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HOMOTOPY ANALYSIS METHOD FOR THE SOLUTION OF FUZZY FRACTIONAL TELEGRAPH EQUATION BY USING LAPLACE TRANSFORM

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Abstract. The purpose of this article is to present a numerical method to find an approximate solution for fuzzy fractional differential type equation. This method is applied for linear and non-linear equations and has been examined on two examples. Their solutions compared with the exact solutions. The result show that the proposed method is very simple and effective.

1. Introduction

The telegraph equation of hyperbolic equations is proved to be better model the suspension flows [1, 5, 3]. The time fractional telegraph equations have recently been considered by many authors. B-spline differential quadrature method [15]. Mittal et al, have applied radial basis functions for solving fractional telegraph equation [7, 2, 4].

Several work has been done on analyzing numerical methods for solving fuzzy fractional differential equations. [12, 13]. Mikaeilvand in [16] have applied a fuzzy differential transform method to solve fuzzy partial differential equations. Recently, Salahshour have applied fuzzy differential transform method (FDTM) for solving fuzzy Volterra integral equations [14].

On the other hand, there are many numerical methods for solving fractional equations, but on fuzzy fractional partial equations, few works have been done [16, 17].

In this paper, we consider the fuzzy space-fractional telegraph equation as follows

$$\frac{\partial^{2\alpha}\tilde{U}(x,t)}{\partial x^{2\alpha}} = \frac{\partial^{2}\tilde{U}(x,t)}{\partial t^{2}} + a\frac{\partial\tilde{U}(x,t)}{\partial t} + b\tilde{U}^{n}(x,t) + \tilde{f}(x,t), 0 < \alpha \leq 1. \tag{1.1}$$

With given supplementary initial conditions. Where a, b and n are given constants, $f(x,t)$ is known functions and the function u is unknown. We give a new Homotopy

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analysis transform method. The application of this method in linear and nonlinear differential and integral equations has been devoted by scientists and engineers. The fundamental work was done by Liao and He. This method has a significant advantage in that it provides an analytical approximate solution to a wide range of nonlinear problems.

2. Preliminaries

We now recall some definitions and symbols needed through the paper

Definition 2.1. A fuzzy number is a fuzzy set $u : R \to [0,1]$ which satisfies the following properties:
a) u is upper semicontinuous on R,
b) $u(x) = 0$ outside of some interval [c,d],
c) there are the real numbers a and b with $c \leq a \leq b \leq d$, such that u is increasing on $[c,a]$, decreasing on $[b,d]$ and $u(x) = 1$ for each $x \in [a,b]$,
d) u is fuzzy convex set (that is $u(\lambda x+(1-\lambda)y) \geq min\,\{u(x), u(y)\}$, $\forall x, y \in R, \lambda \in [0,1]$).

The set of all fuzzy numbers is denoted by E.

Definition 2.2. For any $u \in E$ the α-cut set of u is denoted by $[u]^{\alpha}$ and define by $[u]^{\alpha} = \{x \in R \mid u(x) \geq \alpha\}$, where $0 \leq \alpha \leq 1$. The notation $[u]^{\alpha} = [\underline{u}^{\alpha}, \overline{u}^{\alpha}], 0 \leq \alpha \leq 1$ refers to the lower and upper branches on u.

A fuzzy number u in parametric form is a pair $(\underline{\mathrm{u}}, \bar{u})$, of functions $\underline{\mathrm{u}}(r), \bar{u}(r)$, $0 \leq \alpha \leq 1$, which satisfy the following requirements:
1. $\underline{\mathrm{u}}(r)$ is a bounded non-decreasing left continuous function in $(0,1]$, and right continuous at 0,
2. $\bar{u}(r)$ is a bounded non-increasing left continuous function in $(0,1]$, and right continuous at 0,
3. $\underline{\mathrm{u}}(r) \leq \bar{u}(r)$, $0 \leq r \leq 1$.
For arbitrary $u = (\underline{\mathrm{u}}, \bar{u})$, $\nu = (\underline{\mathrm{v}}, \bar{\nu})$ and $k \geq 0$, addition $(u + \nu)$ and multiplication by k as $(\underline{u+\nu}) = \underline{u}(r)+\underline{\nu}(r)$, $(\overline{u+\nu}) = \bar{u}(r)+\bar{\nu}(r)$, $(\underline{ku}(r)) = k\underline{u}(r)$, $\overline{ku}(r) = k\bar{u}(r)$ are defined.
Since each $y \in R$ can be regarded as a fuzzy number y defined by

$$\tilde{y}(k) = \begin{cases} 1 & t = y \\ 0 & t \neq y. \end{cases}$$

Definition 2.3. The Hausdorff distance between fuzzy numbers given by $D : E \times E \to R_{+}U\{0\}$,

$$D(u, v) = sup_{r \in [0,1]} \max\{|\underline{u}(r) - \underline{v}(r)|, |\bar{u}(r) - \bar{v}(r)|\}.$$

It is easy to see that D is a metric in E and has the following properties (see [18]).
(i). $D(u \oplus w, \nu \oplus w) = D(u, \nu), \forall u, \nu, w \in E$,
(ii). $D(k \odot u, k \odot \nu) = |k| D(u, \nu), \forall \in R, u, \nu \in E$,
(iii). $D(u \oplus \nu, w \oplus e) \leq D(u, w) + D(\nu, e), \forall u, \nu, w, e \in E$,
(iv). (D, E) is a complete metric space.

Definition 2.4. The function $f : T \to E$ is called a fuzzy function and the α-cut set of f is represented by $f(t, \alpha) = \left[\underline{f}(t, \alpha), \overline{f}(t, \alpha)\right], \quad \forall \alpha \in [0,1]$. A fuzzy function may have domain and fuzzy range. so the function $f : E \to E$ is also a fuzzy function.

Definition 2.5. Let $f : R \to E$ be a fuzzy valued function. If for arbitrary fixed $t_0 \in R$ and $\varepsilon > 0$, $\ni \delta > 0$ such that

$$| t - t_0 |< \delta \Rightarrow D(f(t), f(t_0)) < \epsilon,$$

f is said to be continuous.
It is well-known that the H-derivative (differentiability in the sense of Hukuhara) for fuzzy mappings was initially introduced by Puri and Ralescu[19] it is based on the H-difference of sets, as follows:

Definition 2.6. Let $x, y \in E$. If there exists $z \in E$ such that $x = y + z$, then z is called the H-difference of x and y, and it is denoted by $x \ominus y$.

Definition 2.7. Let $f : (a, b) \to E$ and $x_0 \in (a, b)$. We say that f is differential at x_0, If there exists an element $\acute{f}(x_0) \in E$, such that

(1). for all $h > 0$ sufficiently near to 0, $\exists f(x_0 + h) \ominus f(x_0)$, $\exists f(x_0) \ominus f(x_0 - h)$ and the limits (in the metric D)

$$\lim_{h\to 0^+} \frac{f(x_0 + h) \ominus f(x_0)}{h} = \lim_{h\to 0^+} \frac{f(x_0) \ominus f(x_0 - h)}{h} = \acute{f}(x_0)$$

or
(2). for all $h < 0$ sufficiently near to 0, $\exists f(x_0 + h) \ominus f(x_0)$, $\exists f(x_0) \ominus f(x_0 - h)$ and the limits (in the metric D)

$$\lim_{h\to 0^-} \frac{f(x_0 + h) \ominus f(x_0)}{h} = \lim_{h\to 0^-} \frac{f(x_0) \ominus f(x_0 - h)}{h} = \acute{f}(x_0)$$

Definition 2.8. Let y be a real valued function on $[0, a]$. The Riemann-Liouville fractional integral $I^\alpha y$ of order $\alpha > 0$ is defined by

$$I^\alpha y(t) = \frac{1}{\Gamma} \int_0^t (t - s)^{\alpha-1} y(s)ds, 0 < t < a,$$

The Riemann-Liouville fractional derivative $D^\alpha y$ of y of order $0 < \alpha < 1$ is defined by $D^\alpha y(t) = \frac{d}{dt} I^{1-\alpha} y(t), \quad 0 < t < a.$

3. Applications

We consider the parametric form of equation (1.1) as follows

$$\frac{\partial^{2\alpha}\overline{U}(x,t)}{\partial x^{2\alpha}} = \frac{\partial^2\overline{U}(x,t)}{\partial t^2} + a\frac{\partial\overline{U}(x,t)}{\partial t} + b\overline{U}^n(x,t) + \overline{f}(x,t), \quad 0 < \alpha \leq 1,$$

$$\frac{\partial^{2\alpha}\underline{U}(x,t)}{\partial x^{2\alpha}} = \frac{\partial^2\underline{U}(x,t)}{\partial t^2} + a\frac{\partial\underline{U}(x,t)}{\partial t} + b\underline{U}^n(x,t) + \underline{f}(x,t), \quad 0 < \alpha \leq 1,$$

subject to the initial conditions

$$\tilde{U}(0,t) = \tilde{f}_1(t), \quad \tilde{U}(0,t) = \tilde{f}_2(t), t \geq 0, \quad 0 < \alpha \leq 1.$$

This equation can be written as

$$\begin{aligned}\frac{\partial\overline{U}(x,t)}{\partial x} \quad &= \quad \frac{\partial^{1-2\alpha}}{\partial x^{1-2\alpha}}\frac{\partial^2\overline{U}(x,t)}{\partial t^2} + a\frac{\partial^{1-2\alpha}}{\partial x^{1-2\alpha}}\frac{\partial\overline{U}(x,t)}{\partial t} \\ &\quad +b\frac{\partial^{1-2\alpha}}{\partial x^{1-2\alpha}}\overline{U}^n(x,t) \\ &\quad +\frac{\partial^{1-2\alpha}}{\partial x^{1-2\alpha}}\overline{f}(x,t).\end{aligned} \tag{3.1}$$

We apply the Laplace transform to both sides of Eq. (3.1) and by using linearity property of Laplace transforms, we have

$$\begin{aligned} L[\overline{U}(x,t)] - \frac{\overline{U}(0,t)}{s} - \frac{\overline{U}_x(x,t)}{s^2} &= \frac{1}{s^{2\alpha}} L[\frac{\partial^{1-2\alpha}}{\partial x^{1-2\alpha}} \frac{\partial^2 \overline{U}(x,t)}{\partial t^2}] \\ &+ \frac{1}{s^{2\alpha}} L[a \frac{\partial^{1-2\alpha}}{\partial x^{1-2\alpha}} \frac{\partial \overline{U}(x,t)}{\partial t}] + \frac{1}{s^{2\alpha}} L[b \frac{\partial^{1-2\alpha} \overline{U}^n(x,t)}{\partial x^{1-2\alpha}}] \\ &+ \frac{1}{s^{2\alpha}} L[\frac{\partial^{1-2\alpha} \overline{f}(x,t)}{\partial x^{1-2\alpha}}]. \end{aligned} \tag{3.2}$$

We have a nonlinear operator form as

$$N[\overline{U}(x,t)] = 0, \tag{3.3}$$

where N is nonlinear operator and $\overline{U}(x,t)$ is unknown function. we have a nonlinear operator

$$\begin{aligned} N[\overline{\Phi}(x,t,q)] &= L[\overline{\Phi}(x,t,q)] - \frac{\overline{f_1}}{s} - \frac{\overline{f_2}}{s^2} - \frac{1}{s^{2\alpha}} L[\frac{\partial^{1-2\alpha}}{\partial x^{1-2\alpha}} \frac{\partial^2 \overline{\Phi}(x,t,q)}{\partial t^2}] \\ &- \frac{1}{s^{s^{2\alpha}}} L[a \frac{\partial^{1-2\alpha}}{\partial x^{1-2\alpha}} \frac{\partial \overline{\Phi}(x,t,q)}{\partial t}] - \frac{1}{s^{2\alpha}} L[b \frac{\partial^{1-2\alpha}}{\partial x^{1-2\alpha}} \overline{\Phi}^n(x,t,q)] \\ &- \frac{1}{s^{2\alpha}} L[\frac{\partial^{1-2\alpha}}{\partial x^{1-2\alpha}} \overline{f}(x,t)], \end{aligned}$$

where $q \in [0,1]$ be an embeding parameter and $\overline{\Phi}(x,t,q)$ is the real function of x,t and q. By means of generalizing the traditional homotopy methods construct the zero order deformation equation

$$(1-q)L[\overline{\Phi}(x,t,q) - u_0(x,t)] = qhN[\overline{\Phi}(x,t,q)], \tag{3.4}$$

where h is a nonzero auxilary parameter, when $q=0$ and $q=1$, it hold

$$\overline{\Phi}(x,t,0) = \overline{U_0}(x,t), \quad \overline{\Phi}(x,t,1) = \overline{U}(x,t).$$

Expanding $\overline{\Phi}_i(x,t,q)$ in Taylor's series with respect to q, we have

$$\overline{\Phi}(x,t,q) = \overline{U}_0(x,t) + \sum_{m=1}^{\infty} \overline{U}_m(x,t) q^m, \tag{3.5}$$

where

$$\overline{U}_m(x,t) = \frac{1}{m!} \frac{\partial^m \overline{\Phi}(x,t,q)}{\partial q^m} \mid_{q=0} . \tag{3.6}$$

If the auxiliary linear operator, the initial guess, the auxiliary parameter h are properly chosen, the series (3.5) converges at $q=1$, we have

$$\tilde{U}(x,t) = \tilde{U}_0(x,t) + \sum_{m=1}^{\infty} \tilde{U_m}(x,t), \tag{3.7}$$

which must be one of the solutions of the original nonlinear equations. differentiating equation (3.4) m times with respect to parameter q and then setting $q=0$ and finally dividing them by $m!$, we obtain the mth order deformation equation

$$L[\tilde{U}_m(x,t) - \chi_m \tilde{U}_{m-1}(x,t)] = hqR_m \tilde{U}_{m-1}(x,t)$$

where

$$R_m(\tilde{U}_{m-1}) = \frac{1}{(m-1)!} \frac{\partial^{m-1} N[\tilde{\Phi}(x,t,q)]}{\partial q^{m-1}} \mid_{q=0},$$

$$\chi_m = \begin{cases} 1, & m > 1, \\ 0, & m \le 1, \end{cases}$$

In this way, it is easily to obtain $\tilde{U}_m(x,t)$ for $m \ge 1$, at mth order, we get an accurate approximation of the original equation (1.1).

4. Numerical examples

In order to assess the accuracy of the homotopy analysis transform method presented in this paper for fuzzy fractional differential equations, we solve some examples with proposed techniques in this section.

Example 1. We consider the following fuzzy fractional equation with the initial condition.

$$\frac{\partial^{2\alpha}\tilde{U}}{\partial x^{2\alpha}} = \frac{\partial^2\tilde{U}}{\partial t^2} + \frac{\partial\tilde{U}}{\partial t} + \tilde{U}, \quad t \ge 0, \quad 0 < \alpha \le 1,$$
$$\tilde{U}(0,t) = \tilde{K}e^{-t}, \quad \tilde{U}_x(0,t) = \tilde{K}e^{-t}, \quad t \ge 0.$$

The general terms of the equation

$$\begin{aligned} \frac{\partial^{2\alpha}\overline{U}}{\partial x^{2\alpha}} &= \frac{\partial^2\overline{U}}{\partial t^2} + \frac{\partial\overline{U}}{\partial t} + \overline{U}, \quad t \ge 0, \quad 0 < x < 1, \\ \frac{\partial^{2\alpha}\underline{U}}{\partial x^{2\alpha}} &= \frac{\partial^2\underline{U}}{\partial t^2} + \frac{\partial\underline{U}}{\partial t} + \underline{U}, \quad t \ge 0, \quad 0 < x < 1, \end{aligned}$$
$$\overline{U}_0(x,t,\beta) = \overline{k}(\beta)e^{-t}, \quad \overline{U}_0^{'}(x,t,\beta) = \overline{k}(\beta)e^{-t}.$$

By using our proposed method, we get the solution of the fuzzy differential equations as follows:

$$\begin{aligned} \overline{U}_1(x,t,\beta) &= \overline{k}(\beta)xe^{-t} + \frac{\overline{k}(\beta)x^{2\alpha}e^{-t}}{\Gamma(2\alpha+1)}, \\ \overline{U}_2(x,t,\beta) &= \frac{\overline{k}(\beta)x^{2\alpha+1}e^{-t}}{\Gamma(2\alpha+2)} + \frac{\overline{k}(\beta)x^{4\alpha}e^{-t}}{\Gamma(4\alpha+1)}, \\ \overline{U}_3(x,t,\beta) &= \frac{\overline{k}(\beta)x^{4\alpha+1}e^{-t}}{\Gamma(4\alpha+2)} + \frac{\overline{k}(\beta)x^{6\alpha}e^{-t}}{\Gamma(6\alpha+1)}, \\ \overline{U}_4(x,t,\beta) &= \frac{\overline{k}(\beta)x^{6\alpha+1}e^{-t}}{\Gamma(6\alpha+2)} + \frac{\overline{k}(\beta)x^{8\alpha}e^{-t}}{\Gamma(8\alpha+1)}, \\ &\vdots \end{aligned}$$

Procceding in this manner, the rest of the components $\overline{U}_n(x,t,\beta)$ for $n \ge 5$ can be completely obtained. Therefore, the exact solution is given by

$$\overline{U}(x,t,\beta) = \sum_{r=0}^{\infty} \frac{x^{2r\alpha}}{\Gamma(2r\alpha+1)}\overline{k}(\beta)e^{-t} + \sum_{r=0}^{\infty} \frac{x^{2r\alpha+1}}{\Gamma(2r\alpha+2)}\overline{k}(\beta)e^{-t}, \tag{4.1}$$

$$\underline{U}(x,t,\beta) = \sum_{r=0}^{\infty} \frac{x^{2r\alpha}}{\Gamma(2r\alpha+1)}\underline{k}(\beta)e^{-t} + \sum_{r=0}^{\infty} \frac{x^{2r\alpha+1}}{\Gamma(2r\alpha+2)}\underline{k}(\beta)e^{-t}. \tag{4.2}$$

Let $\overline{K}(\beta) = 0.25\beta$ and $\underline{K}(\beta) = -0.25\beta$ for $0 \le \beta \le 1$.

Example 2. Consider the following fuzzy fractional equation with the initial condition.

$$\frac{\partial^{2\alpha}\tilde{U}}{\partial x^{2\alpha}} = \frac{\partial^2\tilde{U}}{\partial t^2} + \frac{\partial\tilde{U}}{\partial t} + \tilde{U} - x^2 - \tilde{k}t + 1 \quad t \ge 0 \quad 0 \le x \le 1$$

$$\tilde{U}(0,t)=\tilde{k}t, \quad \tilde{u}_x(0,t)=0, \quad 0\leq x\leq 1$$

The general terms of the equation

$$\begin{aligned}
\frac{\partial^{2\alpha}\overline{U}}{\partial x^{2\alpha}} &= \frac{\partial^2\overline{U}}{\partial t^2}+\frac{\partial\overline{U}}{\partial t}+\overline{U}-x^2-\overline{k}t+1,\\
\frac{\partial^{2\alpha}\underline{U}}{\partial x^{2\alpha}} &= \frac{\partial^2\underline{U}}{\partial t^2}+\frac{\partial\underline{U}}{\partial t}+\underline{U}-x^2-\underline{k}t+1,
\end{aligned}$$

and

$$\overline{U}_0(x,t,\beta)=\overline{K}(\beta)t, \quad \overline{U_0'}(x,t,\beta)=0.$$

By using homotopy transform method, we have

$$\begin{aligned}
\overline{u}_1(x,t,\beta) &= -\left(\frac{2x^{2\alpha+2}}{\Gamma(2\alpha+3)}\right)-\frac{x^{2\alpha}}{\Gamma(2\alpha+1)}-\frac{\overline{k}(\beta)x^{2\alpha}}{\Gamma(2\alpha+1)},\\
\overline{u}_2(x,t,\beta) &= \frac{-2x^{4\alpha+2}}{\Gamma(4\alpha+3)}+\frac{x^{4\alpha}}{\Gamma(4\alpha+1)}+\frac{\overline{k}(\beta)x^{4\alpha}}{\Gamma(4\alpha+1)},\\
\overline{u}_3(x,t,\beta) &= \frac{-2x^{6\alpha+2}}{\Gamma(6\alpha+3)}+\frac{x^{6\alpha}}{\Gamma(6\alpha+1)}+\frac{\overline{k}(\beta)x^{6\alpha}}{\Gamma(6\alpha+1)},\\
\overline{u}_4(x,t,\beta) &= \frac{-2x^{8\alpha+2}}{\Gamma(8\alpha+3)}+\frac{x^{8\alpha}}{\Gamma(8\alpha+1)}+\frac{\overline{k}(\beta)x^{8\alpha}}{\Gamma(8\alpha+1)},\\
\overline{u}_5(x,t,\beta) &= \frac{-2x^{10\alpha+2}}{\Gamma(10\alpha+3)}+\frac{x^{10\alpha}}{\Gamma(10\alpha+1)}+\frac{\overline{k}(\beta)x^{10\alpha}}{\Gamma(10\alpha+1)},\\
&\vdots
\end{aligned}$$

Procceding in this manner, the rest of the components $\overline{U}_n(x,t,\beta)$ for $n\geq 6$ can be completely obtained. Therefore, the exact solution is given by

$$\begin{aligned}
\overline{u}(x,t) &= \overline{k}(\beta)t+(1+\overline{k}(\beta))\left(\sum_{r=1}^{\infty}\frac{x^{2r\alpha}}{\Gamma(2r\alpha+1)}\right)-2\sum_{r=1}^{\infty}\frac{x^{2r\alpha+2}}{\Gamma(2r\alpha+3)},\\
\underline{u}(x,t) &= \underline{k}(\beta)t+(1+\underline{k}(\beta))\left(\sum_{r=1}^{\infty}\frac{x^{2r\alpha}}{\Gamma(2r\alpha+1)}\right)-2\sum_{r=1}^{\infty}\frac{x^{2r\alpha+2}}{\Gamma(2r\alpha+3)}.
\end{aligned}$$

Let $\overline{K}(\beta)=0.25\beta$ and $\underline{K}(\beta)=-0.25\beta$ for $0\leq\beta\leq 1$.

In the following tables comparison between the exact solution and the different terms of approximation solution for $\overline{U},\underline{U}$ is given by the homotopy analysis transform method at $\alpha=\frac{1}{8}$.
If we increase the computational process, the approximation solution will be closer to the exact solution.

Table. 1. Numerical results for example 1

(x,t,β)	$\overline{U}_{approx[5]}$	$\overline{U}_{approx[15]}$	$\overline{U}_{exact}$
(0.2,0.2,0.2)	0.1366826352	0.1430736220	0.1430787868
(0.4,0.4,0.4)	0.3314834136	0.3709771230	0.3711424567
(0.6,0.6,0.6)	0.5455609982	0.6577745142	0.6589806176
(0.8,0.8,0.8)	0.7571237640	0.9875152418	0.9923035310

Table. 2. Numerical results for example 1

(x,t,β)	$\underline{U}_{approx[5]}$	$\underline{U}_{approx[15]}$	$\underline{U}_{exact}$
(0.2,0.2,-0.2)	-0.1366826352	-0.1430736220	-0.1430787868
(0.4,0.4,-0.4)	-0.3314834136	-0.3709771230	-0.3711424567
(0.6,0.6,-0.6)	-0.5455609982	-0.6577745142	-0.6589806176
(0.8,0.8,-0.8)	-0.7571237640	-0.9875152418	-0.9923035310

Table. 3. Numerical results for example 2

(x,t,β)	$\overline{U}_{approx[5]}$	$\overline{U}_{approx[15]}$	$\overline{U}_{exact}$
(0.2,0.2,0.2)	1.946996365	2.098551258	2.098678495
(0.4,0.4,0.4)	2.887174141	3.438033996	3.440530642
(0.6,0.6,0.6)	3.548765283	4.764609749	4.779423339
(0.8,0.8,0.8)	3.866933055	6.024998314	6.078554584

Table. 4. Numerical results for example 2

(x,t,β)	$\underline{U}_{approx[5]}$	$\underline{U}_{approx[15]}$	$\underline{U}_{exact}$
(0.2,0.2,-0.2)	1.742520521	1.879641614	1.879756733
(0.4,0.4,-0.4)	2.289506115	2.740209634	2.742252344
(0.6,0.6,-0.6)	2.466478687	3.365146337	3.376095511
(0.8,0.8,-0.8)	2.311288703	3.749998876	3.785703056

5. Conclusion

In this paper, the homotopy analysis transform method is clearly a very efficient technique for finding the solutions of fuzzy fractional differential equations. We give the expression of exact solution for the fuzzy time-fractional telegraph equation with initial conditions. Finally, illustrative examples are acquired to demonstrate the validity and applicability of the proposed method.

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