, classical set theory falls short of expressing intermediate state values. This inadequate situation in classical set theory was first tried to be overcome with fuzzy set theory [21]. A fuzzy set A is characterized with the help of a membership function $\mu_A(x)$, a mapping from the universal set X to the unit interval [0,1], where x in the fuzzy set A has a certain degree of membership. Although a phenomenon can be represented with only one of 0 and 1 values in

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classical set theory, it can take infinite values in fuzzy logic. Thus, a phenomenon can have uncertain values in the fuzzy approach. Fuzzy logic controllers have been applied many areas from electrical household appliances to auto electronics, from business machines we use daily to production engineering, from industrial control technologies to automation. Atanassov [5] put forward the intuitionistic fuzzy set theory which is a generalized type of the fuzzy set theory. In intuitionistic fuzzy sets, unlike fuzzy sets, the elements have degrees of non-membership. However, in these theories, the uncertainty of an element is not discussed, although the values of an element such as whether it is a member or not. Based on this, Smarandache [19] introduced the neutrosophic set theory which is an extended and special case of fuzzy set theory. The neutrosophic sets are stated by three functions. These are truth, indeterminacy and falsity membership function. The neutrosophic models produce more suitable solutions for the complex systems. In addition, an individual may not always be fully informed about a subject. In this case, the indeterminacy membership function comes into play and it provides a very large place for modeling events involving many uncertainties. Maji [14] defined the neutrosophic soft set concept and examined the properties of this concept. Broumi [7] worked on generalized netrosophic soft sets. Deli [11] gave the notion of interval-valued neutrosophic soft set and also applied this concept to the a decision-making problem.

Some great scientific theories grew out of answers to simple questions. Graf theory is one of them. Graf theory was first put forward by Euler [12]. Graph theory which is an important mathematical tool for solving complicated problems in many different fields. Graphs are used to put forth a rellevance between elements in a given set, where every element can be expressed with the help of vertices and their relation edges. Since graph theory provides conveniences in modeling complicated systems, it has many number of applications. A simple graph is showed by $G^* = (\nu, \varepsilon)$ where ν and ε represents sets of vertices and edges, respectively. After Euler's introduction of the graph concept, Rosenfeld [17] introduced the fuzzy graph theory. Bhattacharya [6] gave some properties of fuzzy graphs. Mordeson and Peng [16] have defined some operations on fuzzy graphs. Later, many researchers discussed the concept of fuzzy sets on the graph theory and defined different structures. Akram and Dudek [1] gave the concept of interval-valued fuzzy graphs and examined their related properties. Broumi et al. [9] gave the concept of interval valued pentapartitioned neutrosophic graphs. Broumi et al. [8] defined interval-valued fermatean neutrosophic graphs and presented some operations on this. Thumbakara and George [20] gave the concepts of soft graph and soft subgraph, and examined the properties of these structures. Akram and Nawaz [2] described some new algebraic operations on soft graphs. Mohinta and Samanta [15] defined the concept of fuzzy soft graph. Later, Akram and Nawaz [3] studied different types of fuzzy soft graphs. Zihni et al. [22] gave the concept of interval-valued fuzzy soft graph and examined its basic properties. Çelik [10] gave the concept of bipolar fuzzy soft graph and investigated some operations on this concept. Kandasamy et al. [13]

gave the concept of neutrosophic graph and made various applications of neutrosophic graphs. Akram et al. [4] combined neutrosophic soft set concept and graph theory, then define notion of neutrosophic soft graph (ns-graph). They also applied the ns-graphs to the a decision-making problem. Shah and Hussain [18] gave new features on ns-graphs.

In the current study, the renewed concept of ns-graphs is defined and some new operations not previously defined such us extended union, restricted union, extended intersection, restricted intersection and complement are presented. Also illustrative examples related these operations are given. Hence an application of ns-graphs for a decision-making problem is examined with the method we developed. Moreover a comparative analysis between proposed method and existing method given in [4] are revealed.

2. Preliminaries

Definition 1. *[19] Let $X \neq \emptyset$ be an universe. Then a neutrosophic set A on X is given by $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X\}$, where the functions T_A, I_A and F_A are fuzzy sets on X under the conditions $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for all $x \in X$. The family of all neutrosophic sets on X is denoted by $\mathcal{N}(X)$.*

Definition 2. *[14] Let X be an initial universe. Let E be a set of parameters. Then, a pair (f, E) is called a neutrosophic soft set (briefly ns-set) over X representing a mapping by $f : E \to \mathcal{N}(X)$.*

Definition 3. *[14] Let $(f, E) \in \mathcal{N}(X)$. Then, for all $e \in E$ and $x \in X$,*

(a) (f, E) is called a null ns-set if $T_{f(e)}(x) = 0$, $I_{f(e)}(x) = 1$ and $F_{f(e)}(x) = 1$.
(b) (f, E) is called a whole ns-set if $T_{f(e)}(x) = 1$, $I_{f(e)}(x) = 0$ and $F_{f(e)}(x) = 0$.

Definition 4. *[14] Let (f, E_1) and (g, E_2) be two ns-sets over X, then (f, E_1) is said to be a ns-subset of (g, E_2) if*

i. $E_1 \subseteq E_2$
ii. $T_{f(e)}(x) \leq T_{g(e)}(x)$, $I_{f(e)}(x) \geq I_{g(e)}(x)$, $F_{f(e)}(x) \geq F_{g(e)}(x)$ *for all* $e \in E_1$ *and* $x \in X$.

3. Renewed Structure of Ns-graphs with Some New Operations

Definition 5. *A ns-graph is an order 4-tuple $G_N = (G^*, f, g, E)$ such that*

i. $G^* = (\nu, \varepsilon)$ *is a simple graph*
ii. $E \neq \emptyset$ *is a set of parameters*
iii. (f, E) *is a ns-set over* ν
iv. (g, E) *is a ns-set over* ε
v. $h(e) = (f(e), g(e))$ *is a neutrosophic graph for all* $e \in E$*. That is, for all* $e \in E$ *and* $xy \in \varepsilon$,

$$T_{ge}(xy) \leq min\{T_{fe}(x), T_{fe}(y)\}$$
$$I_{ge}(xy) \geq max\{I_{fe}(x), I_{fe}(y)\}$$

$$F_{ge}(xy) \geq max\{F_{fe}(x), F_{fe}(y)\}$$

Note that (f, E) is called a ns-vertex and (g, E) is called a ns-edge.

Example 1. *Let $G^* = (\nu, \varepsilon)$ be a simple graph with $\nu = \{v_1, v_2, v_3\}$ and $\varepsilon = \{v_1v_2, v_1v_3, v_2v_3\}$. Let $E = \{e_1, e_2, e_3\}$ be a set of parameters. Let consider ns-sets f and g over ν and ε, respectively, as given in Table 1.*

TABLE 1. Ns-sets (f, E) and (g, E)

f	v_1	v_2	v_3
e_1	*(0.2,0.4,0.5)*	*(0.4,0.5,0.6)*	*(0,1,1)*
e_2	*(0.2,0.7,0.8)*	*(0.3,0.5,0.6)*	*(0.5,0.6,0.7)*
e_3	*(0.3,0.3,0.5)*	*(0.2,0.2,0.3)*	*(0.3,0.4,0.9)*
g	v_1v_2	v_2v_3	v_1v_3
e_1	*(0.1,0.6,0.7)*	*(0,1,1)*	*(0,1,1)*
e_2	*(0.1,0.7,0.9)*	*(0,1,1)*	*(0.2,0.8,0.9)*
e_3	*(0.2,0.4,0.6)*	*(0.1,0.5,0.9)*	*(0.3,0.5,0.9)*

Clearly $G_N = (G^*, f, g, E)$ is a ns-graph over G^*.

Definition 6. *Let $G^* = (\nu, \varepsilon)$ be a simple graph. A ns-graph $G_N^{'} = (G^*, f^1, g^1, E_1)$ is called a ns-subgraph of $G_N = (G^*, f, g, E_2)$ if*

i. $E_1 \subseteq E_2$
ii. $f_e^1 \subseteq f_e$, *that is* $T_{f^1e}(x) \leq T_{fe}(x)$, $I_{f^1e}(x) \geq I_{fe}(x)$, $F_{f^1e}(x) \geq F_{fe}(x)$
iii. $g_e^1 \subseteq g_e$, *that is,* $T_{g^1e}(x) \leq T_{ge}(x)$, $I_{g^1e}(x) \geq I_{ge}(x)$, $F_{g^1e}(x) \geq F_{ge}(x)$

for all $e \in E_1, x \in \nu$.

Example 2. *Let consider a ns-graph $G_N = (G^*, f, g, E)$ as taken in Example 1. Let consider another ns-graph $G_N^{'} = (G^*, f^1, g^1, E_1)$ as in the Table 2 with the parameter set $E_1 = \{e_1, e_2\}$.*

TABLE 2. Ns-sets (f^1, E_1) and (g^1, E_1)

f^1	v_1	v_2	v_3
e_1	*(0.2,0.5,0.6)*	*(0.3,0.6,0.8)*	*(0,1,1)*
e_2	*(0.1,0.7,0.9)*	*(0.1,0.5,0.7)*	*(0.2,0.8,0.9)*
g^1	v_1v_2	v_2v_3	v_1v_3
e_1	*(0.1,0.7,0.8)*	*(0,1,1)*	*(0,1,1)*
e_2	*(0.1,0.9,0.9)*	*(0,1,1)*	*(0.1,0.8,0.9)*

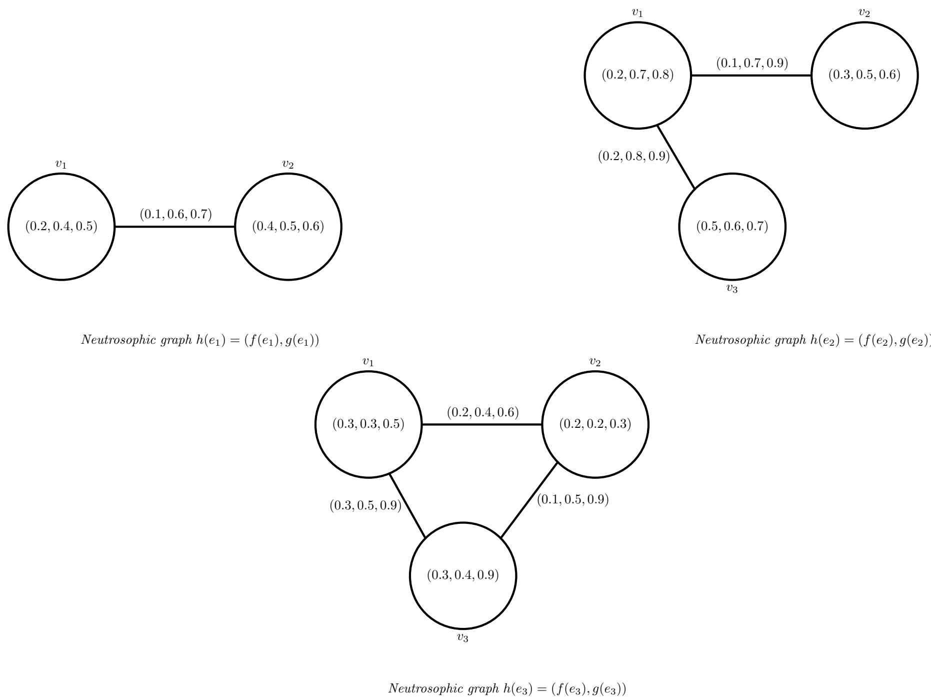

FIGURE 1. Ns-graph $G_N = (G^*, f, g, E)$

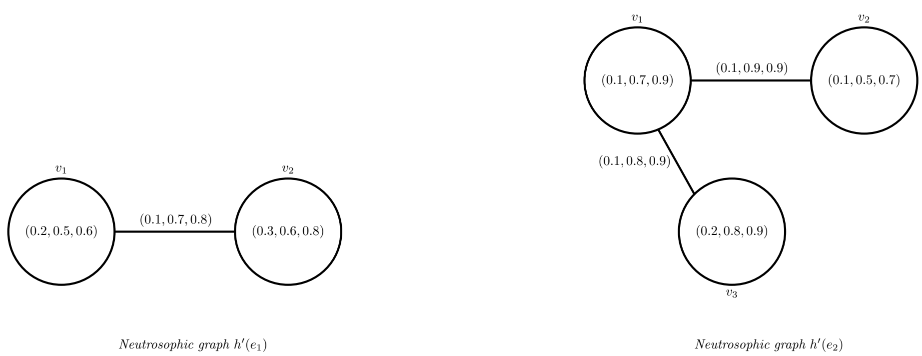

FIGURE 2. Ns-graph $G_N^{'} = (G^*, f^1, g^1, E_1)$

It is evident that $G_N^{'} = (G^*, f^1, g^1, E_1)$ is a ns-subgraph of $G_N = (G^*, f, g, E)$.

Definition 7. *Let $G_N = (G^*, f, g, E)$ be a ns-graph over $G^* = (\nu, \varepsilon)$. Then G_N is called strong ns-graph iff*

$$T_{g_e}(xy) = min\{T_{f_e}(x), T_{f_e}(y)\}$$
$$I_{g_e}(xy) = max\{I_{f_e}(x), I_{f_e}(y)\}$$
$$F_{g_e}(xy) = max\{F_{f_e}(x), F_{f_e}(y)\}$$

for all $e \in E$ and $xy \in \varepsilon$.

Example 3. *Let $G^* = (\nu, \varepsilon)$ be a simple graph with $\nu = \{v_1, v_2, v_3\}$ and $\varepsilon = \{v_1v_2, v_1v_3, v_2v_3\}$. Let $E = \{e_1, e_2, e_3\}$ be a set of parameters. Let consider ns-sets f and g over ν and ε, respectively, as given in Table 3.*

TABLE 3. Ns-sets (f, E) and (g, E)

f	v_1	v_2	v_3
e_1	(0.2,0.4,0.5)	(0.4,0.5,0.6)	(0,1,1)
e_2	(0.2,0.7,0.8)	(0.3,0.5,0.6)	(0.5,0.6,0.7)
e_3	(0.3,0.3,0.5)	(0.2,0.2,0.3)	(0.3,0.4,0.9)

g	v_1v_2	v_2v_3	v_1v_3
e_1	(0.2,0.5,0.6)	(0,1,1)	(0,1,1)
e_2	(0.2,0.7,0.8)	(0.3,0.6,0.7)	(0.2,0.7,0.8)
e_3	(0.2,0.3,0.5)	(0.2,0.4,0.9)	(0.3,0.4,0.9)

Clearly $G_N = (G^*, f, g, E)$ is strong ns-graph.

Definition 8. *Let $G_N = (G^*, f, g, E)$ be a ns-graph over $G^* = (\nu, \varepsilon)$. Then the complement of $G_N = (G^*, f, g, E)$ is denoted by $\overline{G_N} = (G^*, \bar{f}, \bar{g}, E)$ and is defined by*

i. $T_{\bar{f}_e}(x) = T_{f_e}(x)$, $I_{\bar{f}_e}(x) = I_{f_e}(x)$, $F_{\bar{f}_e}(x) = F_{f_e}(x)$

ii. $T_{\bar{g}_e}(x, y) = min\{T_{f_e}(x), T_{f_e}(y)\} - T_{g_e}(x, y)$
$I_{\bar{g}_e}(x, y) = max\{I_{f_e}(x), I_{f_e}(y)\} - I_{g_e}(x, y)$
$F_{\bar{g}_e}(x, y) = max\{F_{f_e}(x), F_{f_e}(y)\} - F_{g_e}(x, y)$

for all $e \in E$ and $xy \in \varepsilon$.

Definition 9. *Let $G_N = (G^*, f^1, g^1, E_1)$ and $G_N^{'} = (G^*, f^2, g^2, E_2)$ be two ns-graphs over the simple graph $G^* = (\nu, \varepsilon)$. The extended union of G_N and $G_N^{'}$ is denoted by $G_N \bigcup G_N^{'} = (G^*, f, g, E)$, where $E = E_1 \cup E_2$. T, I and F membership values of vertices and edges of $G_N \bigcup G_N^{'}$ are defined by as follow.*

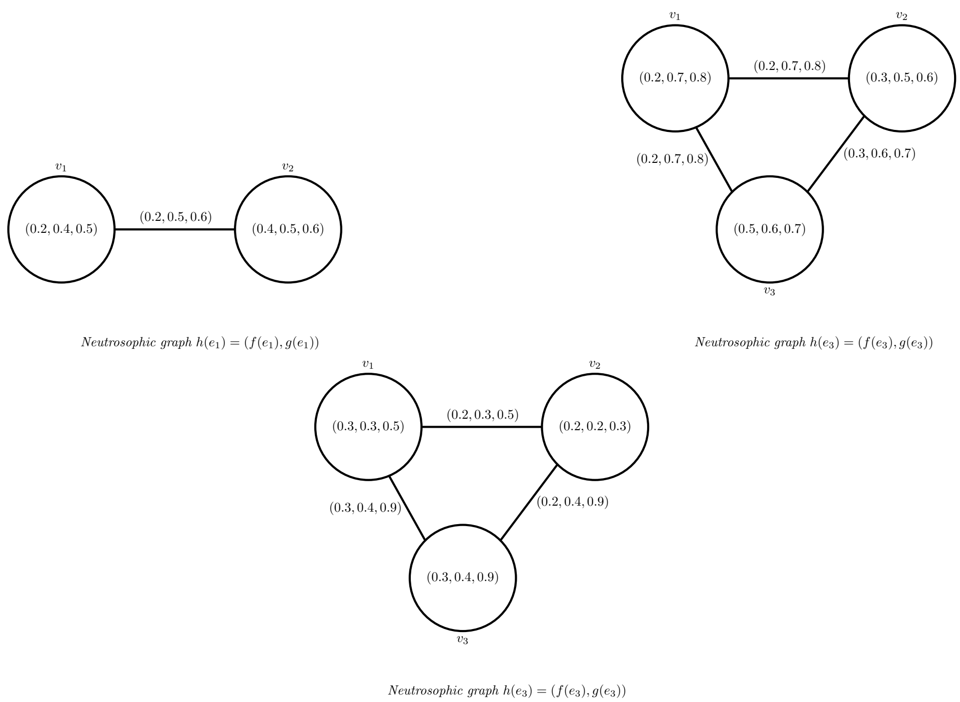

FIGURE 3. Strong ns-graph $G_N = (G^*, f, g, E)$

i. For all $e \in E$ *and* $x \in \nu$

$$T_{f_e}(x) = \begin{cases} T_{f_e^1}(x) & e \in E_1 \backslash E_2 \\ T_{f_e^2}(x) & e \in E_2 \backslash E_1 \\ max\{T_{f_e^1}(x), T_{f_e^2}(x)\} & e \in E_1 \cap E_2 \end{cases}$$

$$I_{f_e}(x) = \begin{cases} I_{f_e^1}(x) & e \in E_1 \backslash E_2 \\ I_{f_e^2}(x) & e \in E_2 \backslash E_1 \\ min\{I_{f_e^1}(x), I_{f_e^2}(x)\} & e \in E_1 \cap E_2 \end{cases}$$

$$F_{f_e}(x) = \begin{cases} F_{f_e^1}(x) & e \in E_1 \backslash E_2 \\ F_{f_e^2}(x) & e \in E_2 \backslash E_1 \\ min\{F_{f_e^1}(x), F_{f_e^2}(x)\} & e \in E_1 \cap E_2 \end{cases}$$

ii. For all $e \in E$ *and* $xy \in \varepsilon$

$$T_{g_e}(xy) = \begin{cases} T_{g_e^1}(xy) & e \in E_1 \backslash E_2 \\ T_{g_e^2}(xy) & e \in E_2 \backslash E_1 \\ max\{T_{g_e^1}(xy), T_{g_e^2}(xy)\} & e \in E_1 \cap E_2 \end{cases}$$

$$I_{g_e}(xy) = \begin{cases} I_{g_e^1}(xy) & e \in E_1 \backslash E_2 \\ I_{g_e^2}(xy) & e \in E_2 \backslash E_1 \\ min\{I_{g_e^1}(xy), I_{g_e^2}(xy)\} & e \in E_1 \cap E_2 \end{cases}$$

$$F_{g_e}(xy) = \begin{cases} F_{g_e^1}(xy) & e \in E_1 \backslash E_2 \\ F_{g_e^2}(xy) & e \in E_2 \backslash E_1 \\ min\{F_{g_e^1}(xy), F_{g_e^2}(xy)\} & e \in E_1 \cap E_2 \end{cases}$$

Example 4. *Let* $G^* = (\nu, \varepsilon)$ *be a simple graph with* $\nu = \{v_1, v_2, v_3, v_4, v_5\}$*. Let consider a ns-graph* $G_N = (G^*, f^1, g^1, E_1)$ *with the parameter set* $E_1 = \{e_1, e_2, e_3\}$ *as in the Table 4.*

Table 4. Ns-sets (f^1, E_1) and (g^1, E_1)

f^1	v_1	v_2	v_3	v_4	v_5
e_1	(0.1,0.2,0.3)	(0,1,1)	(0.2,0.3,0.4)	(0.2,0.5,0.7)	(0,1,1)
e_2	(0.1,0.3,0.7)	(0,1,1)	(0.4,0.6,0.7)	(0.1,0.2,0.3)	(0,1,1)
e_3	(0.5,0.6,0.7)	(0,1,1)	(0.6,0.8,0.9)	(0.3,0.4,0.6)	(0,1,1)

g^1	v_1v_2	v_1v_3	v_1v_4	v_1v_5	v_2v_3
e_1	(0,1,1)	(0.1,0.4,0.5)	(0.1,0.6,0.7)	(0,1,1)	(0,1,1)
e_2	(0,1,1)	(0.1,0.7,0.8)	(0.1,0.4,0.8)	(0,1,1)	(0,1,1)
e_3	(0,1,1)	(0,1,1)	(0.2,0.7,0.9)	(0,1,1)	(0,1,1)

g^1	v_2v_4	v_2v_5	v_3v_4	v_3v_5	v_4v_5
e_1	(0,1,1)	(0,1,1)	(0.1,0.6,0.8)	(0,1,1)	(0,1,1)
e_2	(0,1,1)	(0,1,1)	(0.1,0.8,0.9)	(0,1,1)	(0,1,1)
e_3	(0,1,1)	(0,1,1)	(0.3,0.8,0.9)	(0,1,1)	(0,1,1)

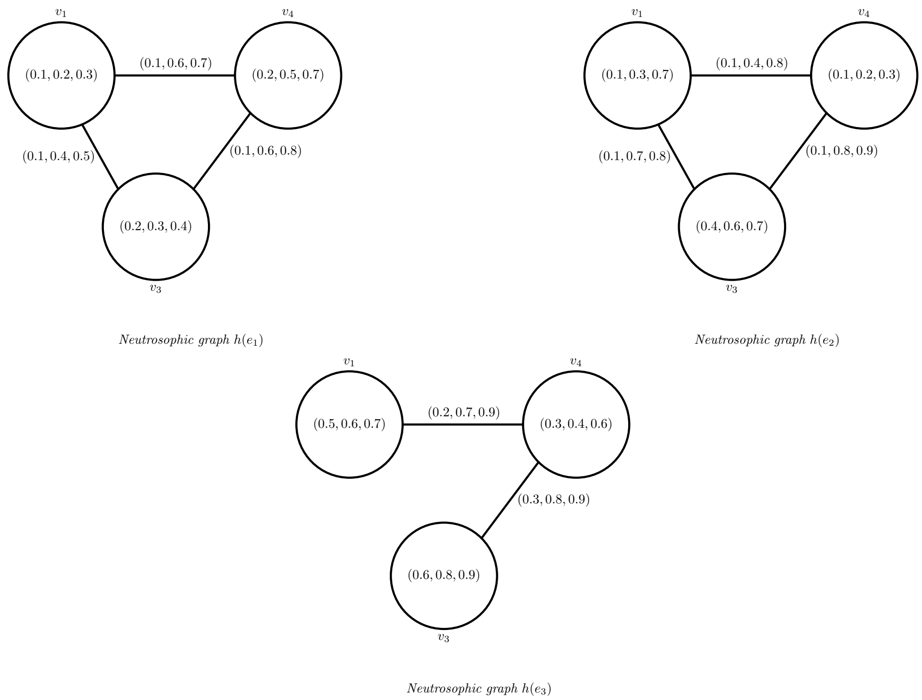

FIGURE 4. Ns-graph $G_N = (G^*, f^1, g^1, E_1)$

Now let consider another ns-graph $G_N^{'} = (G^*, f^2, g^2, E_2)$ *with the parameter set* $E_2 = \{e_2, e_4\}$ *as in the Table 5.*

TABLE 5. Ns-sets (f^2, E_2) and (g^2, E_2)

f^2	v_1	v_2	v_3	v_4	v_5
e_2	(0,1,1)	(0.1,0.2,0.4)	(0.2,0.3,0.4)	(0,1,1)	(0.4,0.6,0.7)
e_4	(0,1,1)	(0.3,0.6,0.8)	(0.5,0.7,0.9)	(0,1,1)	(0.3,0.4,0.5)

g^2	v_1v_2	v_1v_3	v_1v_4	v_1v_5	v_2v_3
e_2	(0,1,1)	(0,1,1)	(0,1,1)	(0,1,1)	(0.1,0.4,0.8)
e_4	(0,1,1)	(0,1,1)	(0,1,1)	(0,1,1)	(0.2,0.7,0.9)

g^2	v_2v_4	v_2v_5	v_3v_4	v_3v_5	v_5v_6
e_2	(0,1,1)	(0,1,1)	(0,1,1)	(0.2,0.8,0.9)	(0,1,1)
e_4	(0,1,1)	(0.2,0.6,0.8)	(0,1,1)	(0.3,0.9,0.9)	(0,1,1)

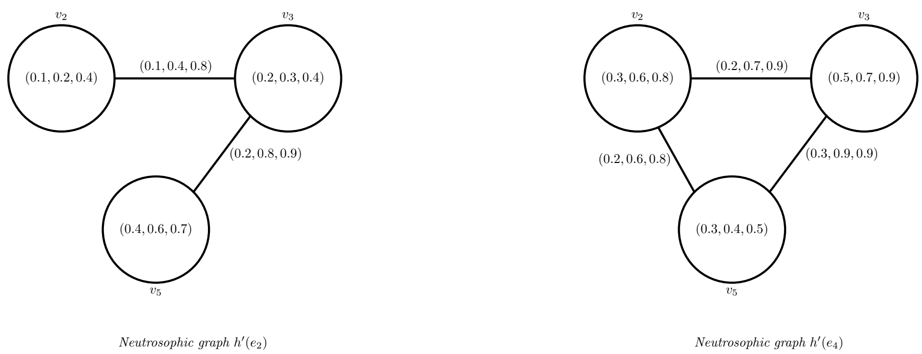

FIGURE 5. Ns-graph $G_N^{'} = (G^*, f^2, g^2, E_2)$

The parameter set of $G_N \bigcup G_N^{'} = (G^*, f, g, E)$ *is* $E = E_1 \cup E_2 = \{e_1, e_2, e_3, e_4\}$. *Moreover the ns-graph* $G_N \bigcup G_N^{'} = (G^*, f, g, E)$ *is obtained as in the Table 6 and Table 7.*

TABLE 6. Ns-set (f, E)

f	v_1	v_2	v_3	v_4	v_5
e_1	(0.1,0.2,0.3)	(0,1,1)	(0.2,0.3,0.4)	(0.2,0.5,0.7)	(0,1,1)
e_2	(0.1,0.3,0.7)	(0.1,0.2,0.4)	(0.2,0.4,0.4)	(0.1,0.2,0.3)	(0.4,0.6,0.7)
e_3	(0.5,0.6,0.7)	(0,1,1)	(0.6,0.8,0.9)	(0.3,0.4,0.6)	(0,1,1)
e_4	(0,1,1)	(0.3,0.6,0.8)	(0.5,0.7,0.9)	(0,1,1)	(0.3,0.4,0.5)

TABLE 7. Ns-set (g, E)

g	v_1v_2	v_1v_3	v_1v_4	v_1v_5	v_2v_3
e_1	(0,1,1)	(0.1,0.4,0.5)	(0.1,0.6,0.7)	(0,1,1)	(0,1,1)
e_2	(0,1,1)	(0.1,0.7,0.8)	(0.1,0.4,0.8)	(0,1,1)	(0.1,0.4,0.8)
e_3	(0,1,1)	(0,1,1)	(0.2,0.7,0.9)	(0,1,1)	(0,1,1)
e_4	(0,1,1)	(0,1,1)	(0,1,1)	(0,1,1)	(0.2,0.7,0.9)

g	v_2v_4	v_2v_5	v_3v_4	v_3v_5	v_4v_5
e_1	(0,1,1)	(0,1,1)	(0.1,0.6,0.8)	(0,1,1)	(0,1,1)
e_2	(0,1,1)	(0,1,1)	(0.1,0.8,0.9)	(0.2,0.8,0.9)	(0,1,1)
e_3	(0,1,1)	(0,1,1)	(0.2,0.8,0.9)	(0,1,1)	(0,1,1)
e_4	(0,1,1)	(0.2,0.6,0.8)	(0,1,1)	(0.3,0.9,0.9)	(0,1,1)

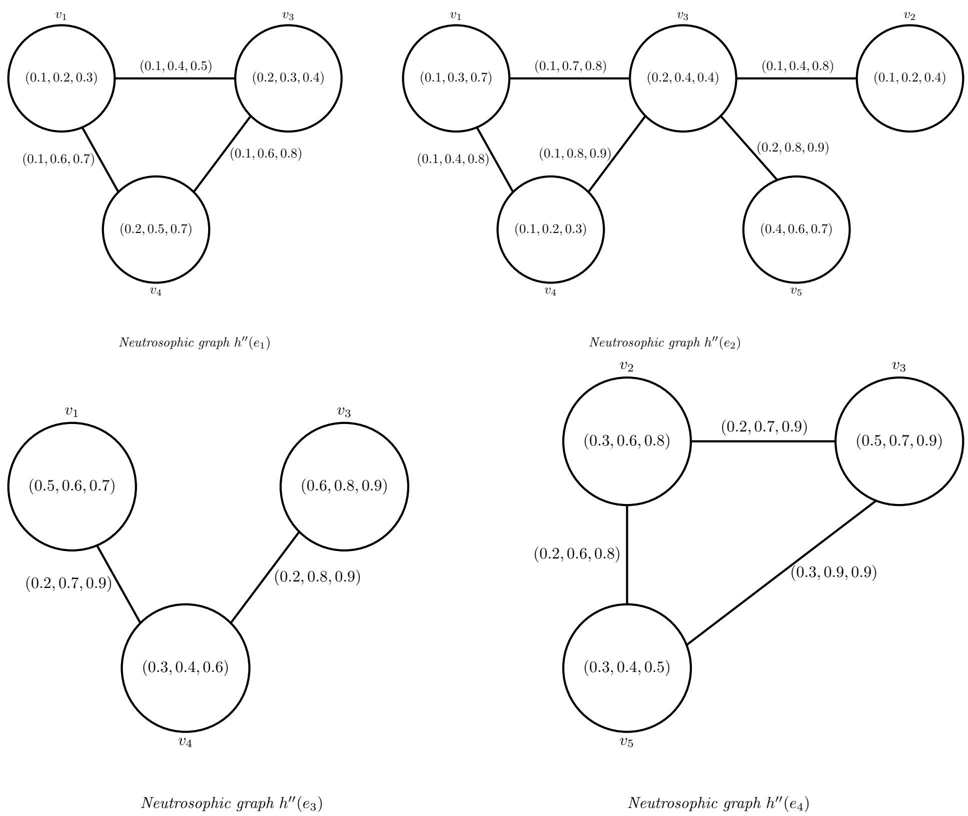

FIGURE 6. Ns-graph $G_N \bigcup G_N^{'} = (G^*, f, g, E)$

Definition 10. *Let* $G_N = (G^*, f^1, g^1, E_1)$ *and* $G_N^{'} = (G^*, f^2, g^2, E_2)$ *be two ns-graph over* $G^* = (\nu, \varepsilon)$. *The restricted union of* G_N *and* $G_N^{'}$ *is denoted by* $G_N \bigsqcup G_N^{'} = (G^*, f, g, E)$, *where* $E = E_1 \cap E_2$. *T, I and F membership values of vertices and edges of* $G_N \bigsqcup G_N^{'}$ *are defined by as follow.*

i. For all $e \in E$ *and* $x \in \nu$

$$T_{f_e}(x) = max\{T_{f_e^1}(x), T_{f_e^2}(x)\}$$
$$I_{f_e}(x) = min\{I_{f_e^1}(x), I_{f_e^2}(x)\}$$
$$F_{f_e}(x) = min\{F_{f_e^1}(x), F_{f_e^2}(x)\}$$

ii. For all $e \in E$ *and* $xy \in \varepsilon$

$$T_{g_e}(xy) = max\{T_{g_e^1}(xy), T_{g_e^2}(xy)\}$$
$$I_{g_e}(xy) = min\{I_{g_e^1}(xy), I_{g_e^2}(xy)\}$$

$F_{g_e}(xy) = min\{F_{g_e^1}(xy), F_{g_e^2}(xy)\}$

Example 5. *Let consider ns-graphs* $G_N = (G^*, f^1, g^1, E_1)$ *and* $G_N^{'} = (G^*, f^2, g^2, E_2)$ *as taken in Example 4. Clearly* $E = E_1 \cap E_2 = \{e_2\}$*. Also the restricted union of* G_N *and* $G_N^{'}$ *is obtained as follow.*

TABLE 8. Ns-sets (f, E) and (g, E)

f	v_1	v_2	v_3	v_4	v_5
e_2	(0.1,0.3,0.7)	(0.1,0.2,0.4)	(0.4,0.3,0.4)	(0.1,0.2,0.3)	(0.4,0.6,0.7)
g	v_1v_4	v_3v_4	v_1v_3	v_2v_3	v_3v_5
e_2	(0.1,0.4,0.8)	(0.1,0.8,0.9)	(0.1,0.7,0.8)	(0.1,0.4,0.8)	(0.2,0.8,0.9)

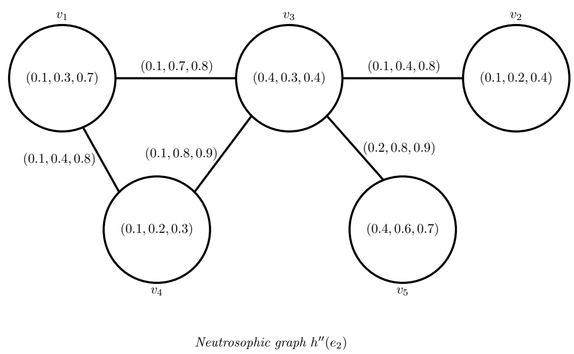

FIGURE 7. Ns-graph $G_N \bigsqcup G_N^{'} = (G^*, f, g, E)$

Definition 11. *Let* $G_N = (G^*, f^1, g^1, E_1)$ *and* $G_N^{'} = (G^*, f^2, g^2, E_2)$ *be two ns-graph over* $G^* = (\nu, \varepsilon)$*. The extended intersection of* G_N *and* $G_N^{'}$ *is denoted by* $G_N \bigcap G_N^{'} = (G^*, f, g, E)$*, where* $E = E_1 \cup E_2$*. T, I and F membership values of vertices and edges of* $G_N \bigcap G_N^{'}$ *are defined by as follow.*

i. For all $e \in E$ *and* $x \in \nu$

$$T_{f_e}(x) = \begin{cases} T_{f_e^1}(x), & e \in E_1 \backslash E_2 \\ T_{f_e^2}(x), & e \in E_2 \backslash E_1 \\ min\{T_{f_e^1}(x), T_{f_e^2}(x)\}, & e \in E_1 \cap E_2 \end{cases}$$

$$I_{f_e}(x) = \begin{cases} I_{f_e^1}(x), & e \in E_1 \backslash E_2 \\ I_{f_e^2}(x), & e \in E_2 \backslash E_1 \\ max\{I_{f_e^1}(x), I_{f_e^2}(x)\}, & e \in E_1 \cap E_2 \end{cases}$$

$$F_{f_e}(x) = \begin{cases} F_{f_e^1}(x), & e \in E_1 \backslash E_2 \\ F_{f_e^2}(x), & e \in E_2 \backslash E_1 \\ max\{F_{f_e^1}(x), F_{f_e^2}(x)\}, & e \in E_1 \cap E_2 \end{cases}$$

ii. For all $e \in E$ and $xy \in \varepsilon$

$$T_{g_e}(xy) = \begin{cases} T_{g_e^1}(xy) & e \in E_1 \backslash E_2 \\ T_{g_e^2}(xy) & e \in E_2 \backslash E_1 \\ min\{T_{g_e^1}(xy), T_{g_e^2}(xy)\}, & e \in E_1 \cap E_2 \end{cases}$$

$$I_{g_e}(x) = \begin{cases} I_{g_e^1}(xy) & e \in E_1 \backslash E_2 \\ I_{g_e^2}(xy) & e \in E_2 \backslash E_1 \\ max\{I_{g_e^1}(xy) I_{g_e^2}(xy)\}, & e \in E_1 \cap E_2 \end{cases}$$

$$F_{g_e}(xy) = \begin{cases} F_{g_e^1}(xy) & e \in E_1 \backslash E_2 \\ F_{g_e^2}(xy) & e \in E_2 \backslash E_1 \\ max\{F_{g_e^1}(xy), F_{g_e^2}(xy)\}, & e \in E_1 \cap E_2 \end{cases}$$

Example 6. *Let $G^* = (\nu, \varepsilon)$ be a simple graph with $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1v_2, v_1v_4, v_2v_4\}$. Let $E_1 = \{e_1, e_2\}$ be a set of parameters. Consider a ns-graph $G_N = (G^*, f^1, g^1, E_1)$ over $G^* = (\nu, \varepsilon)$ as taken in the Table 9.*

TABLE 9. Ns-sets (f^1, E_1) and (g^1, E_1)

f^1	v_1	v_2	v_3	v_4
e_1	*(0.1,0.2,0.3)*	*(0.2,0.4,0.5)*	*(0,1,1)*	*(0.1,0.5,0.7)*
e_2	*(0.2,0.3,0.7)*	*(0.4,0.6,0.7)*	*(0,1,1)*	*(0.3,0.4,0.6)*

g^1	v_1v_2	v_1v_3	v_1v_4	v_2v_3	v_2v_4	v_3v_4
e_1	*(0.1,0.5,0.6)*	*(0,1,1)*	*(0.1,0.5,0.7)*	*(0,1,1)*	*(0,1,1)*	*(0,1,1)*
e_2	*(0.2,0.7,0.8)*	*(0,1,1)*	*(0.1,0.6,0,7)*	*(0,1,1)*	*(0.2,0.7,0.9)*	*(0,1,1)*

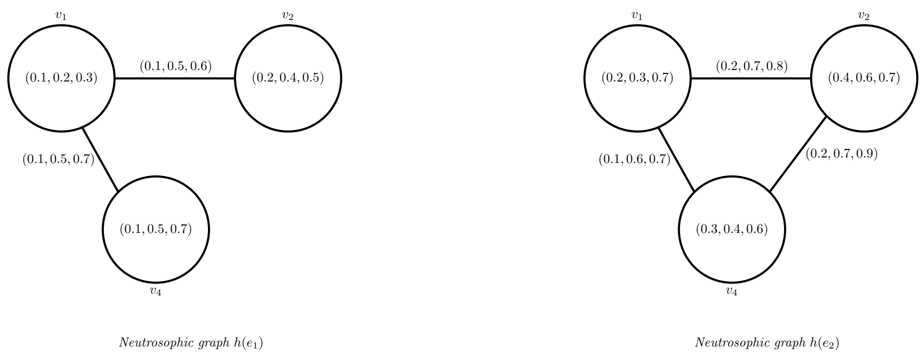

FIGURE 8. Ns-graph $G_N = (G^*, f^1, g^1, E_1)$

Now let consider another ns-graph $G_N^{'} = (G^*, f^2, g^2, E_2)$ *with the parameter set* $E_2 = \{e_2, e_3\}$ *as taken in the Table 10.*

TABLE 10. Ns-sets (f^2, E_2) and (g^2, E_2)

f^2	v_1	v_2	v_3	v_4
e_2	(0,1,1)	(0.3,0.5,0.6)	(0.2,0.4,0.5)	(0.4,0.5,0.9)
e_3	(0,1,1)	(0.2,0.4,0.5)	(0.1,0.2,0.6)	(0.1,0.5,0.7)

g^2	v_1v_2	v_1v_3	v_1v_4	v_2v_3	v_2v_4	v_3v_4
e_2	(0,1,1)	(0,1,1)	(0,1,1)	(0.1,0.6,0.7)	(0.2,0.6,0.9)	(0.2,0.7,0.9)
e_3	(0,1,1)	(0,1,1)	(0,1,1)	(0.1,0.5,0.8)	(0.1,0.7,0.8)	(0.1,0.8,0.9)

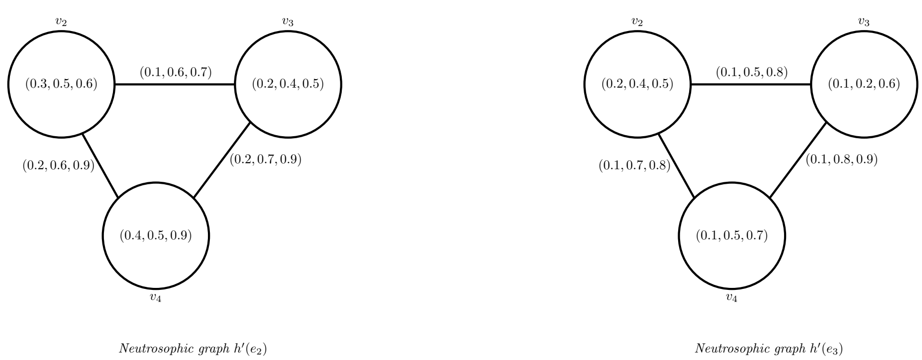

FIGURE 9. Ns-graph $G_N^{'} = (G^*, f^2, g^2, E_2)$

Clearly the parameter set of $G_N \bigcap G_N^{'}$ *is* $E = E_1 \cup E_2 = \{e_1, e_2, e_3\}$*. If the membership values of vertices and edges of* $G_N \bigcap G_N^{'}$ *are calculated, then the ns-graph* $G_N \bigcap G_N^{'} = (G^*, f, g, E)$ *is obtained as in the Table 11.*

TABLE 11. Ns-sets (f, E) and (g, E)

f	v_1	v_2	v_3	v_4
e_1	(0.1,0.2,0.3)	(0.2,0.4,0.5)	(0,1,1)	(0.1,0.5,0.7)
e_2	(0,1,1)	(0.3,0.6,0.7)	(0,1,1)	(0.3,0.5,0.9)
e_3	(0,1,1)	(0.2,0.4,0.5)	(0.1,0.2,0.6))	(0.1,0.5,0.7)

g	v_1v_2	v_1v_3	v_1v_4	v_2v_3	v_2v_4	v_3v_4
e_1	(0.1,0.5,0.6)	(0,1,1)	(0.1,0.5,0.7)	(0,1,1)	(0,1,1)	(0,1,1)
e_2	(0,1,1)	(0,1,1)	(0,1,1)	(0,1,1)	(0.2,0.7,0.9)	(0,1,1)
e_3	(0,1,1)	(0,1,1)	(0,1,1)	(0.1,0.5,0.8)	(0.1,0.7,0.8)	(0.1,0.8,0.9)

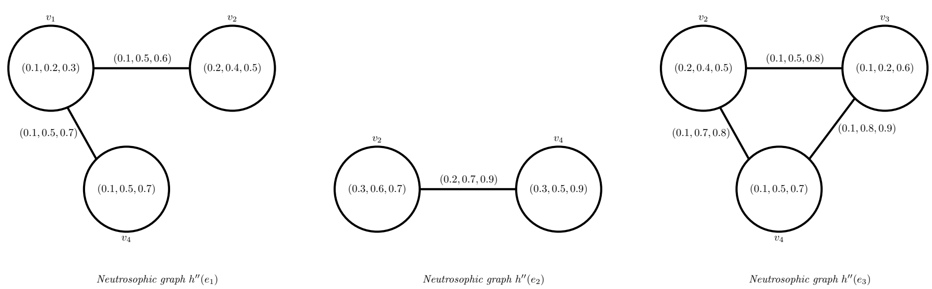

FIGURE 10. Ns-graph $G_N \bigcap G_N^{'} = (G^*, f, g, E)$

Definition 12. *Let* $G_N = (G^*, f^1, g^1, E_1)$ *and* $G_N^{'} = (G^*, f^2, g^2, E_2)$ *be two ns-graph over* $G^* = (\nu, \varepsilon)$*. The restricted intersection of* G_N *and* $G_N^{'}$ *is denoted by* $G_N G_N^{'} = (G^*, f, g, E)$*, where* $E = E_1 \cap E_2$*. T, I and F membership values of vertices and edges of* $G_N G_N^{'}$ *are defined by as follow.*

i. For all $e \in E$ *and* $x \in \nu$

$T_{f_e}(x) = min\{T_{f_e^1}(x), T_{f_e^2}(x)\}$

$I_{f_e}(x) = max\{I_{f_e^1}(x), I_{f_e^2}(x)\}$

$F_{f_e}(x) = max\{F_{f_e^1}(x), F_{f_e^2}(x)\}$

ii. For all $e \in E$ *and* $xy \in \varepsilon$

$T_{g_e}(xy) = min\{T_{g_e^1}(xy), T_{g_e^2}(xy)\}$

$I_{g_e}(xy) = max\{I_{g_e^1}(xy), I_{g_e^2}(xy)\}$

$F_{g_e}(xy) = max\{F_{g_e^1}(xy), F_{g_e^2}(xy)\}$

Example 7. *Let consider ns-graphs $G_N = (G^*, f^1, g^1, E_1)$ and $G_N^{'} = (G^*, f^2, g^2, E_2)$ as taken in Example 6. Clearly $E = E_1 \cap E_2 = \{e_2\}$. Also the restricted intersection of G_N and $G_N^{'}$ is obtained as follow.*

TABLE 12. Ns-sets (f, E) and (g, E)

f	v_1	v_2	v_3	v_4
e_2	(0,1,1)	(0.3,0.6,0.7)	(0,1,1)	(0.3,0.5,0.9)

g	(v_1v_2)	(v_1v_3)	(v_1v_4)	(v_2v_3)	(v_2v_4)	(v_3v_4)
e_2	(0,1,1)	(0,1,1)	(0,1,1)	(0,1,1)	(0.2,0.7,0.9)	(0,1,1)

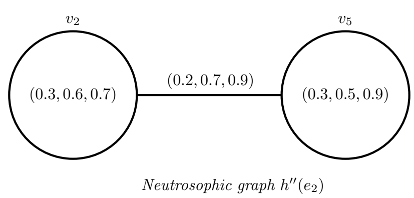

FIGURE 11. Ns-graph $G_N G_N^{'} = (G^*, f, g, E)$

Definition 13. *Let $G_N = (G^*, f, g, E)$ be an neutrosophic soft graph of $G^* = (\nu, \varepsilon)$ and $E = \{e_1, e_2, \ldots, e_n\}$ be a set of parameters. The $\vee-$union of subgraphs of G_N is denoted by $h(e) = h(e_1) \vee h(e_2) \vee \ldots \vee h(e_n)$ and for all $xy \in \varepsilon$*

$T_{h(e)}(xy) = max\{T_{g(e_1)}(xy), T_{g(e_2)}(xy), \ldots, T_{g(e_n)}(xy)\}$
$I_{h(e)}(xy) = min\{I_{g(e_1)}(xy), I_{g(e_2)}(xy), \ldots, I_{g(e_n)}(xy)\}$
$F_{h(e)}(xy) = min\{F_{g(e_1)}(xy), F_{g(e_2)}(xy), \ldots, F_{g(e_n)}(xy)\}$

Definition 14. *Let $G_N = (G^*, f, g, E)$ be an neutrosophic soft graph of $G^* = (\nu, \varepsilon)$ and $E = \{e_1, e_2, \ldots, e_n\}$ be a set of parameters. The $\wedge-$intersection of subgraphs of G_N is denoted by $h(e) = h(e_1) \wedge h(e_2) \wedge \ldots \wedge h(e_n)$ and for all $xy \in \varepsilon$*

$T_{h(e)}(xy) = min\{T_{g(e_1)}(xy), T_{g(e_2)}(xy), \ldots, T_{g(e_n)}(xy)\}$
$I_{h(e)}(xy) = max\{I_{g(e_1)}(xy), I_{g(e_2)}(xy), \ldots, I_{g(e_n)}(xy)\}$
$F_{h(e)}(xy) = max\{F_{g(e_1)}(xy), F_{g(e_2)}(xy), \ldots, F_{g(e_n)}(xy)\}$

4. An Application of Ns-Graphs in a Decision-Making Problem

Ns-graphs are important mathematical tool to cope with uncertainties occurs in real life problems. In this section, we have applied the concept of ns-graph to a decision-making problem and then we have gave an algorithm for optimal object selection by using given data. Suppose that $\nu = \{v_1, v_2, v_3, v_4, v_5\}$ be the set of five mobile phones under consideration. A customer is going to purchase a mobile phone on the basis certain parameters set $E = \{e_1 = \text{performance}, e_2 = \text{material quality}, e_3 = \text{price}\}$. Let (f, E) and (g, E) be two neutrosophic soft sets on ν and $\varepsilon = \{v_1v_2, v_1v_3, v_1v_4, v_1v_5, v_2v_3, v_2v_4, v_2v_5, v_3v_4, v_3v_5, v_4v_5\}$, respectively, as in the table 13. Where (f, E) describes the value of mobile phones according to given parameters, (g, E) describes the value obtained by comparing two mobile phones based upon each parameter.

Table 13. Ns-sets (f, E) and (g, E)

f	v_1	v_2	v_3	v_4	v_5
e_1	(0.1,0.5,0.3)	(0.2,0.4,0.5)	(0.5,0.6,0.1)	(0.3,0.2,0.1)	(0.5,0.7,0.1)
e_2	(0.3,0.5,0.1)	(0.3,0.2,0.1)	(0.7,0.5,0.4)	(0.2,0.1,0.8)	(0.4,0.3,0.6)
e_3	(0.2,0.3,0.2)	(0.4,0.3,0.5)	(0.6,0.4,0.3)	(0.3,0.2,0.6)	(0.1,0.4,0.5)

g	v_1v_2	v_1v_3	v_1v_4	v_1v_5	v_2v_3
e_1	(0.1,0.6,0.6)	(0,1,1)	(0,1,1)	(0.1,0.8,0.4)	(0.1,0.8,0.6)
e_2	(0.2,0.5,0.3)	(0,1,1)	(0,1,1)	(0.1,0.6,0.8)	(0.3,0.6,0.5)
e_3	(0.1,0.4,0.6)	(0,1,1)	(0,1,1)	(0.1,0.5,0.6)	(0.3,0.5,0.7)

g	v_2v_4	v_2v_5	v_3v_4	v_3v_5	v_4v_5
e_1	(0.2,0.7,0.7)	(0.1,0.8,0.5)	(0.2,0.7,0.2)	(0.3,0.7,0.5)	(0,1,1)
e_2	(0,1,1)	(0.3,0.4,0.7)	(0.2,0.6,0.8)	(0,1,1)	(0,1,1)
e_3	(0.2,0.4,0.8)	(0.1,0.7,0.8)	(0.1,0.6,0.6)	(0,1,1)	(0.1,0.5,0.7)

The matrice representations of neutrosophic graphs $h(e_1)$, $h(e_2)$ and $h(e_3)$ corresponding to the parameters e_1, e_2 and e_3, respectively, are represented by as follows.

$$h(e_1) = \begin{bmatrix} (0,1,1) & (0.1,0.6,0.6) & (0,1,1) & (0,1,1) & (0.1,0.8,0.4) \\ (0.1,0.6,0.6) & (0,1,1) & (0.1,0.8,0.6) & (0.2,0.7,0.7) & (0.1,0.8,0.5) \\ (0,1,1) & (0.1,0.8,0.6) & (0,1,1) & (0.2,0.7,0.2) & (0.3,0.7,0.5) \\ (0,1,1) & (0.2,0.7,0.7) & (0.2,0.7,0.2) & (0,1,1) & (0,1,1) \\ (0.1,0.8,0.4) & (0.1,0.8,0.5) & (0.3,0.7,0.5) & (0,1,1) & (0,1,1) \end{bmatrix}$$

$$h(e_2) = \begin{bmatrix} (0,1,1) & (0.2,0.5,0.3) & (0,1,1) & (0,1,1) & (0.1,0.6,0.8) \\ (0.1,0.6,0.6) & (0,1,1) & (0.3,0.6,0.5) & (0,1,1) & (0.3,0.4,0.7) \\ (0,1,1) & (0.3,0.6,0.5) & (0,1,1) & (0.2,0.6,0.8) & (0,1,1) \\ (0,1,1) & (0,1,1) & (0.2,0.6,0.8) & (0,1,1) & (0,1,1) \\ (0.1,0.6,0.8) & (0.3,0.4,0.7) & (0,1,1) & (0,1,1) & (0,1,1) \end{bmatrix}$$

$$h(e_3) = \begin{bmatrix} (0,1,1) & (0.1,0.4,0.6) & (0,1,1) & (0,1,1) & (0.1,0.5,0.6) \\ (0.1,0.4,0.6) & (0,1,1) & (0.3,0.5,0.7) & (0.2,0.4,0.8) & (0.1,0.7,0.8) \\ (0,1,1) & (0.3,0.5,0.7) & (0,1,1) & (0.1,0.6,0.6) & (0,1,1) \\ (0,1,1) & (0.2,0.4,0.8) & (0.1,0.6,0.6) & (0,1,1) & (0.1,0.5,0.7) \\ (0.1,0.5,0.6) & (0.1,0.7,0.8) & (0,1,1) & (0.1,0.5,0.7) & (0,1,1) \end{bmatrix}$$

If the operations $\vee$ and $\wedge$ are applied, we get resultant neutrosophic graphs $h(e)$ and $h'(e)$. Their incidence matrice are given by as follows.

$$h(e) = \begin{bmatrix} (0,1,1) & (0.2,0.4,0.3) & (0,1,1) & (0,1,1) & (0.1,0.5,0.4) \\ (0.1,0.4,0.6) & (0,1,1) & (0.3,0.5,0.5) & (0.2,0.4,0.7) & (0.3,0.4,0.5) \\ (0,1,1) & (0.3,0.5,0.5) & (0,1,1) & (0.2,0.6,0.2) & (0.3,0.7,0.5) \\ (0,1,1) & (0.2,0.4,0.7) & (0.2,0.6,0.2) & (0,1,1) & (0.1,0.5,0.7) \\ (0.1,0.5,0.4) & (0.3,0.4,0.5) & (0.3,0.7,0.5) & (0.1,0.5,0.7) & (0,1,1) \end{bmatrix}$$

$$h'(e) = \begin{bmatrix} (0,1,1) & (0.1,0.6,0.6) & (0,1,1) & (0,1,1) & (0.1,0.8,0.8) \\ (0.1,0.6,0.6) & (0,1,1) & (0.1,0.8,0.7) & (0,1,1) & (0.1,0.8,0.8) \\ (0,1,1) & (0.1,0.8,0.7) & (0,1,1) & (0.1,0.7,0.8) & (0,1,1) \\ (0,1,1) & (0,1,1) & (0.1,0.7,0.8) & (0,1,1) & (0,1,1) \\ (0.1,0.8,0.8) & (0.1,0.8,0.8) & (0,1,1) & (0,1,1) & (0,1,1) \end{bmatrix}$$

For a given neutrosophic set $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle\}$, the possible membership degree of an element x is calculated by $S(x) = \frac{1}{3}\left[T_A(x) + 1 - I_A(x) + 1 - F_A(x)\right]$.

Based on this formula, we construct the tabular representation of score value of incidence matrices and calculate choice value for each mobile phone v_k for $k = 1, 2, 3, 4, 5$ as follows.
If the arithmetic average of v'_k and v''_k are calculated, we find the average score values of $h(e)$ and $h'(e)$ as follow.
It is evident that the maximum score value is 1.150. Then the best choice for customer is mobile phone v_2.

Algorithm

1. Input the set E which express choice of parameters.
2. Determine the ns-sets (f, E) and (g, E).

TABLE 14. Score value of incidence matrice $h(e)$

	v_1	v_2	v_3	v_4	v_5	v'_k
v_1	0.0	0.500	0.0	0.0	0.400	0.900
v_2	0.367	0.0	0.434	0.366	0.467	1.634
v_3	0.0	0.434	0.0	0.467	0.366	1.267
v_4	0.0	0.366	0.467	0.0	0.300	1.500
v_5	0.400	0.467	0.366	0.300	0.0	1.533

TABLE 15. Score value of incidence matrice $h'(e)$

	v_1	v_2	v_3	v_4	v_5	v''_k
v_1	0.0	0.300	0.0	0.0	0.167	0.467
v_2	0.300	0.0	0.200	0.0	0.167	0.667
v_3	0.0	0.200	0.0	0.200	0.0	0.400
v_4	0.0	0.0	0.200	0.0	0.0	0.200
v_5	0.167	0.167	0.0	0.0	0.0	0.334

TABLE 16. Avarage score values of $h(e)$ and $h'(e)$

	v'_k	v''_k	v_k
v_1	0.900	0.467	0.683
v_2	1.634	0.667	1.150
v_3	1.267	0.400	0.833
v_4	1.500	0.200	0.850
v_5	1.533	0.334	0.933

3. Construct the ns-graph $G_N = (G^*, f, g, E)$.
4. Compute the resultant neutrosophic graphs $h(e)$ and $h'(e)$ with $h(e) = \underset{k\in\Lambda}{\vee} h(e_k)$ and $h'(e) = \underset{k\in\Lambda}{\wedge} h(e_k)$, respectively, for all $k \in \Lambda$.
5. Construct incidence matrice forms of $h(e)$ and $h'(e)$.
6. Calculate the score S_k of v_k for all $k \in \Lambda$.
7. Determine decision as v_k if $v_k^{'} = max\ v_k$.

5. Comparative Study and Discussion

In this section, for determining of optimal object, a comparative study based on the results of numerical computation is discussed. For this, based on the application discussed above, we present a comparative analysis between the our proposed method and the existing method in [4]. The method given in [4] just takes into consideration "AND" or "OR" operations to obtain resultant ns-graph which is used determine of optimal object. However if these two operations are applied separately, different results occur in the selection of the optimal object. So this leads to uncertainty in determining the most appropriate choice. Our proposed approach just takes into consideration both of $\wedge$-intersection and $\vee$-union operations to obtain resultant ns-graphs, because we observe that these two operations should be interdependent in determining the possible choice. When the existing method given in [4] is compared with the current proposed method, ranking orders both of them are appeared as in Table 17.

Table 17. Comparison

Model	Ranking order
Existing method given in [4]	$v_1 > v_3 > v_2 > v_4 > v_5$
	$v_5 > v_3 > v_2 > v_4 > v_1$
Proposed method	$v_2 > v_5 > v_4 > v_3 > v_1$

In existing method, the obtained ranking results are quite close where the first and fifth ranking order are changed but the second, third and fourth ranks are consistent. Nevertheless, obtaining two different rankings causes problems in decision-making process. In proposed method, the obtained ranking result is unique and more effective in determining the appropriate choice. Clearly, the proposed method consider the problem in all aspects and reveals a final result although existing method provides a set of alternatives as a final selection to consider the problem.

6. Conclusions

When compared with soft graph and fuzzy soft graph models, ns-graphs are more useful mathematical tools. Ns-graphs can be used in many areas with uncertainty. We have introduced the concept of ns-graphs of a simple graph with some new notions such as union and intersection, and gave illustrative examples related to these notions. Also we have applied the concept of ns-graph to a decision-making problem, and then a case study has been given to show the application of the technique. Hence a comparative analysis is conducted to show the applicability and validity of the proposed approach. The proposed method can be used in dealing with decision making process involving uncertainty especially in solving the real scientific and engineering problems. For future research, another algorithm can be developed by incorporate the complement of ns-graphs. Therefore we will work for the extension of the this method in different neutrosophic structures and decision-making applications. We plan to extend this research work to (i) Vague ns-graphs, (ii) Intuitionistic ns-graphs, and (iii) Bipolar ns-graphs.

Declaration of Competing Interests The author declares that he has no competing interest.

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