



## Connection of Ciric type F-Contraction Involving Fixed Point on Closed Ball

Aftab HUSSAIN<sup>1,2,\*</sup>, Muhammad ARSHAD<sup>1</sup>, Muhammad NAZAM<sup>1</sup>

<sup>1</sup> Department of Mathematics, International Islamic University, H-10, Islam-abad - 44000, Pakistan

<sup>2</sup> Department of Basic Sciences & Humanities, Khwaja Fareed University of Engineering and Information Technology, Rahim Yar Khan Pakistan

### Article Info

Received: 22/04/2016

Revised: 13/05/2016

Accepted: 30/05/2016

### Keywords

Metric space

Fixed point

F contraction

Closed ball

### Abstract

This paper is a continuation of the investigations of F-contractions. The aim of this article is to extend the concept of Ciric type F-contraction on a closed ball. We presented the notion of F-contraction on a closed ball and introduced a new approach of fixed point theorems for F-contraction on a closed ball in a complete metric space. Our results are very useful for the contraction of the mapping only on closed ball instead of the whole space. Some comparative examples are constructed which illustrate the superiority of our results. Our results provide extension as well as substantial generalizations and improvements of several well-known results in the existing comparable literature.

## 1. INTRODUCTION

In metric fixed point theory the contractive conditions on underlying functions play an important role for finding solutions of fixed point problems. Banach contraction principle [11] is a fundamental result in metric fixed point theory. Due to its importance and simplicity, several authors have generalized/extended it in different directions. From the application point of view the situation is not yet completely satisfactory because there are many situations in which the mappings are not contractive on the whole space but instead they are contractive on its subsets. However, by imposing a subtle restriction, one can establish the existence of a fixed point of such mappings.

In 1971, Ciric [12] introduced generalized contractions and proved some fixed point theorems. Then a lot of generalization has been given in the literature see [13] and many others. Shoaib et al. [38] proved significant results concerning the existence of fixed points of the dominated self-mappings satisfying some contractive conditions on closed ball in a 0-complete quasi-partial metric space. Other results on closed ball can be seen in [6, 7, 9, 10]. Over the years, Fixed Point Theory has been generalized in different ways by several mathematicians (see [2, 5, 8, 15-17, 20, 22-26, 28, 32, 36-37]).

For  $x \in X$  and  $\varepsilon > 0$ ;  $\overline{B}(x; \varepsilon) = \{y \in X : d(x, y) \leq \varepsilon\}$  is a closed ball in  $(X, d)$ .

Definition 1 ([39]). Let  $T: X \rightarrow X$  and  $\alpha: X \times X \rightarrow [0, +\infty)$ . We say that  $T$  is  $\alpha$ -admissible if  $x, y \in X$ ,  $\alpha(x, y) \geq 1$  implies that  $\alpha(Tx, Ty) \geq 1$ .

Definition 2 ([33]). Let  $T: X \rightarrow X$  and  $\eta: X \times X \rightarrow [0, +\infty)$  be two functions. We say that  $T$  is  $\alpha$ -admissible mapping with respect to  $\eta$  if  $x, y \in X$ ,  $\alpha(x, y) \geq \eta(x, y)$  implies that  $\alpha(Tx, Ty) \geq \eta(Tx, Ty)$ .

If  $\eta(x, y) = 1$ ; then above Definition 2 reduces to Definition 1. If  $\alpha(x, y) = 1$ ; then  $T$  is called an  $\eta$ -sub admissible mapping.

Definition 3 [21] Let  $(X, d)$  be a metric space. Let  $T: X \rightarrow X$  and  $\eta: X \times X \rightarrow [0, +\infty)$  be two functions. We say that  $T$  is  $\alpha - \eta$ -continuous mapping on  $(X, d)$  if for given  $x \in X$ , and sequence  $\{x_n\}$  with  $x_n \rightarrow x$  as  $n \rightarrow \infty$ ,  $\alpha(x_n, x_{n+1}) \geq \eta(x_n, x_{n+1})$  for all  $n \in \mathbb{N} \Rightarrow$

$$Tx_n \rightarrow Tx.$$

In 2012, Wardowski [40] introduce a new type of contractions called  $F$ -contraction and proved new fixed point theorems concerning  $F$ -contraction. He generalized the Banach contraction principle in a different way than as it was done by different investigators see [1, 3, 4, 14, 18, 19, 29, 30, 34, 35].

Piri et al. [31] defined the  $F$ -contraction as follows.

Definition 4 [31] Let  $(X, d)$  be a metric space. A mapping  $T: X \rightarrow X$  is said to be an  $F$  contraction if there exists  $\tau > 0$  such that

$$\forall x, y \in X, d(Tx, Ty) > 0 \quad \tau + F(d(Tx, Ty)) \leq F(d(x, y)). \quad (1.1)$$

Where  $F: \mathbb{R}_+ \rightarrow \mathbb{R}$  is a mapping satisfying the following conditions:

(F1)  $F$  is strictly increasing, i.e. for all  $x, y \in \mathbb{R}_+$  such that  $x < y$ ,  $F(x) < F(y)$ ;

(F2) For each sequence  $\{\alpha_n\}_{n=1}^{\infty}$  of positive numbers,  $\lim_{n \rightarrow \infty} \alpha_n = 0$  if and only if  $\lim_{n \rightarrow \infty} F(\alpha_n) = -\infty$ .

(F3) There exist  $k \in (0, 1)$  such that  $\lim_{\alpha \rightarrow +0} \alpha^k F(\alpha) = 0$ .

We denote by  $\Delta_F$ , the set of all functions satisfying the conditions (F1)-(F3).

Example 5 [40] Let  $F: \mathbb{R}_+ \rightarrow \mathbb{R}$  be given by the formula  $F(\alpha) = \ln \alpha$ . It is clear that  $F$  satisfied (F1)-(F2)-(F3) for any  $k \in (0, 1)$ . Each mapping  $T: X \rightarrow X$  satisfying (1.1) is an  $F$ -contraction such that  $d(Tx, Ty) \leq e^{-\tau} d(x, y)$ , for all  $x, y \in X$  such that  $Tx \neq Ty$ .

It is clear that for  $x, y \in X$  such that  $Tx = Ty$  the inequality  $d(Tx, Ty) \leq e^{-\tau} d(x, y)$ , also holds, i.e.  $T$  is a Banach contraction.

Example 6 [40] If  $F(\alpha) = \ln \alpha + \alpha$ ,  $\alpha > 0$  then  $F$  satisfies (F1)-(F3) and the condition (1.1) is of the form  $\frac{d(Tx, Ty)}{d(x, y)} \leq e^{d(Tx, Ty) - d(x, y)} \leq e^{-\tau}$  for all  $x, y \in X$ ;  $Tx \neq Ty$ .

Remark 7 From (F1) and (1.1) it is easy to conclude that every  $F$ -contraction is necessarily continuous.

Wardowski [40] stated a modified version of the Banach contraction principle as follows.

Theorem 8 [40] Let  $(X, d)$  be a complete metric space and let  $T: X \rightarrow X$  be an  $F$  contraction. Then  $T$  has a unique fixed point  $x^* \in X$  and for every  $x \in X$  the sequence  $\{T^n x\}_{n \in \mathbb{N}}$  converges to  $x^*$ .

Hussain et al. [21] introduced the following family of new functions.

Let  $\Delta_G$  denotes the set of all functions  $G: \mathbb{R}^4 \rightarrow \mathbb{R}^+$  satisfying:

(G) for all  $t_1, t_2, t_3, t_4 \in \mathbb{R}^+$  with  $t_1 t_2 t_3 t_4 = 0$  there exists  $\tau > 0$  such that  $G(t_1, t_2, t_3, t_4) = \tau$ .

Definition 9 [21] Let  $(X, d)$  be a metric space and  $T$  be a self-mapping on  $X$ . Also suppose that  $\eta: X \times X \rightarrow [0, +\infty)$  be two function. We say that  $T$  is  $\alpha$ - $\eta$ -GF-contraction if for  $x, y \in X$ , with  $\eta(x, Tx) \leq \alpha(x, y)$  and  $d(Tx, Ty) > 0$ , we have

$$G(d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)) + F(d(Tx, Ty)) \leq F(d(x, y)),$$

where  $G \in \Delta_G$  and  $F \in \Delta_F$ .

The following result regarding the existence of the fixed point of the mapping satisfying a contractive condition on the closed ball is given in [27, Theorem 5.1.4]. The result is very useful in the sense that it requires the contraction of the mapping only on the closed ball instead on the whole space.

Theorem 10 [27] Let  $(X, d)$  be a complete metric space,  $T: X \rightarrow X$  be a mapping,  $r > 0$  and  $x_0$  be an arbitrary point in  $X$ . Suppose there exists  $k \in [0, 1)$  with

$$d(Tx, Ty) \leq kd(x, y); \text{ for all } x, y \in Y = \overline{B(x_0, r)}$$

and  $d(x_0, Tx_0) < (1-k)r$ . Then there exists a unique point  $x$  in  $\overline{B(x_0, r)}$  such that  $x = Tx$ .

## 2. FIXED POINT THEOREM OF CIRIC TYPE F-CONTRACTION ON CLOSED BALL

In this section, we introduce Banach fixed point theorem for modified  $F$  contraction on closed ball in complete metric spaces. Following theorem not only extend above theorem to metric spaces but also rectifies this mistake especially for those researchers who are utilizing the style of the proof of [27, Theorem 5.1.4] to study more general result. We define Ciric type  $F$ -contraction as follows:

Definition 11 Let  $(X, d)$  be a metric space and  $T$  be a self-mapping on  $X$ . Also suppose that  $\eta : X \times X \rightarrow [0, +\infty)$  be two function. We say that  $T$  is Ciric type  $\alpha$ - $\eta$ -GF-contraction if for  $x, y \in X$ , with  $\eta(x, Tx) \leq \alpha(x, y)$  and  $d(Tx, Ty) > 0$  we have

$$G(d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)) + F(d(Tx, Ty)) \leq F(M(x, y)),$$

$$\text{Where } M(x, y) = \max\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2}\},$$

$$G \in \Delta_G \text{ and } F \in \Delta_F.$$

Now we state our main result.

Theorem 12 Let  $T$  be a continuous self-map in a complete metric space  $(X, d)$  and  $x_0$  be an arbitrary point in  $X$ . Assume that  $\tau > 0$  and  $F \in \Delta_F$  for all  $x, y \in \overline{B(x_0, r)}$  with  $d(Tx, Ty) > 0$  such that

$$\tau + F(d(Tx, Ty)) \leq F(M(x, y)), \tag{2.1}$$

where

$$M(x, y) = \max\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2}\}.$$

$$\text{Moreover } \sum_{j=0}^N d(x_0, Tx_0) \leq r \text{ for all } j \in N \text{ and } r > 0. \tag{2.2}$$

Then there exist a point  $x$  in  $\overline{B(x_0, r)}$  such that  $Tx = x$ .

Proof. Choose a point  $x_1$  in  $X$  such that  $x_1 = Tx_0$ , continuing in this way, so we get  $x_{n+1} = Tx_n$  for all  $n > 0$  and this implies that  $(x_n)$  is a non increasing sequence. First we show that  $x_n \in \overline{B(x_0, r)}$  for all  $n \in N$  by using mathematical induction. Since from (2.2), we have

$$d(x_0, x_1) = d(x_0, Tx_0) \leq r. \tag{2.3}$$

thus,  $x_1 \in \overline{B(x_0, r)}$ . Suppose  $x_2 \dots x_j \in \overline{B(x_0, r)}$  for some  $j \in N$ . Thus from (2.1), we obtain

$$F(d(x_j, x_{j+1})) = F(d(Tx_{j-1}, Tx_j)) \leq F(M(x_{j-1}, x_j)) - \tau$$

$$M(x_{j-1}, x_j) = \max\{d(x_{j-1}, x_j), d(x_{j-1}, x_j), d(x_j, x_{j+1}), \frac{d(x_{j-1}, x_{j+1}) + d(x_j, x_j)}{2}\}$$

$$= \max\{d(x_{j-1}, x_j), d(x_{j-1}, x_j), d(x_j, x_{j+1}), \frac{d(x_{j-1}, x_{j+1})}{2}\}$$

$$= \max\{d(x_{j-1}, x_j), d(x_j, x_{j+1}), \frac{d(x_{j-1}, x_j) + d(x_j, x_{j+1})}{2}\}$$

$$= \max\{d(x_{j-1}, x_j), d(x_j, x_{j+1})\}$$

So, we have

$$F(d(x_j, x_{j+1})) = F(d(Tx_{j-1}, Tx_j)) \leq F(\max \{d(x_{j-1}, x_j), d(x_j, x_{j+1})\}).$$

In this case  $\max \{d(x_{j-1}, x_j), d(x_j, x_{j+1})\} = d(x_j, x_{j+1})$  is impossible, because

$$F(d(x_j, x_{j+1})) \leq F(d(x_j, x_{j+1}))$$

which implies  $\tau \leq 0$ , a contradiction. So

$$\max \{d(x_{j-1}, x_j), d(x_j, x_{j+1})\} = d(x_{j-1}, x_j).$$

As  $F$  is strictly increasing, we have

$$d(x_j, x_{j+1}) < d(x_{j-1}, x_j) \tag{2.4}$$

Now,

$$\begin{aligned} d(x_0, x_{j+1}) &\leq d(x_0, x_1) + \dots + d(x_j, x_{j+1}) \\ &\leq \sum_{j=0}^N d(x_0, x_1) \leq r. \end{aligned}$$

Thus  $x_{j+1} \in \overline{B(x_0, r)}$ . Hence  $x_n \in \overline{B(x_0, r)}$  for all  $n \in \mathbb{N}$ . Continuing this process, we get

$$\begin{aligned} F(d(x_n, x_{n+1})) &\leq F(d(x_{n-1}, x_n)) - \tau \leq F(d(Tx_{n-2}, Tx_{n-1})) - \tau \\ &\leq F(d(x_{n-2}, x_{n-1})) - 2\tau \\ &\leq F(d(Tx_{n-3}, Tx_{n-2})) - 2\tau \\ &\dots \\ &\leq F(d(x_0, x_1)) - n\tau. \end{aligned}$$

This implies that

$$F(d(x_n, x_{n+1})) \leq F(d(x_0, x_1)) - n\tau. \tag{2.5}$$

From (2.5), we obtain  $\lim_{n \rightarrow \infty} d(x_n, x_{n+1}) = -\infty$ . Since  $F \in \Delta_F$ , we have

$$\lim_{n \rightarrow \infty} d(x_n, x_{n+1}) = 0. \tag{2.6}$$

From (F3), there exists  $k \in (0, 1)$  such that

$$\lim_{n \rightarrow \infty} ((d(x_n, x_{n+1})) F(d(x_n, x_{n+1}))) = 0. \tag{2.7}$$

From (2.5), for all  $n \in \mathbb{N}$ , we obtain

$$\begin{aligned} d(x_n, x_{n+1})^k (F(d(x_n, x_{n+1})) F(d(x_0, x_1))) \\ \leq d(x_n, x_{n+1})^k n\tau. \end{aligned} \tag{2.8}$$

By using (2.6), (2.7) and letting  $n \rightarrow \infty$  in (2.8), we have

$$\lim_{n \rightarrow \infty} (n(d(x_n, x_{n+1})^k)) = 0. \tag{2.9}$$

We observe that from (2.9), then there exists  $n_1 \in \mathbb{N}$ ; such that  $n(d(x_n, x_{n+1}))^k \leq 1$  for all  $n \geq n_1$ , we get

$$d(x_n, x_{n+1}) \leq \frac{1}{n^k} \text{ for all } n \geq n_1. \quad (2.10)$$

Now,  $m, n \in \mathbb{N}$  such that  $m > n \geq n_1$ . Then, by the triangle inequality and from (2.10) we have

$$\begin{aligned} d(x_n, x_m) &\leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \dots + d(x_{m-1}, x_m) \\ &\leq \sum_{i=0}^{m-1} d(x_i, x_{i+1}) \leq \sum_{i=n}^{\infty} \frac{1}{i^k} \end{aligned} \quad (2.11)$$

The series  $\sum_{i=n}^{\infty} \frac{1}{i^k}$  is convergent. By taking limit as  $n \rightarrow \infty$  in (2.11), we have  $\lim_{n \rightarrow \infty} d(x_n, x_m) = 0$ . Hence  $\{x_n\}$  is a Cauchy sequence. Since  $X$  is a complete metric space there exists  $x \in \overline{B(x_0, r)}$  such that  $x_n \rightarrow x$  as  $n \rightarrow \infty$ .  $T$  is a continuous then  $x_{n+1} = Tx_n \rightarrow Tx$  as  $n \rightarrow \infty$ . That is,  $x = Tx$ .

Hence  $x$  is a fixed point of  $T$ . To prove uniqueness, let  $x, y \in \overline{B(x_0, r)}$  and  $x \neq y$  be any two fixed point of  $T$ , then from (2.1), we have

$$\tau + F(d(Tx, Ty)) F(M(x, y))$$

we obtain

$$\tau + F(d(x, y)) F(d(x, y)).$$

which is a contradiction. Hence,  $x = y$ . Therefore,  $T$  has a unique fixed point

in a closed ball  $\overline{B(x_0, r)}$ .

Example 13 Let  $X = \mathbb{R}^+$  and  $d(x, y) = |x - y|$ . Then  $(X, d)$  is a complete metric space. Define the mapping  $T : X \rightarrow X$

$$by, T(x) = \begin{cases} \frac{x}{4} & \text{if } x \in [0, 1] \\ x - \frac{1}{2} & \text{if } x \in (1, \infty) \end{cases}$$

$x_0 = 1, r = 2, \overline{B(x_0, r)} = [0, 1]$ . If  $F(\alpha) = \ln \alpha > 0$  and  $\tau > 0$ , then  $d(1, T1) = |1 - 1/4| = 3/4 < r$ . If  $x, y \in \overline{B(x_0, r)}$ , then

$$\frac{1}{4}|x - y| < |x - y|$$

$$\left| \frac{x}{4} - \frac{y}{4} \right| < |x - y|$$

$$d(Tx, Ty) < d(x, y) \leq M(x, y).$$

This implies that

$$\tau + F(d(Tx, Ty)) = \tau + \ln(d(Tx, Ty)) \leq \ln M(x, y) = F(M(x, y)). \text{ If } x, y \in (1, \infty), \text{ then}$$

$$\left| x - \frac{1}{2} - y + \frac{1}{2} \right| = |x - y|$$

$$\tau + |Tx - Ty| > |x - y|$$

$$\tau + F(d(Tx, Ty)) > F(d(x, y)).$$

Then the contractive condition does not hold on  $X$ .

### 3. FIXED POINT THEOREM FOR CIRIC TYPE GF-CONTRACTION ON CLOSED BALL

In this section, we define a new contraction called  $\alpha$ -GF-contraction on closed ball and obtained a new Banach fixed point theorems for such contraction in the setting of complete metric spaces. We define Ciric type  $\alpha$ -GF-contraction on a closed ball as follows:

**Definition 14** Let  $T$  be a self-mapping in a metric space  $(X, d)$  and let  $x_0$  be an arbitrary point in  $X$ . Also suppose that  $\eta : X \times X \rightarrow [0, +\infty)$  two functions. We say that  $T$  is called Ciric type  $\alpha$ -GF-contraction on a closed ball if for all  $x, y \in \overline{B}(x_0, r) \subseteq X$ , with  $\eta(x, Tx) \leq \alpha(x, y)$  and  $d(Tx, Ty) > 0$ , we have

$$G(d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)) + F(d(Tx, Ty)) \leq F(M(x, y)), \quad (3.1)$$

Where  $M(x, y) = \max\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2}\}$  and

$$\sum_{j=0}^N d(x_0, Tx_0) \leq r \text{ for all } j \in \mathbb{N} \text{ and } r > 0. \quad (3.2)$$

$G \in \Delta_G$  and  $F \in \Delta_F$ .

**Theorem 15** Let  $(X, d)$  be a complete metric space. Let  $T : X \rightarrow X$  be a Ciric type  $\alpha$ -GF-contraction mapping on a closed ball satisfying the following assertions:

$T$  is an  $\alpha$ -admissible mapping with respect to  $\eta$ ;

there exists  $x_0 \in X$  such that  $\alpha(x_0, Tx_0) \geq \eta(x_0, Tx_0)$ ;

$T$  is  $\alpha$ - $\eta$ -continuous.

Then there exist a point  $x$  in closed ball  $\overline{B}(x_0, r)$  such that  $Tx = x$ .

**Proof.** Let  $x_0$  in  $X$  such that  $\alpha(x_0, Tx_0) \geq \eta(x_0, Tx_0)$ . For  $x_0 \in X$ , we construct a sequence  $\{x_n\}_{n=1}^{\infty}$  such that  $x_1 = Tx_0$ ,  $x_2 = Tx_1 = T^2x_0$ . Continuing this way,  $x_{n+1} = Tx_n = T^{n+1}x_0$ , for all  $n \in \mathbb{N}$ : Now since,  $T$  is an  $\alpha$ -admissible mapping with respect to  $\eta$  then  $\alpha(x_0, x_1) = \alpha(x_0, Tx_0) \geq \eta(x_0, Tx_0) = \eta(x_0, x_1)$ . By continuing in this process we have,

$$\eta(x_{n-1}, Tx_{n-1}) = \eta(x_{n-1}, x_n) \leq \alpha(x_{n-1}, x_n), \text{ for all } n \in \mathbb{N}. \quad (3.3)$$

If there exists  $n \in \mathbb{N}$  such that  $d(x_n, Tx_n) = 0$ , there is nothing to prove. So,

we assume that  $x_n \neq x_{n+1}$  with

$$d(Tx_{n-1}, Tx_n) = d(x_n, Tx_n) > 0; \forall n \in \mathbb{N}:$$

First we show that  $x_n \in \overline{B}(x_0, r)$  for all  $n \in \mathbb{N}$ . Since  $T$  be a Ciric type  $\alpha$ -GF-contraction on closed ball, we have

$$d(x_0, x_1) = d(x_0, Tx_0) \leq r. \quad (3.4)$$

Thus,  $x_1 \in \overline{B}(x_0, r)$ . Suppose  $x_2, \dots, x_j \in \overline{B}(x_0, r)$  for some  $j \in \mathbb{N}$ , such that

$$G(d(x_{j-1}, Tx_{j-1}), d(x_j, Tx_j), d(x_{j-1}, Tx_j), d(x_j, Tx_{j-1})) + F(d(Tx_{j-1}, Tx_j)) \leq F(M(x_{j-1}, x_j))$$

which implies

$$G(d(x_{j-1}, x_j), d(x_j, x_{j+1}); d(x_{j-1}, x_{j+1}), 0) + F(d(Tx_{j-1}, Tx_j)) \leq F(M(x_{j-1}, x_j)). \quad (3.5)$$

Now by definition of  $G$ ,  $d(x_{j-1}, x_j), d(x_j, x_{j+1}), d(x_{j-1}, x_{j+1}), 0 = 0$ , so there exists  $\tau > 0$  such that,

$$G(d(x_{j-1}, x_j), d(x_j, x_{j+1}), d(x_{j-1}, x_{j+1}), 0) = \tau.$$

Therefore

$$F(d(x_j, x_{j+1})) = F(d(Tx_{j-1}, Tx_j)) F(M(x_{j-1}, x_j)): \quad (3.6)$$

Rest of the proof follows the similar lines of Theorem 12. Since  $X$  is a complete metric space there exists  $x \in \overline{B(x_0, r)}$  such that  $x_n \rightarrow x$  as  $n \rightarrow \infty$ .  $T$  is an  $\alpha$ - $\eta$ -continuous and  $\eta(x_{n-1}, x_n) \leq \alpha(x_{n-1}, x_n)$ , for all  $n \in \mathbb{N}$ , then  $x_{n+1} = Tx_n \rightarrow Tx$  as  $n \rightarrow \infty$ . That is,  $x = Tx$ . Hence  $x$  is a fixed point of  $T$ . ■

Example 16 Let  $X = \mathbb{R}^+$  and  $d$  be the usual metric on  $X$ . Define  $T : X \rightarrow X$ ,  $\alpha, \eta : X \times X \rightarrow [0, +\infty)$ ,  $(\mathbb{R}^+)^4 \rightarrow \mathbb{R}^+$  and  $F : \mathbb{R}^+ \rightarrow \mathbb{R}$  by

$$Tx = \begin{cases} \sqrt{x} & \text{if } x \in [0, 1] \\ 2x & \text{if } x \in (1, \infty) \end{cases} \quad \alpha(x, y) = \begin{cases} e^{x+y} & x \in [0, 1] \\ \frac{1}{3} & \text{otherwise.} \end{cases}$$

$\eta(x, y) = \frac{1}{2}$  for all  $x, y \in X$ .  $G(t_1, t_2, t_3, t_4) = \tau > 0$  and  $F(t) = \ln t$  with  $t > 0$ .  $x_0 = \frac{1}{2}$ ,  $r = 1$ ,  $\overline{B(x_0, r)} = [0, 1]$ , then

$$d\left(\frac{1}{2}, T\frac{1}{2}\right) = \left|\frac{1}{2} - \frac{1}{\sqrt{2}}\right| = 0.30710 < r.$$

$$\text{If } x, y \in \overline{B(x_0, r)} \text{ then } \alpha(x, y) = e^{x+y} \geq \frac{1}{2} = \eta(x, y).$$

On the other hand,

$Tx \in [0, 1]$  for all  $x \in [0; 1]$ . Then  $\alpha(Tx, Ty) \geq \eta(x, Tx)$  with  $d(Tx, Ty) = |\sqrt{x} - \sqrt{y}| > 0$ : clearly  $\alpha(0, T0) \geq \eta(0, T0)$ : Hence we have

$$d(Tx, Ty) = \left| \frac{\sqrt{x} - \sqrt{y} \times \sqrt{x + \sqrt{y}}}{\sqrt{x + \sqrt{y}}} \right| = \left| \frac{x - y}{\sqrt{x + \sqrt{y}}} \right| < |x - y| \leq M(x, y).$$

Consequently,

$$\begin{aligned} \tau + F(d(Tx, Ty)) &= \tau + \ln d(Tx, Ty) \leq \ln M(x; y) = F(M(x; y)). \\ 2|x - y| &\geq |x - y| \\ |2x - 2y| &\geq |x - y| \\ |Tx - Ty| &\geq |x - y| \\ \tau + F(d(Tx, Ty)) &\geq F(d(x, y)). \end{aligned}$$

Then the contractive condition does not hold on  $X$ .

#### 4. CONCLUSION

In this connection, the main aim of our paper is to present new concepts of Ciric type F-contraction on closed ball and different from F-contractions given in [21, 31, 40]. Existence of fixed point results of such type of F-contraction on closed ball in complete metric space are established. The study of results are very useful in the sense that it requires the F-contraction mapping only on the closed ball instead of the whole space. The new concepts lead to further investigations and applications. It will be also interesting to apply these concepts in a different metric spaces.

#### CONFLICT OF INTEREST

No conflict of interest was declared by the authors

## REFERENCES

- [1] M. Abbas, B. Ali and S. Romaguera, Fixed and periodic points of generalized contractions in metric spaces, *Fixed Point Theory Appl.* 2013, 2013:243.
- [2] T. Abdeljawad, Meir-Keeler contractive fixed and common fixed point theorems, *Fixed Point Theory Appl.* 2013 doi:10.1186/1687-1812-2013-19.
- [3] . Acar and I. Altun, A Fixed Point Theorem for Multivalued Mappings with  $\phi$ -Distance, *Abstr. Appl. Anal.*, Volume 2014, Article ID 497092, 5 pages.
- [4] M. Arshad, E. Ameer and A. Hussain , Hardy-Rogers-Type Fixed Point Theorems for  $\alpha$ -GF-Contractions, *Archivum Mathematicum*, 51 (2015), 129 141.
- [5] M. Arshad , Fahimuddin, A. Shoaib and A. Hussain, Fixed point results for  $\alpha$ - $\psi$  -locally graphic contraction in dislocated qusai metric spaces, *Math Sci.*, (2014) doi 10.1007/s40096-014-0132, 7 pages.
- [6] M. Arshad , A. Shoaib, I. Beg, Fixed point of a pair of contractive dominated mappings on a closed ball in an ordered complete dislocated metric space, *Fixed Point Theory and Appl.* (2013), 2013:115, 15 pp.
- [7] M. Arshad, A. Shoaib, and P. Vetro, Common Fixed Points Of A Pair Of Hardy Rogers Type Mappings On A Closed Ball In Ordered Dislocated Metric Spaces, *Journal of Function Spaces and Appl.* 2013 (2013), article ID 638181, 9 pages.
- [8] H. Aydi, W. Shatanawi, M. Postolache, Z. Mustafa, and N. Tahat, Theorems for Boyd-Wong type contractions in ordered metric spaces, *Abstr. Appl. Anal.*, 2012 (2012), Article ID 359054, 14 pages.
- [9] Azam, S. Hussain and M. Arshad, Common fixed points of Chatterjee type fuzzy mappings on closed balls, *Neural Computing & Applications*, (2012) 21 (Suppl 1):S313 S317.
- [10] Azam, M. Waseem, M. Rashid, Fixed point theorems for fuzzy contractive mappings in quasi-pseudo-metric spaces, *Fixed Point Theory Appl.*, 2013:2013:27.
- [11] S.Banach, Sur les opØrations dans les ensembles abstraits et leur application aux equations itegrales, *Fund. Math.*, 3 (1922) 133 181.
- [12] Lj.B. Ćirić, Generalized contractions and fixed point theorems, *Publ. Inst. Math.*, 12 (26) (1971), 19 26.
- [13] LB. Ćirić, A generalization of Banach s contraction principle. *Proc. Am. Math. Soc.*, 45 (1974) 267-273.
- [14] M. Cosentino, P. Vetro, Fixed point results for F-contractive mappings of Hardy-Rogers-Type, *Filomat* 28:4(2014), 715-722. doi:10.2298/FIL1404715C
- [15] M. Edelstein, On fixed and periodic points under contractive mappings. *J. Lond. Math. Soc.*, 37, 74-79 (1962).
- [16] Fisher, Set-valued mappings on metric spaces, *Fundamenta Mathematicae*, 112 (2) (1981) 141 145.
- [17] M. Geraghty, On contractive mappings, *Proc. Amer. Math. Soc.*, 40 (1973) 604-608.
- [18] Hussain, M. Arshad and S. U. Khan, -Generalization of Fixed Point Results for F-Contractions, *Bangmod Int. J. Math & Comp. Sci.* 1(1) (2015), 136-146.
- [19] N. Hussain, M. Arshad, M. Abbas and A. Hussain, Generalized Dynamic Process for Generalized (f,L)-almost F-Contraction with Applications, *J. Nonlinear Sci. Appl.* 9(3) (2016), 1702 1715.
- [20] N. Hussain, M. Arshad, A. Shoaib and Fahimuddin, Common fixed point results for  $\phi$ -contractions on a metric space endowed with graph, *J. Inequal. Appl.*, (2014) 2014:136.
- [21] N. Hussain and P. Salimi, suzuki-wardowski type xed point theorems for  $\alpha$ -GF-contractions, *Taiwanese J. Math.*, 20 (20) (2014), doi: 10.11650/tjm.18.2014.4462



- [22] N. Hussain, E. Karapınar, P. Salimi and F. Akbar,  $\alpha$ -admissible mappings and related fixed point theorems, *J. Inequal. Appl.*, 114 (2013) 1-11.
- [23] N. Hussain, P. Salimi and A. Latif, Fixed point results for single and set-valued  $\alpha$ - $\psi$ - $\eta$ -contractive mappings, *Fixed Point Theory Appl.* 2013, 2013:212.
- [24] N. Hussain, E. Karapınar, P. Salimi, P. Vetro, Fixed point results for  $G^m$ -Meir-Keeler contractive and  $G$ - $(\alpha, \psi)$ -Meir-Keeler contractive mappings, *Fixed Point Theory Appl.* 2013, 2013:34.
- [25] N. Hussain, S. Al-Mezel and P. Salimi, Fixed points for  $\alpha$ - $\psi$ -graphic contractions with application to integral equations, *Abstr. Appl. Anal.*, (2013) Article 575869.
- [26] E. Karapınar and B. Samet, Generalized  $(\alpha$ - $\psi)$ -contractive type mappings and related fixed point theorems with applications, *Abstr. Appl. Anal.*, (2012) Article id:793486.
- [27] E. Kryszyż, *Introductory Functional Analysis with Applications*, John Wiley & Sons, New York, (Wiley Classics Library Edition) 1989.
- [28] M.A. Kutbi, M. Arshad and A. Hussain, On Modified Contractive mappings, *Abstr. Appl. Anal.*, (2014) Article ID 657858, 7 pages.
- [29] M. A. Kutbi, M. Arshad and A. Hussain, Fixed Point Results for Ciric type  $\alpha$ - $\eta$ -GF-Contractions, *J. Comp. Anal. Appl.*, 21 (3) 2016, 466-481.
- [30] G. Minak, A. Halvacı and I. Altun, Ciric type generalized  $F$ -contractions on complete metric spaces and fixed point results, *Filomat*, 28 (6) (2014), 1143-1151.
- [31] H. Piri and P. Kumam, Some fixed point theorems concerning  $F$ -contraction in complete metric spaces, *Fixed Point Theory Appl.* (2014) 2014:210.
- [32] J. Roshan, V. Parvaneh, S. Sedghi, N. Shobkolaei and W. Shatanawi, Common Fixed Points of Almost Generalized  $(\psi, \varphi)$ - $s$ -Contractive Mappings in Ordered  $b$ -Metric Spaces, *Fixed Point Theory Appl.*, 2013, 2013:159, 1-23.
- [33] P. Salimi, A. Latif and N. Hussain, Modified  $\alpha$ - $\psi$ -Contractive mappings with applications, *Fixed Point Theory Appl.* (2013) 2013:151.
- [34] N.A. Secelean, Iterated function systems consisting of  $F$ -contractions, *Fixed Point Theory Appl.* 2013, Article ID 277 (2013). doi:10.1186/1687-18122013-277
- [35] M. Sgroi, C. Vetro, Multi-valued  $F$ -Contractions and the Solution of certain Functional and integral Equations, *Filomat* 27:7, (2013), 1259-1268.
- [36] W. Shatanawi, M. Postolache, Common fixed point results of mappings for nonlinear contraction of cyclic form in ordered metric spaces, *Fixed Point Theory Appl.* 2013, 2013:60, 1-13.
- [37] W. Shatanawi, Some fixed point results for a generalized  $\alpha$ -weak contraction mappings in orbitally metric spaces, *Chaos, Solitons and Fractals* 45 (2012) 520526.
- [38] Shoab, M. Arshad and J. Ahmad, Fixed point results of locally contractive mappings in ordered quasi-partial metric spaces, *The Scientific World Journal*, 2013 (2013), Article ID 194897, 8 pp.
- [39] Samet, C. Vetro and P. Vetro, Fixed point theorems for  $\alpha$ - $\psi$ -contractive type mappings, *Nonlinear Anal.* 75 (2012) 2154-2165.
- [40] Wardowski, Fixed point theory of a new type of contractive mappings in complete metric spaces. *Fixed Point Theory Appl.* (2012) Article ID 94.