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Common Fixed Point Theorems in Relatively Intuitionistic Fuzzy Metric Spaces

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Abstract

Fixed points theorem in relatively two intuitionistic fuzzy metric spaces is obtained by generalizing a theorem of [6] in fuzzy metric space.

1. INTRODUCTION

The concept of fuzzy sets was introduced initially by Zadeh [12] in 1965. George and Veeramani [7] slightly modified the concept of fuzzy metric space introduced by Kramosil and Michalek [9]. In 1986, Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets by generalizing fuzzy sets. In 2004, Park [8] defined the concept of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms. Recently, in 2006, Alaca et al. [1] defined the concept of intuitionistic fuzzy metric space with the help of continuous t–norms and continuous t-conorms as a generalization of fuzzy metric space which is introduced by Kramosil and Michalek [7]. The aim of this paper is to obtain a common fixed point theorem for a pair of maps intuitionistic fuzzy metric space. Our theorem extend and generalize a theorem of Hamaizia and Aliouche [6].

2. PRELIMINARIES

First of all we recall the following basic properties of fuzzy metric space:

Definition 1. [8]. A binary operation $*: [0,1] \times [0,1] \longrightarrow [0,1]$ is called a continuous t-norm if it satisfies the following conditions:

1) $*$ is associative and commutative,

2) $*$ is continuous,

3) $a * 1 = 1$ for all $a \in [0,1]$,

4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c \in [0,1]$.

Two typical examples of a continuous t-norm are $a * b = ab$ and $\min\{a, b\}$.

Definition 2. [8]. A binary operation $\Diamond: [0,1] \times [0,1] \longrightarrow [0,1]$ is called a continuous t-norm if it satisfies the following conditions:

1) $\Diamond$ is associative and commutative,

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2) $\lozenge$ is continuous,

3) $a \lozenge 0 = 0$ for all $a \in [0,1]$,

4) $a \lozenge b \leq c \lozenge d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c \in [0,1]$.

Alaca et al. [1] introduced the notion of intuitionistic fuzzy metric space which follows:

Definition 3. [1]. A 5-tuple $(X,M,N,*,\lozenge)$ is called an intuitionistic fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm, $\lozenge$ is a continuous t-conorm and M,N are a fuzzy sets on $X^2 \times (0,1)$ satisfying the following conditions :

(1) $M(x,y,t) + N(x,y,t) \leq 1$ for all $x, y \in X$ and $t > 0$;

(2) $M(x,y,0) = 0$ for all $x, y \in X$;

(3) $M(x,y,t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(4) $M(x,y,t) = M(y,x,t)$ for all $x, y \in X$ and $t > 0$;

(5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for each $x, y, z \in X$ and $t, s > 0$;

(6) For all $x, y \in X$, $M(x, y, .) : (0,1) \longrightarrow [0, 1]$ is continuous;

(7) $\lim_{n\to\infty} M(x,y,t) = 1$ for all $x, y \in X$ and $t > 0$;

(8) $N(x, y, 0) = 1$ for all $x, y \in X$;

(3) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(9) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(10) $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s)$,for each $x, y, z \in X$ and $t, s > 0$;

(11) For all $N(x, y, .) : (0,1) \longrightarrow [0, 1]$ is continuous;

(12) $\lim_{n\to\infty} N(x,y,t) = 0$ for all $x, y \in X$ and $t > 0$.

Then (M,N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ respectively denote the degree of nearness and degree of nonnearness between x and y with respect to t.

Remark 1. [2]Every fuzzy metric space $(X,M, *)$ is an intuitionistic fuzzy metric space of the form $(X,M, 1 - M, *,\lozenge)$ such that t-norm $*$ and t-conorm $\lozenge$,are associated as $x \lozenge y = 1 - ((1 - x) * (1 - y))$ for all $x, y \in X$.

Remark 2. [2]In the intuitionistic fuzzy metric space $(X,M,N, *,\lozenge)$, $M(x, y, \cdot)$ is nondecreasing and $N(x, y, \cdot)$ is non-increasing for all $x, y \in X$.

Definition 4. [1]. Let $(X,M, 1 - M, *,\lozenge)$ be an intuitionistic fuzzy metric space. Then

(a) A sequence $\{x_n\}$ in X is said to be convergent to a point x in X if and only if

$$\lim_{n\to\infty} M(x_n, x, t) = 1 \text{ and } \lim_{n\to\infty} N(x_n, x, t) = 0 \text{ for each } t > 0.$$

(b) A sequence $\{x_n\}$ in X is called Cauchy sequence if

$$\lim_{n\to\infty} M\left(x_{n+p}, x, t\right) = 1 \text{ and } \lim_{n\to\infty} N\left(x_{n+p}, x, t\right) = 0 \text{ for each } p > 0 \text{ and } t > 0.$$

Definition 5. [1] An intuitionistic fuzzy metric space $(X,M,N, *,\lozenge)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Lemma 1. [2] Let $\{x_n\}$ is a sequence in a intuitionistic fuzzy metric space

$(X,M,N,*,\Diamond)$. If there exists a constant $k \in (0, 1)$ such that

$$M(x_{n+1}, x_n, k\,t) \geq M(x_{n-1}, x_n, t)$$
$$N(x_{n+1}, x_n, k\,t) \leq N(x_{n-1}, x_n, t)$$

Then $\{x_n\}$ is a Cauchy sequence in X.

Lemma 2. [2]Let $(X,M,N,*,\Diamond)$ be an intuitionistic fuzzy metric space and for all

x, y in X, $t > 0$ and if there exists a number $k \in (0, 1)$

$M(x,y,k\,t) \geq M(x,y,t)$ and $N(x,y,k\,t) \leq N(x,y,t)$

then $x = y$.

In the interest of our main result we shall recall a theorem proved by Hamaizia and A. Aliouche [6]:

Theorem 1. Let (X, M_1, θ_1) and (Y, M_2, θ_2) be complete fuzzy metric spaces with $M_1(x, x', t) \to 1$ as $t \to 1$ for all $x, x' \in X$ and $M_2(y, y', t) \to 1$ as $t \to 1$ for all $y, y' \in Y$. Let $T : X \to Y$, $S : Y \to X$ be mappings satisfying:

$M_1(STx, STx', kt) \geq \min\{M_1(x,x',t), M_1(x,ST\,x,t), M_1(x',ST\,x',t), M_2(T\,x,T\,x',t)\}$

$M_2(TSy, TSy', kt) \geq \min\{M_2(y,y',t), M_2(y,TS\,y,t), M_2(y',TS\,y',t), M_1(Sy,Sy',t)\}$

for all $x, x' \in X$, $y, y' \in Y$ and for all $t > 0$, where $0 < k < 1$. Then ST has a unique fixed point z in X and TS has a unique fixed point w in Y . Further, $Tz = w$ and $Sw = z$.

3. MAIN RESULT

We prove our main theorem (2) which is an extension of Theorem(1) of fuzzy metric space in to intuitionistic fuzzy metric space.

Theorem 2. Let $(X,M_1,N_1,*,\Diamond)$ and $(Y,M_2,N_2,*,\Diamond)$ be complete intuitionistic fuzzy metric spaces with $M_1(x, x', t) \to 1$ as $t \to 1$ for all $x, x' \in X$ and $M_2(y, y', t) \to 1$ as $t \to 1$ for all $y, y' \in Y$. Let $T : X \to Y$, $S : Y \to X$ be mappings satisfying:

(3.1) $M_1(STx, STx', kt) \geq \min\{M_1(x,x',t), M_1(x,ST\,x,t), M_1(x',ST\,x',t), M_2(T\,x,T\,x',t)\}$

(3.2) $N_1(STx, STx', kt) \leq \max\{N_1(x,x',t), N_1(x,ST\,x,t), N_1(x',ST\,x',t), N_2(T\,x,T\,x',t)\}$

(3.3) $M_2(TSy, TSy', kt) \geq \min\{M_2(y,y',t), M_2(y,TS\,y,t), M_2(y',TS\,y',t), M_1(Sy,Sy',t)\}$

(3.4) $N_2(TSy, TSy', kt) \leq \max\{N_2(y,y',t), N_2(y,TS\,y,t), N_2(y',TS\,y',t), N_1(Sy,Sy',t)\}$

for all $x, x' \in X$, $y, y' \in Y$ and for all $t > 0$, where $0 < k < 1$. Then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Indeed $Tz = w$ and $Sw = z$, whenever T is continuous.

Proof. Let x be an arbitrary point in X. We define the sequences $\{x_n\}$ and $\{y_n\}$ in X and Y respectively by:

$$Sy_n = x_n,\ Tx_{n-1} = y_n,$$

for n=1, 2, ... Putting $x = x_n$ and $y = y_n$ for all n. Applying inequality (3.1),(3.2) we get

(3.5) $M_1(x_{n+1}, x_n, kt)\,) \geq \min\{M_1(x_n,x_{n-1},t), M_1(x_n,x_{n+1},t), M_1(x_{n-1},x_n,t), M_2(y_{n+1},y_n,t)\}$,

(3.6) $N_1(x_{n+1}, x_n, kt)\,) \leq \max\{N_1(x_n,x_{n-1},t), N_1(x_n,x_{n+1},t), N_1(x_{n-1},x_n,t), N_2(y_{n+1},y_n,t)\}$,

Using inequalities (3.3),(3.4) we have

(3.7) $M_2(y_{n+1}, y_n, kt)\,) \geq \min\{M_2(y_n,y_{n-1},t), M_2(y_n,y_{n+1},t), M_2(y_{n-1},y_n,t), M_1(x_n,x_{n-1},t)\}$,

(3.8) $N_2(y_{n+1}, y_n, kt)\,) \leq \max\{N_2(y_n,y_{n-1},t), N_2(y_n,y_{n+1},t), N_2(y_{n-1},y_n,t), N_1(x_n,x_{n-1},t)\}$,

involve, respectively

(3.9) $M_1(x_{n+1}, x_n, kt)\,) \geq \min\{M_1(x_n,x_{n-1},t), M_2(y_{n+1},y_n,t)\}$, (3.10) $N_1(x_{n+1}, x_n, kt)) \leq \max\{N_1(x_n, x_{n-1}, t), N_2(y_{n+1}, y_n, t)\}$,

And

(3.11) $M_2(y_{n+1}, y_n, kt)) \geq \min\{M_2(y_n, y_{n-1}, t), M_1(x_n, x_{n-1}, t)\}$,

(3.12) $N_2(y_{n+1}, y_n, kt)) \leq \max\{N_2(y_n, y_{n-1}, t), N_1(x_n, x_{n-1}, t)\}$,

using inequalities (3.1), (3.2) again, it follows that

(3.13) $M_1(x_{n+1}, x_n, kt)) \geq \min\{M_1(x_n, x_{n-1}, t), M_2(y_{n+1}, y_n, t)\}$,

(3.14) $N_1(x_{n+1}, x_n, kt)) \leq \max\{N_1(x_n, x_{n-1}, t), N_2(y_{n+1}, y_n, t)\}$,

In the similar way, using inequality (3.3),(3.4) we get

(3.15) $M_2(y_{n+1}, y_n, kt)) \geq \min\{M_2(y_n, y_{n-1}, t), M_1(x_n, x_{n-1}, t)\}$,

(3.16) $M_2(y_n, y_{n-1}, kt)) \geq \min\{M_2(y_{n-1}, y_{n-2}, t), M_1(x_{n-1}, x_{n-2}, t)\}$,

And

(3.17) $N_2(y_{n+1}, y_n, kt)) \leq \max\{N_2(y_n, y_{n-1}, t), N_1(x_n, x_{n-1}, t)\}$,

(3.18) $N_2(y_n, y_{n-1}, kt)) \leq \max\{N_2(y_{n-1}, y_{n-2}, t), N_1(x_{n-1}, x_{n-2}, t)\}$,

Using inequalities (3.9),(3.15) and (3.10),(3.17), we have

(3.19) $M_1(x_{n+1}, x_n, kt)) \geq \min\{M_1(x_n, x_{n-1}, t), M_2(y_n, y_{n-1}, t)\}$,

(3.20) $N_1(x_{n+1}, x_n, kt)) \leq \max\{N_1(x_n, x_{n-1}, t), N_2(y_n, y_{n-1}, t)\}$,

In a similar way by using inequalities (3.13),(3.16) and (3.14),(3.18), we get

(3.21) $M_1(x_{n+1}, x_n, kt)) \geq \min\{M_1(x_n, x_{n-1}, t), M_2(y_n, y_{n-1}, t)\}$,

(3.22) $N_1(x_{n+1}, x_n, kt)) \leq \max\{N_1(x_n, x_{n-1}, t), N_2(y_n, y_{n-1}, t)\}$,

It now follows inequalities (3.15),(3.16),(3.19),(3.21) and (3.17),(3.18),(3.20),(3.22) that

(3.23) $M_1(x_{n+1}, x_n, kt)) \geq M_2(y_n, y_{n-1}, t)$,

(3.24) $M_2(y_{n+1}, y_n, kt)) \geq M_1(x_n, x_{n-1}, t)$,

and

(3.25) $N_1(x_{n+1}, x_n, kt)) \geq N_2(y_n, y_{n-1}, t)$,

(3.26) $N_2(y_{n+1}, y_n, kt)) \geq N_1(x_n, x_{n-1}, t)$.

Using (3.23),(3.24) and(3.25),(3.26) we have for n=1, 2,

$$M_1(x_{n+1}, x_n, t)) \geq M_2\left(y_n, y_{n-1}, \frac{t}{k^2}\right),$$

$$M_2(y_{n+1}, y_n, t)) \geq M_1\left(x_n, x_{n-1}, \frac{t}{k^2}\right),$$

and

$$N_1(x_{n+1}, x_n, t)) \geq N_2\left(y_n, y_{n-1}, \frac{t}{k^2}\right),$$

$$N_2(y_{n+1}, y_n, kt)) \geq N_1\left(x_n, x_{n-1}, \frac{t}{k^2}\right).$$

From lemma 2, it follows that x_n and y_n are cauchy sequences in X and Y respec-tively. Hence x_n converges to z in X and y_n converges to w in Y . Now, suppose that T is continuous, then

$$\lim Tx_{n-1} = Tz = \lim y_n = w$$

and so Tz = w. Applying inequalities (3.1) and (3.2),we have

$$M_1(STz, STx_{n-1}, kt) \geq \min\{M_1(z, x_{n-1}, t), M_1(z, ST\,z, t), M_1(x_{n-1}, ST\,x_{n-1}, t), M_2(T\,z, T\,x_{n-1}, t)\},$$

$$N_1(STz, STx_{n-1}, kt) \leq \max\{N_1(z, x_{n-1}, t), N_1(z, ST\,z, t), N_1(x_{n-1}, ST\,x_{n-1}, t), N_2(T\,z, T\,x_{n-1}, t)\},$$

letting n tend to infinity, we have

$$M_1(Sw, z, kt) \geq \min\{1, M_1(z, Sw, t), 1\},$$

$$N_1(Sw, z, kt) \leq \max\{0, N_1(z, Sw, t), 0\},$$

so Sw = z. In the same manner we can show that T z = w. Finally we show that the fixed point is unique. Suppose that ST has a second fixed point z' in X Then, using inequalities (3.1) and (3.2), we have

$$M_1(z, z', kt) \geq \min\{M_1(z, z', t), M_2(T\,z, T\,z', t)\},$$

$$N_1(z, z', kt) \leq \max\{N_1(z, z', t), N_2(T\,z, T\,z', t)\},$$

Again, using inequality (3.3) and (3.4) we have

(3.29) $$M_2(T\,z, T\,z', kt) \geq \min\{M_2(T\,z, T\,z', t), M_2(T\,z', T\,z', t), M_2(T\,z, T\,z, t), M_1(z, z', t)\}$$

(3.30) $$N_2(T\,z, T\,z', kt) \leq \max\{N_2(T\,z, T\,z', t), N_2(T\,z', T\,z', t), N_2(T\,z, T\,z, t), N_1(z, z', t)\}$$

It now follows easily from inequalities (3.27), (3.28 and (3.29), (3.30) that

$$M_1(z, z', kt) \geq M_2(T\,z, T\,z', t)\ ,$$

$$N_1(z, z', kt) \leq N_2(T\,z, T\,z', t),$$

and

$$M_2(T\,z, T\,z', kt) \geq M_1(z, z', t)$$

$$N_2(T\,z, T\,z', kt) \leq N_1(z, z', t).$$

Thus, we see that ,

$$M_1(z, z', kt) \geq M_1\left(z, z', \frac{t}{k^2}\right)\ ,$$

$$N_1(z, z', kt) \leq N_1\left(z, z', \frac{t}{k^2}\right),$$

and so $z = z'$. The uniqueness of w follows in a similar manner.

Now we shall establish a theorem involving quadratic terms and proof the theorem is basically depends on Theorem(2) of this paper.

Theorem 3. Let $(X,M_1,N_1, *,\lozenge)$ and $(Y,M_2,N_2, *,\lozenge)$ be complete intuitionistic fuzzy metric spaces with $M_1(x, x', t) \to 1$ as $t \to 1$ for all $x, x' \in X$ and $M_2(y, y', t) \to 1$ as $t \to 1$ for all $y, y' \in Y$. Let $T : X \to Y$, $S : Y \to X$ be mappings satisfying:

(3.31) $$M_1^2(STx, STx', kt) \geq \min\{M_1^2(x, x', t), M_1(x, ST\,x, t) * M_1(x', ST\,x', t), M_2^2(T\,x, T\,x', t)\}$$

(3.32) $$N_1^2(STx, STx', kt) \leq \max\{N_1^2(x, x', t), N_1(x, ST\,x, t) \lozenge N_1(x', ST\,x', t), N_2^2(T\,x, T\,x', t)\}$$

(3.33) $$M_1^2(TSy, TSy', kt) \geq \min\{M_2^2(y, y', t), M_2(y, TS\,y, t) * M_2(y', TS\,y', t), M_1^2(Sy, Sy', t)\}$$

(3.34) $$N_1^2(TSy, TSy', kt) \leq \max\{N_2^2(y, y', t), N_2(y, TS\,y, t) \lozenge N_2(y', TS\,y', t), N_1^2(Sy, Sy', t)\}$$

for all $x, x' \in X$, $y, y' \in Y$ and for all $t > 0$, where $0 < k < 1$. Then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Indeed Tz = w and Sw = z, whenever T is continuous.

Proof. Let x be an arbitrary point in X. We define the sequences $\{x_n\}$ and $\{y_n\}$ in X and Y respectively by:

$$Sy_n = x_n,\ T\,x_{n-1} = y_n,$$

for n=1, 2, ... Putting $x = x_n$ and $y = y_n$ for all n. Applying inequality (3.31),(3.32) we get

(3.35) $$M_1^2(x_{n+1}, x_n, kt) \geq \min\{M_1^2(x_n, x_{n-1}, t), M_1(x_n, x_{n+1}, t) * M_1(x_{n-1}, x_n, t), M_2^2(y_{n+1}, y_n, t)\},$$

(3.36) $N_1^2(x_{n+1}, x_n, kt) \leq \max\{N_1^2(x_n, x_{n-1}, t), N_1(x_n, x_{n+1}, t) \lozenge N_1(x_{n-1}, x_n, t), N_2^2(y_{n+1}, y_n, t)\}$,

Using inequalities (3.33),(3.34) we have

(3.37) $M_1^2(y_{n+1}, y_n, kt) \geq \min\{M_2^2(y_n, y_{n-1}, t), M_2(y_n, y_{n+1}, t) * M_2(y_{n-1}, y_n, t), M_1^2(x_n, x_{n-1}, t)\}$,

(3.38) $N_1^2(y_{n+1}, y_n, kt) \leq \max\{N_2^2(y_n, y_{n-1}, t), N_2(y_n, y_{n+1}, t) \lozenge N_2(y_{n-1}, y_n, t), N_1^2(x_n, x_{n-1}, t)\}$,

involve, respectively

(3.39) $M_1^2(x_{n+1}, x_n, kt) \geq \min\{M_1^2(x_n, x_{n-1}, t), M_2^2(y_{n+1}, y_n, t)\}$,

(3.40) $N_1^2(x_{n+1}, x_n, kt) \leq \max\{N_1^2(x_n, x_{n-1}, t), N_2^2(y_{n+1}, y_n, t)\}$,

and

(3.41) $M_2^2(y_{n+1}, y_n, kt) \geq \min\{M_2^2(y_n, y_{n-1}, t), M_1^2(x_n, x_{n-1}, t)\}$,

(3.42) $N_2^2(y_{n+1}, y_n, kt) \leq \max\{N_2^2(y_n, y_{n-1}, t), N_1^2(x_n, x_{n-1}, t)\}$,

using inequalities (3.31), (3.32) again, it follows that

(3.43) $M_1^2(x_{n-1}, x_n, kt) \geq \min\{M_1^2(x_{n-2}, x_{n-1}, t), M_2^2(y_n, y_{n-1}, t)\}$,

(3.44) $N_1^2(x_{n-1}, x_n, kt) \leq \max\{N_1^2(x_{n-2}, x_{n-1}, t), N_2^2(y_n, y_{n-1}, t)\}$,

In the similar way, using inequality (3.33),(3.34) we get

(3.45) $M_2^2(y_{n+1}, y_n, kt) \geq \min\{M_2^2(y_n, y_{n-1}, t), M_1^2(x_n, x_{n-1}, t)\}$,

(3.46) $M_2^2(y_n, y_{n-1}, kt) \geq \min\{M_2^2(y_{n-1}, y_{n-2}, t), M_1^2(x_{n-1}, x_{n-2}, t)\}$,

and

(3.47) $N_2^2(y_{n+1}, y_n, kt) \leq \max\{N_2^2(y_n, y_{n-1}, t), N_1^2(x_n, x_{n-1}, t)\}$,

(3.48) $N_2^2(y_n, y_{n-1}, kt) \leq \max\{N_2^2(y_{n-1}, y_{n-2}, t), N_1^2(x_{n-1}, x_{n-2}, t)\}$,

Using inequalities (3.39),(3.45) and (3.40),(3.47), we have

(3.49) $M_1^2(x_{n+1}, x_n, kt) \geq \min\{M_1^2(x_n, x_{n-1}, t), M_2^2(y_n, y_{n-1}, t)\}$,

(3.50) $N_1^2(x_{n+1}, x_n, kt) \leq \max\{N_1^2(x_n, x_{n-1}, t), N_2^2(y_n, y_{n-1}, t)\}$,

In a similar way by using inequalities (3.43),(3.46) and (3.44),(3.48), we get

(3.51) $M_1^2(x_{n+1}, x_n, kt) \geq \min\{M_1^2(x_n, x_{n-1}, t), M_2^2(y_n, y_{n-1}, t)\}$,

(3.52) $N_1^2(x_{n+1}, x_n, kt) \leq \max\{N_1^2(x_n, x_{n-1}, t), N_2^2(y_n, y_{n-1}, t)\}$,

It now follows inequalities (3.45),(3.46),(3.49),(3.51) and (3.47),(3.48),(3.50),(3.52) that

(3.53) $M_1^2(x_{n+1}, x_n, kt) \geq M_2^2(y_n, y_{n-1}, t)$,

(3.54) $M_2^2(y_{n+1}, y_n, kt) \geq M_1^2(x_n, x_{n-1}, t)$,

and

(3.55) $N_1^2(x_{n+1}, x_n, kt) \geq N_2^2(y_n, y_{n-1}, t)$,

(3.56) $N_2^2(y_{n+1}, y_n, kt) \geq N_1^2(x_n, x_{n-1}, t)$.

Using (3.53),(3.54) and(3.55),(3.56) we have for n=1, 2,

$$M_1^2(x_{n+1}, x_n, t) \geq M_2^2\left(y_n, y_{n-1}, \frac{t}{k^2}\right),$$

$$M_2^2(y_{n+1}, y_n, t) \geq M_1^2\left(x_n, x_{n-1}, \frac{t}{k^2}\right),$$

and

$$N_1^2(x_{n+1}, x_n, t) \geq N_2^2\left(y_n, y_{n-1}, \frac{t}{k^2}\right),$$

$$N_2^2(y_{n+1}, y_n, kt) \geq N_1^2\left(x_n, x_{n-1}, \frac{t}{k^2}\right).$$

By lemma 2, it follows that x_n and y_n are cauchy sequences in X and Y respectively. Hence x_n converges to z in X and y_n converges to w in Y . Now, suppose that T is continuous, then

$$\lim T\, x_{n-1} = T\, z = \lim y_n = w$$

and so T z = w. Applying inequalities (3.31) and (3.32),we have

$$M_1^2(STz, STx_{n-1}, kt) \geq \min\{M_1^2(z, x_{n-1}, t), M_1(z, ST\, z, t) * M_1(x_{n-1}, ST\, x_{n-1}, t), M_2^2(T\, z, T\, x_{n-1}, t)\},$$

$$N_1^2(STz, STx_{n-1}, kt) \leq \max\{N_1^2(z, x_{n-1}, t), N_1(z, ST\, z, t) \lozenge N_1(x_{n-1}, ST\, x_{n-1}, t), N_2^2(T\, z, T\, x_{n-1}, t)\},$$

letting n tend to infinity, we have

$$M_1^2(Sw, z, kt) \geq \min\{1, M_1^2(z, Sw, t), 1\},$$

$$N_1^2(Sw, z, kt) \leq \max\{0, N_1^2(z, Sw, t), 0\},$$

so Sw = z. In the same manner we can show that T z = w. Finally we show that the fixed point is unique. Suppose that ST has a second fixed point z′ in X Then, using inequalities (3.31) and (3.32), we have

(3.57) $$M_1^2(z, z', kt) \geq \min\{M_1^2(z, z', t), M_2^2(T\, z, T\, z', t)\},$$

(3.58) $$N_1^2(z, z', kt) \leq \max\{N_1^2(z, z', t), N_2^2(T\, z, T\, z', t)\},$$

Again, using inequality (3.33) and (3.34) we have

(3.59) $$M_2^2(T\, z, T\, z', kt) \geq \min\{M_2^2(T\, z, T\, z', t), M_2(T\, z', T\, z', t) * M_2(T\, z, T\, z, t), M_1^2(z, z', t)\}$$

(3.60) $$N_2^2(T\, z, T\, z', kt) \leq \max\{N_2^2(T\, z, T\, z', t), N_2(T\, z', T\, z', t) \lozenge N_2(T\, z, T\, z, t), N_1^2(z, z', t)\}$$

It now follows easily from inequalities (3.57), (3.58 and (3.59), (3.60 that

$$M_1^2(z, z', kt) \geq M_2^2(T\, z, T\, z', t)\ ,$$

$$N_1^2(z, z', kt) \leq N_2^2(T\, z, T\, z', t),$$

and

$$M_2^2(T\, z, T\, z', kt) \geq M_1^2(z, z', t)$$

$$N_2^2(T\, z, T\, z', kt) \leq N_1^2(z, z', t).$$

Thus, we see that ,

$$M_1^2(z, z', kt) \geq M_1^2\left(z, z', \frac{t}{k^2}\right)\ ,$$

$$N_1^2(z, z', kt) \leq N_1^2\left(z, z', \frac{t}{k^2}\right),$$

and so z = z′. The uniqueness of w follows in a similar manner.

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CONFLICT OF INTEREST

No conflict of interest was declared by the authors

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