

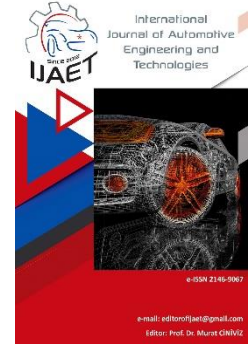


e-ISSN: 2146 - 9067

International Journal of Automotive Engineering and Technologies

journal homepage:

<https://dergipark.org.tr/en/pub/ijaet>



Original Research Article

Estimation of the shape factor (m) in Wiebe function at different operating conditions for a SI engine



Abdurrahman Demirci^{1,*}, Hüseyin Emre Doğan² and Osman Akın Kutlar³

^{1,*}, ² Karamanoğlu Mehmetbey University, Engineering Faculty, Department of Mechanical Engineering, 70200, Karaman, Türkiye.

³ Istanbul Technical University, Mechanical Engineering Faculty, Department of Mechanical Engineering, 34469, İstanbul, Türkiye.

ARTICLE INFO

Orcid Numbers

1. 0000-0002-2569-5174
2. 0000-0002-9445-3697
3. 0000-0002-4795-3541

Doi: 10.18245/ijaet.1234678

* Corresponding author

arahmandemirci@kmu.edu.tr

Received: Jan 16, 2023

Accepted: Jun 13, 2023

Published: 30 Sep 2023

Published by Editorial Board Members of IJAET

© This article is distributed by Turk Journal Park System under the CC 4.0 terms and conditions.

ABSTRACT

The burning process is one of the most important periods, which affects thermal efficiency and exhaust gas emissions, in internal combustion engines. The combustion process in internal combustion engines is modeled with one-dimensional or multi-dimensional software because it is cheaper, faster, and more practical than experiment. One of these methods, which is used to model the combustion period, is the Wiebe function. The Wiebe equation is an approach used in calculating the mass fraction burned and the heat release rate. The selection of Wiebe parameters is one of the most important factors affecting the accuracy of the mass fraction burned. In this study, the measured cylinder pressure of a spark ignition engine was directly used to calculate the heat released rate. The experiments were conducted at different brake mean effective pressures, engine speeds and relative air/fuel ratios, which were called independent variables. The shape factor (m) was determined by fitting the Wiebe equation to the heat release rate curves, which were extracted from the experimental results. The relationship between determined shape factor and independent variables was analyzed with a statistical approach. Eventually, a linear regression model, which explains 80% of the change in the shape factor, was created.

Keywords: Mass fraction burned, Combustion process, Heat release rate, Spark ignition engine, Wiebe function.

1. Introduction

The emission legislations are getting increasingly more stringent, and consumers are demanding improving of engine performance and decreasing of fuel consumption. Hence, many investigations on the internal combustion engines have been conducted for decades [1–3]. Engine emissions and fuel consumption are directly related to the combustion process and

burning speeds [4–6]. One of the most important variables in the combustion process is the burning speed or mass fraction burned rate, which directly affects the combustion process. In the theoretical studies, Wiebe and similar functions are used to determine the mass fraction burned of fuel [7, 8]. The Wiebe function is widely used in computer aided models to determine the optimum burning process. The choosing of parameters in the

Wiebe function is the most critical issue, which is important in the precise results of the function [7]. Numerous studies have been conducted on predictive Wiebe functions [9, 10].

Giglio and Gaeta studied the regression analysis for calculating the 50% mass fraction burned and to calculate the combustion duration in a 1.6 L spark-ignition engine [9]. Hu et al. [10] analyzed Wiebe parameters for an engine operating in biodiesel and diesel mode. They suggested a formula for Wiebe function, which has high correction ($R^2 > 0.997$). In another study, the Wiebe function parameters were estimated using the least squares method. More accurate Wiebe parameters were selected by making comparisons with the experimental results [11]. Arslan et al. [12] conducted a study about Wiebe parameters used for calculating mass fraction burned in a diesel engine for comparison of emission and efficiency parameters. They stated that the noise and NO_x emissions were reduced at the obtained maximum engine power. In addition, the maximum burn pressure decreased 23% in the same experimental condition.

In this study, experiments were carried out with gasoline fuel in a single-cylinder research engine at fourteen different operating conditions. A formula was created for calculating of the shape factor (m) in Wiebe function.

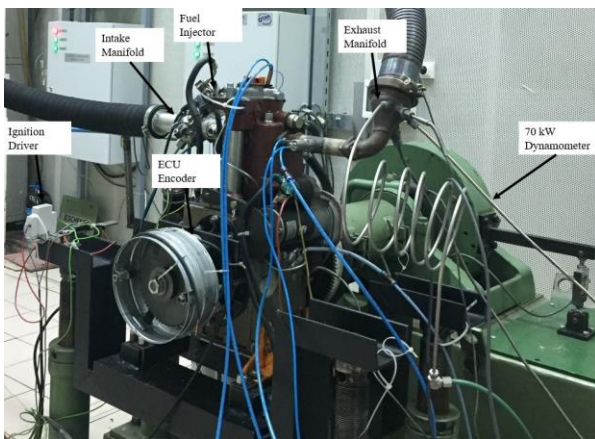


Figure 1: Research engine and experimental setup.

2. Experimental Method

A single cylinder, spark-ignition engine was used in the experiments. This engine was based on a direct-injection and water-cooled diesel engine, which was converted to a port fuel spark ignition engine (Figure 1) [15]. The engine has one intake and one exhaust valve and a central

spark plug. The technical specification of this engine was given in Table 1. In addition, ignition and injection systems of this engine were controlled by Arduino Mega 2560 board [16]. The gasoline fuel line pressure was selected 1.0 bar. Fuel was started to inject to the intake manifold at the 50 °CA after the TDC during the intake process.

Table 1: Research Engine Specifications.

Engine specifications	Description
Cylinder number	1
Fuel	Gasoline
Compression ratio	10.5
Engine volume, cm ³	454
Bore, mm	85
Stroke, mm	80
Fuel system	Electronically controlled, PFI
Ignition system	Electronically controlled, spark ignition
Charging	Naturally aspirated

In this study, a 70-kW eddy-current engine brake dynamometer was used. Torque measurement is conducted with a load cell after the mechanical revision of dynamometer. All the measured physical data such as temperature, speed, mean pressure was recorded during 90 seconds by the laboratory control system. An AVL GU13Z-24 piezoelectric transducer was used to measure the cylinder pressure. Cylinder pressure was recorded by a Kistler Kibox high speed data acquisition system.

Table 2: Experimental design and determined shape factors.

BMEP (bar)	Speed (rpm)	Relative Air/Fuel Ratio (-)	Shape Factor (m)
3	1500	1.0	2.2
3	1500	1.4	1.8
5	1500	1.0	2.7
5	1500	1.4	2.2
7	1500	1.0	2.7
7	1500	1.4	2.4
3	2000	1.0	2.2
3	2000	1.4	1.9
5	2000	1.0	2.3
5	2000	1.4	2.2
7	2000	1.0	2.4
7	2000	1.4	2.3
9.6	2500	1.0	2.6
7.6	2500	1.4	2.2

The cylinder pressure and the volume were recorded during 200 cycles in every 0.1° crank angle (°CA). The mass flow rate of gasoline was measured with the AVL 733S device and air flow rate was measured by a Dresser G65 flow

meter. Exhaust gas emissions were measured with a Horiba Mexa 7500. The inlet temperature of the cooling water was fixed at approximately 70°C. The experiments were carried out at the two different relative air/fuel ratios ($\lambda=1.0$ and $\lambda=1.4$), different brake mean effective pressure (BMEP=3.0, BMEP=5.0 and BMEP=7.0 bar, for 1500 and 2000 rpm and BMEP=7.6 and 9.6 bar for 2500 rpm). The spark advance was set for maximum brake torque (MBT) at each experimental condition (Table 2).

2.1 Wiebe function

The Wiebe function explains the mass fraction of burned fuel according to the crank angle during the combustion process in theoretical models [1,13,14]. Wiebe function was given as:

$$x_b = \left\{ 1 - e \left[-a \left(\frac{\theta - \theta_0}{\Delta\theta} \right)^{m+1} \right] \right\} \quad (1)$$

where x_b is the mass fraction burned, a is the efficiency parameter, m is the shape factor, θ is the crank angle in degrees, θ_0 is the start of combustion, $\Delta\theta$ is the burn duration. The Wiebe function has an S-shaped curve and is used to characterize the combustion process. The mass fraction burned in the combustion process ranges from 0 to 1. The zero fraction indicates that the combustion has not started yet. If x_b is equal to one, it means that all of the fuel burned in the process. The duration between these two cases might be defined as the combustion period. Although the Wiebe equation is a useful function in describing the combustion process, the selection of Wiebe parameters correctly is a challenge. In this study, the variation of the shape factor (m), which is the variable in determining the burning rate in the Wiebe function, for different loads at different engine speeds will be examined.

3. Regression Analysis

Regression is a concept that describes the form of the relationship between the independent and response (dependent) variables and helps to explain this interaction. On the other hand, regression analysis is summarizing the relationship between at least two variables (one dependent and the other independent) with the help of a mathematical function [17]. This function aims to define a statistical probability between the independent and dependent

variables.

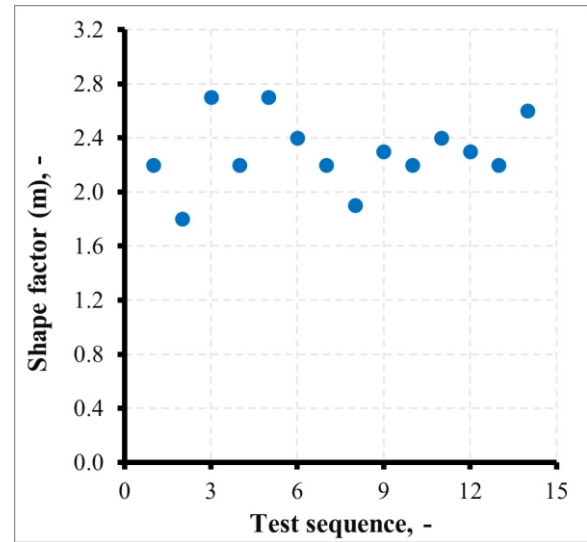


Figure 2: Scatter diagram of Wiebe shape factor.

In this study, statistical methods were used to determine the independent variables that are effective on the change in the response or dependent parameter, which was shown in equation 1 and directly describes mass fraction burned. Multiple linear regression analysis was performed to determine the relationship between the shape factor and the least two independent variables. There are three different independent variables, which were engine speed (n), brake mean effective pressure (BMEP), and relative air/fuel ratio (λ). The “ m ” parameter in the Wiebe function was selected as a dependent or response variable. According to the scatter diagram seen in Figure 2, it is possible to establish a linear relationship between the independent variables and the response parameter (shape factor). The least squares method is the most common approach used in regression analysis [18]. In this study, a linear relationship between dependent and independent variables was established using the least squares approach. The regression model expressing this relationship was given in equation 2 in basic form. Regression coefficients and independent variables affecting the response variable were obtained by regression analysis and ANOVA.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon \quad (2)$$

4. Results

Heat release rate, which was obtained from the experimentally measured cylinder pressure data with Rassweiler-Withrow approach [19], was

calculated to determine the value of the dependent variable (shape factor). Detailed description of this calculation can be found at reference [4]. In addition, the burn duration (5%-90% mass fraction burned) for each operating condition was calculated with the heat release rate to use in equation 1. Using these data, the shape factor in equation 1 was changed and the results of the Wiebe equation were matched to the experimentally calculated heat release rate curves. The result of the fitting process for an operating point was given in Figure 3. The shape factor was determined by this method for 14 different experimental conditions (Figure 2). Then, the relationship between changing of shape factor and the independent variables was determined. Microsoft Excel software was used in ANOVA and Regression analysis.

In each experimental conditions given in Table 2, the spark advances were selected according to the MBT. Pressure data obtained at a BMEP of 5 bar were given in Figure 4. Since the spark advance was selected according to MBT, the maximum pressure occurred 13-15 °CA after TDC. In lean mixtures, pressure values were higher at compression period. Because the throttle valve must be opened slightly, which was increased the inducted air into the cylinder, while reducing the amount of the fuel to obtain constant BMEP conditions in the lean mixture.

According to the experimental results in this study, operating with lean mixture instead of stoichiometric mixture increased efficiency at constant BMEP and engine speed.

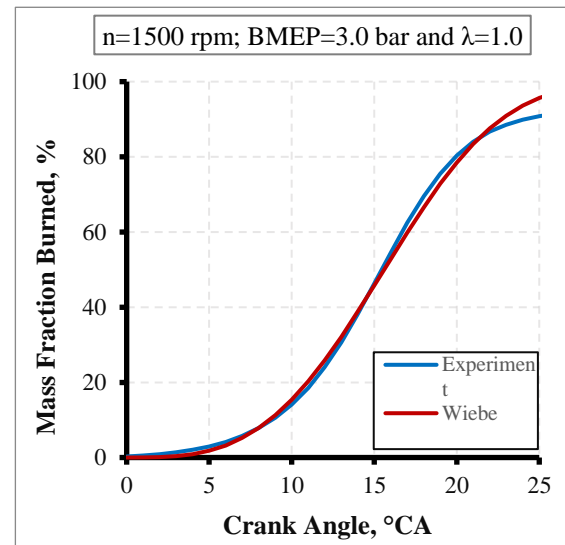


Figure 3: The mass burned fraction for experimental result and Wiebe function.

The heat release rates, which were calculated with the Rassweiler-Withrow method, using the pressure data shown in Figure 4. As can be seen in Figure 5, the heat release rates with the leaning of the mixture decreased. However, the integrated heat releases were the same levels in the stoichiometric and lean conditions. In addition, it was seen that the burn rate decreases in the lean mixture.

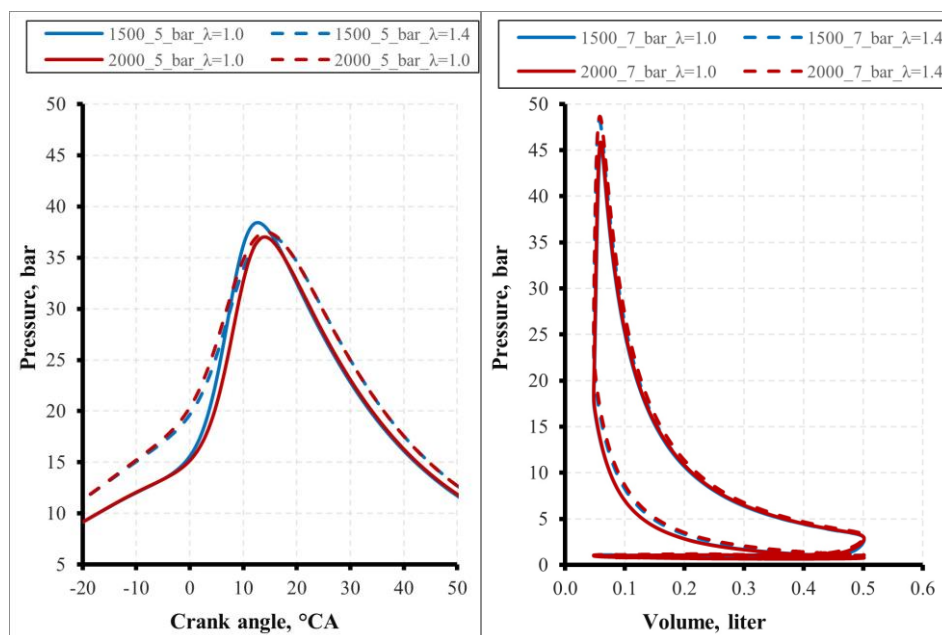


Figure 4: Cylinder pressure of different operating conditions.

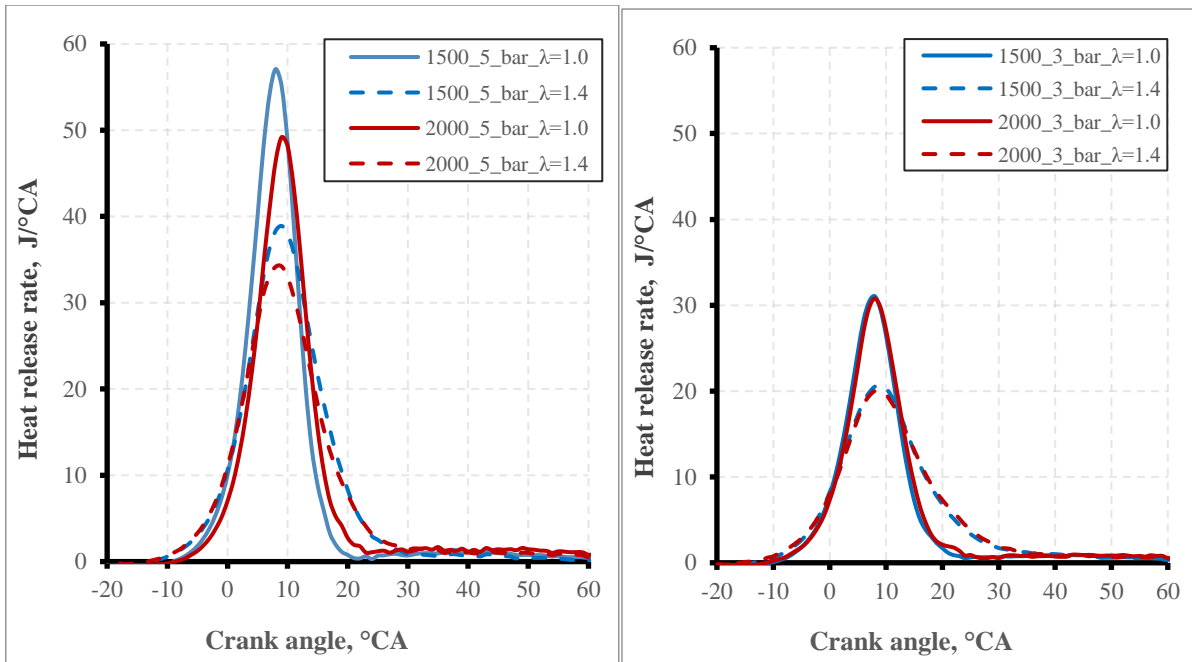


Figure 5: Heat release rate of different operating conditions at a BMEP of 5 and 3 bar.

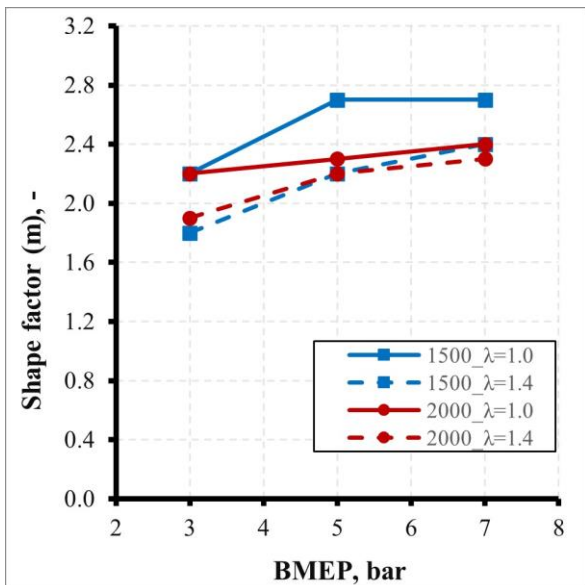


Figure 6: Variation of shape factor depending on the operating conditions.

The Wiebe function given in equation 1 is an empirical expression used in the theoretical calculation of the heat release rate or mass fraction burned. This study aims to determine the most appropriate shape factors in the Wiebe function for calculating the heat release rates more correctly. If a functional relationship is established between shape factor and the independent engine variables, some basic calculations can be made about the combustion process before the experimental studies. The variation of the shape factor according to the independent variables for 14 different experimental points used in this study was given in Table 2 and Figure 6. While shape factor took

smaller values at low loads, shape factor increased with increasing load (BMEP). The increase in shape factor depending on the load was continuous in lean mixtures (Figure 6). As given in the scatter diagram (Figure 2), it is seen from this graph that a linear relationship might be established between the shape factor and the independent variables.

The ANOVA results for the shape factor, which was determined at 14 experimental points, were given in Table 3. A null (H_0) hypothesis was suggested in here. All tests were evaluated according to the 95% confidence interval. According to claim in the null hypothesis, all three independent variables have no effect on the change of shape factor. The accuracy of this model was tested with ANOVA. The F value was calculated 13.0709 as seen in Table 3. It is found that $F_{0.05,3,10} = 3.71$ from the F distribution tables. The H_0 hypothesis was rejected according to these results. That is, at least one of the three independent variables influences the variation of shape factor.

Table 3: ANOVA results.

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	3	0.7809	0.2603	13.0709	0.0009
Residual	10	0.1991	0.0199		
Total	13	0.9800			

However, it is difficult to understand which one of these three variables has an effect or not with

the F test. Because of that, the effect of each independent variable on the shape factor was determined with the t test. Here, the null hypothesis (H_0) is again established as $\beta_1 = \beta_2 = \beta_3 = 0$. Statistical t and P -value were given along with the regression coefficients in Table 4. If the P value of each independent variable is less than 0.05, the null hypothesis was rejected. Accordingly, all three independent variables are effective on the change of the “ m ” parameter ($\beta_1, \beta_2, \beta_3 \neq 0$) due to its very low P value. The regression model (equation 3) was created using the regression coefficients in Table 4. According to the calculated coefficients, there is a positive relationship between BMEP and m parameter. However, engine speed and relative air/fuel ratio have a negative relationship with “ m ” parameter.

$$m = 3.16 + 0.1BMEP - 0.0003n - 0.71\lambda \quad (3)$$

Table 4: Regression coefficients and P-values

	Coefficients	Standard Error	t Stat	P-value
Intercept	3.16	0.3058	10.3284	0.0000011
BMEP	0.1	0.0214	4.6668	0.0008852
Engine speed	-0.0003	0.0001	-2.4402	0.0348391
Relative air/fuel ratio	-0.71	0.1892	-3.7716	0.0036519

The R square, which is a measure of the success of the regression equation, was calculated as 0.797 (Table 5). Accordingly, approximately 80% of the variation of the shape factor is explained by the established regression model (equation 3). There is a slight difference between the adjusted R square and the R square (Table 5). Thus, it can be stated again that all the independent variables in the regression model have a non-ignorable effect on the shape factor. Therefore, it is not necessary to remove any independent variable from the regression model. However, interactions of independent variables might be included in the model to make the model stronger. In this way, it is possible to increase the R square values even more. The shape factors obtained according to the regression model and the experimentally determined shape factors were shown in Figure 7. As it can be seen from the Figure 7, these values are compatible with each other.

The shape factors were determined and

calculated with the created formula for different experiment points ($\lambda=1.2$ and $\lambda=1.6$ for at a BMEP of 5 bar at the different engine speeds), which were not used in the regression model. The calculated shape factors were shown in **Hata! Başvuru kaynağı bulunamadı.**8. Thus, the accuracy of the created formula was tested. It has been observed that the created formula calculates the shape factor with an accuracy of over 96% at each experiment points for not described in the model.

Table 5: Results of regression analysis.

Regression statistics	
Multiple R	0.893
R Square	0.797
Adjusted R Square	0.736
Standard Error	0.141
Observation	14

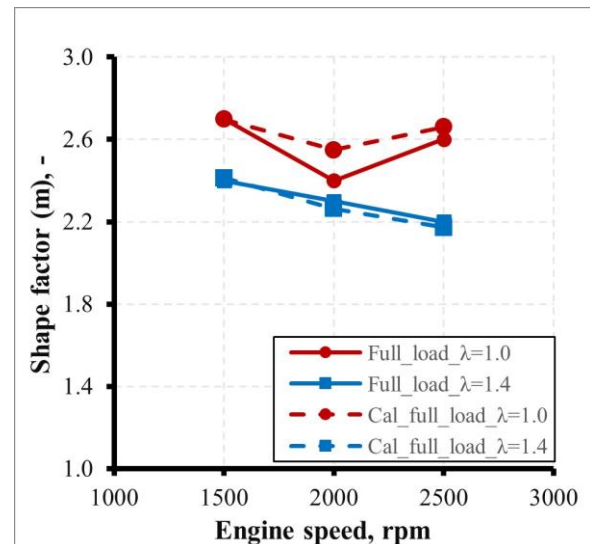


Figure 7: Comparison of experimental and fitting regression model results.

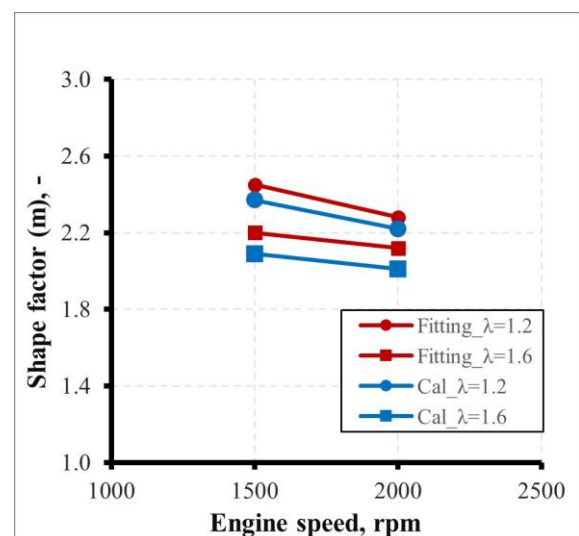


Figure 8: Comparison of shape factors for experiment and fitting.

points different from regression model.

5. Conclusion

In this study, experiments were carried out with gasoline fuel in a single-cylinder research engine for fourteen different operating conditions. By using the measured cylinder pressure data, the heat release rates were calculated. Using these data, the change of shape factor in equation 1 was obtained. The results of the Wiebe equation, which was widely used in theoretical calculations, were matched to the experimentally calculated heat release rate curves. Therefore, the most appropriate value of the shape factor in the Wiebe equation was investigated for each experimental condition. The values of the response variable (shape factor) were transferred to the scatter diagram, and it was seen that there might be a linear relationship between the independent variables and the response variable (shape factor). As a result of the ANOVA analysis, it was determined that the independent variables, which are engine speed, relative air/fuel ratio and BMEP, were effective on the change in the shape factor in the established regression equation/model. The established regression model with these three variables explained 80% of the change in the response parameter (shape factor). It has been observed that the created formula calculates the shape factor with an accuracy of over 96% at each experiment points, which were not used in the model. By using the created equation together with the Wiebe function, it is possible to obtain more precise and easy results in theoretical combustion analysis. In future studies, to improve the regression model, the interactions between independent variables can be examined. In addition, the flexibility of the model can be greatly expanded by examining the effect of different combustion chamber designs on the shape factor.

Nomenclature

ANOVA	: Analysis of variation
BMEP	: Brake mean effective pressure
λ	: Relative air/fuel ratio
H_0	: Null hypothesis
m	: Shape factor
MBT	: Maximum brake torque
n	: Engine speed

TDC	: Top dead center
x_B	: Mass fraction burned

Acknowledgment

The authors thank to the undergraduate students who are Abdullah Said Kalın, Tuğrul Çengel and Harun Dayı because of their invaluable helps.

CRedit authorship contribution statement

Abdurrahman Demirci: Conceptualization, Investigation, Validation, Writing-Original Draft. **Hüseyin Emre Doğan:** Conceptualization, Investigation, Formal analysis, Writing – Review & Editing. **Osman Akin Kutlar:** Conceptualization, Methodology.

Declaration of conflicting interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

6. References

- Heywood JB., “Internal Combustion Engine Fundamentals”, NewYork: McGraw-Hill Education; 1988. <https://doi.org/10.5860/choice.26-0943>.
- Pulkrabek WW., “Engineering Fundamentals of the Internal Combustion Engines”, USA: Pearson Prentice Hall; 2004.
- Demirci A., “The effects of different combustion chamber geometries on the performance and emissions of an internal combustion engine”, Istanbul Technical University, Ph.D. Thesis; 2017.
- Doğan HE, Kutlar OA, Javadzadehkalkhoran M, Demirci A., “Investigation of burn duration and NO emission in lean mixture with CNG and gasoline”, *Energies*, Volume:12 Issue: 4432. DOI: <https://doi.org/10.3390/en12234432>.
- Pischinger F, Walzer P. Future trends in automotive engine technology. XXVI'th Congress FISITA, vol. 3, Prague: 1996.
- Menacer B, Bouchetara M., “The automotive engine modeling for steady-state condition using FORTRAN/GT-POWER”, *International Journal of Automotive Engineering and Technologies*, Volume:4 Issue:3, pp:118–29. 2015.
- Kutlar OA, Cihan Ö., “Investigation of

Parameters Affecting Rotary Engine by Means of a One Zone Thermodynamic Model”, *J Energy Resour Technol*, Volume: 144, Issue:4, 042304. 2022. DOI:

<https://doi.org/10.1115/1.4052615>.

8. Kutlar OA, Malkaz F., “Two-Stroke Wankel Type Rotary Engine: A New Approach for Higher Power Density” *Energies*, Volume:12, 4096, 2019. DOI:

<https://doi.org/10.3390/en12214096>.

9. Giglio V, di Gaeta A., “Novel regression models for Wiebe parameters aimed at 0D combustion simulation in spark ignition engines”, *Energy*, Volume: 210, 118442, 2020. DOI:

<https://doi.org/10.1016/j.energy.2020.118442>.

10. Hu D, Wang H, Wang B, Shi M, Duan B, Wang Y, et al., “Calibration of 0-D combustion model applied to dual-fuel engine”, *Energy*, Volume: 261 Issue: Part B, 125251, 2022. DOI:

<https://doi.org/10.1016/J.ENERGY.2022.125251>.

11. Yeliana Y, Cooney C, Worm J, Michalek DJ, Naber JD., “Estimation of double-Wiebe function parameters using least square method for burn durations of ethanol-gasoline blends in spark ignition engine over variable compression ratios and EGR levels”, *Appl Therm Eng*, Volume: 31, Issues: 14-15, 2011. DOI:

<https://doi.org/10.1016/j.applthermaleng.2011.01.040>.

12. Arslan H, Kutlar OA, Mehdiyev R., “Development of a combustion chamber for optimum combustion process in diesel engines”, *Uluslararası Yakıtlar Yanma Ve Yangın Dergisi*, Volume:1, pp. 50–58, 2019.

13. Yontar AA., “Effects of ethanol, methyl tert-butyl ether and gasoline-hydrogen blend on performance parameters and HC emission at Wankel engine”, *Biofuels*, Volume: 11, Issue: 3, pp: 377-388, 2020. DOI:

<https://doi.org/10.1080/17597269.2019.1613765>.

14. Yontar AA., “Numerical Comparative Mapping Study to Evaluate Performance of a Dual Sequential Spark Ignition Engine Fuelled with Ethanol and E85”, *International Journal of Automotive Engineering and Technologies*, Volume: Issue:3, pp. 98–106, 2018. DOI:

<https://doi.org/10.18245/ijaet.486405>.

15. Kutlar OA., “A New method to decrease the fuel consumption at part load conditions of four stroke ottocycle (rochas) engine (skipperiod engine)”, *Istanbul Technical University, Ph. D. Thesis*, 1999.

16. Tekeli Ö., “Designing and Production Ignition and Injection Units of a Gasoline Engine with Skip Cycle”, *Istanbul Technical University, Master Thesis* 2013.

17. Güler F., “Istatistik Methods and Applications”, *Türkiye, Beta Basım Yayım*, 2012.

18. Montgomery DC., “Design and analysis of experiments”, 8th ed. *John wiley & sons*, 2017.