



On Random Coincidence & Fixed Points for a Pair of Hybrid Measurable Mappings

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Abstract

Let (X, d) be a Polish space, $\mathcal{CB}(X)$ the family all nonempty closed and bounded subsets of X and (Ω, Σ) be a measurable space. In this paper, a pair of hybrid measurable mappings $f: \Omega \times X \rightarrow X$ and $T: \Omega \times X \rightarrow \mathcal{CB}(X)$, satisfying non-expansive type condition;

$$H(T(\omega, x), T(\omega, y)) \leq a(\omega)d(f(\omega, x), f(\omega, y)) + b(\omega)\max\{d(f(\omega, x), T(\omega, x)), d(f(\omega, y), T(\omega, y))\} \\ + c(\omega)[d(f(\omega, x), T(\omega, y)) + d(f(\omega, y), T(\omega, x))]$$

for every $x, y \in X$, where $a, b, c: \Omega \rightarrow [0, 1]$ are measurable mappings such that for all $\omega \in \Omega$, $b(\omega) > 0$, $c(\omega) > 0$ and $a(\omega) + b(\omega) + 2c(\omega) = 1$, are introduced and investigated. It is proved that if X is complete, $T(\cdot, \cdot), f(\cdot, \cdot)$ are continuous for all $\omega \in \Omega$, $T(\cdot, x), f(\cdot, x)$ are measurable for all $x \in X$ and $T(\omega, \xi(\omega)) \subseteq f(\omega \times X)$ and $f(\omega \times X) = X$ for each $\omega \in \Omega$, then there is a measurable mapping $\xi: \Omega \rightarrow X$ such that $f(\omega, \xi(\omega)) \in T(\omega, \xi(\omega))$ for all $\omega \in \Omega$. This result generalizes and extends the fixed point theorems of Papageorgiou [21], Cirić et al.[8], Jhade et al. [16] and many classical fixed point theorems. We also discuss an illustrative example to highlight the realized improvements

1. INTRODUCTION

Let (X, d) be a metric space and let T be a self-mappings on X . If T is such that for all x, y in X ;

$$d(Tx, Ty) \leq \lambda d(x, y) \quad (1.1)$$

where $0 \leq \lambda < 1$, then T is said to be a contraction mapping. If T satisfies $d(Tx, Ty) < d(x, y)$, then T is called contractive mapping. If T satisfies (1.1) with $\lambda = 1$, then T is called a non-expansive mapping. If T satisfies any conditions of type

$$d(Tx, Ty) \leq a_1 d(x, y) + a_2 d(x, Tx) + a_3 d(y, Ty) + a_4 d(x, Ty) + a_5 d(y, Tx) \quad (1.2)$$

where a_i ($i = 1, 2, 3, 4, 5$) are nonnegative real numbers such that $a_1 + a_2 + a_3 + a_4 + a_5 < 1$, then T is said to be a contractive type mapping. If T satisfies (1.2) with $a_1 + a_2 + a_3 + a_4 + a_5 = 1$, then T is said to be a non-expansive type mapping. Similar terminology is used for multi-valued mappings. Fixed point theorems for contractive, non-expansive, contractive type and non-expansive type mappings provide techniques for solving a variety of applied problems in mathematical and engineering sciences. It is one of the reason that many authors have studied various classes of contractive type or non-expansive type mappings. For Banach spaces the famous is Gregus's Fixed Point Theorem [10] for non-expansive type single-valued mappings, which satisfy (1.2) with $a_4 = a_5 = 0, a_1 < 1$. The class of mappings T satisfying the following non-expansive type condition:

$$d(Tx, Ty) \leq a \max \left\{ d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Tx) + d(y, Ty)}{2} \right\} + b \max \{d(x, Tx), d(y, Ty)\} \\ + c [d(x, Ty) + d(y, Tx)] \quad (1.3)$$

for all $x, y \in X$, where

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$$a = a(x, y), b = b(x, y), c = c(x, y)$$

are nonnegative real numbers such that $b > 0, c > 0$ and $a + b + 2c = 1$, was introduced and investigated by Cirić [7]. Cirić proved that in a complete metric space such mappings have a unique fixed point. Chandra et al. [3] consider the following generalization of (1.3), let $T, f: X \rightarrow X$ satisfying:

$$d(Tx, Ty) \leq a(x, y)d(fx, fy) + b(x, y)\max\{d(fx, Tx), d(fy, Ty)\} + c(x, y)[d(fx, Ty) + d(fy, Tx)] \quad (1.4)$$

where

$$\begin{aligned} a(x, y) &\geq 0, \quad \beta = \inf_{x, y \in X} b(x, y) > 0, \\ \gamma &= \inf_{x, y \in X} c(x, y) > 0 \end{aligned}$$

with

$$\sup_{x, y \in X} (a(x, y) + b(x, y) + 2c(x, y)) = 1$$

Random nonlinear analysis is an important mathematical discipline which is mainly concerned with the study of random nonlinear operators and their properties and is much needed for the study of various classes of random equations. Of course famously random methods have revolutionized the financial markets. Random fixed point theorems for random contraction mappings on separable complete metric spaces were first proved by Spacek [29] and Hans [11-12]. Random fixed point theorems for contraction mappings on separable complete metric spaces have been proved by several authors (Chang and Huang [4], Huang [14], Itoh [15], Liu [19], Papageorgiou [20-21] Shahzad and Latif [26], Shahzad and Hussain [27], Spacek [29], Tan and Yuan [30]). The stochastic version of the well-known Schauder's fixed point theorem was proved by Sehgal and Singh [25].

In this paper, we introduced a new class of non-expansive type mappings for a pair of multi-valued and single valued mappings which is a stochastic version of Chandra et al. [3] fixed point theorem to find the coincidence and fixed points for such class of mappings. This result generalizes and extends the fixed point theorems of Papageorgiou [21], Cirić et al. [8], Jhade et al. [16] and many classical fixed point theorems.

2. PRELIMINARIES

Let (Ω, Σ) be a measurable space with Σ a sigma algebra of subsets of Ω and let (X, d) be a metric space. We denote by 2^X the family of all subsets of X , by $\mathcal{CB}(X)$ the family of all nonempty closed and bounded subsets of X and by H the Hausdorff metric on $\mathcal{CB}(X)$, induced by the metric d . For any $x \in X$ and $A \subseteq X$, by $d(x, A)$ we denote the distance between x and A , i.e.

$$d(x, A) = \inf\{d(x, a): a \in A\}.$$

A mapping $T: \Omega \rightarrow 2^X$ is called Σ -measurable if for any open subset U of X , $T^{-1}(U) = \{\omega: T(\omega) \cap U \neq \emptyset\} \in \Sigma$. In what follows, when we speak of measurability we will mean Σ -measurability. A mapping $f: \Omega \times X \rightarrow X$ is called a random operator if for any $x \in X$, $f(\cdot, x)$ is measurable. A mapping $T: \Omega \times X \rightarrow \mathcal{CB}(X)$ is called a multi-valued random operator if for every $x \in X$, $T(\cdot, x)$ is measurable. A mapping $s: \Omega \rightarrow X$ is called a measurable selector of a measurable multifunction $T: \Omega \rightarrow 2^X$ if s is measurable and $s(\omega) \in T(\omega)$ for all $\omega \in \Omega$.

A measurable mapping $\xi: \Omega \rightarrow X$ is called a random fixed point of a random multifunction $T: \Omega \times X \rightarrow \mathcal{CB}(X)$ if $\xi(\omega) \in T(\omega, \xi(\omega))$ for every $\omega \in \Omega$. A mapping $\xi: \Omega \rightarrow X$ is called a random coincidence of $T: \Omega \times X \rightarrow \mathcal{CB}(X)$ and $f: \Omega \times X \rightarrow X$ if $f(\omega, \xi(\omega)) \in T(\omega, \xi(\omega))$ for every $\omega \in \Omega$.

The aim of this paper is to prove a stochastic analogue of the Chandra et al. [3] fixed point theorem for single valued mappings, extended to a coincidence point theorem for a pair of a random operator $f: \Omega \times X \rightarrow X$ and a multi-valued random operator $T: \Omega \times X \rightarrow \mathcal{CB}(X)$, satisfying the following non-expansive type condition: for each $\omega \in \Omega$,

$$H(T(\omega, x), T(\omega, y)) \leq a(\omega)d(f(\omega, x), f(\omega, y))$$

$$\begin{aligned} &+b(\omega) \max\{d(f(\omega, x), T(\omega, x)), d(f(\omega, y), T(\omega, y))\} \\ &+c(\omega)[d(f(\omega, x), T(\omega, y))+d(f(\omega, y), T(\omega, x))] \end{aligned} \quad (2.1)$$

for every $x, y \in X$, where $a, b, c: \Omega \rightarrow [0,1]$ are measure-able mappings such that for all $\omega \in \Omega$,

$$b(\omega) > 0, c(\omega) > 0 \quad (2.2)$$

$$a(\omega) + b(\omega) + 2c(\omega) = 1 \quad (2.3)$$

3. MAIN RESULTS

Now, we give our main result.

Theorem 3.1 Let (X, d) be a complete metric space, (Ω, Σ) be a measurable space and $T: \Omega \times X \rightarrow \mathcal{CB}(X)$ & $f: \Omega \times X \rightarrow X$ be mappings such that

1. $T(\omega, .)$ and $f(\omega, .)$ are continuous for all $\omega \in \Omega$,
2. $T(. , x)$ and $f(. , x)$ are measurable for all $x \in X$,
3. They satisfy (2.1), where $a(\omega), b(\omega), c(\omega): \Omega \rightarrow X$ satisfy (2.2) and (2.3).

If $T(\omega, \xi(\omega)) \subseteq f(\omega \times X)$ and $f(\omega \times X) = X$ for each $\omega \in \Omega$, then there is a measurable mapping $\xi: \Omega \rightarrow X$ such that $f(\omega, \xi(\omega)) \in T(\omega, \xi(\omega))$ for all $\omega \in \Omega$, i.e. T and f have a random coincidence point.

Proof. Let $\psi = \{\xi: \Omega \rightarrow X\}$ be a family of measurable mappings. Define a function $g: \Omega \times X \rightarrow \mathbb{R}^+$ as follows:

$$g(\omega, x) = d(x, T(\omega, x)). \quad (3.1)$$

Since $x \rightarrow T(\omega, x)$ is continuous for all $\omega \in \Omega$, we conclude that $g(\omega, .)$ is continuous for all $\omega \in \Omega$. Also, since $\omega \rightarrow T(\omega, x)$ is measurable for all $x \in X$, we conclude that $g(. , x)$ is measurable (see Wagner [31], p 868) for all $\omega \in \Omega$. Thus $g(\omega, x)$ is the Caratheodory function. Therefore, if $\xi: \Omega \rightarrow X$ is a measurable mapping, then $\omega \rightarrow g(\omega, \xi(\omega))$ is also measurable (see Rockafellar [24]). Now we shall construct a sequence of measurable mappings $\{\xi_n\}$ in ψ and a sequence $\{f(\omega, \xi_n(\omega))\}$ in X as follows. Let $\xi_0 \in \psi$ be arbitrary. Then the multifunction $G: \Omega \rightarrow \mathcal{CB}(X)$ defined by $G(\omega) = T(\omega, \xi_0(\omega))$ is measurable. From the Kuratowski-Nardzewski [18] Selector Theorem, there is a measurable selector $\mu_1: \Omega \rightarrow X$ such that $\mu_1(\omega) \in T(\omega, \xi_0(\omega))$ for all $\omega \in \Omega$. Since $\mu_1(\omega) \in T(\omega, \xi_0(\omega)) \subseteq X = f(\omega \times X)$, let $\xi_1 \in \psi$ be such that $(\omega, \xi_1(\omega)) = \mu_1(\omega)$. Thus $f(\omega, \xi_1(\omega)) \in T(\omega, \xi_0(\omega))$ for all $\omega \in \Omega$. Let $k: \Omega \rightarrow (1, +\infty)$ defined by

$$k(\omega) = 1 + \frac{b(\omega)c(\omega)}{2} \quad (3.2)$$

for all $\omega \in \Omega$. Then $k(\omega)$ is measurable. Since $k(\omega) > 1$ and $f(\omega, \xi_1(\omega))$ is a selector of $T(\omega, \xi_0(\omega))$, from Lemma 2.1 of Papageorgiou [21] there is a measurable selector $\mu_2(\omega) = f(\omega, \xi_1(\omega))$; $\xi_2 \in \psi$, such that for all $\omega \in \Omega$, $f(\omega, \xi_2(\omega)) \in T(\omega, \xi_1(\omega))$ and

$$d(f(\omega, \xi_1(\omega)), f(\omega, \xi_2(\omega))) \leq k(\omega)H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) \quad (3.3)$$

Similarly, as $f(\omega, \xi_2(\omega))$ is a selector of $T(\omega, \xi_1(\omega))$, there is a measurable selector $\mu_3(\omega) = f(\omega, \xi_3(\omega))$ of $T(\omega, \xi_2(\omega)) \subseteq f(\Omega \times X)$ such that

$$d(f(\omega, \xi_2(\omega)), f(\omega, \xi_3(\omega))) \leq k(\omega)H(T(\omega, \xi_1(\omega)), T(\omega, \xi_2(\omega)))$$

Continuing in this way, we can construct a sequence of measurable mappings $\mu_n: \Omega \rightarrow X$, $\mu_n(\omega) = f(\omega, \xi_n(\omega))$; $\xi_n \in \psi$, such that for all $\omega \in \Omega$,

$$f(\omega, \xi_{n+1}(\omega)) \in T(\omega, \xi_n(\omega))$$

and

$$d(f(\omega, \xi_n(\omega)), f(\omega, \xi_{n+1}(\omega))) \leq k(\omega)H(T(\omega, \xi_{n-1}(\omega)), T(\omega, \xi_n(\omega))) \quad (3.4)$$

Applying (2.1), we have

$$\begin{aligned} H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) &\leq a(\omega)d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) \\ &\quad + b(\omega) \max \{d(f(\omega, \xi_0(\omega)), T(\omega, \xi_0(\omega))), d(f(\omega, \xi_1(\omega)), T(\omega, \xi_1(\omega)))\} \\ &\quad + c(\omega) [d(f(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) + d(f(\omega, \xi_1(\omega)), T(\omega, \xi_0(\omega)))] \end{aligned} \quad (3.5)$$

Since $f(\omega, \xi_1(\omega)) \in T(\omega, \xi_0(\omega))$, then

$$\begin{aligned} d(f(\omega, \xi_1(\omega)), T(\omega, \xi_0(\omega))) &= 0, \\ d(f(\omega, \xi_0(\omega)), T(\omega, \xi_0(\omega))) &\leq d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))), \\ d(f(\omega, \xi_1(\omega)), T(\omega, \xi_1(\omega))) &\leq H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))). \end{aligned}$$

Thus from (3.5), we have

$$\begin{aligned} H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) &\leq a(\omega)d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) \\ &\quad + b(\omega) \max \{d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))), H(T(\omega, \xi_1(\omega)), T(\omega, \xi_1(\omega)))\} \\ &\quad + c(\omega) [d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) + H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega)))] \end{aligned} \quad (3.6)$$

If assume that

$$H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) > d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))).$$

Then from (3.6) and (2.3), we obtain, as $c(\omega) > 0$ and

$$\begin{aligned} H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) &\leq a(\omega)H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) \\ &\quad + b(\omega)H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) \\ &\quad + 2c(\omega)H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) \\ &= (a(\omega) + b(\omega) + 2c(\omega)) \times H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) \\ &= H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) \end{aligned}$$

a contradiction. Therefore, we have

$$H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) \leq d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega)))$$

Since

$$d(f(\omega, \xi_1(\omega)), T(\omega, \xi_1(\omega))) \leq H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega)))$$

we have

$$d(f(\omega, \xi_1(\omega)), T(\omega, \xi_1(\omega))) \leq d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega)))$$

By induction, we can show that

$$H(T(\omega, \xi_n(\omega)), T(\omega, \xi_{n+1}(\omega))) \leq d(f(\omega, \xi_n(\omega)), f(\omega, \xi_{n+1}(\omega))), \quad (3.7)$$

$$d(f(\omega, \xi_n(\omega)), T(\omega, \xi_n(\omega))) \leq d(f(\omega, \xi_{n-1}(\omega)), f(\omega, \xi_n(\omega))) \quad (3.8)$$

for all $n \geq 1$ and for all $\omega \in \Omega$.

From (3.4) and (3.7), we have

$$d(f(\omega, \xi_n(\omega)), f(\omega, \xi_{n+1}(\omega))) \leq k(\omega)d(f(\omega, \xi_{n-1}(\omega)), f(\omega, \xi_n(\omega))) \quad (3.9)$$

From (3.6), we get

$$\begin{aligned} d(f(\omega, \xi_0(\omega)), f(\omega, \xi_2(\omega))) &\leq d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) + d(f(\omega, \xi_1(\omega)), f(\omega, \xi_2(\omega))) \\ &\leq (1 + k(\omega))d(f(\omega, \xi_1(\omega)), f(\omega, \xi_2(\omega))) \end{aligned} \quad (3.10)$$

Again from (2.1), we have

$$\begin{aligned} H(T(\omega, \xi_0(\omega)), T(\omega, \xi_2(\omega))) &\leq a(\omega)d(f(\omega, \xi_0(\omega)), f(\omega, \xi_2(\omega))) \\ &\quad + b(\omega) \max \{d(f(\omega, \xi_0(\omega)), T(\omega, \xi_0(\omega))), d(f(\omega, \xi_2(\omega)), T(\omega, \xi_2(\omega)))\} \\ &\quad + c(\omega) [d(f(\omega, \xi_0(\omega)), T(\omega, \xi_2(\omega))) + d(f(\omega, \xi_2(\omega)), T(\omega, \xi_0(\omega)))] \end{aligned} \quad (3.11)$$

Using (3.7), (3.8), (3.9), (3.10) and triangle inequality, we have

$$d(f(\omega, \xi_2(\omega)), T(\omega, \xi_0(\omega))) \leq H(T(\omega, \xi_1(\omega)), T(\omega, \xi_0(\omega))) \leq d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega)))$$

and

$$\begin{aligned} d(f(\omega, \xi_0(\omega)), T(\omega, \xi_2(\omega))) &\leq d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) + d(f(\omega, \xi_1(\omega)), f(\omega, \xi_2(\omega))) \\ &\quad + d(f(\omega, \xi_2(\omega)), T(\omega, \xi_2(\omega))) \\ &\leq (1 + k(\omega))d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) \\ &\quad + d(f(\omega, \xi_1(\omega)), f(\omega, \xi_2(\omega))) \\ &\leq (1 + 2k(\omega))d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) \end{aligned}$$

Now from (3.11), (3.10), (3.9), and (2.3), we have

$$\begin{aligned} H(T(\omega, \xi_0(\omega)), T(\omega, \xi_2(\omega))) &\leq a(\omega)(1 + 2k(\omega))d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) \\ &\quad + b(\omega)k(\omega)d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) + 2c(\omega)(1 + 2k(\omega))d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) \\ &= [a(\omega)(1 + 2k(\omega)) + b(\omega)k(\omega) + 2c(\omega)(1 + 2k(\omega))]d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) \\ &= [(1 + k(\omega))(a(\omega) + b(\omega) + 2c(\omega)) - b(\omega)]d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) \\ &= [1 + k(\omega) - b(\omega)]d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) \end{aligned}$$

As $1 + k(\omega) \leq 2k(\omega)$, we have

$$H(T(\omega, \xi_0(\omega)), T(\omega, \xi_2(\omega))) \leq [2k(\omega) - b(\omega)]d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) \quad (3.12)$$

From (2.3) and (2.1), as $f(\omega, \xi_2(\omega)) \in T(\omega, \xi_1(\omega))$, we have

$$H(T(\omega, \xi_1(\omega)), T(\omega, \xi_2(\omega))) \leq a(\omega)d(f(\omega, \xi_1(\omega)), f(\omega, \xi_2(\omega)))$$

$$\begin{aligned}
& + b(\omega) \max \left\{ d(f(\omega, \xi_1(\omega)), T(\omega, \xi_1(\omega))), d(f(\omega, \xi_2(\omega)), T(\omega, \xi_2(\omega))) \right\} \\
& + c(\omega) [d(f(\omega, \xi_1(\omega)), T(\omega, \xi_2(\omega))) + d(f(\omega, \xi_2(\omega)), T(\omega, \xi_1(\omega)))] \\
& \leq a(\omega) d(f(\omega, \xi_1(\omega)), f(\omega, \xi_2(\omega))) \\
& + b(\omega) \max \left\{ d(f(\omega, \xi_1(\omega)), T(\omega, \xi_1(\omega))), d(f(\omega, \xi_2(\omega)), T(\omega, \xi_2(\omega))) \right\} \\
& + c(\omega) d(f(\omega, \xi_1(\omega)), T(\omega, \xi_2(\omega)))
\end{aligned} \tag{3.13}$$

Also by (3.12), since $f(\omega, \xi_1(\omega)) \in T(\omega, \xi_0(\omega))$, we have

$$\begin{aligned}
d(f(\omega, \xi_1(\omega)), T(\omega, \xi_2(\omega))) & \leq H(f(\omega, \xi_0(\omega)), T(\omega, \xi_2(\omega))) \\
& \leq [2k(\omega) - b(\omega)] d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega)))
\end{aligned}$$

Thus from (3.13), (3.8) and (2.3), we have

$$\begin{aligned}
H(T(\omega, \xi_1(\omega)), T(\omega, \xi_2(\omega))) & \leq a(\omega) k(\omega) d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) \\
& + b(\omega) k(\omega) d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) + c(\omega) [2k(\omega) - b(\omega)] d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) \\
& = [k(\omega)(a(\omega) + b(\omega) + 2c(\omega)) - b(\omega)c(\omega)] \times d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega)))
\end{aligned}$$

implies that

$$H(T(\omega, \xi_1(\omega)), T(\omega, \xi_2(\omega))) \leq [k(\omega) - b(\omega)c(\omega)] d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) \tag{3.14}$$

From (3.4) and (3.14), we get

$$\begin{aligned}
d(f(\omega, \xi_2(\omega)), f(\omega, \xi_3(\omega))) & \leq k(\omega) H(T(\omega, \xi_1(\omega)), T(\omega, \xi_2(\omega))) \\
& \leq k(\omega)[k(\omega) - b(\omega)c(\omega)] \times d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega)))
\end{aligned} \tag{3.15}$$

As $k(\omega) = 1 + \frac{b(\omega)c(\omega)}{2}$, we have

$$\begin{aligned}
k(\omega)[k(\omega) - b(\omega)c(\omega)] & = \left(1 + \frac{b(\omega)c(\omega)}{2}\right) \left[1 + \frac{b(\omega)c(\omega)}{2} - b(\omega)c(\omega)\right] \\
& = 1 + \frac{b^2(\omega)c^2(\omega)}{4}
\end{aligned}$$

Thus from (3.15), we obtain

$$d(f(\omega, \xi_2(\omega)), f(\omega, \xi_3(\omega))) \leq \left(1 + \frac{b^2(\omega)c^2(\omega)}{4}\right) d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega)))$$

Similarly

$$d(f(\omega, \xi_3(\omega)), f(\omega, \xi_4(\omega))) \leq \left(1 + \frac{b^2(\omega)c^2(\omega)}{4}\right) d(f(\omega, \xi_1(\omega)), f(\omega, \xi_2(\omega)))$$

Hence by induction

$$\begin{aligned}
d(f(\omega, \xi_n(\omega)), f(\omega, \xi_{n+1}(\omega))) & \leq \left(1 + \frac{b^2(\omega)c^2(\omega)}{4}\right)^{\left[\frac{n}{2}\right]} \\
& \max \left\{ d(f(\omega, \xi_1(\omega)), f(\omega, \xi_2(\omega))), d(f(\omega, \xi_0(\omega)), f(\omega, \xi_1(\omega))) \right\}
\end{aligned} \tag{3.16}$$

where $\left[\frac{n}{2}\right]$ stands for the greatest integer not exceeding $\frac{n}{2}$. Also, since $b(\omega)c(\omega) > 0$ for all $\omega \in \Omega$, from (3.16), we have $\{f(\omega, \xi_n(\omega))\}$ is a Cauchy sequence in $f(\omega \times X)$. Since $f(\omega \times X) = X$ is complete, there is measurable mapping $f(\omega, \xi(\omega)) \in f(\omega \times X)$ such that

$$\lim_{n \rightarrow +\infty} f(\omega, \xi_n(\omega)) = f(\omega, \xi(\omega)) \quad (3.17)$$

Now, we prove that $f(\omega, \xi(\omega)) \in T(\omega, \xi(\omega))$. By triangle inequality, we have

$$\begin{aligned} d(f(\omega, \xi(\omega)), T(\omega, \xi(\omega))) &\leq d(f(\omega, \xi(\omega)), f(\omega, \xi_{n+1}(\omega))) + d(f(\omega, \xi_{n+1}(\omega)), T(\omega, \xi(\omega))) \\ &\leq d(f(\omega, \xi(\omega)), f(\omega, \xi_{n+1}(\omega))) + H(T(\omega, \xi_n(\omega)), T(\omega, \xi(\omega))) \end{aligned}$$

Taking limit as $n \rightarrow +\infty$ in above inequality, we have

$$d(f(\omega, \xi(\omega)), T(\omega, \xi(\omega))) \leq \lim_{n \rightarrow +\infty} H(T(\omega, \xi_n(\omega)), T(\omega, \xi(\omega))) \quad (3.18)$$

Again from (2.1), we have

$$\begin{aligned} H(T(\omega, \xi_n(\omega)), T(\omega, \xi(\omega))) &\leq a(\omega)d(f(\omega, \xi_n(\omega)), f(\omega, \xi(\omega))) \\ &\quad + b(\omega) \max \{d(f(\omega, \xi_n(\omega)), T(\omega, \xi_n(\omega))), d(f(\omega, \xi(\omega)), T(\omega, \xi(\omega)))\} \\ &\quad + c(\omega) [d(f(\omega, \xi_n(\omega)), T(\omega, \xi(\omega))) + d(f(\omega, y), T(\omega, \xi_n(\omega)))] \\ &\leq a(\omega)d(f(\omega, \xi_n(\omega)), f(\omega, \xi(\omega))) \\ &\quad + b(\omega) \max \{d(f(\omega, \xi_n(\omega)), f(\omega, \xi_{n+1}(\omega))), d(f(\omega, \xi(\omega)), T(\omega, \xi(\omega)))\} \\ &\quad + c(\omega) [d(f(\omega, \xi_n(\omega)), T(\omega, \xi(\omega))) + d(f(\omega, \xi(\omega)), f(\omega, \xi_{n+1}(\omega)))] \end{aligned}$$

Taking limit as $n \rightarrow +\infty$ in above inequality, we have

$$\lim_{n \rightarrow +\infty} H(T(\omega, \xi_n(\omega)), T(\omega, \xi(\omega))) \leq [b(\omega) + c(\omega)]d(f(\omega, \xi(\omega)), T(\omega, \xi(\omega))) \quad (3.19)$$

Hence from (3.18) and (3.19), we have

$$\begin{aligned} d(f(\omega, \xi(\omega)), T(\omega, \xi(\omega))) &\leq [b(\omega) + c(\omega)]d(f(\omega, \xi(\omega)), T(\omega, \xi(\omega))) \\ &\leq [1 - a(\omega) - c(\omega)]d(f(\omega, \xi(\omega)), T(\omega, \xi(\omega))) \end{aligned}$$

Implies that $d(f(\omega, \xi(\omega)), T(\omega, \xi(\omega))) = 0$ as $1 - a(\omega) - c(\omega) < 1$ and for all $\omega \in \Omega$. Hence as $T(\omega, \xi(\omega))$ is closed, $f(\omega, \xi(\omega)) \in T(\omega, \xi(\omega))$ for all $\omega \in \Omega$.

Corollary 3.2 (Theorem 1, Cirić et al. [8]) Let (X, d) be a complete metric space, (Ω, Σ) be a measurable space and $T: \Omega \times X \rightarrow \mathcal{CB}(X)$ & $f: \Omega \times X \rightarrow X$ be mappings such that

1. $T(\omega, .)$ and $f(\omega, .)$ are continuous for all $\omega \in \Omega$,
2. $T(., x)$ and $f(., x)$ are measurable for all $x \in X$,
3. They satisfy the following condition;

$$\begin{aligned} H(T(\omega, x), T(\omega, y)) &\leq a(\omega) \max \{d(f(\omega, x), f(\omega, y)), d(f(\omega, x), T(\omega, x)), d(f(\omega, y), T(\omega, y)) \\ &\quad , \frac{1}{2}[d(f(\omega, x), T(\omega, y)) + d(f(\omega, y), T(\omega, x))]\} \\ &\quad + b(\omega) \max \{d(f(\omega, x), T(\omega, x)), d(f(\omega, y), T(\omega, y))\} \\ &\quad + c(\omega) [d(f(\omega, x), T(\omega, y)) + d(f(\omega, y), T(\omega, x))] \end{aligned} \quad (3.20)$$

where $a(\omega), b(\omega), c(\omega): \Omega \rightarrow X$ satisfy (2.2) and (2.3). If $T(\omega, \xi(\omega)) \subseteq f(\omega \times X)$ and $f(\omega \times X) = X$ for each $\omega \in \Omega$, then there is a measurable mapping $\xi: \Omega \rightarrow X$ such that $f(\omega, \xi(\omega)) \in T(\omega, \xi(\omega))$ for all $\omega \in \Omega$, i.e. T and f have a random coincidence point.

Proof We shall show that (3.20) is contained in (2.1). Define

$$\begin{aligned}\Delta_{f,T}(x, y, \omega) &= \max\{d(f(\omega, x), f(\omega, y)), d(f(\omega, x), T(\omega, x)), d(f(\omega, y), T(\omega, y)) \\ &\quad , \frac{1}{2}[d(f(\omega, x), T(\omega, y)) + d(f(\omega, y), T(\omega, x))]\}\end{aligned}$$

For each $x, y \in X$ such that

$$\Delta_{f,T}(x, y, \omega) = d(f(\omega, x), f(\omega, y)),$$

define $a(\omega), b(\omega), c(\omega): \Omega \rightarrow (0, 1)$.

For each $x, y \in X$ such that

$$\Delta_{f,T}(x, y, \omega) = \max\{d(f(\omega, x), T(\omega, x)), d(f(\omega, y), T(\omega, y))\},$$

define $a(\omega) = 0, b(\omega) = a(\omega) + b(\omega), c(\omega) = b(\omega)$.

For each $x, y \in X$ such that

$$\Delta_{f,T}(x, y, \omega) = \frac{1}{2}[d(f(\omega, x), T(\omega, y)) + d(f(\omega, y), T(\omega, x))],$$

define $a(\omega) = 0, b(\omega) = 0, c(\omega) = a(\omega) + 2c(\omega)$.

Thus, condition (2.1) is an extension of condition (3.20). All conditions of Theorem 3.1 hold and T and f have a random coincidence point.

Corollary 3.3 (Theorem 3.1, Jhade et al. [16]) Let (X, d) be a complete metric space, (Ω, Σ) be a measurable space and $T: \Omega \times X \rightarrow \mathcal{CB}(X)$ & $f: \Omega \times X \rightarrow X$ be mappings such that

1. $T(\omega, .)$ and $f(\omega, .)$ are continuous for all $\omega \in \Omega$,
2. $T(., x)$ and $f(., x)$ are measurable for all $x \in X$,
3. They satisfy the following condition;

$$\begin{aligned}H(T(\omega, x), T(\omega, y)) &\leq a(\omega) \max\{d(f(\omega, x), f(\omega, y)), d(f(\omega, x), T(\omega, y))\} \\ &\quad + b(\omega) \max\{d(f(\omega, x), T(\omega, x)), d(f(\omega, y), T(\omega, y)), d(f(\omega, y), T(\omega, x))\} \\ &\quad + c(\omega)[d(f(\omega, x), T(\omega, y)) + d(f(\omega, y), T(\omega, x))] \end{aligned} \quad (3.21)$$

where $a(\omega), b(\omega), c(\omega): \Omega \rightarrow X$ satisfy (2.2) and (2.3). If $T(\omega, \xi(\omega)) \subseteq f(\omega \times X)$ and $f(\omega \times X) = X$ for each $\omega \in \Omega$, then there is a measurable mapping $\xi: \Omega \rightarrow X$ such that $f(\omega, \xi(\omega)) \in T(\omega, \xi(\omega))$ for all $\omega \in \Omega$, i.e. T and f have a random coincidence point.

Proof Since

$$d(f(\omega, x), f(\omega, y)) \leq \max\{d(f(\omega, x), f(\omega, y)), d(f(\omega, x), T(\omega, y))\}$$

and

$$\begin{aligned}&\max\{d(f(\omega, x), T(\omega, x)), d(f(\omega, y), T(\omega, y))\} \\ &\leq \max\{d(f(\omega, x), T(\omega, x)), d(f(\omega, y), T(\omega, y)), d(f(\omega, y), T(\omega, x))\}\end{aligned}$$

All conditions of Theorem 3.1 hold and T and f have a random coincidence point.

If in Theorem 3.1, $f(\omega, x) = x$ for all $\omega \in \Omega$, then we get the following random fixed point theorem.

Corollary 3.4 Let (X, d) be a separable complete metric space, (Ω, Σ) be a measurable space and $T: \Omega \times X \rightarrow \mathcal{CB}(X)$ be mapping such that $T(\omega, .)$ is continuous for all $\omega \in \Omega$, $T(. , x)$ is measurable for all $x \in X$ and

$$\begin{aligned} H(T(\omega, x), T(\omega, y)) &\leq a(\omega)d(x, y) + b(\omega)\max\{d(x, T(\omega, x)), d(y, T(\omega, y))\} \\ &\quad + c(\omega)[d(x, T(\omega, y)) + d(y, T(\omega, x))] \end{aligned} \quad (3.22)$$

$\forall x, y \in X$, where $a, b, c: \Omega \rightarrow [0, 1]$ are measurable mappings satisfying (2.2) and (2.3) for all $\omega \in \Omega$, then there is a measurable mapping $\xi: \Omega \rightarrow X$ such that $\xi(\omega) \in T(\omega, \xi(\omega))$ for all $\omega \in \Omega$.

Corollary 3.5 (Corollary 1, Cirić et al. [8]) Let (X, d) be a separable complete metric space, (Ω, Σ) be a measurable space and $T: \Omega \times X \rightarrow \mathcal{CB}(X)$ be mapping such that $T(\omega, .)$ is continuous for all $\omega \in \Omega$, $T(. , x)$ is measurable for all $x \in X$ and

$$\begin{aligned} H(T(\omega, x), T(\omega, y)) &\leq a(\omega)\max\{d(x, y), d(x, T(\omega, x)), d(y, T(\omega, y))\} \\ &\quad + \frac{1}{2}[d(x, T(\omega, y)) + d(y, T(\omega, x))] \\ &\quad + b(\omega)\max\{d(x, T(\omega, x)), d(y, T(\omega, y))\} \\ &\quad + c(\omega)[d(x, T(\omega, y)) + d(y, T(\omega, x))] \end{aligned} \quad (3.23)$$

for every $x, y \in X$, where $a, b, c: \Omega \rightarrow [0, 1]$ are measurable mappings satisfying (2.2) and (2.3) for all $\omega \in \Omega$, then there is a measurable mapping $\xi: \Omega \rightarrow X$ such that $\xi(\omega) \in T(\omega, \xi(\omega))$ for all $\omega \in \Omega$.

Corollary 3.6 (Corollary 3.3, Jhade et al. [16]) Let (X, d) be a separable complete metric space, (Ω, Σ) be a measurable space and $T: \Omega \times X \rightarrow \mathcal{CB}(X)$ be mapping such that $T(\omega, .)$ is continuous for all $\omega \in \Omega$, $T(. , x)$ is measurable for all $x \in X$ and

$$\begin{aligned} H(T(\omega, x), T(\omega, y)) &\leq a(\omega)\max\{d(x, y), d(x, T(\omega, y))\} \\ &\quad + b(\omega)\max\{d(x, T(\omega, x)), d(y, T(\omega, y)), d(y, T(\omega, x))\} \\ &\quad + c(\omega)[d(x, T(\omega, y)) + d(y, T(\omega, x))] \end{aligned} \quad (3.24)$$

$\forall x, y \in X$, where $a, b, c: \Omega \rightarrow [0, 1]$ are measurable mappings satisfying (2.2) and (2.3) for all $\omega \in \Omega$, then there is a measurable mapping $\xi: \Omega \rightarrow X$ such that $\xi(\omega) \in T(\omega, \xi(\omega))$ for all $\omega \in \Omega$.

Corollary 3.7 (Corollary 2, Cirić et al. [8]) Let (X, d) be a complete metric space, (Ω, Σ) be a measurable space and $T: \Omega \times X \rightarrow \mathcal{CB}(X)$ & $f: \Omega \times X \rightarrow X$ be mappings such that

1. $T(\omega, .)$ and $f(\omega, .)$ are continuous for all $\omega \in \Omega$,
2. $T(. , x)$ and $f(. , x)$ are measurable for all $x \in X$,
3. They satisfy the following condition;

$$\begin{aligned} H(T(\omega, x), T(\omega, y)) &\leq \lambda(\omega)\max\{d(f(\omega, x), f(\omega, y)), d(f(\omega, x), T(\omega, x)), d(f(\omega, y), T(\omega, y)) \\ &\quad , \frac{1}{2}[d(f(\omega, x), T(\omega, y)) + d(f(\omega, y), T(\omega, x))]\} \end{aligned} \quad (3.25)$$

where $\lambda(\omega): \Omega \rightarrow X$ is a measurable function. If $T(\omega, \xi(\omega)) \subseteq f(\omega \times X)$ and $f(\omega \times X) = X$ for each $\omega \in \Omega$, then there is a measurable mapping $\xi: \Omega \rightarrow X$ such that $f(\omega, \xi(\omega)) \in T(\omega, \xi(\omega))$ for all $\omega \in \Omega$, i.e. T and f have a random coincidence point.

Proof It is clear that if f and T satisfy (3.25), then f and T satisfy (3.20) with

$$a(\omega) = \lambda(\omega), b(\omega) = \frac{1 - \lambda(\omega)}{2}, c(\omega) = \frac{1 - \lambda(\omega)}{4}$$

Remark 3.8

1. The non-expansive type condition (2.1) includes (3.20) (condition 1.2 of Cirić et al. [8]) and (3.21) (condition 2.1 of Jhade et al. [16]). Thus, Theorem 3.1 is an extension of Theorem 1 of Cirić et al. [8] and Theorem 3.1 of Jhade et al. [16].
2. The non-expansive type condition (3.22) includes (3.23) (condition 2.17 of Cirić et al. [8]) and (3.24) (condition 3.16 of Jhade et al. [16]). Thus, Corollary 3.4 is an extension of Corollary 3.5 (Corollary 1 of Cirić et al. [8]) and Corollary 3.6 (Corollary 3.3 of Jhade et al. [16]).
3. If in Corollary 3.7, $f(\omega, x) = x$ for all $\omega \in \Omega$ then we obtain the corresponding theorems of Hadžić [32] and Papageorgiou [21].
4. Corollary 3.7 is a stochastic generalization and improvement of the corresponding fixed point theorems for contractive type multi-valued mappings of Cirić [5], Cirić and Ume [9], Kubiacyk [33], Kubiak [34], Ray [35] and several other authors.
5. Theorem 3.1 generalizes and extends the corresponding fixed point theorems for non-expansive type single-valued mappings of Cirić [7] and Rhoades [22].

Finally, we give a simple example in support of Theorem 3.1 and Corollary 3.4 which shows that these results are actually an improvement of the result of Itoh [15].

Example 3.9 Let (X, d) be any measurable space and $K = \{0, 1, 2, 4, 6\}$ be the subset of the real line. Let the mappings $f: \Omega \times K \rightarrow K$ and $T: \Omega \times K \rightarrow K$ be defined such that for each $\omega \in \Omega$;

$$f(\omega, 0) = 2, \quad f(\omega, 1) = 4, \quad f(\omega, 2) = 6, \quad f(\omega, 4) = 0, \quad f(\omega, 6) = 1,$$

and

$$T(\omega, 0) = 1, \quad T(\omega, 1) = 2, \quad T(\omega, 2) = 4, \quad T(\omega, 4) = 0, \quad T(\omega, 6) = 0.$$

Then f and T do not satisfy the contractive type condition (2.18). Indeed, for $x = 1$ and $y = 2$, we have

$$d(T(\omega, 1), T(\omega, 2)) = \|2 - 4\| = 2$$

and

$$\begin{aligned} & \max\{d(f(\omega, 1), f(\omega, 2)), d(f(\omega, 1), T(\omega, 1)), d(f(\omega, 2), T(\omega, 2)) \\ & , \frac{1}{2}[d(f(\omega, 1), T(\omega, 2)) + d(f(\omega, 2), T(\omega, 1))]\} \\ & = \max\{\|4 - 6\|, \|4 - 2\|, \|6 - 4\|, \frac{1}{2}[0 + \|6 - 2\|]\} = 2 \end{aligned}$$

Hence for any $\lambda(\omega) < 1$, we have

$$\begin{aligned} d(T(\omega, 1), T(\omega, 2)) & > \lambda(\omega) \max\{d(f(\omega, 1), f(\omega, 2)), d(f(\omega, 1), T(\omega, 1)), d(f(\omega, 2), T(\omega, 2)) \\ & , \frac{1}{2}[d(f(\omega, 1), T(\omega, 2)) + d(f(\omega, 2), T(\omega, 1))]\} \end{aligned}$$

On the other hand, if we take

$$a(\omega) = \frac{4}{5}, b(\omega) = \frac{1}{10}, c(\omega) = \frac{1}{20},$$

we have

$$\begin{aligned} & \frac{4}{5}d(f(\omega, 1), f(\omega, 2)) + \frac{1}{10}\max\{d(f(\omega, 1), T(\omega, 1)), f(\omega, 2), T(\omega, 2)\} \\ & + \frac{1}{20}[d(f(\omega, 1), T(\omega, 2)) + d(f(\omega, 2), T(\omega, 1))] \\ & = \frac{4}{5}\|4 - 6\| + \frac{1}{20}\max\{\|4 - 2\|, \|6 - 4\|\} + \frac{1}{20}[0 + \|6 - 2\|] \\ & = \frac{4}{5} \cdot 2 + \frac{1}{10} \cdot 2 + \frac{1}{20} \cdot 4 = 2 \end{aligned}$$

$$= d(T(\omega, 1), T(\omega, 2))$$

Thus, for $x = 1$ and $y = 2$, f and T satisfy (2.1) with $a(\omega) + b(\omega) + 2c(\omega) = 1$. It is easy to show that f and T satisfy (2.1) for all $x, y \in K$ with the same $a(\omega), b(\omega)$ and $c(\omega)$.

Also, the rest of the assumptions of Theorem 3.1 is satisfied and for $\xi(\omega) = 4$, we have $f(\omega, \xi(\omega)) = 0 = T(\omega, \xi(\omega))$.

Note that if $f(\omega, x) = x$, T does not satisfy (3.17) either, as for instance, for $x = 2$ and $y = 4$, we have

$$\begin{aligned} & a(\omega)d(2,4) + b(\omega)\max\{d(2, T(\omega, 2)), d(4, T(\omega, 4))\} + c(\omega)[d(2, T(\omega, 4)) + d(4, T(\omega, 2))] \\ &= a(\omega)\|2 - 4\| + b(\omega)\max\{\|2 - 4\|, \|4 - 0\|\} + c(\omega)[\|2 - 0\| + \|4 - 4\|] \\ &= 2a(\omega) + 4b(\omega) + 2c(\omega) < 4(a(\omega) + b(\omega) + 2c(\omega)) = 4 = \|4 - 0\| \\ &= d(T(\omega, 2), T(\omega, 4)) \end{aligned}$$

CONFLICT OF INTEREST

No conflict of interest was declared by the authors

AUTHOR'S CONTRIBUTIONS

All authors contributed equally and significantly to writing this paper. All authors read and approved the final manuscript.

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