Fermi Problemlerinde Öğretmen Adayları Tarafından Kullanılan Modelleme Stratejileri: Otomatik Sulama Sistemi Görevi

Zeynep ÇAKMAK GÜREL, Erzincan Binali Yıldırım Üniversitesi, ORCID: 0000-0003-0913-3291

Öz

Bu çalışmanın amacı matematik öğretmeni adaylarının tahmin içeren matematiksel modelleme etkinliklerinde ortaya çıkan model stratejilerinin ve çözüm uzaylarının incelenmesidir. Araştırmada durum çalışması yöntemi kullanılmıştır. Çalışmaya 119 dördüncü sınıf matematik öğretmeni adayı katılmıştır. Veriler iki haftada toplanmıştır. Öğretmen adayları dörder ya da beşer kişilik olacak şekilde grup oluşturmuşlardır. Toplamda 26 farklı grup bulunmaktadır. Fermi problemi, matematiksel modelleme kriterleri dikkate alınarak araştırmacı tarafından tasarlanmıştır. Veri toplama araçlarını çalışma kağıtları ve sunum yapılan esnada alınan kayıtlar oluşturmaktadır. Verilerin analizinde ise tümdengelim ve tümevarım analiz yöntemleri birlikte kullanılmıştır. Elde edilen bulgulara göre, öğretmen adaylarının beş farklı modelleme stratejisi kullandıkları belirlenmiştir. Kapalı olmayan bir yüzeye sığabilecek eleman sayısı bağlamında verilen problemde en fazla "referans noktası" en az "konsantrasyon ölçümü" stratejisi kullanılmıştır. Çözüm uzaylarında fıskiyelerin kapladığı alan yerine bir fıskiye modelinin kapladığı alanın referans alınmasının daha gerçekçi sonuçlar verdiği görülmüştür. Bu stratejiler bağlamında oluşturulan çözüm uzayları modelleme sürecini ayrıntılandırmaktadır. Benzer bir modelleme problemi uygulayacak olan öğretmenlerin oluşan farklı modelleme stratejilerini ve çözüm uzaylarını önceden bilmesi, öğrencilere destek olması açısından önem arz etmektedir. Çözüm uzaylarının öğretmenlere öğrencilerinin bilgilerini takip etmede bir kaynak sağlayacağı düşünülmektedir.

Anahtar Kelimeler: çözüm uzayları, Fermi problemi, matematiksel modelleme, model stratejileri

İnönü Üniversitesi Eğitim Fakültesi Dergisi
Vol 24, No 2, 2023
pp. 980-1003
DOI
10.17679/inuefd.1235549

Makale Türü
Research Article

Gönderim Tarihi
15.01.2023

Kabul Tarihi
31.08.2023

Önerilen Atıf

Çakmak Gürel, Zeynep. (2023). Fermi Problemlerinde öğretmen adayları tarafından kullanılan modelleme stratejileri: otomatik sulama sistemi görevi, *İnönü Üniversitesi Eğitim Fakültesi Dergisi*, *24*(2), 980-1003. DOI: 10.17679/inuefd.1235549

GENİŞLETİLMİŞ ÖZET

Giriş

Matematiksel modelleme ve tahmin becerisi önemli matematiksel düşünme becerileri arasında yer almaktadır (Sriraman & Lesh, 2006). Özellikle matematik eğitiminde Fermi problemlerinden matematiksel modelleme ve tahmin becerisi şeklinde iki bağlamda bahsedilmektedir (Bergman & Bergsten, 2010). Fermi problemi tahmin içeren bir gerçekliğin basitleştirilmesini ve matematikselleştirilmesini gerektiren bir problem türüdür (Gallart, vd., 2017). Fermi problemleri, ilk olarak 1938 yılında Nobel ödülü kazanan fizikçi Enrico Fermi tarafından ortaya atılmıştır (Bergman & Bergsten, 2010). Bu problemler, bir problemde ortaya çıkabilecek değişkenler ve nicelikler hakkında tahminler yapmak için pedagojik amaçla kullanılan tahmin problemleridir (Sriraman & Lesh, 2006). Matematiksel modellemeye girişte Fermi problemlerinin potansiyelini araştıran çalışmalarda, Fermi problemlerinin öğrencilerin modelleme döngüsü boyunca çalışmalarına fırsat sunduğu tespit edilmiştir (Bergman & Bergsten, 2010; Borromeo Ferri, 2018). Matematiksel modelleme günümüzde git gide daha popüler hale gelmekte ve uluslararası okul müfredatlarında yerini almaktadır (Blomhøj & Kjeldsen, 2006). Matematiksel modelleme gerçek hayat problemlerini çözmek için matematiksel yöntemlerin kullanılması şeklinde açıklanmaktadır (Stender & Kaiser, 2015). Matematiksel modelleme sürecini inceleyen birçok çalışma olmasına rağmen nihai oluşan matematiksel modele yeterince odaklanılmamıştır (Gallart, vd. 2017). Nihai model stratejilerinin incelendiği çalışmalarda öğrencilerin, Fermi problemlerine yönelik kapsamlı sayma, dış kaynak, indirgeme ve orantı kullanımı, konsantrasyon ölçümü, referans noktası ve ızgara dağılımı stratejilerini kullandıkları belirlenmiştir (Albarracin & Gorgorio, 2019; Albarracin, Ferrando & Gorgorio, 2021). Yapılan çalışmalarda ilkokuldan lise seviyesine kadar öğrencilerin modelleme stratejileri tespit edilmiştir. Bu çalışmada ise matematik öğretmen adaylarının modelleme stratejilerinin ortaya konulması amaçlanmıştır. Modelleme stratejilerinin geniş bir yelpazede belirlenmesi, öğretmenlerin öğrencilerin modelleme stratejilerini önceden fark etmelerini sağlayacaktır. Özellikle öğretmenlerin öğrencilerin farklı modelleme stratejilerini önceden bilmesi, onlara destek olması açısından önemlidir. Ayrıca öğretmenlerin fark ettiği farklı modelleme stratejilerini öğrenciler de fark edecek ve kendi öğrenmelerini yönetebileceklerdir (Ferrando & Albarracín, 2019). Tüm bunlar modelleme stratejilerinin ortaya konulması açısından önemlidir. Fakat stratejiler sadece ortaya çıkan model hakkında bir fikir sunmaktadır. Bu stratejiler bağlamında çoklu çözüm yollarının belirlenmesi de oldukça önemlidir (Albarracin, vd., 2021). Böylece çözüm sürecini yansıtan çözüm uzayları sadece sonucun değil aynı zamanda sürecin de analiz edilmesine fırsat sunmaktadır. Bu çalışmada Leikin (2007) tarafından ortaya konan çözüm uzayları fikri kullanılmıştır. Çözüm uzayları, çoklu çözüm yollarının birleştirilmesi ile bilginin aynası niteliği taşımaktadır (Leikin, 2007; Leikin & Levav-Waynberg (2008).

Amaç

Bu çalışmanın amacı öğretmen adaylarının matematiksel modelleme etkinlikleri sırasında ortaya çıkan modelleme stratejilerinin ve çözüm uzaylarının incelenmesidir.

1. Öğretmen adaylarının matematiksel modelleme etkinlikleri sırasında ortaya çıkan modelleme stratejileri nelerdir?
2. Öğretmen adaylarının stratejilere yönelik ortaya çıkan çözüm uzayları nasıldır?

Yöntem

Bu çalışmada nitel araştırma desenlerinden durum çalışması yöntemi kullanılmıştır. Durum çalışması yöntemi, araştırmaya dair durumun belirlenmesi ve belirlenen durumun derinlemesine incelenmesidir (Bogdan & Biklen, 2007). Durum olarak stratejiler kullanılırken, bu stratejilere ait çözüm uzayları derinlemesine inceleme yapmayı sağlamaktadır.

Bu araştırmanın çalışma grubunu ilköğretim matematik öğretmenliği okuyan 119 dördüncü sınıf öğretmen adayı oluşturmaktadır. Çalışmada öğretmen adayları kendi isteklerine göre dört ya da beş kişilik olacak şekilde gruplar oluşturmuştur. Böylece toplamda 26 farklı grup bulunmaktadır. Öğretmen adayları Fermi problemleri ile ilk defa bu araştırma kapsamında karşılaşmışlardır

Veri toplama aracında yer alan Fermi problemi: "Bulunduğunuz üniversite kampüsüne otomatik sulama sistemi kurulması planlanmaktadır. En uygun sulama sistemi için kaç adet fıskiye alınması gerektiğini belirlemek için bir model geliştiriniz ve sonuçlarınızı paylaşınız" şeklindedir. Veri toplama süreci, ilk hafta problemin çözümü ve ikinci hafta çözümlerin sunumu olacak şekilde toplam iki hafta sürmüştür. Sunum esnasında araştırmacı çözüm süreçlerine açıklık getirmek amacıyla sorular yöneltmiştir. Bu süreç gözlem notları ile kayıt altına alınmıştır. Çalışmada 26 grubun çalışma kâğıdı veri kaynağı olarak kullanılmıştır.

Veri analizinde Mayring (2015) tarafından tanımlanan nitel veri analizi yöntemlerinden tümdengelim ve tümevarım analiz yöntemleri birlikte kullanılmıştır. Her iki yöntemin birlikte kullanılmasını sağlayan karma yönteme içerik yapılandırma yöntemi denilmektedir (Mayring, 2014). Öncelikle öğretmen adaylarının modelleme stratejileri Albarracin, Ferrando ve Gorgorio (2021) tarafından ortaya konan modelleme stratejileri dikkate alınarak analize tabi tutulmuştur. Ardından aynı stratejileri kullanan gruplar bir araya getirilerek ortak çözüm uzayları elde edilmiştir.

Bulgular ve Tartışma

Bu çalışmada Fermi problemi çözümünde öğretmen adaylarının matematiksel modelde kullandıkları stratejileri ortaya konulmuştur. Kapalı olmayan bir yüzeye sığabilecek eleman sayısı bağlamında verilen problemde en fazla "referans noktası" en az "konsantrasyon ölçümü" stratejisi kullanılmıştır. Ferrando, Segura & Pla-Castells (2020; 2021) çalışmalarında, belirli bağlamsal özellikler ile kullanılan strateji arasında anlamlı bir ilişki bulunmuştur. Özellikle tahmin edilecek elemanların düzenli bir şekil ile geniş bir alanı kapladığı Fermi problemlerinin referans noktası stratejisi ile ilişkili olduğu görülmüştür. Bu durumda, sunulan problemin bağlamsal özellikleri, fıskiyelerin kapsadığı alan geniş ve düzenli olduğundan (daireler) referans noktası stratejisiyle ilişkiyi doğrular. Ayrıca öğrencilerin üst sınıflara doğru ilerledikçe daha çok referans noktası stratejilerini kullanmaya başladıkları da belirlenmiştir (Ferrando ve Albarracín, 2019).

Bu çalışmada öğretmen adaylarının kullandıkları stratejiler incelenirken, aynı zamanda bu stratejilere ilişkin çözüm uzayları açığa çıkarılmıştır. Engellerle kaplı olan bir alanı sulama görevine ait çözüm uzayları, bir alana sığabilecek ağaç sayısını veya insan sayısını belirlemek için oluşturulan çözüm uzaylarından (Ör: Albarracin, vd., 2021) farklılaşmaktadır. Tüm yeşil alanların toplam değerinin bulunması sulanması gereken alanın bir bütünmüş gibi algılanmasına neden olmuştur. Fakat ayrıştırılmış yeşil alanların hesaplanması ve her bir ayrıştırılmış yeşil alana sığacak fıskiye sayısının belirlenmesi daha fazla gerçeği yansıtan bir çözüm uzayını göstermektedir. Ayrıca sadece yerleştirilen fıskiye sayısının değil bu fıskiyelerin nasıl yerleştirilmesi gerektiği de (fıskiye modeli) önemli bir modelleme becerisi gerektirmektedir. Ayrıştırılmış alanlarla fıskiye modelini birlikte kullanan bir grubun olmaması dikkat çeken bir sonuçtur. Böylece çözüm uzaylarının, her bir çözümü bir araya getirerek göreve ait geçerli bir çözüm yolu sunması, çoklu çözüm içeren problemlerin öğretimi için önemlidir. Bu çözüm uzayının öğretmenlere öğrencilerinin bilgilerini takip etmede bir kaynak sağlayacağı düşünülmektedir.

Sonuç

Bu çalışmada problemin bağlamının stratejiler üzerinde etkisi olduğu tespit edilmiştir. Böylece referans noktası stratejisinin en çok kullanılmasının nedeni, sorunun bağlamından kaynaklanmıştır. Ayrıca bu çalışma önceki çözüm uzaylarını genişletmekte ve referans noktası stratejisini detaylı olarak incelemektedir. Çalışmada kullanılan problemdeki bir elemanın kapladığı alan yerine 'sulama modelinin' kapladığı alanı referans almanın daha gerçekçi sonuçlar verdiği görülmüştür. Bu problemde sulanmayacak alan kalmaması veya fazla sulama yapılmayacak olması referans noktası stratejisinde 'sulama modelinin' kullanılmasını zorunlu kılmıştır. Bu stratejide, boş bir tarlanın sulanmasının, engelli bir arazinin sulanması kadar gerçekçi sonuçlar vermediği bulunmuştur. Fakat öğretmen adaylarının birçoğu ne sulama modelini ne de engelleri olan bir arazinin sulanması ile ilgili detayları dikkate almamışlardır.

The Modelling Strategies Used by the Pre-service Teachers in Fermi Problems: The Task of Automatic Irrigation System

Zeynep ÇAKMAK GÜREL, Erzincan Binali Yıldırım University, ORCID: 0000-0003-0913-3291

Abstract

This study aims to explore pre-service mathematics teachers' solution spaces and model strategies in the mathematical modelling tasks involving estimations. A case study method was employed in the research. 119 pre-service mathematics teachers participated in the study. The data were collected in two weeks. The pre-service teachers created groups consisting of four or five individuals. 26 different groups emerged in total. The Fermi problem was designed by the researcher by considering mathematical modelling task criteria. The data collection tools consisted of the worksheets and presentation records of the pre-service teachers. The deduction and induction analysis methods were used together in the data analysis. According to the obtained findings, it was found that the pre-service teachers used five different modelling strategies. In the problem given within the scope of the number of elements that could fit in a surface, the "reference point" was used most; on the other hand, the "concentration measures" were used least. It is found that taking the area covered by a sprinkler model as a reference rather than the area occupied by sprinklers in the solution spaces produces more realistic results. The solutions spaces created within these strategies was used to detail the modelling process. It is significant for teachers who will apply a similar modelling problem to know the created different modelling strategies and solution spaces. It is thought that the solution spaces will provide a source for teachers to keep track of the information related to their students.

Keywords: Fermi problem, mathematical modelling, model strategies, solution spaces

Inonu University Journal of the Faculty of Education
Vol 24, No 2, 2023
pp. 980-1003
DOI
10.17679/inuefd.1235549

Article Type
Research Article

Received
15.01.2023

Accepted
31.08.2023

Suggested Citation

Çakmak Gürel, Z. (2023). The Modelling Strategies Used by the Pre-service Teachers in Fermi Problems: The Task of Automatic Irrigation System. *Inonu University Journal of the Faculty of Education*, *24*(2), 980-1003. DOI: 10.17679/inuefd.1235549

The Modelling Strategies Used by the Pre-service Teachers in Fermi Problems: The Task of Automatic Irrigation System

Mathematical modeling is increasingly becoming more popular in recent decades (Blomhøj & Kjeldsen, 2006; Common Core State Standards Initiative [CCSI], 2010) and is currently being introduced into the curriculum at different educational levels (Ärlebäck, 2009; Vorhölter, Kaiser & Ferri, 2014). The mathematical modelling is defined as the process of using mathematical methods to solve real-life problems (Stender & Kaiser, 2015). In this study, it is focused on mathematical modelling activities. Specifically, in this research, it was based on a Fermi problem as a modelling activity. Studies which investigated the introduction to mathematical modelling have found that Fermi problems provide opportunities for students to work through the modelling cycle (Ärlebäck & Bergsten, 2010; Borromeo Ferri, 2018; Peter-Koop, 2004). So, the Fermi problems can be suitable for introduction to mathematical modeling at all education levels (Ferrando & Albarracín, 2019). Pre-service teachers who are inexperienced in mathematical modeling were included in this study. This had two purposes. The first was to know how they solve problems and what strategies they use. The second was to provide the opportunity for them to experience the Fermi problems and begin to develop competency in modeling. Indeed, the future teachers' competence is important in interpreting student responses or in understanding the implications of using certain strategies in a problem (Chapman, 2015; Ferrando, Segura & Pla-Castells, 2020).

Theoretical Framework

Fermi Problems in Mathematics Education

Mathematical modelling and estimation skill are among the significant types of mathematical thinking (Sriraman & Lesh, 2006). Fermi problems, specifically in mathematics teaching, are mentioned in two contexts as mathematical modelling and estimation skills (Ärlebäck & Bergsten, 2010). Fermi problem is a problem type that necessitates simplifying and mathematising of a reality which consists of prediction (Gallart, Ferrando, García-Raffi, Albarracín and Gorgorió, 2017).

Fermi problems were first put forth by the famous physician Enrico Fermi who won the Nobel prize in 1938 (Ärlebäck & Bergsten, 2010). The scientist Fermi was known for the problem of stating how many piano tuners there were in Chicago. Fermi problems were described as open-ended and non-standard problems that require making assumptions about the problem situation and estimating relevant quantities by Ärlebäck (2009). These problems are the estimation problems which are used for pedagogical purposes to determine initial conditions and to make estimations about variables and quantities that may arise in a problem (Sriraman & Lesh, 2006). For instance;

- How many people fit in the schoolyard? (Albarracín, Segura, Ferrando, & Gorgorió, 2022)
- Estimate the number of vehicles in a 3 km of traffic queue on the highway (Peter-Koop, 2009).

- How many bagels are needed to be bought for a Sunday breakfast at school? (Haberzettl, Klett & Schukajlow, 2018)
- How many trees are there in Central Park? (Gallart, et al. 2017)

The most significant advantage of the Fermi problems is that they provide an opportunity to simplify estimations, demonstrate multiple approaches to generating predictions, and discuss these variations. In addition, it provides an environment for analyzing different approaches and questioning the accuracy of estimations, while also realizing (setting on) the key components of a modelling cycle (Sriraman & Lesh, 2006).

Mathematical Modelling in Mathematics Education

Mathematical modeling is described as a process in which students consider and make sense of a real-world situation that will be used mathematics for the purpose of understanding, explaining, or predicting something (Anhalt, Cortez & Bennett, 2018). Niss (2015) explained the aim of mathematical modelling as using mathematical phenomena as a tool to answer, understand, analyze and represent practical, intellectual and scientific questions. The mathematical modelling process is represented by a cycle designed according to the different approaches (Blum & Leiss, 2007; Greefrath & Vorhölter, 2016; Perrenet & Zwaneveld, 2012). Vos and Fredj (2022) define the modelling cycles as a schematic diagram that indicates the mathematical modelling process. There are five stages in the mathematical modelling cycle (Kaiser & Stender, 2013). These are “real situation”, “real world model”, “mathematical model”, “mathematical results” and “real results". On the other hand, different modelling competencies are needed during these stages. These competencies are structuring and simplification in the transition from the real to the real model, mathematising in the transition from the real model to the mathematical model, solving the model in the transition from the mathematical model to the mathematical results, interpreting the results in the transition from mathematical results to real results. One of the most significant stages of mathematical modelling is to create a mathematical model (Niss, 2010). Mathematical models are the conceptual systems that are used to predict, explain, think or interpret a real-life situation with representation, such as a graph, formula, equation, table etc. (Doerr & Tripp, 1999). Although several studies investigate the mathematical modelling process, the final mathematical model was not focused adequately (Gallart, et al. 2017). This study specifically focuses on the strategies used in the final mathematical model created by the pre-service teachers. The reason for focusing on the mathematical modeling stage is that preservice teachers (Deniz & Akgün, 2018; Kol, 2014; Ural, 2014) and middle school students (Baran Bulut & Türker, 2022; Özkan, 2021) often encounter difficulties during this stage. Specifically, it has been observed that prospective teachers provide the least accurate answers during the mathematical modeling stage of Fermi problems (Abay & Gökbulut, 2017). It is important for teachers to be aware of appropriate and diverse strategies to enhance this skill. This study aims to uncover different strategies in mathematical modeling.

Modelling Strategies for Fermi problems

In a study, in which the modelling strategies were investigated, it was determined that the students used the strategies of *the exhaustive count*, *external source, reduction and use of proportion, concentration measures, reference point* and *grid distribution* for the Fermi problems (Albarracín & Gorgorió, 2014; Albarracin & Gorgorio, 2019; Albarracin, Ferrando &

Gorgorio, 2021; Brunet-Biarnes & Albarracin, 2022; Ferrando, Segura & Pla-Castells, 2020; 2021). It was determined that the students at the primary school level used the exhaustive count strategy most; however, as they continued into higher classes, they used concentration measures and reference points (Ferrando & Albarracín, 2019). In another study, it was observed that the experienced modelers used the concentration measures in closer modelling tasks, such as calculating school area; on the other hand, inexperienced modelers preferred this strategy less. However, it was realized that the experienced and inexperienced modellers used similar strategies to predict areas farther from students (Gallart, et al., 2017). Albarracin, et al. (2021) and Ferrando, Segura and Pla-Castells (2020; 2021) determined that the context of modelling tasks affected students' modelling strategies. In addition, it was determined that the students applied digital tools such as "Google maps", experimental measurements or the decomposition of the surface into simpler geometric shapes and then applied to the known formulas while using the modelling strategies (Albarracin, et al., 2021). Specifically, it was noticed that some students counted the tiles on the surface while calculating the surface area, and some students made measurements with a tape measure in modelling tasks (Gallart, et al., 2017).

Gallart, et al., (2017), who conducted more comprehensive research, investigated the 16-year-old students' Fermi problems solving process in three stages. The first stage is the conceptual system. These consist of a conceptual analysis of the strategies students use in the mathematical model. For instance, *grid distribution, concentration measures, reference point*s etc. The second stage is the analysis related to the procedure. It is an operational process related to how the students obtain data. The methods which are used in the operations related to the data collection are the calculation of areas and estimations. The former can be realized based on measurements (experimental in situ measurements or by digital tools such as Google maps) and in some cases, by decomposing the surface area into simpler geometric shapes and then applying known formulas. The second, as an example of estimation, is to calculate the estimation of the surface area of a tile or to find the proportion of unused surface in a larger area. The last stage is the analysis related to the language that they use in expressing these. These are graph use, arithmetic language, algebraic language, etc.

This study's aim is to identify the strategies used by pre-service teachers in solving the modelling task and then to obtain the solution spaces associated with the strategies. In this study, the problem consists of modeling “how many sprinklers are necessary to design an effective irrigation system on the campus”. The problem follows the structure of previous work focused on the analysis of cognitive aspects in the written productions of element estimation problems in bounded enclosures (Albarracín, Ferrando & Gorgorió, 2021; Ferrando & Albarracín, 2019; Gallart et al., 2017). However, the problem posed has some particularities that differentiate it from those analyzed in previous works. This problem requires estimating the radius of action of a sprinkler, the characteristics of the sprinkler. Then, the turf area needs to be covered with circles with the fixed radius(es) so that the entire surface can be watered, the areas watered by more than one sprinkler are minimized, and areas without turf are not watered unnecessarily to avoid wasting water. In this sense, the problem is different from the Fermi problems in the previous works, since the difficulty lies not so much in estimating many elements in an enclosure, but rather in identifying and considering the variables of the context (the conditions for distributing the sprinklers) and then, based on these, obtaining an optimal distribution.

In the study, it was aimed to put forth the modelling strategies of the pre-service mathematics teachers. Identifying the modelling strategies in a wider range will provide teachers realize their students' modelling strategies earlier. It is suggested in the literature that the solution to open-ended problems like Fermi problems can vary considerably (Albarracín & Gorgorió, 2014). Specifically, it is significant for teachers to know their students' various modelling strategies earlier in terms of supporting them. In addition, like their teacher, the students will also recognize the different modelling strategies and direct their learning (Ferrando & Albarracín, 2019). All these are critical in terms of putting the modelling strategies forth. However, strategies offer an idea solely about the emerging model. It is also considered significant to determine how these strategies are obtained (Albarracin, et al., 2021). Thus, the solution spaces that reflect the solution process allow the analysis of not only the result but also the process. In this study, the idea of solution spaces put forth by Leikin (2007) was followed. The solution spaces are a mirror of knowledge by combining multiple solutions (Leikin, 2007). Solution spaces are needed to combine and elaborate the multiple solution processes of strategies.

This study aims to investigate the modelling strategies of the pre-service teachers during the mathematical modelling activities.

3. What are the pre-service teachers ' modelling strategies emerging during the mathematical modelling activities?
4. How are the pre-service teachers' solution spaces emerged related to the strategies?

Method

Research Model

This study, it was aimed to explore the pre-service teachers' modelling strategies and solution spaces during the modelling activities. The method of this study was based on the qualitative-interpretative research paradigm. For this reason, a case study method, among the qualitative approach, was employed in the study. The case study method is to determine the case related to the research and explore the determining case in depth (Bogdan & Biklen, 2007). The strategies were used as the case and the solution spaces related to these strategies provided a deep investigation.

Study Group

The study group of this research consisted of 119 pre-service teachers studying in the fourth class of the Secondary School Mathematics Teacher Training Department of the Faculty of Education. The average age of the participants was 22. The pre-service teachers had attended courses related to the mathematics field (i.e: algebra, arithmetic, statistics) and mathematics education (problem-solving, material development, teaching methods) for four years. It was in this research that they first encountered Fermi problems. Although the pre-service teachers had taken three hours of education related to the model, modelling and modelling cycle concepts, their experiences in solving the modelling problems were quite new. In the study, the pre-service teachers constituted groups consisting of four or five members according to their wishes. Thus, 26 different groups were created. The worksheets of each group were collected and coded from Group 1 to group 26.

Data Collection Tools and Collecting the Data

The worksheet that included the mathematical modelling problem was used as the data collection tool in this study. The modelling task was developed by the researcher. The modelling problem given to the pre-service teachers is as follows.

"It is planned to build an automatic irrigation system for your university campus. Develop a model to determine how many sprinklers are needed for this irrigation system and share the results."

This task was developed suitable for the criteria related to the modelling tasks given by Wess and Greefrath (2019). The task in this study consists of the campus in which the pre-service teachers studied. Therefore, the task is considerably close to the pre-service teachers physically. This campus was settled as a new settlement, the irrigation system has not been established yet. Considering all these reasons, the task given was realistic, authentic and suitable for the closeness criteria for the pre-service teachers. In addition, the task was open-ended and its addressing modelling sub-competencies ensured the criteria by Wess and Greefrath (2019).

The data collection process took two weeks in the form of problem-solving and presentation of solutions. Three hours of practice were held each week. The researcher had the task of guidance in this process. In the first week, the pre-service teachers solved their problems by dividing them into groups according to their wishes. In the second week, the groups made their presentations. The researcher asked questions to clarify the solution processes during the presentation. This process was recorded with the observation notes.

Data Analysis

This study is based on a qualitative analysis of the participants' worksheets. The worksheets of the 26 groups were used as the data source in the study. In the data analysis, the "exhaustive count, external source, reduction and use of proportion, concentration measures, reference point and grid distribution" modelling strategies put forth by Albarracin and Gorgorio, (2019), and Albarracin, et al. (2021) were taken into consideration. They are not strategies for any Fermi problem, but only for those consisting in estimating many elements in a bounded enclosure. The indicators related to these strategies are presented in Table 1.

Table 1.
Mathematical Modelling Strategies

Strategies	Indicator
A strategy cannot be determined.	Misunderstanding the task and inability to determine a strategy appropriate to the task
Exhaustive Count	Counting one by one without any order
Effective Count	Counting one by one in a certain order
External Source	Asking someone who knows
Reduction and use of proportion	An initial problem is handled. Smaller numbers are used to solve the initial problem. The main problem is solved using the proportionality factor.

Reference point	Finding the number of elements that can fit in the area by dividing the area occupied by a unit by the specified area; in short, the number of elements obtained because of repetition of a unit
Grid distribution	Finding the total number of elements by multiplying the number of elements that will fit on the edges.
Concentration measures	Finding the total number of elements by estimating the number of elements that fit in a unit, multiplying by the value of the specified area

Among the qualitative data analysis methods, the deduction and induction analysis methods, which were defined by Mayring (2015) were employed together in the data analysis. The mixed method which enables using both methods is called "content structuring" (Mayring, 2014). Content structuring is deductively ordered into strategies, and within each strategy's solution spaces, an inductive process is performed (Mayring, 2015). Firstly, the modelling strategies of the pre-service teachers were coded according to the modelling strategies given in Table 1. After the qualitative analysis of the solutions of the pre-service teachers in the study, it was found that five strategies appear.

Then, the groups using the same strategies were brought together and a common solution space was obtained. The solution spaces were proposed as a tool for the analysis of multiple solution problems by Leikin & Levav-Waynberg (2008). The solution space related to the Fermi problems was represented with a tree diagram by Albarracin, et al. (2021). The data in this study were analyzed with a tree diagram based on the solution space. The strategies which were commonly used and the solution methods that took place in the solution space were determined with the frequencies. Frequencies are one of the methods that are used in the analysis of qualitative data (Mayring, 2014). In the reliability of coding, the same researcher analyzed the 26 solutions and then re-analyzed them 3 months later and there was only disagreement in 3 of them. Cohen's Kappa coefficient which is inter-rater reliability was found as .88. In this case, the inter-rater reliability was satisfying (Cohen, 1960). Sample worksheets related to strategies are included in the findings.

Findings

The first research question is "What are the pre-service teachers ' modelling strategies emerging during the mathematical modelling activities?" The strategies are presented in Table 2.

Table 2.
The strategies that the groups use in modelling

Strategy	Groups	*f*
A strategy cannot be determined.	G14, G15, G20	3
Only counting	G12, G24 (Exhaustive count) G1, G19, G23, G26 (Effective count)	6
Reference point	G2, G4, G6, G8, G9, G10, G17, G18, G22, G25	10
Grid distribution	G5, G7, G11, G16	4
Concentration measures	G3, G13, G21	3
Total		26

In the study, it was determined that 3 groups (G14, G15, G20), among the 26 groups, could not determine a strategy for calculating the number of sprinklers to fit in a shapeless area. The students in this group modelled the problem by transforming it into 3-dimensional shapes or completed the solutions only by sharing the types of sprinklers. The solution papers of the rest 23 groups were analyzed according to their strategies given in the data analysis. The *"reference point"* was used most; on the other hand, the "concentration measures" were used least.

The "*Only counting*" strategy refers to counting the number of sprinklers that will be placed in an area. In this scope, two sub-strategies were determined. These were exhaustive counting" and "effective count". It was seen that the groups in the first category placed the sprinklers randomly and completely intuitively and expressed their total by counting the sprinklers placed one by one while calculating the number of sprinklers that would fit in an area. For example, an image of the study paper related to the G12 Coded group is presented in Figure 1.

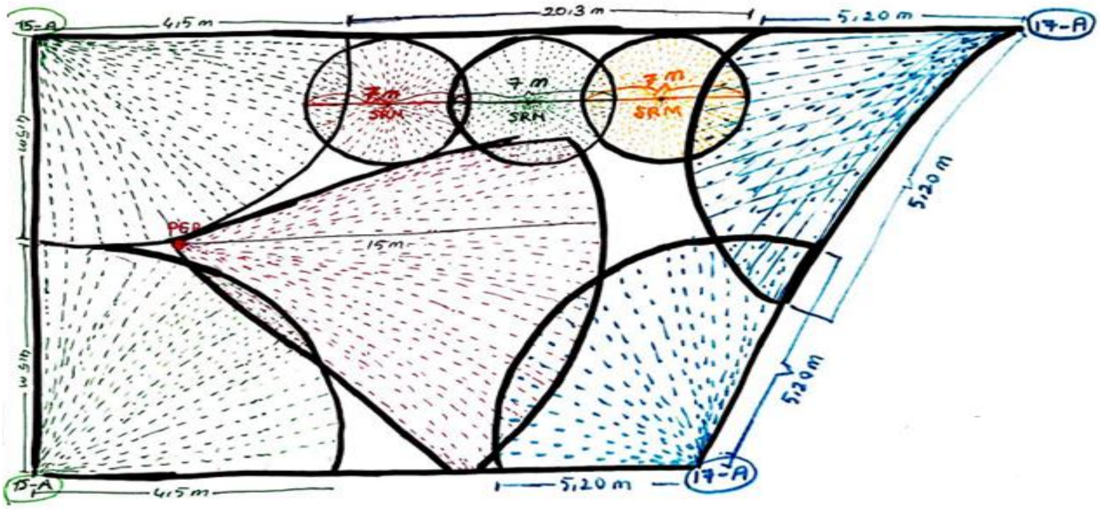


Fig. 1. *The citation of exhaustive count for the Group 12*

This group, which is in the category of the exhaustive count, covered the whole area with fountains without any order. When it was asked if there was an order, it was stated that they only paid attention to coating the entire surface.

The students in the category of the effective count divided the area into sub-areas and calculated the number of sprinklers that would fit into similar spaces. It was seen that the groups in this category form a more regular sprinkler placement even though they were counted one by one. For example, the citation of G1 is as in Figure 2.

Fig. 2. *The citation of Group 1 for the effective count*

The students in this group, who were in the category of the effective count, developed a more effective solution than the solution of the former group. The reason why it was called the effective count was that they were placed in a certain order and at a fixed interval. The students in this group took a sketch of the entire green area and placed the fountains one by one in the whole area at regular intervals. They concluded by counting the number of sprinklers. In this solution, they determined the distance between the sprinklers by calculating the radius of the sprinklers. Instead of generalizing the model, the solution of this group was evaluated in the category of "only counting" as they counted the sprinklers one by one.

The second research problem is “How are the pre-service teachers’ solution spaces emerged related to the strategies?” Solution spaces related to reference point, grid distribution, concentration measures were presented as tree diagrams.

Reference Point

The pre-service teachers, who used this strategy, tended to find the number of sprinklers by dividing the entire area that can be irrigated by the area occupied by a sprinkler. Thus, the pre-service teachers developed a solution strategy because of repeating the area of a sprinkler. The tree diagram for the reference point strategy is presented in Figure 3.

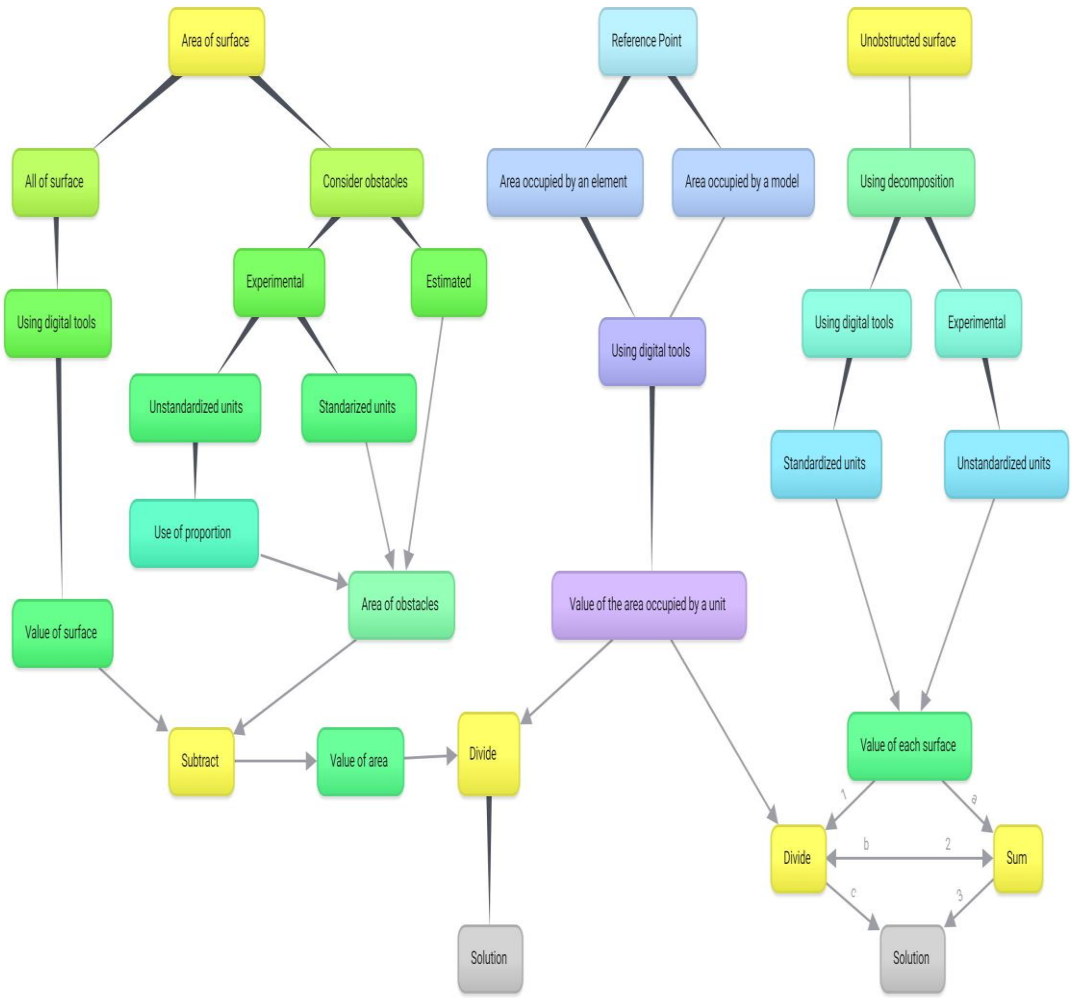


Fig 3. *Tree diagram for solution space related to the reference point strategy*

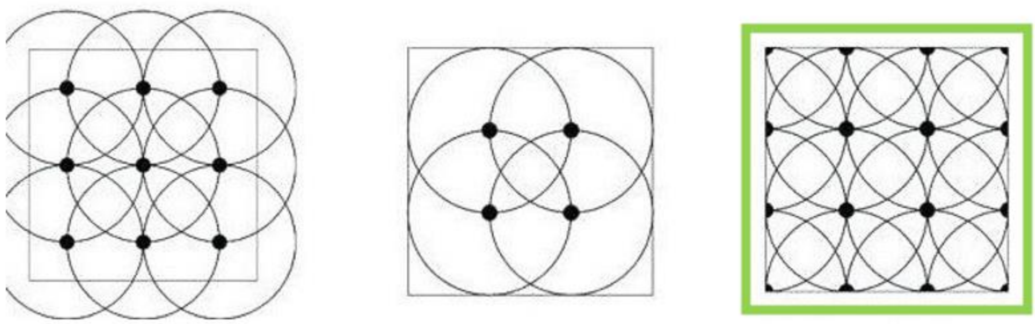
The pre-service teachers, who had the *reference point* repetition strategy, used two different methods. The first was that they determined the green area that would be irrigated by subtracting the area of the blocks (buildings and the areas that cannot be irrigated) from the whole land (the whole settlement area of the university). Then they reached the result by dividing the obtained value by the area of the sprinklers. The second was that they calculated the green areas directly and divided the obtained value by the area of the sprinkler. It was determined that the number of groups that applied the first method was equal to the number of groups that applied the second method.

The pre-service teachers who used the first method (*f:5*) used the Google search engine as a digital tool and searched the value of the whole area. Then, they estimated the block areas either experimentally (*f:4*) or completely intuitively (*f:1*). The pre-service teachers who estimated experimentally calculated the areas of blocks either with non-standard units such as calculating the area of buildings based on windows, the area of cars parks based on vehicles in the parking lot, the area of roads based on the number of steps, and the area of buildings based on tiles (*f:3*) or by using the formula *"actual length/denominator scale = sketch length"* in standard units *(f:1).* It is remarkable that all groups using non-standard units have to estimate the area of other buildings from the area of one building using the *use of* *proportion.* Here, it was seen that the proportionality constant was created by eye decision. Thus, all blocked areas obtained were added and subtracted from the whole area. Therefore, the area that can be irrigated was calculated. The number of sprinklers was determined by dividing the obtained value by the area of a sprinkler.

The pre-service teachers who used the second method decomposed all green areas (*f:5*). The decomposed green areas were calculated either with standard units *(f:3)* using digital tools such as Google Earth, parcel query, or experimentally with non-standard units *(f:2).* Then the groups followed two ways: the total area obtained by adding the surface areas of the separated regions was divided by the area covered by the sprinklers *(f: 2)* or each separated area was divided by the area covered by the sprinklers, and the number of sprinklers belonging to the regions was determined and then the number of sprinklers was added *(f: 3).*

Related to the *reference point* strategy, the pre-service teachers determined either the area of a sprinkler or the value of the land covered by a unit from the area of a sprinkler model. For instance, the citation belonging to Group 10 is as in Figure 5.

1 2 3



1-Unnecessary and over-irrigation (wrong) 2-Having dry areas (wrong) 3-Equal water distribution (right)

Fig. 5. *The sprinkler model that Group 10 used for the reference point*

In the study, it is important relevant variables, such as distribution efficiency, the choice of different types of sprinklers, etc. The group estimated the radius of action of a sprinkler and the distribution of the sprinkler. This group noticed that the turf area needs to be covered with circles with the fixed radius so that the entire surface can be watered, the areas watered by more than one sprinkler are minimized, and areas without turf are not watered unnecessarily. So, the group that determined the third model, calculated the area of 16 sprinklers in the model and then divided the entire area by the area of this sprinkler model. However, a lot of groups determined the number of sprinklers by dividing the area irrigated into a sprinkler over the entire area. The sprinkler model proved that there was no excessive or unnecessary irrigation and that there are no dry areas left, ensuring more valid results. All of the groups determined the area where the sprinklers would irrigate by using digital tools (Google search). Therefore, they used standard units of measure.

Grid Distribution

The pre-service teachers, who used this strategy, first likened the whole area or a sample area to regular geometric shapes. Thus, they created a grid in an area by multiplying side lengths and determining the number of sprinklers. A tree diagram created for the grid distribution strategy is presented in Figure 6.

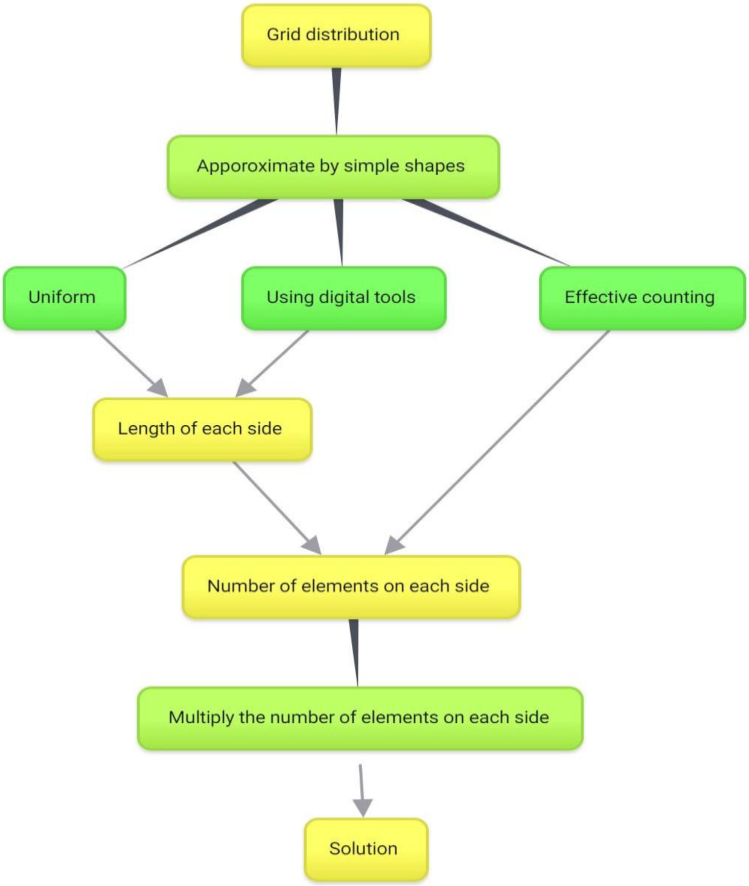


Fig 6. *A tree diagram for solution space related to the grid distribution strategy*

The grid distribution strategy was used by four groups. Since the green areas given in the problem were created from real data, there was not a regular rectangle, triangle or circle in the area. But these shapes were likened to regular polygons with a slight error. This method is possible in the grid distribution strategy obtained by multiplying two sides. Then one of the groups determined the side lengths by drawing a sample rectangular uniform. One of the groups, on the other hand, determined the edge lengths of the rectangular-shaped regions using digital tools (with the application of parcel search). These two groups obtained the number of sprinklers that could fit on the edge (side length/diameter of the sprinkler) using the + 1 model. Then, they reached the number of sprinklers that could fit in the area they compared to a rectangle by multiplying the number of sprinklers that fit on the sides. On the other hand, two of the groups determined the number of sprinklers on the sides by using the effective count strategy.

Concentration measures

After the pre-service teachers in this category had estimated the number of sprinklers that can fit in an area, they determined the number of sprinklers that could fit into the total irrigable area by using the ratio. The groups in this category developed the *concentration measures* strategy considering not the area that the sprinkler cover but the number of sprinklers. A tree diagram related to the *Concentration measures* strategy is presented in Figure 7.

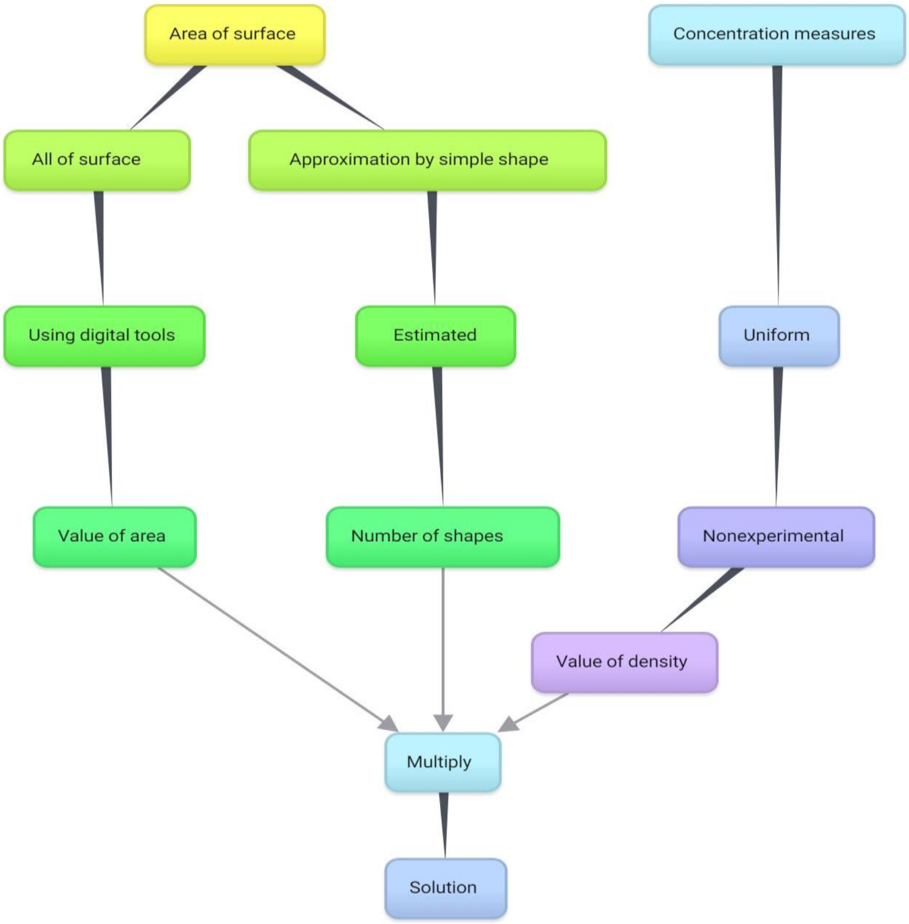


Fig. 7. *A tree diagram for solution space related to the concentration measures strategy*

There are three groups in the category of *concentration measure*. One of these groups found the entire area by using digital tools (Google search). On the other hand, two of the groups divided the whole area into rectangles by likening all irrigable areas to rectangles. For instance, the G21-coded pre-service teachers constituted 12 rectangular areas from the whole area by purely intuitive estimation. There was not any proof or basis related to this topic. The value of the area reached or the number of rectangles was multiplied by the number of sprinklers fit in a single unit. The number of sprinklers that fit in a single unit was obtained by non-experimental estimation from a sample. The lack of any basis for the estimates reduced the validity of the solutions of the groups who applied this strategy.

Discussion & Conclusion

In this study, the strategies that the pre-service teachers used in the mathematical model were put forth in solving Fermi problems. It was found that five strategies appear, which coincides with what was also found for pre-service teachers in Ferrando, Segura & Pla-Castells (2021). In the problem given within the scope of the number of elements that could fit in a surface, the *"reference point"* was used most; on the other hand, the "concentration measures" were used least. Results with pre-service teachers (Ferrando, Segura & Pla-Castells, 2020, 2021) show that they do use concentration measure very frequently (they already have enough mathematical maturity), but it depends on the contextual characteristics of the problem. In these studies, a significant correlation was found between certain contextual features and the strategy used. In particular, it was found that Fermi problems in which the elements to be estimated occupy a large area with a regular shape are related to the reference point strategy. On the contrary, when the elements occupy a small area with an irregular shape, concentration measure strategies increase. In this case, the contextual characteristics of the problem presented confirm the relationship with the reference point strategy, since the area covered by a sprinkler is large and regular (a circle that is easy to measure). It was also determined that the students began mostly to use *reference point* strategies more as they progressed towards the upper classes (Ferrando & Albarracín, 2019). On the other hand, Albarracin, et al. (2021) observed that students use the *reference point* strategy, but they do not use the concentration measurement and grid distribution strategies when the tasks are physically far from their environment. This result is partially different from the present study. The task in this study consists of the pre-service teachers' campus. Therefore, the task is considerably close to the pre-service teachers physically. However, it is thought that the pre-service teachers used the *reference point* strategy in the task as the settlement of the campus was quite big. Thus, the task in the study by Albarracin, et al. (2021) was related to estimating the area of the central park which is a large area.

In the *reference point* strategy, the result can be found by dividing the area which can be irrigated into the area of a sprinkler. In the solution space of this strategy, the estimation of two variables the area that can be irrigated and the area that a sprinkler covers are needed. Experimental methods, digital tools or solely intuitive estimations are used in estimating the areas. Even Albarracin, et al. (2021) and Gallart, et al. (2017) determined that similar methods were applied. It is seen that mostly non-standard units are used in experimental methods (area estimation with non-standard units such as steps, windows, and tiles) and the ratio is used for the estimation of the entire irrigable area. It was used to estimate larger areas using an area proportionality calculated in non-standard units. Reducing the problem and using proportion in area calculation are among the strategies determined even by Albarracin and Gorgorio (2019). It was determined that intuitive estimation was used during the use of proportion in the present study. On the other hand, in digital tools, digital environments such as Parcel search, and Google earth were used and standard units were obtained. Similarly, digital tools were also used to determine the area that a sprinkler covers. The basic problem is to determine the validity of the *reference point* strategy in an area with blocks. Particularly, this validity problem comes forth when pre-service teachers calculate the value of the area obtained by subtracting the area of blocks from the entire area. This value caused the area to be irrigated to be perceived. On the other hand, the groups calculating directly the green lands created decomposed green lands and divided each green land by the area of a sprinkler. In this strategy, the area that could be irrigated was decomposed and therefore a solution space that reflected more reality was used. The pre-service teachers have to do with the realistic assumptions of them to enrich their models and achieve a more accurate estimation (Krawitz, Schukajlow & Van Dooren, 2018; Segura & Ferrando, 2021).

The strategy which was used high after the reference point was the strategy of "only counting". This strategy was explained with two methods. These are the exhaustive count and effective count. In this problem, given that it is reasonable to count, the “effective count” can be an appropriate strategy. This strategy may appear because the number of sprinklers is not large enough to be uncountable. This is not the case for problems with large numbers in a closed enclosure, which are unachievable. The exhaustive count can be an unsuccessful strategy in any case. It was determined that this method emerged in the low-aged groups more (Ferrando & Albarracín, 2019). On the other hand, in this study, it is thought that a problem arose since pre-service teachers had not experienced this type of problem before. It was revealed that students who were not experienced in mathematical modelling choose more strenuous methods in Fermi problems and did not generalize (Gallart, et al. 2017).

In the grid distribution strategy, groups tried to liken the shapeless area to seem like regularity shapes. This situation is extremely possible for the grid distribution strategy. However, the task is quite effective in the emergence of this strategy. In the study by Albarracin, et al. (2021), it was determined that the grid distribution strategy emerged and the students only used this strategy in the rectangular areas. In regions that are not regular, the grid distribution strategy was not observed. In this study, the preservice teachers use grid distribution because they approximate the grassy areas by forming rectangles. It is not an appropriate strategy for the characteristics of the context. The fact that this strategy was used in the present study is extreme levels of simplification leading to unrealistic situations. This result is in line with Anhalt, Cortez and Bennett (2018).

In the concentration measures strategy, which was applied the least, it was determined that the pre-service teachers used intuitive methods in estimating both the area and the number of sprinklers that can fit in each unit. It is notable that the students in this group also used digital tools in calculating the area. It was seen that this strategy was the type of strategy in which they made intuitive predictions without justification. Intuitive prediction without justification is among the methods even determined by Albarracin, et al. (2021).

While the strategies used by the pre-service teachers in this study were investigated, their solution spaces related to these strategies were also uncovered. In the Reference point strategy, it was seen that mostly digital tools and non-standard measurement units were used with experimental methods. It was determined that the method of likening a known shape was used in the grid distribution strategy. On the other hand, in the strategy of density, intuitive estimation comes forth. All of the strategies determined in this study are among the strategies determined in former studies (i.e: Albarracín & Gorgorió, 2014; Albarracin and Gorgorio, 2019; Albarracin, et al., 2021). This is because the problem used to determine the number of elements that can fit in an area is similar to the problem contexts given in other studies. Although the strategies are similar in the other studies, the created tree diagrams differ from the other studies and give detailed information about the strategies. In the studies, in which the model strategies are investigated (Albarracin, et al., 2021), it is stated that the diagram given in the form of a tree is not yet completed and is open to development. This study presents a more detailed version of the diagram given in other studies.

The solution spaces related to irrigating an area which is covered by blocks differ from the solution spaces created to determine the number of trees that can fit an area or the number of people (i.e: Albarracin, et al., 2021). Finding the total value of the whole green lands caused the area to be irrigated to be perceived. However, calculating the decomposed green areas and determining the number of sprinklers to fit in each decomposed green area shows a more realistic solution space. In addition, not only the number of sprinklers placed but also how these sprinklers should be placed (the sprinkler model) requires an important modelling skill. This problem requires estimating the radius of action of a sprinkler, the characteristics of the sprinkler. Then, the turf area needs to be covered with circles with the fixed radius(es) so that the entire surface can be watered, the areas watered by more than one sprinkler are minimized, and areas without turf are not watered unnecessarily to avoid wasting water. In the reference point-based model, these variables are considered. It is significant that the solution spaces present a valid way of solution related to the task by bringing each solution together in terms of teaching the problems consisting of multiple solutions. It is thought that this solution space will provide a source for teachers to keep track of the information related to their students.

The results of the paper, which show a presence of more realistic assumptions that enrich the reference point-based model, and the absence in the concentration measure model, also coincide with what Segura et al. (2021) found for prospective teachers. In this problem, most realistic assumptions are associated with the reference point strategy. Therefore, the solution space tree is richer and more complex.

The context of problem has an impact on strategies. The reason why the reference point strategy is used most may be due to the context of the problem. This work expands the previous solution spaces and examine the reference point strategy in detail. It is found that taking the area covered by an irrigation model as a reference rather than the area occupied by an element in the problem used in the study produces more realistic results. In this problem, the fact that there is no area that will not be irrigated or that excessive irrigation will not be made necessitated the use of an irrigation model in the reference point strategy. It is found that in this strategy, irrigation of an empty field does not produce the same realistic results as irrigating an area with obstacles.

This study is limited to the final products of pre-service teachers. It does not report the strategies they used in the process. Although this study is limited to a single problem, solution strategies and spaces of different problems are wondered. Although this study expands the solution spaces of previous studies, the solution space is not limited to those in these studies and should be furthered. Knowing the strategies and solution spaces to be put forward with different problems is important for teachers to support their students.

Çıkar Çatışması Bildirimi

Yazar, bu makalenin araştırılması, yazarlığı veya yayınlanmasına ilişkin herhangi bir potansiyel çıkar çatışması beyan etmemiştir.

Destek/Finansman Bilgileri

Yazar, bu makalenin araştırılması, yazarlığı ve yayınlanması için herhangi bir finansal destek almamıştır.

Etik Kurul Kararı

Bu araştırma için Erzincan Binali Yıldırım Üniversitesi İnsan Araştırmaları Eğitim Bilimleri Etik Kurulundan (12.09.2022- 195138) etik izin alınmıştır.

Kaynakça/References

Abay, S. & Gökbulut, Y. (2017). Sınıf öğretmeni adaylarının matematiksel modelleme becerileri: fermi problemleri uygulamaları. *Uluslararası Türk Eğitim Bilimleri Dergisi ,9*, 65-83. Retrieved from https://dergipark.org.tr/en/pub/goputeb/issue/34356/379918.

Albarracín, L., & Gorgorió, N. (2014). Devising a plan to solve Fermi problems involving large numbers. *Educ Stud Math, 86*, 79–96. https://doi.org/10.1007/s10649-013-9528-9.

Albarracín, L., & Gorgorió, N. (2019). Using large number estimation problems in primary education classrooms to introduce mathematical modelling. *International Journal of Innovation in Science and Mathematics Education, 27*(2), 45–47.

Albarracín, L., Ferrando, I. & Gorgorió, N. (2021). The role of context for characterising students' strategies when estimating large numbers of elements on a surface. *International Journal of Science and Mathematics Education, 19*, 1209–1227. https://doi.org/10.1007/s10763-020-10107-4

Albarracín, L., Segura, C., Ferrando, I., & Gorgorió N. (2022). Supporting mathematical modelling by upscaling real context in a sequence of tasks. *Teaching Mathematics and its Applications: An International Journal of the IMA, 41*(3), 183–197. https://doi.org/10.1093/teamat/hrab027.

Anhalt, C. O. Cortez, R. & Bennett, A. (2018). The emergence of mathematical modeling competencies: An investigation of prospective secondary mathematics teachers. *Mathematical Thinking and Learning, 20*(3), 202-221.

Ärlebäck, J. B. (2009). On the use of realistic Fermi problems for introducing mathematical modelling in school. *The Mathematics Enthusiast, 6*(3), 331–364.

Ärlebäck, J. B., Bergsten, C. (2010). On the Use of Realistic Fermi Problems in Introducing Mathematical Modelling in Upper Secondary Mathematics. In: Lesh, R., Galbraith, P., Haines, C., Hurford, A. (eds) *Modeling Students' Mathematical Modeling Competencies*. Springer, Boston, MA. https://doi.org/10.1007/978-1-4419-0561-1_52

Baran Bulut, D. & Türker, M. (2022). Ortaokul öğrencilerinin üslü ifadeler konusunda modelleme yeterliklerinin incelenmesi: Sarmal kitaplık problemi. *Recep Tayyip Erdoğan Üniversitesi Eğitim Fakültesi Dergisi, 2*(2), 39-56.

Blomhøj, M., & Kjeldsen, T. H. (2006). Teaching mathematical modelling through project work. *ZDM Mathematics Education, 38*(2), 163-177. https://doi.org/10.1007/BF02655887

Blum, W., & Borromeo Ferri, R. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application, 1*(1), 45-58.

Blum, W., and Leiss, D. (2007). How do teachers deal with modeling problems? In C. Haines, P. Galbraith, W. Blum and S. Khan (Eds.), *Mathematical modeling (ICTMA 12): Education, engineering and economics* (*pp*. 222–231). Chichester: Horwood Publishing.

Bogdan, R. C. & Biklen, S. K. (2007). *Qualitative research for education: an introduction to theory and methods* (5. bs.). USA: Pearson Education, Inc

Borromeo Ferri, R. (2018). *Theoretical Competency: For Your Practical Work. In: Learning How to Teach Mathematical Modeling in School and Teacher Education*. Springer, Cham. https://doi.org/10.1007/978-3-319-68072-9_2

Brunet-Biarnes, M., Albarracín, L. (2022). Exploring the negotiation processes when developing a mathematical model to solve a Fermi problem in groups. *Math Ed Res J*, . https://doi.org/10.1007/s13394-022-00435-9

Chapman, O. (2015). Mathematics teachers' knowledge for teaching problem solving. *LUMAT: International Journal on Math, Science and Technology Education, 3*(1), 19-36.

Cohen, J. (1960). A coefficient of agreement for nominal scales. *Educational and Psychological Measurement. 20,* 37-46. https://doi.org/10.1177/001316446002000104.

Common Core State Standards Initiative [CCSI]. (2010). Common core state standards for mathematics. National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Deniz, D. & Akgün, L. (2018). İlköğretim matematik öğretmeni adaylarının matematiksel modelleme becerilerinin incelenmesi. *Akdeniz Eğitim Araştırmaları Dergisi, 12*(24), 294-312. doi: 10.29329/mjer.2018.147.16

Doerr, H. M., & Tripp, J. S. (1999). Understanding how students develop mathematical models. *Mathematical Thinking and Learning, 1*(3), 231–254.

Ferrando, I. & Albarracín, L. (2019). Students from grade 2 to grade 10 solving a Fermi problem: analysis of emerging models. *Mathematics Education Research Journal, 33*, 61-78. https://doi.org/10.1007/s13394-019-00292-z.

Ferrando, I., Segura, C., & Pla-Castells, M. (2020). Relations entre contexte, situation et schéma de résolution dans les problèmes d'estimation. *Canadian Journal of Science, Mathematics and Technology Education*, *20*(3), 557-573.

Ferrando, I., Segura, C., & Pla-Castells, M. (2021). Analysis of the relationship between context and solution plan in modelling tasks involving estimations. In *Mathematical Modelling Education in East and West* (pp. 119-128). Springer, Cham.

Gallart, C., Ferrando, I., García-Raffi, L. M., Albarracín, L., & Gorgorió, N. (2017). Design and implementation of a tool for analysing student products when they solve fermi problems. In G. A. Stillman, W. Blum & G. Kaiser (Eds.), *Mathematical Modelling and Applications. Crossing and Researching Boundaries in Mathematics Education*, (pp. 265–275). Cham, Switzerland: Springer.

Greefrath, G., and Vorhölter, K. (2016). Teaching and learning mathematical modelling: approaches and developments from German speaking countries. *ICME-13 Topical Surveys*, 1-42, Switzerland: Springer International Publishing. doi: 10.1007/978-3-319-45004-9_1

Haberzettl, N., Klett, S., & Schukajlow, S. (2018). Mathematik rund um die Schule—Modellieren mit Fermi-Aufgaben. In K. Eilerts, & K. Skutella (Eds.)*, Neue Materialien für einen realitätsbezogenen Mathematikunterricht 5. Ein ISTRON-Band für die Grundschule* (pp. 31–41). Wiesbaden: Springer Spectrum.https://doi.org/10.1007/978-3-658-21042-7_3

Kaiser, G., and Stender, P. (2013). Complex modelling problem in cooperative learning environments self-directed. In G. Stillman, G. Kaiser, W. Blum, & J. Brown (Eds.), *Teaching mathematical modelling: Connecting to research and practice* (pp. 277–294). Dordrecht: Springer.

Kol, M. (2014). İlköğretim matematik öğretmen adaylarının matematikselleştirme sürecinin bir matematiksel modelleme etkinliği süresince incelenmesi (Yüksek Lisans Tezi). Middle East Technical University, Ankara.

Krawitz, J., Schukajlow, S., & Van Dooren, W. (2018). Unrealistic responses to realistic problems with missing information: What are important barriers? *Educational Psychology*, *38*(10), 1221-1238.

Leikin, R. (2007). Habits of mind associated with advanced mathematical thinking and solution spaces of mathematical tasks. In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings of the fifth congress of the European Society for Research in mathematics education* (pp. 2330–2339). Nicosia, Cyprus: Department of Education, University of Cyprus.

Leikin, R., & Levav-Waynberg, A. (2008). Solution spaces of multiple-solution connecting tasks as a mirror of the development of mathematics teachers' knowledge. *Canadian Journal of Science, Mathematics, and Technology Education, 8*(3), 233–251.

Mayring, P. (2014). *Qualitative content analysis. Theoretical foundation, basic procedures and software solution,* http://nbn-resolving.de/urn:nbn:de:0168-ssoar-395173

Mayring, P. (2015). Qualitative content analysis: Theoretical background and procedures. In A. Bikner-Ahsbahs, C. Knipping & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education* (pp. 365–380). Dordrecht: Springer.

Niss, M. (2010). Modeling a crucial aspect of students' mathematical modelling. In R. Lesh, P. L. Galbraith, C. R, Haines, & A. Hurford (Eds.), *Modeling students' mathematical modelling competencies* (pp. 43–59). New York: Springer.

Niss, M. (2015). Prescriptive modelling—Challenges and opportunities. In G. A. Stillman, W. Blum, & M. S. Biembengut (Eds.), *Mathematical modelling in education research and practice: Cultural, social and cognitive influences* (pp. 67–79). Cham: Springer.

Özkan, M. (2021). Ortaokul öğrencilerinin düzgün çokgenler konusunda modelleme yeterlikleri üzerine bir eylem araştırması (Doktora tezi). Marmara Üniversitesi, İstanbul

Perrenet, J., & Zwaneveld, B. (2012). The many faces of the mathematical modeling cycle. *Journal of Mathematical Modelling and Application, 1*(6), 3–21.

Peter-Koop, A. (2004). Fermi problems in primary mathematics classrooms: Pupils' interactive modelling processes. In I. Putt, R. Faragher, and M. McLean (Eds.), *Mathematics Education for the Third Millennium: Towards 2010* (Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia, Townsville) (pp. 454–461). Sydney: MERGA.

Peter-Koop, A. (2009). Teaching and understanding mathematical modelling through Fermi–problems. In B. Clarke, B. Grevholm, & R. Millman (Eds.), *Tasks in primary mathematics teacher education* (pp. 131–146). Dordrecht: Springer.

Segura, C., & Ferrando, I. (2021). Classification and analysis of pre-service teachers' errors in solving Fermi problems. *Education Sciences*, *11*(8), 451.

Sriraman, B. & Lesh, R. (2006). Beyond traditional conceptions of modelling. *Zentralblatt für Didaktik der Mathematik*, *38*(3), 247-254.

Stender, S. and Kaiser, G. (2015). Scaffolding in complex modelling situations. ZDM Mathematics Education, 47(7). doi:10.1007/s11858-015-0741-0

Ural, A. (2014). Matematik öğretmen adaylarının matematiksel modelleme becerilerinin incelenmesi, *Dicle Üniversitesi Ziya Gökalp Eğitim Fakültesi Dergisi, 23*, 110-141.

Vorhölter, K., Kaiser, G. & Borromeo Ferri, R. (2014) Modelling in mathematics classroom instruction: an innovative approach for transforming mathematics education. In Y. Li, E. A. Silver & S. Li (Eds). *Transforming Mathematics Instruction* (pp. 21–36). Cham, Switzerland: Springer.

Vos, P. & Frejd, P. (2022, February). The modelling cycle as analytic research tool and how it can be enriched beyond the cognitive dimension. *Paper accepted for presentation at the 12th Congress of the European Society for Research in Mathematics Education (CERME12)*, withing the Thematic Working Group 6 Applications and Modelling. Bolzano, Italy

Wess, R., & Greefrath, G. (2019). Professional competencies for teaching mathematical modelling—Supporting the modelling-specific task competency of prospective teachers in the teaching laboratory. In U. T. Jankvist, M. Van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *European Research in Mathematics: Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education* (pp. 1274–1283). Utrecht, Netherlands.

İletişim/Correspondence

Dr. Öğr. Üyesi Zeynep ÇAKMAK GÜREL,

zcakmak@erzincan.edu.tr