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SELF CENTERED INTERVAL-VALUED INTUITIONISTIC FUZZY GRAPH WITH AN APPLICATION

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Abstract. In comparison to conventional fuzzy sets, the idea of interval-valued intuitionistic fuzzy sets provides a more accurate definition of uncertainty. Defuzzification is the aspect of fuzzy control that requires the most processing. It has numerous applications in fuzzy control. In this paper, the concepts strength, length, distance, eccentricity, radius, diameter, centred, self-centered, path cover, and edge cover of an interval-valued intuitionistic fuzzy graph (IVIFG) are defined in this work. Further, we introduce the definition of a self-centered IVIFG and the necessary and sufficient conditions for an IVIFG to be self-centered are given. Moreover, we investigate some properties of self-centered IVIFG with an illustration and we have discussed applications in IVIFG.

1. Introduction

L.A. Zadeh [18] developed fuzzy sets in 1965 to solve the challenges of dealing with ambiguity in fuzzy sets. Various scholars have since investigated fuzzy sets and fuzzy logic in attempt to answer other real-world issues involving ambiguous and uncertain situations. Interval-valued fuzzy sets are an extension of fuzzy sets that the author first introduced in Turksen [16] in 1986. Instead than utilising numbers as the membership function, it includes the values of number intervals to account

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for uncertainty. It is typically represented by the symbol $[\mu^-_{AL}(x), \mu^+_{AU}]$. Use the equation $0 \leq \mu^-_{AL}(x) + \mu^+_{AU}(x) \leq 1$ to represent the degree of membership of the fuzzy set A. T. Atanassov added a non-membership function that is represented by intuitionistic fuzzy sets as an additional component to fuzzy sets. Additionally, he added interval valued intuitionistic fuzzy sets to the concept of intuitionistic fuzzy sets [2]. It is preferable to depict uncertainty using interval-valued intuitionistic fuzzy sets as opposed to traditional fuzzy sets. Defuzzification, which has several applications in fuzzy control, is the component that needs the most processing. For the purpose of interpreting the degree of true and false membership functions, it is defined as a pair of intervals $[\mu^-, \mu^+], 0 \leq \mu^- + \mu^+ \leq 1$ and $[\lambda^-, \lambda^+], 0 \leq \lambda^- + \lambda^+ \leq 1$ with $0 \leq \mu^+ + \lambda^+ \leq 1$. Rosenfeld [13] created fuzzy graph theory in 1975. examined the fuzzy graphs for which Kauffmann conceived of the fundamental concept in 1973. The interval-valued neutrosophic sets (IVNS) [14], an extension of the interval-valued intuitionistic fuzzy sets (IVIFS), offer a more accurate representation of uncertainty when compared to ordinary fuzzy sets. The list of components that make up IVIFS was expanded by the addition of the indeterminate-membership function, which is represented by IVNS. Fuzzy control uses it in a variety of ways. Only holding incomplete data is permitted by the aforementioned limitations, but processing uncertain information is still necessary. Let's take a hypothetical situation where there are ten to seventeen patients being tested for a pandemic. In that period, five to seven patients will have positive results, three to six will have negative results, and two to four will still be awaiting results. It can be written as x([0.5,0.7],[0.2, 0.4], [0.3,0.6]) using neutrosophic notions. In this work, self-centered IVIFG analyses the proportion of interval-valued true and false membership functions in our result. If the indeterminate-membership function, which is represented by IVNG, is added to the result, the result can then devolve into self-centered IVNG. Rashmanlou [[9], [10], [11], [12]] researched fuzzy graphs with irregular IVFGs. Furthermore, they defined balanced IVFGs, antipodal IVFGs, and some properties of highly irregular IVFGs. The concept of an interval valued fuzzy subset of a set was created by Zadeh [17] in 1975 as an extension of the idea of a fuzzy set, where the values of the membership degrees are intervals of numbers rather than real numbers. Akram and Dudec [6] proposed the concept IVFGs in 2011. In this article, we present the idea of an IVIFG and analyse the concepts of strength, length, distance, eccentricity, radius, and diameter as well as of self-centered and centered. We also investigate into some of the properties of an illustration of a self-centered interval-valued intuitionistic fuzzy graph. Moreover, IVIFG applications are used to identify instability in various aspects of human life. These applications' purpose is to enhance the country's defences to the degree of its vulnerabilities. We suggested reading this article so that researchers could investigate this idea further using fuzzy graph theory to measure centre point radius distance, eccentricity, radius, and diameter are analyses. An interval-based membership structure is offered by this set theory to handle interval-valued intuitionistic fuzzy data. By recording their hesitation when determining membership values, users are able to more accurately represent the ambiguity and unpredictability of this data. This work advances various areas that are relevant to fuzzy graph architectures across all types of graphs by utilising a number of conceptual frameworks that include vertex point and edge will analysis. this can be used in a wide variety of fuzzy set condition analysis domains. Applications of this principle include spotting instability in all aspects of human life. The structure of this paper is as follows: Introducing the concept is covered in Section 1 of the lesson plan. Preliminaries provide the fundamental definitions required for Section 2. Section 3 important concepts An interval-valued intuitionistic fuzzy graph (IV-IFG)'s strength, length, distance, eccentricity, radius, diameter, self-centeredness, path coverage, and edge coverage are specified in this work.

2. Preliminaries

The discussion of some fundamental definitions and properties in this section will help in the formulation of the research studies.[[3], [8], [4], [7], [6]]
A graph is indeed an ordered pair $G^* = (Q, R)$, where Q is the collection of vertex positions for G^*. If a, b are on an edge of G^*, then two vertices a and b are said to be adjacent in G^*. To represent $\{a, b\}$ in R, we write ab in R. A simple graph is considered complete if an edge connects each pair of unique vertices in it. Path $P : a_1a_2....a_{n+1}(n > 0)$ in G^* has a length of n. If $a_1 = a_{n+1}$ and $n \geq 3$, a path $P : a_1a_2....a_{n+1}$ in G^* is referred to as a cycle. It should be noted that any edge in the cycle graph C_n can be removed to provide the path graph P_n, which has $n-1$ edges. An undirected graph G^* is considered to be linked if there is a path connecting every pair of distinct vertices. If every pair of different vertices in a connected graph G^* has a path between them, then the distance between two vertices a, b is equal to the length of the shortest path that connects them. Eccentricity $e(a) = \max\{d(a, b)/a \in Q\}$. A connected graph's radius is given by the formula $r(G) = \min\{e(a)/a \in Q\}$. The formula $d(G) = \max\{e(a)/a \in Q\}$ is used to determine the diameter of a connected graph G^*. The set of eccentricities in a graph is called the eccentric set (S). A graph's $C(G^*)$ centre is made up of the collection of vertices with the least amount of eccentricity. If all of a graph's vertices are in the middle, the graph is said to be self-centered. As a result, the eccentric set of a self-centered graph only includes one element, meaning that all of the vertices are equally eccentric. A graph with a diameter equal to its radius is equivalently said to be self-centered.
Map $\mu : X \to [0, 1]$ is referred to as a fuzzy subset λ on a set X. Map $\lambda : X \times X \to [0, 1]$ If $\lambda(a, b) \leq \min\{\mu(a), \mu(b)\}$ for all $a, b \in X$, called a fuzzy relation on X. A fuzzy relation λ is symmetric if $\lambda(a, b) = \lambda(b, a)$ for all $a, b \in X$.
An interval number D is an interval $[a^-, a^+]$ with $0 \leq a^- \leq a^+ \leq 1$. The interval

$[a, a]$ is identified with the number $a \in [0,1] D[0,1]$ denotes the set of all interval numbers.

Definition 1. *[1] An IFS R in the universe of discourse X is characterized by two membership functions given by* $\lambda_R : X \to [0,1]$ *and* $\mu_R : X \to [0,1]$ *respectively, such that* $\lambda_R(a) + \mu_R(a) \leq 1$ *for all* $a \in X$. *The IFS R denoted by* $R = \{(a, \lambda_R(a), \mu_R(a))/a \in X\}$

Definition 2. *[15] An IFG is of form* $\tilde{G} = (\mu, \lambda)$ *which* $\mu = (\mu_1, \mu_2)$ *and* $\lambda = (\lambda_1, \lambda_2)$ *so that*
(i) The function $\mu_1 : Q \to [0,1]$ *and* $\mu_2 : Q \to [0,1]$ *denote the degree of membership and non membership of the element* $a \in Q$ *respectively, such that* $0 \leq \mu_1(a) + \mu_2(a) \leq 1$ *for all* $a \in Q$
(ii) The function $\lambda_1 : Q \times Q \to [0,1]$ *and* $\lambda_1 : Q \times Q \to [0,1]$ *are defined by* $\lambda_1(a,b) \leq \min\{\mu_1(a), \mu_1(b)\},\ \lambda_2(a,b) \leq \max\{\mu_2(a), \mu_2(b)\}$ *such that* $0 \leq \lambda_1(a,b) + \lambda_2(a,b) \leq 1,\ \forall\ ab \in R$.

TABLE 1. Abbreviations

Notation	Meaning
$\tilde{G} = (\mu, \lambda)$	IVIFG
μ_1	IVIF degree of membership
μ_1	IVIF degree of non membership
λ_1	IVIF degree of edge membership
λ_2	IVIF degree of edge non membership
S_p	Strength of the strongest path P
l_{λ_1,λ_2}	$\lambda_1\lambda_2$-length of a path P
$\delta_{\lambda_1,\lambda_2}(a_i, a_j)$	$\lambda_1\lambda_2$-distance
$e_{\lambda_1,\lambda_2}(a_i)$	eccentricity of a_i
$r_{\lambda_1,\lambda_2}(a_i)$	radius of $\tilde{G}$
$d_{\lambda_1,\lambda_2}(a_i)$	diameter of $\tilde{G}$

3. SELF CENTERED IVIFG

IVIFG is defined in this section, which also lists helpful terms for the concepts which were used to create the main findings. We use illustrations to discuss some of the self-centered IVIFG's properties.

Definition 3. *An IVIFG of the form* $\tilde{G} = (\mu, \lambda)$ *which* $\mu = (\mu_1, \mu_2) = ([\mu_1^-, \mu_1^+], [\mu_2^-, \mu_2^+])$ *and* $\lambda = (\lambda_1, \lambda_2) = ([\lambda_1^-, \lambda_1^+], [\lambda_2^-, \lambda_2^+])$ *So that*
(1) The function $\mu_1 : Q \to [0,1]$ *and* $\mu_2 : Q \to [0,1]$ *denote the degree of membership and non membership of the element* $a \in Q$ *respectively, such that* $0 \leq \mu_1^+(a) + \mu_2^+(a) \leq 1$ *for all* $a \in Q$

(2) The function $\lambda_1 : Q \times Q \to [0,1]$ and $\lambda_2 : Q \times Q \to [0,1]$ denote the degree of interval-valued membership and interval-valued non-membership of the edge $ab \in R$, respectively, are defined by
(i) $\lambda_1^-(a,b) \le \min\{\mu_1^-(a), \mu_1^-(b)\}$ and $\lambda_1^+(a,b) \le \min\{\mu_1^+(a), \mu_1^+(b)\}$
(ii) $\lambda_2^-(a,b) \le \max\{\mu_2^-(a), \mu_2^-(b)\}$ and $\lambda_2^+(a,b) \le \max\{\mu_2^+(a), \mu_2^+(b)\}$ such that $0 \le \lambda_1^+(a,b) + \lambda_2^+(a,b) \le 1,\ \forall\ ab \in R$.

Definition 4. *An IVIFG $\tilde{G} = (\mu, \lambda)$ of a graph $G^* = (Q,R)$ is called a complete if (i) $\lambda_1^-(a,b) = \min\{\mu_1^-(a), \mu_1^-(b)\}$ and $\lambda_1^+(a,b) = \min\{\mu_1^+(a), \mu_1^+(b)\}$, $(ii) \lambda_2^-(a,b) = \max\{\mu_2^-(a), \mu_2^-(b)\}$ and $\lambda_2^+(a,b) = \max\{\mu_2^+(a), \mu_2^+(b)\}$.*

Example 1. *An IVIFG $\tilde{G} = (\mu, \lambda)$ of a graph $G^* = (Q,R)$ given figure-1 is a complete IVIFG $\tilde{G} = (\mu, \lambda)$ such that $\mu = \{u_1([0.3,0.5][0.2,0.4]), u_2([0.4,0.6][0.3,0.4]), u_3([0.1,0.3][0.3,0.6]),\ u_4([0.3,0.4][0.4,0.5])\}$.*

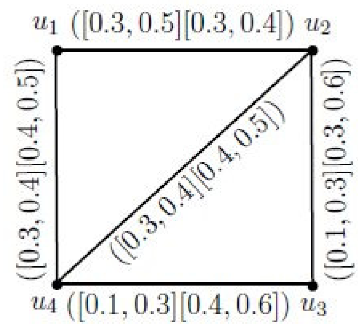

FIGURE 1. $\tilde{G} = (\mu, \lambda)$ is IVIFG of G_1^* is complete

Definition 5. *A path P in IVIFG $\tilde{G} = (\mu, \lambda)$ of a graph $G^* = (Q,R)$ is a sequence of distinct vertices $a_1, a_2, ..., a_n$ such that either one of the following conditions is satisfied:*
(1) $\lambda_1^-(a,b) > 0,\ \lambda_2^-(a,b) = 0$ and $\lambda_1^+(a,b) > 0,\ \lambda_2^+(a,b) = 0$ for some $a, b \in R$
(2) $\lambda_1^-(a,b) = 0,\ \lambda_2^-(a,b) > 0$ and $\lambda_1^+(a,b) = 0,\ \lambda_2^+(a,b) > 0$ for some $a, b \in R$
(3) $\lambda_1^-(a,b) > 0,\ \lambda_1^+(a,b) = 0$ and $\lambda_2^-(a,b) > 0,\ \lambda_2^+(a,b) = 0$ for some $a, b \in R$
(4) $\lambda_1^-(a,b) = 0,\ \lambda_1^+(a,b) > 0$ and $\lambda_2^-(a,b) = 0,\ \lambda_2^+(a,b) > 0$ for some $a, b \in R$
A path $P : a_1 a_2 ... a_{n+1}$ in G^* is called a cycle if $a_1 = a_{n+1}$ and $n \ge 3$.

Definition 6. *Let $P : u_1, u_2, ..., u_n$ be a path in IVIFG $\tilde{G} = (\mu, \lambda)$ of graph $G^* = (Q,R)$. The λ_1^-−strength of all paths joining any two vertices and the expression a_i, a_j is represented by the symbol $(\lambda_{1ij}^-)^\infty$ and is defined as $\max(\lambda_1^-(a_i, a_j))$. The λ_1^+−strength of all paths joining any two vertices and the expression a_i, a_j is represented by the symbol $(\lambda_{1ij}^+)^\infty$ and is defined as $\max(\lambda_1^+(a_i, a_j))$. The λ_2^- −*

strength of all paths joining any two vertices and the expression a_i, a_j *is represented by the symbol* $(\lambda^-_{2ij})^\infty$ *and is defined as* $\max(\lambda^-_2(a_i, a_j))$ *. The* $\lambda^+_2 -$ *strength of all paths joining any two vertices and the expression* a_i, a_j *is represented by the symbol* $(\lambda^+_{2ij})^\infty$ *and is defined as* $\max(\lambda^+_2(a_i, a_j))$ *.If the same edge possesses every value of* $\lambda 1^- -$ *strength,* $\lambda 1^+ -$ *strength,* $\lambda 2^- -$ *strength,* $\lambda 2^+ -$ *strength, then it is the strength of the strongest path* P *and it is denoted by* $S_P = ([(\lambda^-_{1ij})^\infty, (\lambda^+_{1ij})^\infty], [(\lambda^-_{2ij})^\infty, (\lambda^+_{2ij})^\infty])$ *for all* $i, j = 1, 2, ..., k$.

Definition 7. *The IVIFG If any two vertices of* $\tilde{G} = (\mu, \lambda)$ *are connected by a path, they are said to be connected. That is, an IVIFG* $\tilde{G}$ *is connected if* $(\lambda^-_{1ij})^\infty > 0$, $(\lambda^+_{1ij})^\infty > 0$ *and* $(\lambda^-_{2ij})^\infty > 0$, $(\lambda^+_{2ij})^\infty > 0$.

Example 2. *Consider a IVIFG* $\tilde{G} = (\mu, \lambda)$ *as shown in Figure-1 in Example-1. The path* u_1u_4 *has a length of 1 and a strength of* $([0.3, 0.4][0.4, 0.5])$. *The path* $u_1u_2u_4$ *has a length of 2 and a strength of* $([0.3, 0.5][0.4, 0.5])$. *The path* $u_1u_2u_3u_4$ *has a length of 3 and a strength of* $([0.3, 0.5][0.4, 0.6])$.

Definition 8. *Let* $\tilde{G} = (\mu, \lambda)$ *be a connected IVIFG of graph* $G^* = (Q, R)$. *The* $\lambda^-_1 -$ *length of a path* $P : a_1a_2...a_n$ *in* G^*, $l_{\lambda^-_1}(p)$, *is defined as* $l_{\lambda^-_1}(p) = \sum_{i=1}^{n-1} \lambda^-_1(a_i, a_{i+1})$ *and the* $\lambda^+_1 -$ *length of a path* $P : u_1u_2...u_n$ *in* G, $l_{\lambda^+_1}(p)$, *is defined as* $l_{\lambda^+_1}(p) = \sum_{i=1}^{n-1} \lambda^+_1(a_i, a_{i+1})$. *The* $\lambda^-_2 -$ *length of a path* $P : a_1a_2...a_n$ *in* G^*, $l_{\lambda^-_2}(p)$, *is defined as* $l_{\lambda^-_2}(p) = \sum_{i=1}^{n-1} \lambda^-_2(a_i, a_{i+1})$ *and the* $\lambda^+_2 -$ *length of a path* $P : u_1u_2...u_n$ *in* G, $l_{\lambda^+_2}(p)$, *is defined as* $l_{\lambda^+_2}(p) = \sum_{i=1}^{n-1} \lambda^+_2(a_i, a_{i+1})$. *The* $\lambda_1\lambda_2 -$ *length of a path* $P : u_1u_2...u_n$ *in* G, $l_{\lambda_1\lambda_2}(p)$, *is defined as* $l_{\lambda_1\lambda_2}(p) = ([l_{\lambda^-_1}, l_{\lambda^+_1}], [l_{\lambda^-_2}, l_{\lambda^+_2}])$.

Example 3. *Consider a connected IVIFG* $\tilde{G} = (\mu, \lambda)$ *as shown in Figure-1 in Example-1. Here,* u_1u_4 *is a path of length 1 and* $l_{\lambda_1\lambda_2} = ([0.3, 0.4][0.4, 0.5])$, $u_1u_2u_4$ *is a path of length 2 and* $l_{\lambda_1\lambda_2} = ([0.6, 0.9][0.7, 0.9])$, $u_1u_2u_3u_4$ *is a path of length 3 and* $l_{\lambda_1\lambda_2} = ([0.5, 1.1][1.0, 1.6])$.

Definition 9. *Let* $\tilde{G} = (\mu, \lambda)$ *be a connected IVIFG of graph* $G^* = (Q, R)$. *The* $\lambda^-_1 -$*distance,* $\delta_{\lambda^-_{1ij}}$, *is the smallest* $\lambda^-_1 -$*length of any* $a_i - a_j$ *path* P *in* $\tilde{G}$, *where* $a_i, a_j \in Q$. *That is,* $\delta_{\lambda^-_{1ij}} = \delta_{\lambda^-_1}(a_i, a_j) = \min(l_{\lambda^-_1}(p))$ *and* $\lambda^+_1 -$ *distance,* $\delta_{\lambda^+_{1ij}}$, *is the smallest* $\lambda^+_1 -$ *length of any* $a_i - a_j$ *path* P *in* $\tilde{G}$, *where* $a_i, a_j \in Q$. *That is,* $\delta_{\lambda^+_{1ij}} = \delta_{\lambda^+_1}(a_i, a_j) = \min(l_{\lambda^+_1}(p))$. *The* $\lambda^-_2 -$ *distance,* $\delta_{\lambda^-_{2ij}}$, *is the smallest* $\lambda^-_2 -$ *length of any* $a_i - a_j$ *path* P *in* $\tilde{G}$, *where* $a_i, a_j \in Q$. *That is,* $\delta_{\lambda^-_{2ij}} = \delta_{\lambda^-_2}(a_i, a_j) = \min(l_{\lambda^-_2}(p))$ *and* $\lambda^+_2 -$ *distance,* $\delta_{\lambda^+_{2ij}}$, *is the smallest* $\lambda^+_2 -$ *length of any* $a_i - a_j$ *path* P *in* $\tilde{G}$, *where* $a_i, a_j \in Q$. *That is,* $\delta_{\lambda^+_{2ij}} = \delta_{\lambda^+_2}(a_i, a_j) = \min(l_{\lambda^+_2}(p))$. *The distance,* $\delta_{\lambda_1,\lambda_2}(a_i, a_j)$, *is defined as* $\delta_{\lambda_1,\lambda_2}(a_i, a_j) = ([\delta_{\lambda^-_{1ij}}, \delta_{\lambda^+_{1ij}}], [\delta_{\lambda^-_{2ij}}, \delta_{\lambda^+_{2ij}}])$.

Example 4. *Consider a connected IVIFG* $\tilde{G} = (\mu, \lambda)$ *as shown in Figure-1 in Example-1. Here,* $\delta_{\lambda_1^-}(u_1, u_4) = 0.3, \delta_{\lambda_1^+}(u_1, u_4) = 0.4$ *and* $\delta_{\lambda_2^-}(u_1, u_4) = 0.4, \delta_{\lambda_2^+}(u_1, u_4) = 0.5$. *That is* $\delta_{\lambda_1,\lambda_2}(u_1, u_4) = ([0.3, 0.4], [0.4, 0.5])$. *Similarly, we calculate*
$\delta_{\lambda_1,\lambda_2}(u_1, u_2) = ([0.3, 0.5], [0.3, 0.4]),\ \delta_{\lambda_1,\lambda_2}(u_1, u_3) = ([0.4, 0.7], [0.6, 1.0]),$
$\delta_{\lambda_1,\lambda_2}(u_2, u_3) = ([0.1, 0.3], [0.3, 0.6]),\ \delta_{\lambda_1,\lambda_2}(u_2, u_4) = ([0.2, 0.4], [0.4, 0.5]),$
$\delta_{\lambda_1,\lambda_2}(u_3, u_4) = ([0.1, 0.3], [0.4, 0.6])$

Definition 10. *Let* $\tilde{G} = (\mu, \lambda)$ *be a connected IVIFG of graph* $G^* = (Q, R)$. *For each* $a_i \in Q$, *the* λ_1^- *− eccentricity of* a_i, *denoted by* $e_{\lambda_1^-}(a_i)$, *is defined as* $e_{\lambda_1^-}(a_i) = \max\{\delta_{\lambda_1^-}(a_i, a_j)/u_i \in Q\}$ *and for each* $a_i \in Q$, *the* λ_1^+ *− eccentricity of* a_i, *denoted by* $e_{\lambda_1^+}(a_i)$, *is defined as* $e_{\lambda_1^+}(a_i) = \max\{\delta_{\lambda_1^+}(a_i, a_j)/a_i \in Q\}$. *For each* $a_i \in Q$, *the* λ_2^- *− eccentricity of* a_i, *denoted by* $e_{\lambda_2^-}(a_i)$, *is defined as* $e_{\lambda_2^-}(a_i) = \max\{\delta_{\lambda_2^-}(a_i, a_j)/u_i \in Q\}$ *and for each* $a_i \in Q$, *the* λ_2^+ *− eccentricity of* a_i, *denoted by* $e_{\lambda_2^+}(a_i)$, *is defined as* $e_{\lambda_2^+}(a_i) = \max\{\delta_{\lambda_2^+}(a_i, a_j)/a_i \in Q\}$. *For each* $a_i \in Q$, *the eccentricity of* a_i, *denoted by* $e_{\lambda_1,\lambda_2}(a_i)$, *is defined as* $e_{\lambda_1,\lambda_2}(a_i) = ([e_{\lambda_1^-}(a_i), e_{\lambda_1^+}(a_i)], [e_{\lambda_2^-}(a_i), e_{\lambda_2^+}(a_i)])$.

Definition 11. *Let* $\tilde{G} = (\mu, \lambda)$ *be a connected IVIFG of graph* $G^* = (Q, R)$. *The* λ_1^-*−radius of* $\tilde{G}$ *is denoted by* $r_{\lambda_1^-}(G)$ *and is defined as* $r_{\lambda_1^-}(G) = \min\{e_{\lambda_1^-}(a_i)/a_i \in Q\}$ *and the* λ_1^+ *− radius of* $\tilde{G}$ *is denoted by* $r_{\lambda_1^+}(G)$ *and is defined as* $r_{\lambda_1^+}(G) = \min\{e_{\lambda_1^+}(a_i)/a_i \in Q\}$. *The* λ_2^- *− radius of* $\tilde{G}$ *is denoted by* $r_{\lambda_2^-}(G)$ *and is defined as* $r_{\lambda_2^-}(G) = \min\{e_{\lambda_2^-}(a_i)/a_i \in Q\}$ *and the* λ_2^+ *− radius of* $\tilde{G}$ *is denoted by* $r_{\lambda_2^+}(G)$ *and is defined as* $r_{\lambda_2^+}(G) = \min\{e_{\lambda_2^+}(a_i)/a_i \in Q\}$. *The radius of* $\tilde{G}$ *is denoted by* $r_{\lambda_1,\lambda_2}(G)$ *and is defined as* $r_{\lambda_1,\lambda_2}(G) = ([r_{\lambda_1^-}(G), r_{\lambda_1^+}(G)], [r_{\lambda_2^-}(G), r_{\lambda_2^+}(G)])$.

Definition 12. *Let* $\tilde{G} = (\mu, \lambda)$ *be a connected IVIFG of graph* $G^* = (Q, R)$. *The* λ_1^-*−diameter of* $\tilde{G}$ *is denoted by* $d_{\lambda_1^-}(G)$ *and is defined as* $d_{\lambda_1^-}(G) = \max\{e_{\lambda_1^-}(a_i)/a_i \in Q\}$ *and the* λ_1^+ *− diameter of* $\tilde{G}$ *is denoted by* $d_{\lambda_1^+}(G)$ *and is defined as* $d_{\lambda_1^+}(G) = \max\{e_{\lambda_1^+}(a_i)/a_i \in Q\}$. *The* λ_2^- *− diameter of* $\tilde{G}$ *is denoted by* $d_{\lambda_2^-}(G)$ *and is defined as* $d_{\lambda_2^-}(G) = \max\{e_{\lambda_2^-}(a_i)/a_i \in Q\}$ *and the* λ_2^+ *− diameter of* $\tilde{G}$ *is denoted by* $d_{\lambda_2^+}(G)$ *and is defined as* $d_{\lambda_2^+}(G) = \max\{e_{\lambda_2^+}(a_i)/a_i \in Q\}$. *The diameter of* $\tilde{G}$ *is denoted by* $d_{\lambda_1,\lambda_1}(G)$ *and is defined as* $d_{\lambda_1,\lambda_1}(G) = ([d_{\lambda_1^-}(G), d_{\lambda_1^+}(G)], [d_{\lambda_1^-}(G), d_{\lambda_1^+}(G)])$.

Example 5. *From the above Examples-1,3,4. using standard calculations, it is easy to see that:* $e_{\lambda_1^-}$ *− eccentricity,* $e_{\lambda_1^+}$ *− eccentricity and* $e_{\lambda_2^-}$ *− eccentricity,* $e_{\lambda_2^+}$ *− eccentricity of each vertex is*
$e_{\lambda_1^-}(u_1) = 0.4,\ e_{\lambda_1^-}(u_2) = 0.3,\ e_{\lambda_1^-}(u_3) = 0.4, e_{\lambda_1^-}(u_4) = 0.3,\ e_{\lambda_1^+}(u_1) = 0.7,\ e_{\lambda_1^+}(u_2) = 0.5,\ e_{\lambda_1^+}(u_3) = 0.7,\ e_{\lambda_1^+}(u_4) = 0.4$ *and* $e_{\lambda_2^-}(u_1) = 0.6,\ e_{\lambda_2^-}(u_2) = 0.4,\ e_{\lambda_2^-}(u_3) = 0.6, e_{\lambda_2^-}(u_4) = 0.4,\ e_{\lambda_2^+}(u_1) = 1.0,\ e_{\lambda_2^+}(u_2) = 0.6,\ e_{\lambda_2^+}(u_3) = 1.0,\ e_{\lambda_2^+}(u_4) = 0.6$

eccentricity of each vertex is
$e_{\lambda_1,\lambda_2}(u_1) = ([0.4, 0.7], [0.6, 1.0])$, $e_{\lambda_1,\lambda_2}(u_2) = ([0.3, 0.5], [0.4, 0.6])$,
$e_{\lambda_1,\lambda_2}(u_3) = ([0.4, 0.7], [0.6, 1.0])$, $e_{\lambda_1,\lambda_2}(u_4) = ([0.3, 0.4], [0.4, 0.6])$
radius of $\tilde{G}$ *is* $r_{\lambda_1,\lambda_2}(G) = ([0.3, 0.4], [0.4, 0.6])$ *and*
diameter of $\tilde{G}$ *is* $d_{\lambda_1,\lambda_2}(G) = ([0.4, 0.7], [0.6, 1.0])$

Definition 13. *A vertex* $u_i \in Q$ *is called a central vertex of a connected IVIFG* $\tilde{G} = (\mu, \lambda)$ *of graph* $G^* = (Q, R)$, *if* $r_{\lambda_1^-}(G) = e_{\lambda_1^-}(u_i)$, $r_{\lambda_1^+}(G) = e_{\lambda_1^+}(u_i)$ *and* $r_{\lambda_2^-}(G) = e_{\lambda_2^-}(u_i)$, $r_{\lambda_2^+}(G) = e_{\lambda_2^+}(u_i)$ *and* $C(\tilde{G})$ *stands for the set of all central vertices of an IVIFG.*

Definition 14. *IVIFG connected If every vertex in* $\tilde{G}$ *is a central vertex, then the graph* $\tilde{G} = (\mu, \lambda)$ *is a self-centered IVIFG, that is* $r_{\lambda_1^-}(G) = e_{\lambda_1^-}(u_i)$, $r_{\lambda_1^+}(G) = e_{\lambda_1^+}(u_i)$ *and* $r_{\lambda_2^-}(G) = e_{\lambda_2^-}(u_i)$, $r_{\lambda_2^+}(G) = e_{\lambda_2^+}(u_i)$ $\forall$ $u_i \in Q$.

Example 6. *Consider a connected IVIFG* $\tilde{G} = (\mu, \lambda)$ *of graph* $G^* = (Q, R)$ *such that* $\mu = \{u_1([0.2, 0.4], [0.3, 0.5]), u_2([0.4, 0.5], [0.3, 0.4]),$
$u_3([0.3, 0.4], [0.2, 0.5])\}$ *as shown in Figure-2 By routine computations, it is easy to*

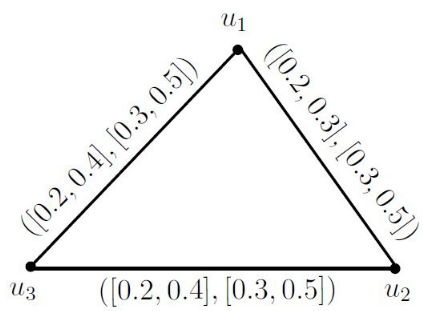

FIGURE 2. $\tilde{G} = (\mu, \lambda)$ is self centered IVIFG of G^*

see that:
(i) Distance $\delta_{\lambda_1,\lambda_2}(u_i, u_j)$ *is*
$\delta_{\lambda_1,\lambda_2}(u_1, u_2) = ([0.2, 0.4], [0.3, 0.5])$,
$\delta_{\lambda_1,\lambda_2}(u_1, u_3) = ([0.2, 0.4], [0.3, 0.5])$,
$\delta_{\lambda_1,\lambda_2}(u_2, u_3) = ([0.2, 0.4], [0.3, 0.5])$
(ii) Eccentricity $e_{\lambda_1,\lambda_2}(u_i)$ *of each vertex is* $([0.2, 0.4], [0.3, 0.5])$ *for* $i = 1, 2, 3$
(iii) radius of $\tilde{G}$ *is* $r_{\lambda_1,\lambda_2}(G) = ([0.2, 0.4], [0.3, 0.5])$ *and*
diameter of $\tilde{G}$ *is* $d_{\lambda_1,\lambda_2}(G) = ([0.2, 0.4], [0.3, 0.5])$
Hence, $\tilde{G}$ *is self centered IVIFG*

Definition 15. *A path cover of an IVIFG* $\tilde{G} = (\mu, \lambda)$ *of graph* $G^* = (Q, R)$ *is a set* P *of paths such that every vertex of* $\tilde{G}$ *is incident to some path of* P.

Example 7. *Consider a connected IVIFG* $\tilde{G} = (\mu, \lambda)$ *of graph* $G^* = (Q, R)$ *such that* $\mu = \{u_1([0.1, 0.3], [0.2, 0.4]), u_2([0.2, 0.4], [0.1, 0.3]), u_3([0.2, 0.3], [0.3, 0.5]), u_4([0.3, 0.4], [0.2, 0.5]), u_5([0.4, 0.5][0.2, 0.3]), u_6([0.2, 0.4], [0.3, 0.5])\}$ *as shown in Figure-3. In this example, the some path covers of an IVIFG* $\tilde{G} = (\mu, \lambda)$ *are* $M_1 =$

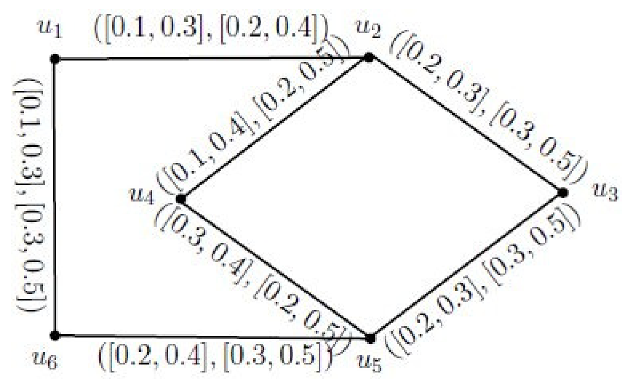

FIGURE 3. $\tilde{G} = (\mu, \lambda)$ is IVIFG of G^*

$\{u_1u_2u_3u_5, u_4u_5u_6\}$,
$M_2 = \{u_1u_2u_4u_5, u_3u_5u_6\}$,
$M_3 = \{u_1u_2u_4u_5u_6, u_5u_3\}$,
$M_4 = \{u_1u_2u_4u_5u_6, u_2u_3\}$,
$M_5 = \{u_1u_2u_3u_5u_6, u_5u_4\}$,
$M_6 = \{u_1u_2u_3u_5u_6, u_2u_3\}$,
$M_7 = \{u_1u_2u_4, u_2u_3u_5u_6\}$,
$M_8 = \{u_1u_2u_3, u_2u_4u_5u_6\}$,
$M_9 = \{u_1u_6, u_2u_3u_5u_4\}$,
$M_{10} = \{u_1u_2, u_3u_5, u_4u_5u_6\}$,
$M_{11} = \{u_1u_2, u_4u_5, u_3u_5u_6\}$

Definition 16. *An edge covers of an IVIFG* $\tilde{G} = (\mu, \lambda)$ *of graph* $G^* = (Q, R)$ *is a set* E *of edge such that every vertex of* $\tilde{G}$ *is incident to some edge of* E.

Example 8. *In above Example-7, as shown in Figure-3. The some of the edge covers of an IVIFG* $\tilde{G} = (\mu, \lambda)$ *are*
$E_1 = \{(u_1, u_2), (u_2, u_3), (u_4, u_5), (u_5, u_6)\}$,
$E_2 = \{(u_1, u_6), (u_2, u_4), (u_3, u_5)\}$,
$E_3 = \{(u_1, u_2), (u_2, u_4), (u_3, u_5), (u_5, u_6)\}$,
$E_4 = \{(u_1, u_6), (u_2, u_3), (u_4, u_5)\}$,
$E_5 = \{(u_1, u_2), (u_2, u_3), (u_2, u_4), (u_5, u_6)\}$,
$E_6 = \{(u_1, u_6), (u_3, u_5), (u_4, u_5)\}$

Theorem 1. *Every complete IVIFG $\tilde{G} = (\mu, \lambda)$ of graph $G^* = (Q, R)$ is a IVIFG and $r_{\lambda_1^-}(G) = \frac{1}{\mu_{1i}^-}$, $r_{\lambda_1^+}(G) = \frac{1}{\mu_{1i}^+}$ and $r_{\lambda_2^-} = \frac{1}{\mu_{2i}^-}$, $r_{\lambda_2^+} = \frac{1}{\mu_{2i}^+}$, where The lowest vertex membership is μ_{1i}^- and The largest vertex membership is μ_{1i}^+ and The lowest vertex membership is μ_{2i}^- and The largest vertex membership is μ_{2i}^+.*

Proof. Let $\tilde{G} = (\mu, \lambda)$ be a complete IVIFG. To prove that $\tilde{G}$ is a self centered IVIFG. Therefore, we must demonstrate that each vertex is a central vertex. First we claim that $\tilde{G}$ is a $\mu 1i$-self centred IVIFG. Then $r_{\lambda_1^-}(G) = \frac{1}{\mu_{1i}^-}$ and $r_{\lambda_1^+}(G) = \frac{1}{\mu_{1i}^+}$, where The lowest vertex membership is μ_{1i}^- and The largest vertex membership is μ_{1i}^+. Fix a vertex u_i in Q so that μ_{1i}^- is the value of $\tilde{G}$ is lowest vertex membership and μ_{1i}^+ is the value of $\tilde{G}$ is largest vertex membership.

Case1: Consider all the $u_i - u_j$ paths P of length n in $\tilde{G}$, $\forall\ u_j \in Q$.

(i) If $n = 1$, then $\lambda_{1ij}^- = \min\{\mu_{1i}^-, \mu_{1j}^-\}$. Therefore, $\lambda_1^- - length$ of $P = l_{\lambda_1^-}(P) = \frac{1}{\mu_{1i}^-}$ and $\lambda_{1ij}^+ = \min\{\mu_{1i}^+, \mu_{1j}^+\}$. Therefore, $\lambda_1^+ - length$ of $P = l_{\lambda_1^+}(P) = \frac{1}{\mu_{1i}^+}$.

(ii) (ii) One of the edges of P has the $\lambda_1^- - strength$ of μ_{1i}^- if $n > 1$ and hence, $\lambda_1^- - length$ of a $u_i - u_j$ path will exceed $\frac{1}{\mu_{1i}^-}$. So that, $\lambda_1^- - length$ of $P = l_{\lambda_1^-}(P) > \frac{1}{\mu_{1i}^-}$.

$$\text{Hence, } \delta_{\lambda_1^-}(u_i, u_j) = \min(l_{\lambda_1^-}(p)) = \frac{1}{\mu_{1i}^-},\ \forall u_j \in Q. \tag{1}$$

Also one of the edges of P possesses the $\lambda_1^+ - strength$ of μ_{1i}^+ and hence, $\lambda_1^+ - length$ of P will exceed $\frac{1}{\mu_{1i}^+}$. that is, $\lambda_1^+ - length$ of $P = l_{\lambda_1^+}(P) > \frac{1}{\mu_{1i}^+}$.

$$\text{Hence,} \delta_{\lambda_1^+}(u_i, u_j) = \min(l_{\lambda_1^+}(p)) = \frac{1}{\mu_{1i}^+},\ \forall u_j \in Q. \tag{2}$$

Case 2: Let $u_k \neq u_i \in Q$. Consider all $u_k - u_j$ paths X of length n in $\tilde{G}$, $\forall\ u_j \in Q$.

(i) If $n = 1$, then $\lambda_{1kj}^- = \min\{\mu_{1k}^-, \mu_{1j}^-\} \geq \mu_{1i}^-$, since μ_{1i}^- is the least. Hence, $\lambda_1^- - length\ (Q) = l_{\lambda_1^-}(X) = \frac{1}{\lambda_1^-(u_k, u_j)} \leq \frac{1}{\mu_{1i}^-}$.

Also $\lambda_{1kj}^+ = \min\{\mu_{1k}^+, \mu_{1j}^+\} \leq \mu_{1i}^+$, since μ_{1i}^+ is the largest. Hence, $\lambda_1^+ - length\ (Q) = l_{\lambda_1^+}(X) = \frac{1}{\lambda_1^+(u_k, u_j)} \geq \frac{1}{\mu_{1i}^+}$.

(ii) If $n = 2$, then $l_{\lambda_1^-}(X) = \frac{1}{\lambda_1^-(u_k, u_{k+1})} + \frac{1}{\lambda_1^-(u_{k+1}, u_j)} \leq \frac{2}{\mu_{1i}^-}$, Since, μ_{1i}^- is the lowest.

Also $l_{\lambda_1^+}(X) = \frac{1}{\lambda_1^+(u_k, u_{k+1})} + \frac{1}{\lambda_1^+(u_{k+1}, u_j)} \geq \frac{2}{\mu_{1i}^+}$, Since, μ_{1i}^+ is the largest.

(iii) If $n > 2$, then $l_{\lambda_1^-}(X) \leq \frac{n}{\mu_{1i}^-}$, since μ_{1i}^- is the lowest.

Also $l_{\lambda_1^+}(X) \geq \frac{n}{\mu_{1i}^+}$, since μ_{1i}^+ is the largest.

Hence, $\delta_{\lambda_1^-}(u_k, u_j) = \min(l_{\lambda_1^-}(X)) \leq \frac{1}{\mu_{1i}^-}$, $\forall u_k, u_j \in Q$. and

$$\delta_{\lambda_1^+}(u_k, u_j) = \min(l_{\lambda_1^+}(X)) \geq \frac{1}{\mu_{1i}^+},\ \forall u_k, u_j \in Q. \tag{3}$$

From equation 1,2 and 3, we have,
$e_{\lambda_1^-}(u_i) = \min(\delta_{\lambda_1^-}(u_i, u_j)) = \frac{1}{\mu_{1i}^-}$, $\forall u_i \in Q$ and

$$e_{\lambda_1^+}(u_i) = \min(\delta_{\lambda_1^+}(u_i, u_j)) = \frac{1}{\mu_{1i}^+}, \ \forall u_i \in Q. \tag{4}$$

Hence, $\tilde{G}$ is a λ_1^- and λ_1^+ self centered IVIFG.
Now, $r_{\lambda_1^-}(G) = \min(e_{\lambda_1^-}(u_i)) = \frac{1}{\mu_{1i}^-}$, since by 4 $r_{\lambda_1^-}(G) = \frac{1}{\mu_{1i}^-}$, where $\mu_{1i}^-(u_i)$ is the lowest and $r_{\lambda_1^+}(G) = \min(e_{\lambda_1^+}(u_i)) = \frac{1}{\mu_{1i}^+}$, since by 4 $r_{\lambda_1^+}(G) = \frac{1}{\mu_{1i}^+}$, where $\mu_{1i}^+(u_i)$ is the largest.
Next, we claim that $\tilde{G}$ is a μ_{2i}-self centered IVIFG. Then $r_{\lambda_2^-}(G) = \frac{1}{\mu_{2i}^-}$ and $r_{\lambda_2^+}(G) = \frac{1}{\mu_{2i}^+}$, where μ_{2i}^- is the lowest and μ_{2i}^+ is the largest. Now fix a vertex $u_i \in Q$ such that μ_{2i}^- is lowest vertex membership value of $\tilde{G}$ and μ_{2i}^+ is largest vertex membership value of $\tilde{G}$.
Case 1: Consider all the $u_i - u_j$ paths P of length n in $\tilde{G}$, $\forall\ u_j \in Q$.
(i) If $n = 1$, then $\lambda_{2ij}^- = \max\{\mu_{2i}^-, \mu_{2j}^-\} = \mu_{2i}^-$. Therefore, $\lambda_2^- - length$ of $P = l_{\lambda_2^-}(P) = \frac{1}{\mu_{2i}^-}$ and $\lambda_{2ij}^+ = \max\{\mu_{2i}^+, \mu_{2j}^+\} = \mu_{2i}^+$. Therefore, $\lambda_2^+ - length$ of $P = l_{\lambda_2^+}(P) = \frac{1}{\mu_{2i}^+}$.
(ii) If $n > 1$, then one of the edges of P possesses the $\lambda_2^- - strength$ of μ_{2i}^- and hence, $\lambda_2^- - length$ of a $u_i - u_j$ path will exceed $\frac{1}{\mu_{2i}^-}$. So that, $\lambda_2^- - length$ of $P = l_{\lambda_2^-}(P) > \frac{1}{\mu_{2i}^-}$.

$$\text{Hence, } \delta_{\lambda_2^-}(u_i, u_j) = \max(l_{\lambda_2^-}(p)) = \frac{1}{\mu_{2i}^-}, \ \forall u_j \in Q. \tag{5}$$

Also one of the edges of P possesses the $\lambda_2^+ - strength$ of μ_{2i}^+ and hence, $\lambda_2^+ - length$ of P will exceed $\frac{1}{\mu_{2i}^+}$. that is, $\lambda_2^+ - length$ of $P = l_{\lambda_2^+}(P) > \frac{1}{\mu_{2i}^+}$.

$$\text{Hence, } \delta_{\lambda_2^+}(u_i, u_j) = \max(l_{\lambda_2^+}(p)) = \frac{1}{\mu_{2i}^+}, \ \forall u_j \in Q. \tag{6}$$

Case 2: Let $u_k \neq u_i \in Q$. Consider all $u_k - u_j$ paths X of length n in $\tilde{G}$, $\forall\ u_j \in Q$.
(i) If $n = 1$, then $\lambda_{2kj}^- = \max\{\mu_{2k}^-, \mu_{2j}^-\} \geq \mu_{2i}^-$, since μ_{2i}^- is the least. Hence, $\lambda_2^- - length\ (Q) = l_{\lambda_2^-}(X) = \frac{1}{\lambda_2^-(u_k, u_j)} \leq \frac{1}{\mu_{2i}^-}$.
Also $\lambda_{2kj}^+ = \max\{\mu_{2k}^+, \mu_{2j}^+\} \leq \mu_{2i}^+$, since μ_{2i}^+ is the greatest. Hence, $\lambda_2^+ - length\ (Q) = l_{\lambda_2^+}(X) = \frac{1}{\lambda_2^+(u_k, u_j)} \geq \frac{1}{\mu_{2i}^+}$.
(ii) If $n = 2$, then $l_{\lambda_2^-}(X) = \frac{1}{\lambda_2^-(u_k, u_{k+1})} + \frac{1}{\lambda_2^-(u_{k+1}, u_j)} \leq \frac{2}{\mu_{2i}^-}$, Since, μ_{2i}^- is the least.
Also $l_{\lambda_2^+}(X) = \frac{1}{\lambda_2^+(u_k, u_{k+1})} + \frac{1}{\lambda_2^+(u_{k+1}, u_j)} \geq \frac{2}{\mu_{2i}^+}$, Since, μ_{2i}^+ is the greatest.
(iii) If $n > 2$, then $l_{\lambda_2^-}(X) \leq \frac{n}{\mu_{2i}^-}$, since μ_{2i}^- is the least.

Also $l_{\lambda_2^+}(X) \geq \frac{n}{\mu_{2i}^+}$, since μ_{2i}^+ is the greatest.
Hence, $\delta_{\lambda_2^-}(u_k, u_j) = \max(l_{\lambda_2^-}(X)) \leq \frac{1}{\mu_{2i}^-},\ \forall u_k, u_j \in Q.$ and

$$\delta_{\lambda_2^+}(u_k, u_j) = \max(l_{\lambda_2^+}(X)) \geq \frac{1}{\mu_{2i}^+},\ \forall u_k, u_j \in Q. \tag{7}$$

From equation 5,6and 7, we have,
$e_{\lambda_2^-}(u_i) = \max(\delta_{\lambda_2^-}(u_i, u_j)) = \frac{1}{\mu_{2i}^-},\ \forall u_i \in Q$ and

$$e_{\lambda_2^+}(u_i) = \max(\delta_{\lambda_2^+}(u_i, u_j)) = \frac{1}{\mu_{2i}^+},\ \forall u_i \in Q. \tag{8}$$

Hence, $\tilde{G}$ is a λ_2^- and λ_2^+ self centered IVIFG.
Now, $r_{\lambda_2^-}(G) = \min(e_{\lambda_2^-}(u_i)) = \frac{1}{\mu_{2i}^-}$, since by 7 $r_{\lambda_2^-}(G) = \frac{1}{\mu_{2i}^-}$, where μ_{2i}^- is the least and $r_{\lambda_2^+}(G) = \max(e_{\lambda_2^+}(u_i)) = \frac{1}{\mu_{2i}^+}$, since by 8 $r_{\lambda_2^+}(G) = \frac{1}{\mu_{2i}^+}$, where μ_{2i}^+ is the largest.
From equation 4 and 8, every vertex of $\tilde{G}$ is a central vertex. Hence $\tilde{G}$ is a self centered IVIFG. □

Corollary 1. *Every complete IVIFG $\tilde{G} = (\mu, \lambda)$ of graph $G^* = (Q, R)$ is a self centered IVIFG and $r_{\lambda_1,\lambda_2}(G) = ([\frac{1}{\mu_{1i}^-}, \frac{1}{\mu_{1i}^+}], [\frac{1}{\mu_{2i}^-}, \frac{1}{\mu_{2i}^+}])$ where, μ_{1i}^- is the lowest vertex membership and μ_{1i}^+ is the largest vertex membership. μ_{2i}^- is the lowest vertex membership and μ_{2i}^+ is the largest vertex membership.*

Proof. By above theorem-1, every complete IVIFG $\tilde{G} = (\mu, \lambda)$ of graph $G^* = (Q, R)$ is a self centered IVIFG.
$r_{\lambda_1,\lambda_2}(G) = (r_{\lambda_1}(G), r_{\lambda_2}(G)) = ([\min\{r_{\lambda_1^-(G)}\}, \min\{r_{\lambda_1^+(G)}\}], [\min\{r_{\lambda_2^-(G)}\},$
$\min\{r_{\lambda_2^+(G)}\}])$. $r_{\lambda_1,\lambda_2}(G) = ([\frac{1}{\mu_{1i}^-}, \frac{1}{\mu_{1i}^+}], [\frac{1}{\mu_{2i}^-}, \frac{1}{\mu_{2i}^+}])$, since μ_{1i}^- is the lowest membership value and μ_{1i}^+ is the largest membership value. μ_{2i}^- is the lowest membership value and μ_{2i}^+ is the largest membership value. □

Remark 1. *Converse of the above theorem-1 is not true. By Example-6. Then $\tilde{G}$ is self centered IVIFG but not complete.*

Lemma 1. *An IVIFG $\tilde{G} = (\mu, \lambda)$ of graph $G^* = (Q, R)$ is a self centered IVIFG if and only if $r_{\lambda_1^-}(G) = d_{\lambda_1^-}(G),\ r_{\lambda_1^+}(G) = d_{\lambda_1^+}(G)$ and $r_{\lambda_2^-}(G) = d_{\lambda_2^-}(G),\ r_{\lambda_2^+}(G) = d_{\lambda_2^+}(G)$.*

Theorem 2. *Let $\tilde{G} = (\mu, \lambda)$ is a connected IVIFG. Then for at least one edge $\max(\lambda_1^-(u_i, u_j)) = \lambda_1^-(u_i, u_j),\ \max(\lambda_1^+(u_i, u_j)) = \lambda_1^+(u_i, u_j)$ and $\max(\lambda_2^-(u_i, u_j)) = \lambda_2^-(u_i, u_j),\ \max(\lambda_2^+(u_i, u_j)) = \lambda_2^+(u_i, u_j)$*

Proof. If $\tilde{G} = (\mu, \lambda)$ be a connected IVIFG. Consider a vertex u_i whose least membership value μ_{1i}^- and greatest membership value is μ_{1i}^+ and least membership value

μ_{2i}^- and greatest membership value is μ_{2i}^+.
Case 1: Let μ_{1i}^- be the least value and $\mu_{1i}^+(u_i)$ be the greatest value and μ_{2i}^- be the least value and μ_{2i}^+ be the greatest value and in the vertex $u_i \in Q$. Let $u_i, u_j \in Q$, then $([\lambda_{1ij}^-, \lambda_{1ij}^+], [\lambda_{2ij}^-, \lambda_{2ij}^+]) = ([\mu_{1i}^-, \mu_{1i}^+], [\mu_{2i}^-, \mu_{2i}^+])$ and $([\max(\lambda_{1ij}^-), \max(\lambda_{1ij}^+)],$ $[\max(\lambda_{2ij}^-), \max(\lambda_{2ij}^+)]) = ([\mu_{1i}^-, \mu_{1i}^+], [\mu_{2i}^-, \mu_{2i}^+])$. The strength of all the edges which are incident on the vertex u_i is $([\mu_{1i}^-, \mu_{1i}^+], [\mu_{2i}^-, \mu_{2i}^+])$. Since $\tilde{G}$ is a connected IVIFG.
Case 2: Let μ_{1k}^- be the least value and μ_{1i}^+ be the greatest value and μ_{2k}^- be the least value and μ_{2i}^+ be the greatest value in the vertex $u_i, u_k \in Q$. Then $([\lambda_{1ik}^-, \lambda_{1ik}^+], [\lambda_{2ik}^-, \lambda_{2ik}^+]) = ([\mu_{1k}^-, \mu_{1i}^+], [\mu_{2k}^-, \mu_{2i}^+])$. Since, it is a connected IVIFG, there will be an edge between u_i and u_k, $\max(\lambda_{1ik}^-) = \mu_{1k}^-$, $\max(\lambda_{1ik}^+) = \mu_{1i}^+$ and $\max(\lambda_{2ik}^-) = \mu_{2k}^-$, $\max(\lambda_{2ik}^+) = \mu_{2i}^+$. □

Theorem 3. *Let $\tilde{G} = (\mu, \lambda)$ be a connected IVIFG of graph $G^* = (Q, R)$ with paths covers P_1 and P_2 of $\tilde{G}$. Then the necessary and sufficient condition for an IVIFG to be self centered IVIFG is $\delta_{\lambda_{1ij}^-} = r_{\lambda_1^-}(G),\ \forall\ (u_i, u_j) \in P_1,\ \delta_{\lambda_{1ij}^+} = d_{\lambda_1^+}(G),\ \forall\ (u_i, u_j) \in P_2$, and*

$$\delta_{\lambda_{2ij}^-} = r_{\lambda_2^-}(G),\ \forall\ (u_i, u_j) \in P_1,\ \delta_{\lambda_{2ij}^+} = d_{\lambda_2^+}(G),\ \forall\ (u_i, u_j) \in P_2. \tag{9}$$

Proof. **Necessary Condition:** We now assume that $\tilde{G} = (\mu, \lambda)$ is a self centered IVIFG and we have to prove that equation 9 holds. Suppose equation 9 does not holds. then we have, $\delta_{\lambda_1^-}(u_i, u_j) \neq r_{\lambda_1^-}(G)$, for some $(u_i, u_j) \in P_1$ and $\delta_{\lambda_1^+}(u_i, u_j) \neq d_{\lambda_1^+}(G)$, for some $(u_i, u_j) \in P_2$ and $\delta_{\lambda_2^-}(u_i, u_j) \neq r_{\lambda_2^-}(G)$, for some $(u_i, u_j) \in P_1$ and $\delta_{\lambda_2^+}(u_i, u_j) \neq d_{\lambda_2^+}(G)$, for some $(u_i, u_j) \in P_2$. By using Lemma-1, the above inequality becomes $\delta_{\lambda_1^-}(u_i, u_j) \neq r_{\lambda_1^-}(G)$, for some $(u_i, u_j) \in P_1$ and $\delta_{\lambda_1^+}(u_i, u_j) \neq d_{\lambda_1^+}(G)$, for some $(u_i, u_j) \in P_2$ and $\delta_{\lambda_2^-}(u_i, u_j) \neq r_{\lambda_2^-}(G)$, for some $(u_i, u_j) \in P_1$ and $\delta_{\lambda_2^+}(u_i, u_j) \neq d_{\lambda_2^+}(G)$, for some $(u_i, u_j) \in P_2$. Then $e_{\lambda_1^-}(u_i) \neq r_{\lambda_1^-}(G)$, $e_{\lambda_1^+}(u_i) \neq r_{\lambda_1^+}(G)$ and $e_{\lambda_2^-}(u_i) \neq r_{\lambda_2^-}(G)$, $e_{\lambda_2^+}(u_i) \neq r_{\lambda_2^+}(G)$ for some $u_i \in Q$, which implies $\tilde{G}$ is not self centered IVIFG, which is contradiction. Hence, $\delta_{\lambda_1^-}(u_i, u_j) = r_{\lambda_1^-}(G),\ \forall\ (u_i, u_j) \in P_1$ and $\delta_{\lambda_1^+}(u_i, u_j) = d_{\lambda_1^+}(G),\ \forall\ (u_i, u_j) \in P_2$ and $\delta_{\lambda_2^-}(u_i, u_j) = r_{\lambda_2^-}(G),\ \forall\ (u_i, u_j) \in P_1$ and $\delta_{\lambda_2^+}(u_i, u_j) = d_{\lambda_2^+}(G),\ \forall\ (u_i, u_j) \in P_2$.
Sufficient Condition: We now assume that equation 9 holds and we have to prove that $\tilde{G}$ is a self centered IVIFG. If equation 9 holds, then we've $e_{\lambda_1^-}(u_i) = \delta_{\lambda_1^-}(u_i, u_j)$, for all $(u_i, u_j) \in P_1$, $e_{\lambda_1^+}(u_i) = \delta_{\lambda_1^+}(u_i, u_j)$, for all $(u_i, u_j) \in P_2$ and $e_{\lambda_2^-}(u_i) = \delta_{\lambda_2^-}(u_i, u_j)$, for all $(u_i, u_j) \in P_1$, $e_{\lambda_2^+}(u_i) = \delta_{\lambda_2^+}(u_i, u_j)$, for all $(u_i, u_j) \in P_2$. Which implies $e_{\lambda_1^-}(u_i) = r_{\lambda_1^-}(G)$, $e_{\lambda_1^+}(u_i) = r_{\lambda_1^+}(G)$ and $e_{\lambda_2^-}(u_i) = r_{\lambda_2^-}(G)$, $e_{\lambda_2^+}(u_i) = r_{\lambda_2^+}(G)$ for all $u_i \in Q$. Hence, $\tilde{G}$ is not self centered IVIFG. □

Corollary 2. *If $\tilde{G} = (\mu, \lambda)$ is a connected IVIFG of graph $G^* = (Q, R)$ with an edge cover E of $\tilde{G}$. Then the necessary and sufficient condition for an IVIFG to be self centered IVIFG is $\delta_{\lambda_1^-}(u_i, u_j) = r_{\lambda_1^-}(G),\ \forall\ (u_i, u_j) \in E_1,\ \delta_{\lambda_1^+}(u_i, u_j) = d_{\lambda_1^+}(G),\ \forall\ (u_i, u_j) \in E_2$ and $\delta_{\lambda_2^-}(u_i, u_j) = r_{\lambda_2^-}(G),\ \forall\ (u_i, u_j) \in E_1$,*

$$\delta_{\lambda_2^+}(u_i, u_j) = d_{\lambda_2^+}(G),\ \forall\ (u_i, u_j) \in E_2. \tag{10}$$

Theorem 4. ***Embedding Theorem:*** *Let $\tilde{H} = (\mu', \lambda')$ is a connected self centered IVIFG. Then there exist a connected IVIFG $\tilde{G}$ such that $< C(\tilde{G}) >$ is isomorphic to $\tilde{H}$. Also $d_{\lambda_1^-}(G) = 2r_{\lambda_1^-}(G),\ d_{\lambda_1^+}(G) = 2r_{\lambda_1^+}(G)$ and $d_{\lambda_2^-}(G) = 2r_{\lambda_2^-}(G),\ d_{\lambda_2^+}(G) = 2r_{\lambda_2^+}(G)$.*

Proof. Given that $\tilde{H} = (\mu', \lambda')$ is a connected self centered IVIFG. Let $d_{\lambda_1^-}(H) = p_1,\ d_{\lambda_1^+}(H) = q_1$ and $d_{\lambda_2^-}(H) = p_2,\ d_{\lambda_2^+}(H) = q_2$. Then construct $\tilde{G} = (\mu, \lambda)$ from $\tilde{H}$ as follows:
Take two vertices $u_i, u_j \in Q$ with $\mu_1^-(u_i) = \mu_1^-(u_j) = \frac{1}{p_1}, \mu_1^+(u_i) = \mu_1^+(u_j) = \frac{1}{2q_1}$ and $\mu_2^-(u_i) = \mu_2^-(u_j) = \frac{1}{p_2}, \mu_2^+(u_i) = \mu_2^+(u_j) = \frac{1}{2q_2}$ and join all the vertices of $\tilde{H}$ to both u_i and u_j with $\lambda_{1ik}^- = \lambda_{1jk}^- = \frac{1}{p_1},\ \lambda_{1ik}^+ = \lambda_{1jk}^+ = \frac{1}{2q_1}$ and $\lambda_{2ik}^- = \lambda_{2jk}^- = \frac{1}{p_2},\ \lambda_{2ik}^+ = \lambda_{2jk}^+ = \frac{1}{2q_2}$ for all $u_k \in Q'$. Put $\mu_{1i}^- = (\mu_{1i}^-)',\ \mu_{1i}^+ = (\mu_{1i}^+)'$ and $\mu_{2i}^- = (\mu_{2i}^-)',\ \mu_{2i}^+ = (\mu_{2i}^+)'$ for all vertices in $\tilde{H}$. and $\lambda_{1ij}^- = (\lambda_{1ij}^-)',\ \lambda_{1ij}^+ = (\lambda_{1ij}^+)'$ for all edges in $\tilde{H}$ and $\lambda_{2ij}^- = (\lambda_{2ij}^-)',\ \lambda_{2ij}^+ = (\lambda_{2ij}^+)'$ for all edges in $\tilde{H}$.
Claim: $\tilde{G}$ is an IVIFG. First note that $\mu_{1i}^- \le \mu_{1k}^-,\ \mu_{2i}^- \le \mu_{2k}^-$ for all $u_k \in \tilde{H}$. If possible, let $\mu_{1i}^- > \mu_{1k}^-$ and $\mu_{2i}^- > \mu_{2k}^-$ for at least one vertex $u_k \in \tilde{H}$. Then $\frac{1}{p_1} > \mu_{1k}^-,\ \frac{1}{p_2} > \mu_{2k}^-$, that is $p_1 < \frac{1}{\mu_{1k}^-} \le \frac{1}{\lambda_{1kl}^-},\ p_2 < \frac{1}{\mu_{2k}^-} \le \frac{1}{\lambda_{2kl}^-}$, where the last inequality holds for every $u_l \in Q'$, since $\tilde{H}$ is an IVIFG. That is $\frac{1}{\lambda_{1kl}^-} > p_1,\ \frac{1}{\lambda_{2kl}^-} > p_2$ for all $u_k \in \tilde{H}$ which contradicts that $d_{\lambda_1^-}(\tilde{H}) = p_1,\ d_{\lambda_2^-}(\tilde{H}) = p_2$. Therefore $\mu_{1i}^- \le \mu_{1k}^-, \mu_{2i}^- \le \mu_{2k}^-$ for all $u_k \in Q'$ and $\lambda_{1ik}^- \le \min\{\mu_{1i}^-, \mu_{1k}^-\} = \frac{1}{p_1},\ \lambda_{2ik}^- \le \max\{\mu_{2i}^-, \mu_{2k}^-\} = \frac{1}{p_2}$, similarly, $\lambda_{1jk}^- \le \min\{\mu_{1j}^-, \mu_{1k}^-\} = \frac{1}{p_1},\ \lambda_{2jk}^- \le \max\{\mu_{2j}^-, \mu_{2k}^-\} = \frac{1}{p_2}$ for all $u_k \in Q'$. Note that $\mu_{1i}^+ \le \mu_{1k}^+,\ \mu_{1j}^+ \le \mu_{1k}^-$ and $\mu_{2i}^+ \le \mu_{2k}^+,\ \mu_{2j}^+ \le \mu_{2k}^-$ for all $u_k \in Q'$, since $d_{\lambda_1^+}(H) = q_1$ and $d_{\lambda_2^+}(H) = q_2$. Therefore $\lambda_{1ik}^+ \le \min\{\mu_{1i}^+, \mu_{1k}^+\} = \frac{1}{2q_1},\ \lambda_{2ik}^+ \le \max\{\mu_{2i}^+, \mu_{2k}^+\} = \frac{1}{2q_2}$, similarly, $\lambda_{1jk}^+ \le \min\{\mu_{1j}^+, \mu_{1k}^+\} = \frac{1}{2q_1}$ and $\lambda_{2jk}^+ \le \max\{\mu_{2j}^+, \mu_{2k}^+\} = \frac{1}{2q_2}$. Hence, $\tilde{G}$ is an IVIFG. Also, $e_{\lambda_1^-}(u_k) = p_1,\ e_{\lambda_1^-}(u_k) = p_2$ for all $u_k \in Q'$ and $e_{\lambda_1^-}(u_i) = e_{\lambda_1^-}(u_j) = \frac{1}{\lambda_{1ik}^-} + \frac{1}{\lambda_{1kl}^-} = 2p_1,\ r_{\lambda_1^-}(G) = p_1,\ d_{\lambda_1^-}(G) = 2p_1$ and $e_{\lambda_2^-}(u_i) = e_{\lambda_2^-}(u_j) = \frac{1}{\lambda_{2ik}^-} + \frac{1}{\lambda_{2kl}^-} = 2p_2,\ r_{\lambda_2^-}(G) = p_2,\ d_{\lambda_2^-}(G) = 2p_2$. Next, $e_{\lambda_1^+}(u_k) = q_1,\ e_{\lambda_2^+}(u_k) = q_2$ for all $u_k \in Q'$ and $e_{\lambda_1^+}(u_i) = e_{\lambda_1^+}(u_j) = \frac{1}{\lambda_{1lk}^+} = 2q_1,\ e_{\lambda_2^+}(u_i) = e_{\lambda_2^+}(u_j) = \frac{1}{\lambda_{2lk}^+} = 2q_2$ for all $u_k \in Q'$. Therefore, $r_{\lambda_1^+}(G) =$

$q_1,\ d_{\lambda_1^+}(G) = 2q_1$ and $r_{\lambda_2^+}(G) = q_2,\ d_{\lambda_2^+}(G) = 2q_2$. Hence, $< C(\tilde{G}) >$ is isomorphic to $\tilde{H}$. $\square$

Theorem 5. *An IVIFG $\tilde{G} = (\mu, \lambda)$ is a self centered if and only if $\delta_{\lambda_1^-}(u_i, u_j) \leq r_{\lambda_1^-}(G),\ \delta_{\lambda_1^+}(u_i, u_j) \geq r_{\lambda_1^+}(G)$ and $\delta_{\lambda_2^-}(u_i, u_j) \leq r_{\lambda_2^-}(G),\ \delta_{\lambda_2^+}(u_i, u_j) \geq r_{\lambda_2^+}(G)$ for all $u_i, u_j \in Q$.*

Proof. We assume that $\tilde{G} = (\mu, \lambda)$ is a self centered IVIFG. That is, $e_{\lambda_1^-}(u_i) = e_{\lambda_1^-}(u_j), e_{\lambda_1^+}(u_i) = e_{\lambda_1^+}(u_j)$ and $e_{\lambda_2^-}(u_i) = e_{\lambda_2^-}(u_j),\ e_{\lambda_2^+}(u_i) = e_{\lambda_2^+}(u_j)$ for all $u_i, u_j \in Q,\ r_{\lambda_1^-}(G) = e_{\lambda_1^-}(u_i),\ r_{\lambda_1^+}(G) = e_{\lambda_1^+}(u_i)$ and $r_{\lambda_2^-}(G) = e_{\lambda_2^-}(u_i),\ r_{\lambda_2^+}(G) = e_{\lambda_2^+}(u_i)$ for all $u_i \in Q$. Now we wish to show that $\delta_{\lambda_1^-}(u_i, u_j) \leq r_{\lambda_1^-}(G),\ \delta_{\lambda_1^+}(u_i, u_j) \geq r_{\lambda_1^+}(G)$ and $\delta_{\lambda_2^-}(u_i, u_j) \leq r_{\lambda_2^-}(G),\ \delta_{\lambda_2^+}(u_i, u_j) \geq r_{\lambda_2^+}(G)$ for all $u_i, u_j \in Q$. By the definition of eccentricity, we obtain, $\delta_{\lambda_1^-}(u_i, u_j) \leq e_{\lambda_1^-}(u_i),\ \delta_{\lambda_1^+}(u_i, u_j) \geq e_{\lambda_1^+}(u_i)$ and $\delta_{\lambda_2^-}(u_i, u_j) \leq e_{\lambda_2^-}(u_i),\ \delta_{\lambda_2^+}(u_i, u_j) \geq e_{\lambda_2^+}(u_i)$ for all $u_i, v_i \in Q$. This is possible only when $e_{\lambda_1^-}(u_i) = e_{\lambda_1^-}(u_j),\ e_{\lambda_1^+}(u_i) = e_{\lambda_1^+}(u_j)$ and $e_{\lambda_2^-}(u_i) = e_{\lambda_2^-}(u_j),\ e_{\lambda_2^+}(u_i) = e_{\lambda_2^+}(u_j)$ for all $u_i, u_j \in Q$. Since, $\tilde{G}$ is a self centered IVIFG, the above inequality becomes $\delta_{\lambda_1^-}(u_i, u_j) \leq r_{\lambda_1^-}(G),\ \delta_{\lambda_1^+}(u_i, u_j) \geq r_{\lambda_1^+}(G)$ and $\delta_{\lambda_2^-}(u_i, u_j) \leq r_{\lambda_2^-}(G),\ \delta_{\lambda_2^+}(u_i, u_j) \geq r_{\lambda_2^+}(G)$.
Conversely, we now assume that $\delta_{\lambda_1^-}(u_i, u_j) \leq r_{\lambda_1^-}(G),\ \delta_{\lambda_1^+}(u_i, u_j) \geq r_{\lambda_1^+}(G)$ and $\delta_{\lambda_2^-}(u_i, u_j) \leq r_{\lambda_2^-}(G),\ \delta_{\lambda_2^+}(u_i, u_j) \geq r_{\lambda_2^+}(G)$ for all $u_i, u_j \in Q$. Then we have to prove that $\tilde{G}$ is a self centered IVIFG. Suppose that $\tilde{G}$ is not self centered IVIFG. Then $r_{\lambda_1^-}(G) \neq e_{\lambda_1^-}(u_i), r_{\lambda_1^+}(G) \neq e_{\lambda_1^+}(u_i)$ and $r_{\lambda_2^-}(G) \neq e_{\lambda_2^-}(u_i), r_{\lambda_2^+}(G) \neq e_{\lambda_2^+}(u_i)$ for some $u_i \in Q$. Let us assume that $e_{\lambda_1^-}(u_i),\ e_{\lambda_1^+}(u_i)$ and $e_{\lambda_2^-}(u_i),\ e_{\lambda_2^+}(u_i)$ is the least value among all other eccentricity. That is, $r_{\lambda_1^-}(G) = e_{\lambda_1^-}(u_i), r_{\lambda_1^+}(G) = e_{\lambda_1^+}(u_i)$ and

$$r_{\lambda_2^-}(G) = e_{\lambda_2^-}(u_i), r_{\lambda_2^+}(G) = e_{\lambda_2^+}(u_i). \tag{11}$$

where $e_{\lambda_1^-}(u_i) < e_{\lambda_1^-}(u_j), e_{\lambda_1^+}(u_i) < e_{\lambda_1^+}(u_j)$ and $e_{\lambda_2^-}(u_i) < e_{\lambda_2^-}(u_j), e_{\lambda_2^+}(u_i) < e_{\lambda_2^+}(u_j)$ for some $u_i, u_j \in Q$ and $\delta_{\lambda_1^-}(u_i, u_j) = e_{\lambda_1^-}(u_j) > e_{\lambda_1^-}(u_i),\ \delta_{\lambda_1^+}(u_i, u_j) = e_{\lambda_1^+}(u_j) > e_{\lambda_1^+}(u_i)$ and $\delta_{\lambda_2^-}(u_i, u_j) = e_{\lambda_2^-}(u_j) > e_{\lambda_2^-}(u_i)$,,

$$\delta_{\lambda_2^+}(u_i, u_j) = e_{\lambda_2^+}(u_j) > e_{\lambda_2^+}(u_i) \text{ for some } u_i, u_j \in Q. \tag{12}$$

Hence, from equation 11 and 12, we have, $\delta_{\lambda_1^-}(u_i, u_j) > r_{\lambda_1^-}(G),\ \delta_{\lambda_1^+}(u_i, u_j) > r_{\lambda_1^+}(G)$, and $\delta_{\lambda_2^-}(u_i, u_j) > r_{\lambda_2^-}(G),\ \delta_{\lambda_2^+}(u_i, u_j) > r_{\lambda_2^+}(G)$, for some $u_i, u_j \in Q$, which is a contradiction to the fact that $\delta_{\lambda_1^-}(u_i, u_j) \leq r_{\lambda_1^-}(G),\ \delta_{\lambda_1^+}(u_i, u_j) \geq r_{\lambda_1^+}(G)$ and $\delta_{\lambda_2^-}(u_i, u_j) \leq r_{\lambda_2^-}(G),\ \delta_{\lambda_2^+}(u_i, u_j) \geq r_{\lambda_2^+}(G)$ for all $u_i, u_j \in Q$. Hence, $\tilde{G} = (\mu, \lambda)$ is a self centered IVIFG. $\square$

4. Application of IVIFG

Application in this paper are applied in detecting instability in human life and in all fields. The findings of this study can be used to prevent crime and foster peace in our nation. Using the security forces of our nation, we will defend the areas where the majority of unlawful operations occur. In order to stop illicit operations in our country, we will first deploy security forces there. Through the use of uncertainty values, we will determine the extent of unlawful activity in the cities of our nation and adjust the deployment of our security troops accordingly. Think of Q as the country and u_1, u_2, u_3, u_4 as the cities within Q. We shall examine this study using IVIFG. We define the most terrible terrorist crimes as μ_1^-, and let's define the special forces protecting the country from terrorist threats as μ_1^+. Other criminal activity will be considered as μ_2^- by those involved. (Examples include the transfer of ill-gotten gains, the smuggling of precious metals, the smuggling of endangered species, and the illegal carrying of weapons). Let's treat troops who guard against other criminal actions as μ_2^+. Let's have a look at the vertex set $Q = \{u_1, u_2, u_3, u_4\}$ in

Table 2.

Cities	[terrorist acts, protection]	[other illegal acts, other protection]
u_1	[0.3,0.5]	[0.2,0.4]
u_2	[0.4,0.6]	[0.3,0.4]
u_3	[0.1,0.3]	[0.3,0.6]
u_4	[0.3,0.4]	[0.4,0.5]

IVIFG. Any two city-connected highway that borders $\tilde{G}$ is defined by R as $G* = (Q, R)$. Let us consider the edge set $R = \{u_1u_2, u_2u_3, u_2u_4, u_3c_4, c_4c_5\}$ in G^* as shown in Figure-1. IVIFG calculates the value of the security forces, the value of the unlawful activities, and the value of the security forces from terrorist acts that take place on highways connecting two cities. used in the definition -9. We

Table 3.

highway	[terrorist acts, protection]	[other illegal acts, other protection]
u_1u_2	[0.3,0.5]	[0.3,0.4]
u_2u_3	[0.1,0.3]	[0.3,0.6]
u_2u_4	[0.3,0.4]	[0.4,0.5]
u_3u_4	[0.1,0.3]	[0.4,0.6]
u_1u_4	[0.3,0.4]	[0.4,0.5]

can estimate the cost of terrorist attacks spreading to other cities by using the symbols $d_{\lambda_{1ij}^-}$ and $d_{\lambda_{1ij}^+}$, respectively. We can also estimate the cost of security forces' protection by using the symbols $d_{\lambda_{2ij}^-}$ and $d_{\lambda_{2ij}^+}$. In the Example-1, we take into consideration that u_1, u_2, u_3, u_4 are cities in Q and R is any two-city connected highway that is located on the boundaries of $\tilde{G}$. Let us consider the edge set $R = \{u_1u_2, u_2u_3, u_2u_4, u_3u_4, u_1u_4\}$ in G^* as shown in Figure-1 in Example-1. We

TABLE 4.

One city relation to distance between others city	$([d_{\lambda^-_{1ij}}, d_{\lambda^+_{1ij}}], [d_{\lambda^-_{2ij}}, d_{\lambda^+_{2ij}}])$
u_1u_4	([0.3,0.4],[0.4,0.5])
u_1u_2	([0.3,0.5],[0.3,0.4])
u_1u_3	([0.4,0.7],[0.6,1.0])
u_2u_3	([0.1,0.3],[0.3,0.6])
u_2u_4	([0.2,0.4],[0.4,0.5])
u_3u_4	([0.1,0.3],[0.4,0.6])

can estimate the impacts and protections a city has on others by comparing it to those cities. From the table-4,5 we find that $d_{\lambda_1,\lambda_2}(G) = ([0.4, 0.7], [0.6, 1.0])$ has

TABLE 5.

$e_{\lambda_1,\lambda_2}(u_i)$	maximum value of impacts and protection
$e_{\lambda_1,\lambda_2}(u_1)$	([0.4,0.7],[0.6,1.0])
$e_{\lambda_1,\lambda_2}(u_2)$	([0.3,0.5],[0.4,0.6])
$e_{\lambda_1,\lambda_2}(u_3)$	([0.4,0.7],[0.6,1.0])
$e_{\lambda_1,\lambda_2}(u_4)$	([0.3,0.4],[0.4,0.6])

the highest number of vulnerabilities and defenses and from the table-4,5 above we find that $r_{\lambda_1,\lambda_2}(G) = ([0.3, 0.4], [0.4, 0.6])$ has the lowest number of vulnerabilities and defenses. The purpose of these applications is to strengthen our country's defenses to the extent of it's vulnerabilities.

5. CONCLUSION

The researcher has developed the idea of an IVIFG in this study article. It has an impact on a lot of different industries. It is common for some features of a graph-theoretical problem to be unclear or ambiguous. This article analyses the concepts of strength, length, distance, eccentricity, radius, and diameter as well as self-centerednes and centeredness and introduces the idea of an IVIFG. We also investigate into some of the properties of a self-centered IVIFG with illustration. Finally, we investigated into an application in IVIFG.

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