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Students' Mathematical Literacy in the Process of Teaching Problem Solving Strategies[1]

Öğrencilerin Problem Çözme Stratejilerinin Öğretildiği Süreçte Matematik Okuryazarlıklarının İncelenmesi[1]

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ABSTRACT: The purpose of the study is to investigate the mathematical competence of 8th grade students in the context of teaching problem solving strategies. In the study, students were taught problem-solving strategies and were asked to solve problems prepared on these strategies using the appropriate strategy. The study was conducted with 32 eighth grade students in an elementary school. The data were analyzed using the content analysis method. The results of the pretest and posttest on mathematical competence suggest that the exercise in which problem-solving strategies were taught increased the level of mathematical competence of the students. In addition, data from the semi-structured interview sheets, student solution sheets, and observation notes identified progress in the students' problem-solving process and implementation of mathematical processes.

Keywords: Mathematical literacy, mathematical processes, problem solving, problem solving strategies

[1] The data reported here were collected as part of the first author's thesis.

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ÖZ: Çalışmanın amacı, 8. sınıf öğrencilerinin problem çözme stratejilerinin öğretildiği süreçte matematik okuryazarlıklarının incelenmesidir. Çalışma sürecinde öğrencilere problem çözme stratejileri öğretilerek bu stratejilerle ilgili hazırlanan problemleri uygun stratejiyi kullanarak çözmeleri istenmiştir. Öğrencilerin çözüm süreçlerindeki ve matematik okuryazarlığındaki değişim, matematiksel süreçler (formüle etme, yürütme, yorumlama ve değerlendirme) bağlamında ele alınmıştır. Çalışma Milli Eğitim Bakanlığına bağlı Ankara ilinde bir ortaokulda öğrenim gören 32 sekizinci sınıf öğrencisi ile gerçekleştirilmiştir. Çalışmanın başında ve sonunda matematik okuryazarlığı testi ön test ve son test olarak uygulanmış ve elde edilen nicel veriler, SPSS paket programında yer alan istatistiksel testler yoluyla analiz edilmiştir. Çalışma sürecinde ise, yarı yapılandırılmış görüşme formu, gözlem formu, yazılı görüş formu ve öğrenci çözüm kâğıtlarından veriler toplanmıştır. Bu veriler içerik analizi yöntemi kullanılarak incelenmiştir. Matematik okuryazarlığı ön test ve son testinden elde edilen bulgulara göre, problem çözme stratejilerinin öğretildiği uygulamanın öğrencilerin matematik okuryazarlığı seviyesini arttırdığı sonucuna ulaşılmıştır. Ayrıca, yarı yapılandırılmış görüşme formları, öğrenci çözüm kâğıtları ve gözlem notlarından elde edilen verilere göre öğrencilerin problem çözüm süreci ve matematiksel süreçleri işe koşmada ilerlemeler tespit edilmiştir.

Anahtar sözcükler: Matematik okuryazarlığı, matematiksel süreçler, problem çözme, problem çözme stratejileri

1. INTRODUCTION

From the past to the present, information and technology have changed rapidly. As a result, the tasks that people and societies have to perform have also changed. The goal is to grow competent people who are able to produce knowledge, use it in daily life, think critically, act entrepreneurially, solve difficulties, etc. (Ministry of National Education [MoNE], 2018; NCTM, 2000). The concept of mathematical literacy comes to the fore when the expected characteristics of people are examined. People who are mathematically literate are those who can apply mathematics in their daily lives, solve problems from different perspectives, think critically, and have mathematical thinking skills (Martin, 2007). In addition, the ability to apply mathematical arguments to understand real-world problems is referred to as mathematical literacy (Machaba, 2017).

In its publications to promote the Programme for International Student Assessment (PISA) the Organisation for Economic Co-Operation and Development (OECD) reported that the mathematics questions in PISA attempt to measure mathematical literacy. The PISA measures how well 15-year-old students have acquired the skills and knowledge needed to succeed in today's society and economy after completing compulsory schooling. The questions in PISA are created considering various personal, social, and cultural contexts that students must solve using their knowledge and skills (She, Stacey, & Schmidt, 2018). The test measures students' ability to apply mathematics to a real-life situation or problem, i.e., their mathematical competence. The concept of mathematical literacy in PISA focuses on measuring students' ability to formulate, apply, and interpret mathematics in different contextual situations (OECD, 2019). PISA aims to assess how students can make sense of what they know and how they can apply their mathematical knowledge when confronted with a new and unusual situation. To this end, the mathematics questions in the PISA tests include questions about real-life situations in which mathematical skills can be applied. According to the PISA assessments, it was found that students in the countries generally know formulas and equations but cannot relate them to their daily lives and cannot imagine how they work. This shows that students have problems with mathematical literacy (Altun, 2016). Mathematical literacy is the transformation (formulation) of a given problem into a mathematical problem, the solution (application) of a mathematical problem using mathematical knowledge and logic, and the evaluation (interpretation and judgment) of the compatibility of the obtained result with real life (MoNE, 2013). These three concepts constitute the mathematical processes that an individual use when solving mathematical problems.

Mathematical processes were first discussed in the PISA 2012 assessment framework PISA. These processes are formulating, applying, interpreting, and evaluating. There are basic mathematical skills that enable the implementation of each of the mathematical processes identified in the Mathematical Literacy Framework (MoNE, 2019). These skills are: Communication, mathematization, reasoning and proof, demonstration, designing a problem-solving strategy, using symbolic and technical language and operations, and using mathematical tools. These skills are used in solving mathematical literacy questions (Canbazoğlu & Tarım, 2021; Çepni, 2019). Mathematics classroom activities should focus on the development of mathematical skills (Hiebert, Morris, & Glass, 2003; Reys, Lindquist, Lambdin, & Smith, 2007). To this end, activities should be conducted primarily in the context of daily life where they can apply problem-solving strategies (Schukajlow et al., 2022; Verschaffel et al., 2020).

A problem is a situation that involves a mental process whose outcome is sought, but the path to be followed in that process is not clear. Where there is no problem, there is no progress. Therefore, problems and problem solving are very important in preparing students for daily life and their spiritual development. Problem solving is not only a goal, but also a fundamental tool in learning mathematics.

Therefore, it should not be considered separately from the mathematics curriculum (Jaeder et al. 2020; Yanık, 2016). Lester and Kehle (2003) describe problem solving as the heart of mathematics. Baki (2019) emphasizes that problem solving is a very important way for students to discover the power of mathematics and to use it. Students have the opportunity to apply the mathematical concepts they have learned and to see the relationship between the concepts by problem solving. In this way, the mathematical concepts and applications learned gain meaning for students.

The stages of problem solving are stated as understanding the problem, developing a strategy (planning) for solving the problem, applying the strategy, and evaluating the solution (Polya, 2004). These steps, especially the first two steps are followed by students in the problem solving process, which allows the creation of a new model for each problem and the use of new expressions. In this way, the individual's ability to understand and use mathematical language develops, and the goal of being mathematically literate is achieved (Baki, 2019). The second step of the problem-solving stages is to determine the problem-solving strategy. In order to choose a solution-based strategy, students must have learned these strategies beforehand. Studies state that problem solving strategies can be learned and that learning different strategies can provide students with a predisposition and facilitation for solving the problems they encounter (Altun & Arslan, 2006; Arslan, 2002; Carpenter & Moser, 2020; Dönmez, 2002; Taşpınar, 2011). In this study, strategies such as making systematic lists, intelligent guessing and testing, working backwards, making a drawing, constructing a table, finding a pattern, solving a simpler equivalent problem, logical reasoning, estimation, and explicit proposition writing were conducted with students. Problem solving strategies should be taught so that they can be applied not only to mathematical situations, but also to experiences in daily life whenever possible. In problem solving, the first goal should be to match students with several problem solving strategies and have them practice using them. Appropriate practice in problem solving enables further goals to be achieved. Thus, students naturally begin to use problem-solving strategies not only to solve mathematical problems, but also to solve problems in daily life (Posamentier & Krulik, 2020). From this point of view, it is very important that students are exposed to problems in daily life and incorporate practices related to problem-solving strategies.

Mathematical literacy refers to the knowledge of understanding and the ability to solve mathematical situations in daily life. Mathematical literacy is a priority need for all countries with development goals. Mathematics education in schools should aim to improve mathematical literacy and develop each student's ability to use and apply mathematical knowledge to solve real-world problems (Sumirattana, Makanong, & Thipkong, 2017). In this way, knowledgeable, reflective citizens who contribute to society are educated (Bolstad, 2020). The need for mathematical thinking arises in a real-world context. When solving problems in a real-world context, part of mathematical literacy is finding logical solutions to complex problems, provided they can define, explain, or predict the problem (Stacey & Turner, 2015). Studies have been conducted in the literature where some factors and mathematical literacy have been discussed together to improve mathematical literacy. However, there are very few studies in the literature that examine problem solving strategies and mathematical competence together (Højgaard, T., 2021; Ukobizaba, F., Nizeyimana, G., & Mukuka, A., 2021). In this study, problem solving strategies are taught to students and their effects on mathematical competence are investigated. Therefore, it is believed that this study is important to increase students' success by improving the level of mathematical literacy. In addition, the use of problem-solving strategies and the encouragement of free thinking in the solution process are considered important to try different solution strategies and discover that there is more than one strategy to solve problems. In this study, students were asked to solve tasks created with real-world problems appropriate to their level, using problem-solving strategies.

In doing so, students' changes in mathematical processes and problem solving processes were examined. For the purpose of the study, the following research questions (RQ) were investigated:

RQ1. How do students' problem solving processes change when they use problem-solving strategies?

RQ2. Is there a significant mean difference between students' mathematical literacy levels before and after the exercise in which problem solving strategies were taught?

RQ3. What is the opinion of 8th grade middle school students about the process of learning problem solving strategies?

1.1. Theoretical Framework

Choosing a problem-solving approach is the second stage of the problem-solving process (Polya, 2004). Students must have previously acquired these methods in order to select a solution-based strategy (Posamentier & Krulik, 2008; 2009). According to the studies, learning different problem-solving strategies could provide students with predisposition and facilitation in coping with difficulties (Alibali, Phillips, and Fischer, 2009; Altun and Arslan, 2006; Arslan, 2002; Dönmez, 2002; Gagne, 1980; Taşpnar, 2011). The ten problem-solving strategies listed below were explored with the students in this study.

Systematic List Making Strategy;

It is the regular recording of all the possibilities associated with the data to solve the problem. When using this strategy, probabilities should be written down systematically without omitting data (Posamentier & Krulik, 2008;2009).

Intelligent Guessing and Testing Strategy;

Starting with a logical assumption, the problem is then solved. The accuracy of the guessed response is verified. If the response is incorrect, a closer estimate is made using the incorrect answer as a base, and so on until the proper answer is found. Here, it's crucial to begin the solution with a logical hypothesis rather than a guess at random. The previous result should be taken into consideration while making future projections. This tactic is also known as the trial-and-error technique (Posamentier & Krulik, 2008; 2009).

Making a Drawing Strategy;

Drawings are created to visually support the solution to the problem. In our daily lives, we often use this tactic, using drawings to help people understand the problem. For example, we prefer pictures to describe things when we are advising someone. Considering that a picture is worth a thousand words, this approach makes the problem more understandable by making the data of the problem more concrete by creating an illustration (Posamentier & Krulik, 2008; 2009).

Finding a Pattern Strategy;

The logic and order that mathematics shows is one of its inherent beauty. Through a number of patterns, we can observe this logic and order in the physical world (Posamentier & Krulik, 2008; 2009). The relationship between the variables is found and the solution is arrived at after listing the specific solutions to the problem. A complex problem is ensured to be reduced to a relation, and a solution is found using this connection (Posamentier & Krulik, 2008; 2009).

Explicit Proposition Writing Strategy;

The relations given in the task are written as equality or inequality. By representing an ambiguous letter or symbol, such as "x" equations or inequalities are created that must be solved to determine the unknown. Not all grade levels are appropriate for this strategy. In particular, it can be used starting in seventh and eighth grade (Posamentier & Krulik, 2008; 2009).

Estimation Strategy;

It is used when an approximate or anticipated solution will suffice instead of an exact one. After rounding and processing the problem-related information in the mind, a rough conclusion is drawn (Posamentier & Krulik, 2008; 2009).

Solving a Simpler Equivalent Problem Strategy;

To solve a problem, it is sometimes necessary to transform the problem into a simpler problem and to gain the understanding needed to solve the main problem by solving this simple problem (Posamentier & Krulik, 2008; 2009). This is mostly used in problems that are difficult and complex due to the size of the numbers or the complexity of the shapes. For this reason, numbers and shapes are treated in the smallest possible form and generalization is achieved by increasing them step by step.

Working Backwards Strategy;

This strategy is applied to problems where we do not know the initial value or state, but we do know the resulting state or value. To solve such problems, one starts from the last step and reverses the operations or situations, going back to the first step and obtaining the initial value (Posamentier & Krulik, 2008; 2009).

Constructing Table Strategy;

This strategy is useful in problems where it is easier to see the relationship between information when the problem data or information obtained during the solution is arranged in a table. In this way, it is easier to recognise relationships and rules to reach a result, correlations are found, and the problem is solved (Altun, 2014; Posamentier & Krulik, 2008; 2009).

Logical Reasoning Strategy;

Although logic and reasoning are involved in the solution process of any problem, in the solution of some problems only logic / reasoning is required. In such problems, a conclusion is drawn by executing logic, then another conclusion is drawn based on that conclusion, and the result is achieved by continuing in this way (Arslan & Yazgan, 2019; Posamentier & Krulik, 2008; 2009).

None of the problem-solving approaches is suitable for solving every problem. However, it is possible to use more than one strategy to solve a particular problem. Alternatively, different approaches can be used at different stages of problem solving to arrive at a solution. The more strategies students know and use, the more practical they become in solving the problems they encounter (Posamentier & Krulik, 2008; 2009). Problem solving has a systematicity, not rules. Students should be taught the systematic nature of problem solving, problem solving strategies should be introduced and they should learn to apply them (Altun, 2014). The reason why problem-solving skills are emphasized in mathematics education programs is to bring up individuals who can apply problem-solving skills to the problems they encounter in daily life and overcome the problems (Özsoy, 2005). Mathematical knowledge can provide solutions to problems in daily life only when combined with problem-solving skills (Temel & Altun, 2020). Problem-solving skills play an important role in developing solutions to real-world problems, that is, in transferring learned information to life (Posamentier & Krulik, 2008; 2009).

It is emphasized that students can acquire problem-solving skills in mathematics education in three ways. (i) According to the problem-solving teaching approach, the student is first taught the skill and then given the task of solving the problem using that skill. As in textbooks, abstract concepts are taught first, and then the learned skills are applied. (ii) In problem-solving instruction, students are first taught the problem-solving process, that is, problem-solving steps and strategies. Then, they are expected to apply these skills in solving the given problems. (iii) In problem-solving approach instruction, the problems are given first and then the skills required to solve the problem are elicited during the problem-solving process. This approach can be called the opposite of problem-solving teaching. Moreover, this approach assumes that students learn mathematics through real contexts, situations, problems, and models (Yanık, 2016).

2. METHOD

For the purposes of this study, a mixed method was used. The Partially Mixed Sequential Dominant Status Design defined by Leech and Onwuegbuzie (2009) was used. This design is a study in which one of the qualitative and quantitative components is more dominant. The purpose of this design is to use qualitative and quantitative methods together, to diversify, compare, and integrate the data obtained, and to obtain data that are distinct but directly related to the research questions. This design seeks to supplement the weaknesses of one method with the strengths of the other by using quantitative and qualitative methods together.

2.1. Participants

The study was conducted in a public elementary school in the Central Anatolia region of Turkey. It was conducted in a single group with a total of 32 students, 17 girls and 15 boys, attending the eighth grade. The participants were selected using the random sampling method.

2.2. Data Collection Tools

A semi-structured interview, observation questionnaire, written opinion survey, and student solution sheets, qualitative data collection instruments, were used to determine the effects of teaching problem-solving strategies to eighth grade students on mathematical literacy. A pretest and a posttest on mathematical literacy were employed as quantitative data collection tools.

The mathematical literacy test used in the study as a pretest and posttest consisted of PISA 2012 main and pilot application questions published by the Ministry of National Education. The Cronbach alpha reliability coefficient of the pretest was calculated as 0.78 and the Cronbach alpha reliability coefficient of the posttest was calculated as 0.82. The questions to be included in this test were determined by the joint opinions of three experts in the field based on the level of 8th grade students.

The opinions of the experts were obtained when determining the questions for the semi-structured interview and the researcher's written questionnaire. Two experts from the field of mathematics and one expert from the field of language teaching were interviewed. Pilot applications were then conducted and changes were made in areas that were not understood, and the final version of the tests was created. The semi-structured interview form was administered by the researcher to each student individually at the end of each week of study. A written questionnaire was administered to 15 students who were randomly selected among the students who participated in the application at the end of the study. In order for the data collected in the study to be systematic and regular, the researcher used an observation form during the study process. The observation form (Table 1) consists of a table in which the mathematical processes and problem-solving strategies used by the observed student are noted.

Table 1: *Observation Form*

Date:	**Implementation Week:**	
Problem Solving Strategies	Utilized Mathematical Process	Observed Student

To determine changes in students' mathematical literacy and problem-solving processes, students' solution sheets were collected at the end of each week of study and analyzed by two researchers.

2.3. Implementation Process

Once the data collection instruments and problem-solving strategy questions to be used in the application were prepared, implementation began. In the first week of the study, the mathematical literacy test was used as a pretest. Considering the mathematical literacy assessment framework based on the definition of mathematical literacy in PISA (Figure 1), three categories emerge: mathematical content, real-world contexts, and mathematical processes (Çepni, 2019; OECD, 2013).

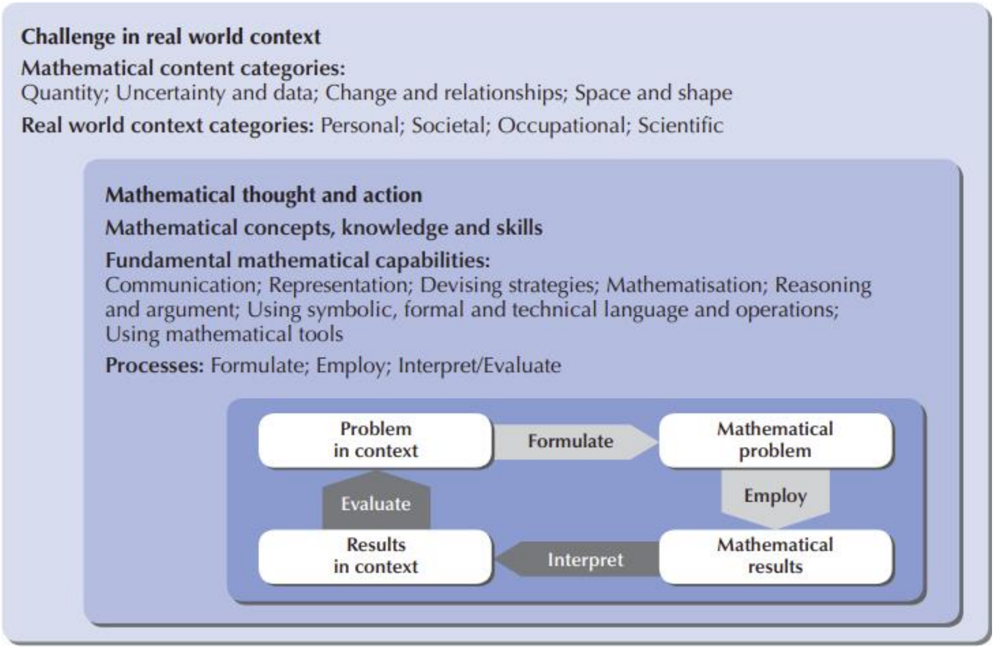


Figure 1: *A model of mathematical literacy in practice* **(***Source: OECD (2013), PISA 2012 Assessment and Analytical Framework, http://dx.doi.org/10.1787/9789264190511-en.***)**

The questions on mathematical literacy in PISA include all mathematical processes. However, on PISA, the question is asked in the category that predominates the process. Table 2 shows to which of the three categories the questions in the mathematical literacy test used in the study belong.

Table 2: *Distribution of Mathematical Literacy Test Questions According to PISA Mathematical Literacy Assessment Framework*

ITEMS	CONTENT	CONTEXT	PROCESSES
Apartment Purchase	Space and Shape	Personal	Formulate
Charts– Item 1	Uncertainty and Data	Societal	Interpret/Evaluate
Charts – Item 2	Uncertainty and Data	Societal	Interpret/Evaluate
Charts – Item 3	Uncertainty and Data	Societal	Interpret/Evaluate
Sauce	Quantity	Personal	Formulate
Ferris Wheel - Item 1	Space and Shape	Societal	Employ
Ferris Wheel - Item 2	Space and Shape	Societal	Employ
Helen the Cyclist – Item 1	Change and Relationships	Personal	Interpret/Evaluate
Helen the Cyclist – Item 2	Change and Relationships	Personal	Interpret/Evaluate
Helen the Cyclist – Item 3	Change and Relationships	Personal	Employ
Which Car? – Item 1	Uncertainty and Data	Personal	Employ
Which Car? – Item 2	Quantity	Personal	Interpret/Evaluate
Which Car? – Item 3	Quantity	Personal	Employ
MP3 Players – Item 1	Change and Relationships	Personal	Employ
MP3 Players – Item 1	Quantity	Personal	Interpret/Evaluate
Holiday Apartment	Quantity	Societal	Employ
DVD Rental – Item 1	Quantity	Personal	Employ
DVD Rental – Item 2	Quantity	Personal	Formulate
Faulty Players– Item 1	Uncertainty and Data	Occupational	Formulate
Faulty Players– Item 2	Uncertainty and Data	Occupational	Interpret/Evaluate

In the following weeks, 10 problem-solving strategies established for the study were explained to the students, one strategy per week, and the students were asked to solve the related problems using the learned strategies. At the end of each week, students' solution sheets were collected and analyzed by two researchers. The implementation process was carried out according to the plan in Table 3.

Table 3: *Weekly Plan of Implementation Process*

Week	Implementation
	Mathematical Literacy Pre-Test
1	Constructing Table Strategy
2	Systematic List Making Strategy
3	Intelligent Guessing and Testing Strategy

4	Working Backwards Strategy
5	Making a Drawing Strategy
6	Finding a Pattern Strategy
7	Solving a Simpler Equivalent Problem Strategy
8	Logical Reasoning Strategy
9	Estimation Strategy
10	Explicit Proposition Writing Strategy
	Mathematical Literacy Post-Test

Before the first week of the study, the Mathematical Literacy Test was administered to the students as a pre-test. This test lasted 80 minutes. In each of the following weeks, the strategies listed in Table 3 were used. The pattern-finding problem addressed in Week 6 is shown in Figure 2.

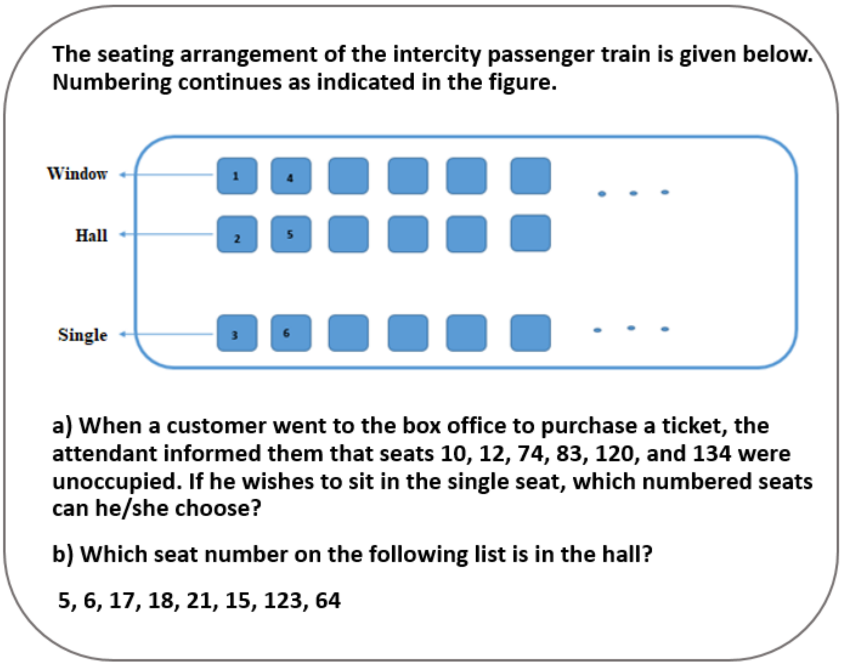


Figure 2: *The problem addressed in the 6th week finding a pattern strategy*

During the implementation process, the tasks prepared for this week's strategy were distributed to the students by the teacher and the students were asked to solve the first task and write their solutions under the task. Then the strategy set for this week was explained to the students. After the strategy was explained, students were asked to solve the distributed task again. It was pointed out that they should not erase their previous solutions even if they were wrong. After the first task was solved by the students, it was solved together with the teacher. Then the second question was solved first by the students and then by the teacher. In this way, the learned strategy was consolidated by applying it to the problem. At the end of the lesson, the students' solution sheets were collected and evaluated by two researchers at the end of the study. Each week, the researcher observed the applied lessons and noted on the observation sheet any noticeable improvement in math skills and problem solving techniques. The lessons were also recorded on camera for later viewing. Following the weekly application, the researcher interviewed each student individually in semi-structured interviews. The following list includes some of the semi-structured interview questions.

1) Did the strategies you learned in the lesson give you a different perspective on solving this problem?

2)What other strategies can be used to solve this problem?

3) Do you think you acted in a logical sequence when solving the problem? How do you think learning problem-solving strategies affects this problem?

4) Can you justify using the language of mathematics when explaining your thinking about this problem to someone? Can you defend the outcome of the task?

5) Do you think you can use the strategies you learned to solve problems in everyday life, in class, and on exams?

In the last week of the study, the Mathematical Literacy Test was administered to the students as a posttest. Then, a written questionnaire was administered to 15 randomly selected students, asking them about their opinions and evaluations of the study and where they could apply the strategies they learned. Some of the questions in the form are listed below.

1) Do you think there is a change in the problem solving process? If so, can you explain this change?

2) Do you think you can apply the strategies you learned in your daily life? How?

3) Do you think you can apply the strategies you have learned in math and other classes? To what extent?

4) Do you think the strategies you have learned will be useful on exams? To what extent? The questions allow students to reflect on their opinion and evaluation of the study and where they can use the strategies they have learned.

2.4. Data Analysis

In analyzing the qualitative data obtained in the study, the content analysis method was used. Content analysis is a systematic technique in which the words of a text are summarized and divided into smaller categories by coding them according to certain rules (Büyüköztürk, Kılıç Çakmak, Akgün, Karadeniz, & Demirel, 2020). When conducting the content analysis, students were coded as S1, S2, S3... codes and these codes were used in the quotations.

The raw qualitative data of the study were coded by two different researchers. The study determined the number of "agreement" and "disagreement" situations for the coding to be done by the two researchers and used the formula "Reliability = (Agreement)/[(Agreement) + (Disagreement)]" from Miles and Huberman (1994). It can be said that the coding reliability is at an acceptable level, since the percentage of agreement is 91.7%, higher than 70%.

The quantitative data obtained from the pre- and post-test of mathematical skills were analyzed by statistical tests using the SPSS package program. In the quantitative part of the study, to determine the effects of the application in which problem-solving strategies were taught on students' mathematical skills, a one-group pre-post test design was used, which belongs to the weak experimental designs. In this design, the experimental procedure was applied to a single group; it is a design in which data are collected through a pretest before the experimental application of the same data collection instrument and a posttest after the application (Büyüköztürk et al., 2020). An application was conducted in which the study group was taught problem-solving strategies. The effect of teaching on students' mathematical competence was determined using the pretest before application and the posttest after application. The questions in the mathematical literacy test were asked by two mathematics teachers alongside the researcher, according to the scoring system in PISA. Scoring was 1 point for complete answers and 0 points for incorrect or blank answers. Partial scoring was done only for the second item of the question "DVD rental". When scored by three different evaluators, the score that resulted from the agreement of responses where there was disagreement was the outcome. The paired- samples t-test was used to determine if there was a significant mean difference in the students' level of mathematical ability between the pretest and posttest. In order to apply the paired samples t-test, some assumptions must be made. The values of the dependent variable should be at least on an interval scale (Büyüköztürk et al., 2020). The other is that the data should have a normal distribution. In this study, the kurtosis and skewness coefficients were examined to determine whether the data were normally distributed. Then, the normality test was used to determine whether the data met the normality assumption. Since the number of participants in the application group was less than 50, the results of the Shapiro-Wilk test were used instead of the Kolmogorov-Smirnov test for the normality assumption. In this study, the posttest data obtained from the Mathematical Literacy Test showed a normal distribution with a value of p=0.501. Since the values obtained by dividing the skewness value (0.828) and the kurtosis value (-0.629) by the standard errors of the pretest data ranged from -1.96 to +1.96, it was concluded that the Mathematical Literacy Test scores were approximately normally distributed (Can, 2013). It is appropriate to use the t-test for related measures to test whether the related measurement results are significantly different from each other (Büyüköztürk et al., 2020). For this reason, the pre-test and post-test means of the students in the study group were compared using the t-test for dependent (paired) samples, one of the parametric tests.

2.5. Research Ethics

In planning the study researchers had the responsibility to evaluate carefully any ethical concerns. Three issues were addressed in the study by the researchers. First of all, there were not any situations that had to be handled about the protection of participants from harm. Secondly, the confidentiality of research data was ensured. Finally, researchers conducted the study using methods that do not require deception.

3. FINDINGS

The purpose of this study is to investigate the mathematical competence of 8th grade students in the context of teaching problem solving strategies. To this end, this section presents the findings obtained from the analysis of the data collected and the comments on these findings, considering the research problems.

3.1. Examining Students' Solution Processes in the Process of Implementing Problem Solving Strategies

The data obtained in the research process from the observation sheet, the students' solution sheets, and the semi-structured interview sheets were used to answer the first research question, "How do students' solution processes change when they use problem-solving strategies?" The analysis of this research question, which aims to determine the change in students' solution processes, is discussed in the context of the change in mathematical processes. The formulation process involves defining the mathematical aspects of the problem situation in a context and identifying its important variables (OECD, 2019). While in the first weeks of instruction it was observed that students had difficulty identifying the important variables in the problem situations and mathematizing the problem situation, in the following weeks it was observed that they could easily identify the variables and express the problems mathematically.

The formulation process involves organizing a contextual problem according to mathematical concepts (OECD, 2019). It was observed that students began to solve the problems more regularly and step by step. In addition, this process involves representing a mathematical situation using mathematical relationships, appropriate variables, symbols, diagrams, and standard models when solving problems (OECD, 2019). During the application process, students indicated that there was a positive change in their use of mathematical language. These changes in the process are evident from both the student solution sheets and the student responses to the semi-structured interview forms used each week. Below are some examples of student responses to the semi-structured interview sheets.

…

S27: “I had no trouble in mathematizing this problem. I have troubles in the beginning. I think, the strategies were useful…” (Week 7 /Solving a Simpler Equivalent Problem Strategy)

S26: “Yes, the strategies allowed me to alter the question after reading it. Thanks to this and the old strategies, I am now more easily mathematized.” (Week 6 / Finding a Pattern Strategy)

S23: “Yes, I solved it in a logical order. I think it has a positive impact. Now I will solve problems neatly, not messily.” (Week 5 / Making a Drawing Strategy)

S21: “After learning the problem solving strategies, I solve the questions step by step and regularly.” (Week 2 /Systematic List Making Strategy)

S26: “I regularly list the data in the questions and apply appropriate action. Before I learned the strategies, I was trying to reach a solution with complex transactions.” (Week 9 / Estimation Strategy)

S32: “It allowed me to better understand the problems and put things in order and act rationally. Now I can understand the questions better and easier.” (Week 9 / Estimation Strategy)

S18: “I first find a strategy and then proceed step by step. The strategies I learned allowed me to act systematically and to minimize the trading error.” (Week 10 / Explicit Proposition Writing Strategy)

…

The application process involves taking the necessary action and reaching the conclusion through reasoning (Altun, 2020). This process includes designing and applying mathematical problem-solving strategies, using mathematical tools and structures, and applying mathematical operations to solve the problem situation (OECD, 2019). During the research process in which the problem-solving strategies were explained, a change in the students' ways of solving the problem was observed. In the first weeks of the study, students began working directly with numbers after reading the problem situation. In the following weeks of the study, after reading the problem situation, students first determined the important variables and organized the data. Then, it was observed that they tried to reach a solution by choosing an appropriate strategy. After they reached the result, they tried to support the solution with another strategy.

In the application process, there is a proposed strategy and an application for solving a problem (Çepni, 2019). Depending on the suitability of the problem situation, it is necessary to use different representations in the problem-solving process and change the strategy as needed (OECD, 2019). During the study, it was found that in the first weeks, students only applied the learned strategy to reach mathematical results, but in the following weeks, they were able to suggest other strategies and thus support the solutions they found with other strategies. In addition, the application process requires skills such as arithmetic calculations, logical reasoning based on mathematical assumptions, solving equations, and data analysis (MoNE, 2013). In analyzing the process, improvements were observed in students during the application process, and this development was reflected in the semistructured interview forms presented to students. The following are some examples.

…

S28: "I solve tasks more regularly using the strategies I learned." (Week 2 / Systematic List Making Strategy)

S23: "When solving a task, you can use more than one strategy. A prediction and control strategy can also be used to solve this task." (Week 4 / Working Backwards Strategy)

S27: "Before this study, I did random and mixed transactions. Now I have started doing more regular transactions." (Week 4 / Working Backwards Strategy)

S27: "With the strategies I learned, I apply the mathematical operations more accurately." (Week 10 / Explicit Proposition Writing Strategy)

S26: "I think that I write down the information given in the task regularly and that I perform the operations logically. I think that my problem solving process has improved after learning the strategies." (Week 10 / Explicit Proposition Writing Strategy)

S24: "Because I learned many strategies, I can explain them in different ways. I can defend the results with strategies." (Week 9 / Estimation Strategy)

…

The interpretation and evaluation process involves relating mathematics to the contextual real-world situation and evaluating the appropriateness of the problem situation or solution in the context of the real world (OECD, 2019). Students should be able to understand mathematical concepts and apply them in everyday life (MoNE, 2018). In the semi-structured interview questionnaires given to students, it was found that students indicated that they were able to apply the strategies they learned in solving problems in everyday life, in class, and on exams. In addition, students indicated that they could relate the problems they solved to real-life strategies. Some examples of student testimonies are presented below.

…

S24: "The problems we solve in class are similar to the problems we may encounter in our daily lives." (Week 8 / Logical Reasoning Strategy)

S25: "Definitely from life." (Week 2 / Systematic List Making Strategy)

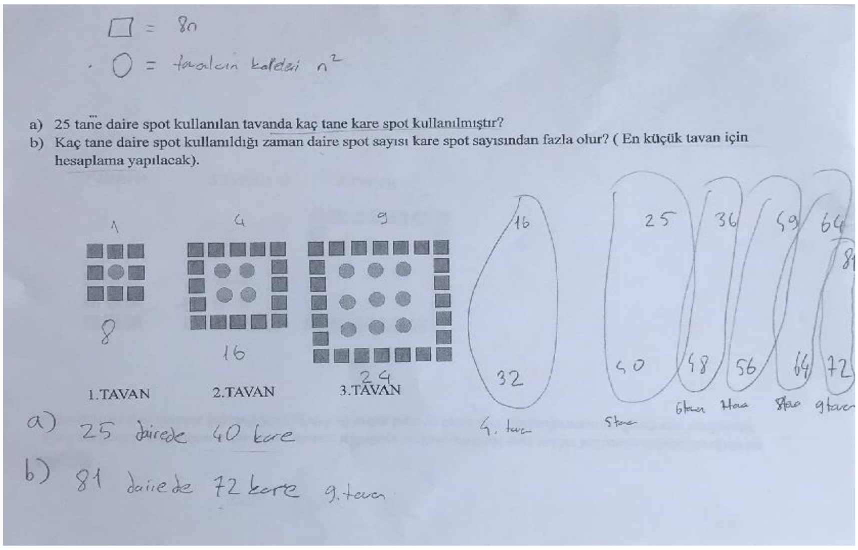
S27: "I used the strategy of making tables in Turkish in today's exam. It saves time and avoids confusion." (Week 4 / Working Backwards Strategy)

S29: "Since I gain a different perspective thanks to the strategies, I can find more logical solutions to the difficulties I encounter in my daily life. I can use them especially in Turkish and science classes." (Week 10 / Explicit Proposition Writing Strategy)

S26: "The problems in the given questions are the problems we face in our daily life. In this strategy, the question was prepared using liveable events." (Week 8 / Logical Reasoning Strategy)

In the process of applying the problem-solving strategies, the students' development in mathematical processes was also reflected in the solution sheets. The data on these developments were recorded in the observation sheets and presented in this section. It was observed that students were able to solve the tasks in the problem-solving strategies study more easily by applying these strategies after learning the strategy. An example of the students' solutions is shown below.

S24 solved the problem related to the Finding Patterns strategy used in Week 6 by finding correlations after learning the strategy. The student's solution is shown in Figure 3.



a) How many square spotlights are used in a ceiling with 25 circle spotlights?
b) How many circle spotlights are used in the situation which the number of circle spotlights is more than the number of square spotlights? (Calculation will be made for the smallest ceiling)

Figure 3: *S24's Solution to the Problem of Finding Pattern Strategy*

The formulation process involves recognizing mathematical structures that involve order, relationships, and patterns in a problem situation (OECD, 2019). It can be seen that in solving the problem, S24 first discovered the relationship between the number of ceilings and the number of circular and square radiators used. This relationship; the number of square radiators was mathematically expressed as "8n" and the number of circular radiators as "n^2". The employment process involves applying mathematical operations and generalizing to find solutions. (OECD, 2019). That the student discovers the mathematical relationship in the solution process shows that (s)he performed the formulation process, and that (s)he expresses this relationship through generalization in terms of "8n" and "n^2" and applies the mathematical operations to arrive at the solution shows that (s)he performs the application process.

When examining the students' solution sheets, it is found that after learning problem solving strategies, they can solve problems easily and propose more than one strategy to solve a problem in the following weeks. At the same time, it was found that they can solve a problem with another strategy or they can solve a problem by using more than one strategy together. The following is an example that reflects this problem.

S24 solved the problem using the strategy "Intelligent Guessing and Testing" while solving the problem in the context of the strategy "Writing Explicit Proposals" which was used in the 10th grade. The problem was given in Figure 4.

Two different branches of a store have separate campaigns. In branch A, a 10% discount is applied to the purchase. In branch B, a discount of 20 TL is applied on the purchase price. Since it is more advantageous for a person who wants to buy three shirts to choose branch B, what is the maximum price of one shirt?

Figure 4: *The problem in the context of the strategy "Writing Explicit Proposals*

The student's solution is shown in Figure 5.

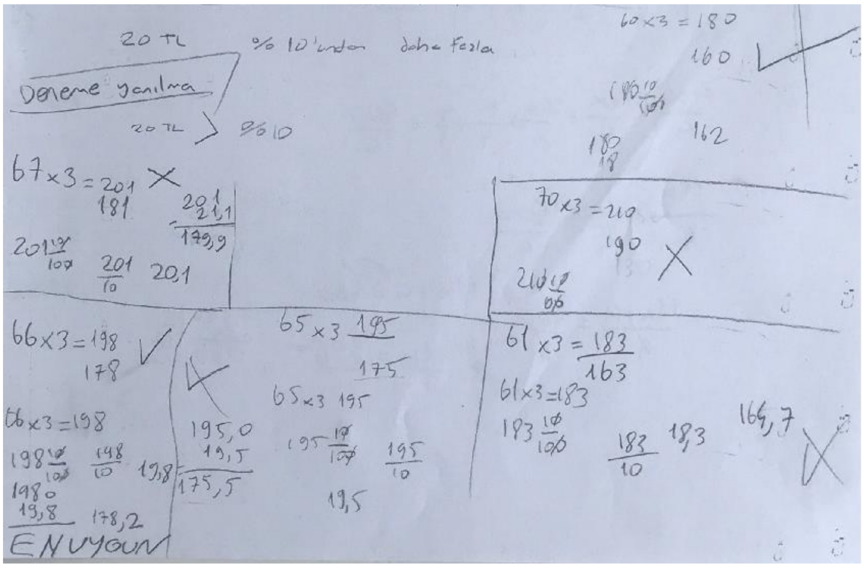


Figure 5: *S24's Solution to the Problem of the Explicit Proposition Writing Strategy*

Looking at the student's solution, S24 solved the problem not with an inequality, but with the previously learned strategy of intelligent guessing and testing. In solving the problem, he tried the numbers 60, 70, 61, 65, 66, and 67 and arrived at the correct result. The employment process involves using different representations in solving the problem and changing those representations when necessary (OECD, 2019). In this context, solving the Explicit Proposition Writing Strategy problem using a different strategy shows that the student performed the employment process.

3.2. Examining Students' Mathematical Literacy Levels Before and After the Implementation of Problem Solving Strategies

To answer the second research problem, "Is there a significant mean difference between the level of students' mathematical literacy before and after the application in which problem-solving strategies were taught?", the pre- and post-study mathematical literacy tests were used. In the following, the statistical results of the data obtained in the mathematical literacy tests and the main points obtained by examining the student solutions in the tests are presented.

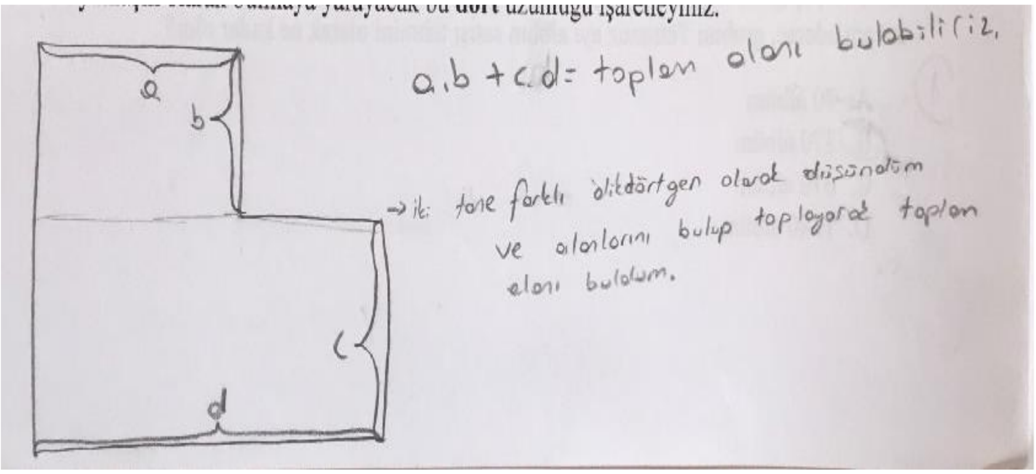
The results of the "paired samples t-test" comparing the students' mathematics literacy level before and after the application are presented in Table 4.

Table 4: *T-Test Results for the mean of Pre and Post-Test*

Mathematical Literacy Test	N	$\bar{X}$	ss	t	p
Pre-test	32	8,00	3,592	11,023	0,000
Post-test	32	14,75	3,162		

In Table 4, the pre-test mean score for mathematical ability is 8.00, while the post-test mean score is 14.75. A statistically significant mean difference was found in the "paired samples t-test" performed to compare the group's pretest and posttest scores; $t(31) = 11.023$, $p< 0.001$. This mean difference was in favor of the posttest. The obtained eta- squared statistic (0.797) shows a large effect size (Pallant, 2011). The results of the mathematical literacy test show that the application in which problem solving strategies are taught increases the mathematical literacy level of students.

The steps of formulating, applying, interpreting, and evaluating are evident when examining student solutions. When we look at examples that students were unable to solve in the pre-test but successfully solved in the post-test, they were unable to express themselves beforehand, that they tried to achieve the result with very short solutions, and that they did not operate the mathematical processes. Figure 6 below shows an example of a student's post-test essay related to the changes in these processes.



I thought of them as two different rectangles and found the total area by finding and adding their areas.

Figure 6: *The Solution of S3 in the Post-Test for the Question of "Apartment Purchase"*

S3 demonstrated this question by marking only the side lengths in the pretest. However, in the final test, S3 supported the solution with a formula. Examining the student's solution, S3 uses spatial reasoning to point out the minimum number of side lengths necessary to determine the area on the given plan. The fact that S3 supports his solution with a formula and applies spatial reasoning shows that he understands the processes of formulating and applying.

3.3. Examining Students' Views on the Process of Learning Problem Solving Strategies

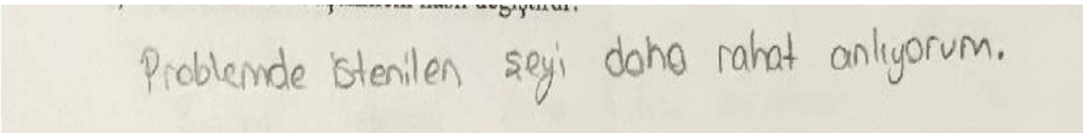
To answer the third research problem, "What are the opinions of eighth grade middle school students about the process of learning problem solving strategies?", a written opinion form was used at the end of the study and given to 15 randomly selected students from the group of students participating in the study. The form asked students questions that asked them to evaluate the process and express what they gained from the process. The data obtained from the interview form was subjected to content analysis. As a result of this analysis, themes were created from the codes obtained from the student opinions. The themes that emerged and the codes that were assigned to those themes are shown in Table 5.

Table 5: *Themes Obtained from Student Opinions and Codes Under Themes*

Theme	Codes Under Themes	Frequency (f)	Percent (%)
Mathematical Thinking	Simple to Comprehend	11	73
	Ability to create equations	4	27
	Finding the solution easily	4	27
	Solving the problem on a regular basis	2	13

Problem Solving	Choosing the most effective strategy	6	40
	Step by step progress	8	53
	Rapid development of solutions	5	33
	Using different strategies	9	60
Transfer to Daily Life	Employing in shopping	6	40
Interdisciplinary Relationship	Usage in verbal logic	11	73
	Usage in science class	5	33
Using in Exams	Facilitating inquiries in daily life questions	4	27
	Quick solution generation	5	33

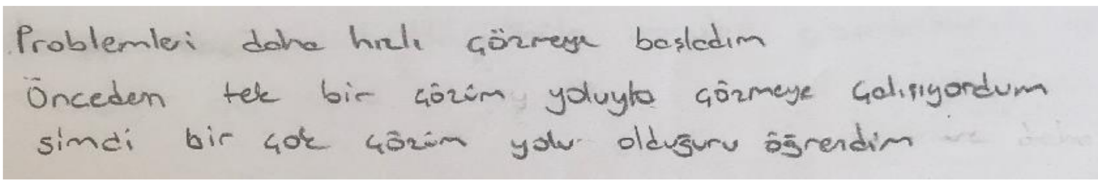
Examination of Table 5 shows that five themes were identified. Under the theme of mathematical thinking, there are four codes: easy to understand, ability to create equations, easy to find the solution, and solving the problem regularly. Under the theme of problem solving, there are codes related to choosing the most effective strategy, progressing step by step, developing solutions quickly, and using different strategies. When examining the first code, "easy to understand", it can be seen that the students indicated that they understood the problem better and were better able to identify what was desired in the task. An example of students' expressions of easy to understand code is shown in Figure 7.



I understand what is required in the problem more easily

Figure 7: *S24's Expression of the simple to comprehend code*

The fourth code under the problem-solving process theme is the code for using different strategies. An example of student utterances that belong to the code for using different strategies is shown in Figure 8.



I started to solve problems faster. Before I was trying to solve them with a single solution and now I learned that there are many solutions.

Figure 8: *Statement of S23 under the Using Different Strategies code*

When examining this code, it can be seen that students indicated that they now use different solutions (strategies) when solving the questions because they have learned different strategies.

4. DISCUSSION and RESULT

The purpose of the study is to investigate the mathematical competence of 8th grade students in the context of teaching problem solving strategies. The results and suggestions for further research based on these findings were provided, and the results were examined in the context of the literature. The results of the first research question were discussed in the context of mathematical processes. There are many different ways to learn about performance in mathematical literacy. One of these ways is to control the mathematical processes in solving mathematical literacy problems (OECD, 2010).

In the formulation process, there is a transfer from the real world to the mathematical world. The variables must be identified in order to calculate the outcomes of a problem situation that arises in the real world, represent them mathematically, and come up with a solution (Çepni, 2019; Heerkens & Van Winden, 2021). In the study, students showed improvements in mathematizing problems during the formulation process. Students were able to determine the variables in the problem situation more easily, they were able to represent them with symbols and formulas, and they found it easier to identify the mathematical relationships, i.e. they made progress in expressing the situation expressed in the problem mathematically. Teaching "mathematization" ought to be the main objective of mathematics education (Martin, 2007). Based on the strategies learned, students found it easy to transfer the problems to the world of mathematics by simplifying them. In addition, students organized the expressions in the task according to mathematical concepts and to solve the task more regularly. Taking all these developments into account, it is concluded that the study enabled significant improvements in the formulation process of mathematical processes.

The application process includes performing mathematical operations and using tools, creating graphs and charts, and drawing mathematical conclusions (Altun, 2014). In the first weeks after reading the task, students worked directly with the given numbers, and in the following weeks, they first determined the variables and then selected the appropriate strategy and started solving the task. Students indicated that this way they were able to solve the tasks more easily and that they were able to detach themselves from the amount of information contained in the task and solve the tasks regularly. During the study, the students could only apply the strategies they had learnt in the first few weeks to get mathematical results. However, in the subsequent weeks, they were allowed to propose new strategies, which helped to validate the solutions they had discovered. Moreover, although students stated that there can be only one solution to a problem at the beginning of the study, they explained that as the study progresses, problems can be solved by more than one solution. Several weeks hence then, students solved the given problems with different strategies, thus supporting the solution and gaining a different perspective with their studies. After expressing the problem situation mathematically, the students were able to perform arithmetic calculations, equation solving, i.e. necessary mathematical operations in a regular way. The students indicated that before the study they performed arbitrary and mixed operations, but with the study they performed more regular and meaningful operations. However, students achieved the solution by using appropriate tables and graphs when needed during the problem solving process. The strategies of creating tables and making systematic lists, which are part of the problem solving strategies, have an impact on this problem. In the study, students were able to use the language of mathematics more effectively and defend the result by explaining the problem solving and results using the learned strategies. This serves one of the general goals of the mathematics education curriculum which is "students should be able to use mathematical terminology and language correctly to explain and communicate their mathematical thoughts in a logical way" (MoNE, 2018, p. 9). At the same time, it is consistent with the finding that non-routine problem study enhances students' creative thinking, relational pattern search, and proof skills (Altun, 2014). These findings suggest that instructing students in strategies for problem solving advances them through the application process. Application process skills are among the fundamental skills that should be acquired by students in mathematics education (MoNE, 2013). Guer and Hanguel (2015) concluded that solving a problem can be done with more than one strategy and not only in one way. Okur (2008) concluded that including problems that require different problem-solving strategies in mathematics lessons, having students try new strategies, and taking risks increase problem-solving success. Taşpınar (2011) found that the number of problem-solving strategies students used in the pretest was quite limited, but after being taught problem-solving strategies, students used more than one strategy when solving problems. The ability of students to suggest more than one strategy to solve a problem and to use more than one problem-solving strategy for a problem supports these studies.

The interpretation and evaluation process consists of finding a solution to the real problem that results from the interpretation of the data obtained by solving the problems (Çepni, 2019). In the study, there are improvements in the interpretation and evaluation process, in linking mathematics to real life and transferring it to real life, and in evaluating the relevance of a problem situation in the context of the real world. Research shows that using real-world situations in the classroom improves student performance in mathematics and fosters the development of mathematically literate people (Gellert, 2004; Karakaş & Ezentaş, 2021). These investigations are supported by the acquisition of real-world problems resolved throughout the application process, which raises the degree of mathematical literacy. The NCTM (1989) defined mathematical literacy as the capacity to investigate, rationalize, forecast, and use mathematical techniques to solve problems successfully. These claims overlap, according to an analysis of student advancement conducted as a result of the study.

Mathematical competency is demonstrated to increase when problem solving strategies are taught within the framework of mathematical procedures. Therefore, there is a positive relationship between problem-solving strategies and mathematical processes and that the use of problem-solving strategies in the classroom contributes to the acquisition of mathematical process skills. Temel and Altun (2020) stated that mathematical process skills are used in the selection and implementation of problem solving strategies, so there is a relationship between problem solving strategies and mathematical process skills. These statements support the finding of the study. At the same time, there are studies that conclude that students and pre-service teachers who are trained problem solving strategies can use these strategies effectively and increase their problem solving success (Arsuk and Sezgin Memnun, 2020; Başdamar. 2019; Cai, 2003; Gallagher and Lisi 1994; Sulak, 2005; Yıldız, 2008). Moreover, this study shows that students learn strategies while using problem-solving strategies and can apply them in case of problems they encounter. In this regard, it also supports the findings of some studies in which problem-solving strategies can be taught (Altun & Arslan, 2006; Arslan, 2002; Dönmez, 2002; Durmaz & Altun, 2014; Taşpınar, 2011; Yazgan & Bintaş 2005).

In examining the second research question, the results of the pretest and posttest were used, which were administered to the students before and after the application. The performance levels of the group in terms of mathematical literacy were compared pretest and posttest, and a statistically significant difference was found in favor of the posttest. These differences were also confirmed qualitatively. The results suggest that teaching problem-solving strategies increases the level of students' mathematical skills. Upon reviewing the literature, little research was discovered that looked at the application of mathematical literacy and direct problem-solving strategies in conjunction. According to particular research, mathematical competency was raised by mathematical modeling training, mathematics application courses, and mathematics instruction (Demirci, 2018; Kaiser & Willander, 2005; Korkmaz, 2016; Mayan, 2019; Şaban, 2019; Taşkın, Ezentaş, & Altun, 2018). Although the implementation processes differ, the findings from the aforementioned studies have similarities with the results of this study.

In the study of the third research question, the written opinion form was collected at the end of the application, and common themes were formed from the data obtained from these forms. From the students' responses on the theme of mathematical reasoning, as a result of this application, students understood the desired situations in the problems more easily, were able to set up equations more easily, arrived at the solution more easily, and solved the problems more regularly. On the topic of problem solving process, students indicated that through the application, they were able to choose the best strategy, take a step-by-step approach when solving problems, solve problems faster, and use different strategies to solve a problem. After explaining problem solving strategies to eighth grade students, Taşpınar (2011) concluded that students can develop different solving strategies for each problem. The result of that study is consistent with the result of the present study. Regarding the topic of application to daily life, students stated that they generally found it easier to buy and employ the problem-solving techniques they had acquired. Gellert (2004) stated that examples from daily life are important for the acquisition of mathematical competencies, Baştürk Şahin and Altun (2019) stated that they care about the students' ability to apply the information learned in class to their daily lives and that the information transferred to daily life is both more remarkable and more lasting. Based on these statements, transferring the knowledge and skills learned in class to daily life demonstrates the importance of studying. Based on the students' responses on the topic of interdisciplinary relationships, it was concluded that the problem-solving strategies learned were used in Turkish classes, especially in the verbal logic questions and in science classes. Based on the students' responses on the topic of "use in exams", this study provided relief in the questions they formulated as new generation questions in the exams they took and helped them solve the problems faster. In line with this result, Kükey (2013) found that there is a positive and high-level relationship between mathematical literacy and mathematics achievement.

Positive feedback was found after reviewing the student expressions from the written opinion forms at the conclusion of the study. Students expressed satisfaction with the study, received a different viewpoint on problem solving, and thought of other strategies when faced with a challenge. Therefore, the study elicited positive results from students in terms of problem-solving processes. The results of this investigation and the investigations that have been done intersect, according to a review of the literature. Yazgan and Bintaş (2005) concluded that teaching problem solving strategies has positive effects on problem solving success and performance. Arslan (2002) found that teaching problem-solving strategies increased students' problem-solving success and gave them a positive attitude toward problem-solving. Thus, these results show that students' mathematical literacy develops as they are taught problem-solving strategies. Researchers are recommended to consider the following ideas for future studies based on the results of this study.

• This study was limited to eighth grade students. It is possible to work with other grade levels and with a larger number of students. In addition, the investigation was conducted on a group. By working with experimental and control groups, it is possible to compare the change in the level of mathematical skills in these groups.

• In the study, two problems of 10 strategies were solved. In future studies, more problems of each strategy can be included and the duration of the investigation can be kept longer.

• The developmental process of students who have been taught problem-solving strategies can be studied longitudinally. In this way, the durability of development and the developmental process can be better tracked.

• Since teaching problem-solving strategies improves the level of mathematical skills, problem-solving strategies can be included in mathematics curricula.

• In order to teach problem-solving strategies to students and work in this area in the classroom, the teacher must first have certain competencies in this area. For this purpose, studies with teachers on problem-solving strategies can be conducted first.

• In order to bring up mathematically literate people and carry out mathematical education correctly, teachers can be trained in this area at the faculties of education.

• In this study, the effects of teaching problem solving strategies on students' solving processes, success in mathematical education, and students' views on the work process were investigated. In addition to the study, the study can be supported by measuring students' attitudes and perceptions.

Researchers’ Contribution Rate

All authors contributed equally rate to the research.

Statements of Publication Ethics

Ethical permission for the research was approved by Niğde Ömer Halisdemir University Ethics Committee. The ethics committee document number is 05/10/2020-44101/09.

Disclosure Statement

No potential conflict of interest was reported by the authors.

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