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ANALYZING MIDDLE SCHOOL STUDENTS' FIGURAL PATTERN GENERATING STRATEGIES CONSIDERING A QUADRATIC NUMBER PATTERN

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ABSTRACT

In this study figural patterns were produced by participants following 2, 6, 12, 20, 30,.. non-linear number pattern and strategies that were used by students while creating figural patterns were investigated. In total, 474 middle school eigth grade students attended the study. Data were collected from a pattern task, in which participants were asked to generate figural patterns based on 2, 6, 12, 20, 30,.. non-linear number pattern and also asked to write the process they followed. The obtained data were analysed at two levels. The results of the study indicated that participants produced different figural patterns using different forms and used different pattern generating strategies such as counting, determining a figure and counting, recursive, drawing, explicit and chunking the numbers strategies. Some participants were found to have difficulties to generate figural patterns

Key Words: Number Pattern, Figural Pattern, Quadratic Pattern, Pattern Generating Strategy

ORTAOKUL ÖĞRENCİLERİNİN LİNEER OLMAYAN SAYI ÖRÜNTÜSÜNE BAĞLI OLARAK ŞEKİL ÖRÜNTÜSÜ OLUŞTURMA STRATEJİLERİNİN ANALİZİ

ÖZ

Bu çalışmada, ortaokul 8. sınıf öğrencilerinin lineer olmayan sayı örüntüsünü şekil örüntüsüne çevirirken ne tür şekil örüntüsü oluşturdukları ve bu şekil örüntülerini oluştururken hangi stratejileri kullandıkları belirlenmeye çalışılmıştır. Çalışmaya toplam 474 ortaokul öğrencisi katılmış olup, çalışmanın verileri 2, 6, 12, 20, 30,…lineer olmayan sayı örüntüsünün yer aldığı örüntü görevi ile toplanmıştır. Görevde öğrencilere bu örüntüyü şekil örüntüsüne çevirmeleri ve bunu yaparken de nasıl bir yol izlediklerini yazılı olarak belirtmeleri istenmiştir. Araştırmadan elde edilen verilerin analizinde hem semantik, hem de betimsel analiz teknikleri kullanılmıştır. Araştırmadan elde edilen sonuçlara bakıldığında, katılımcıların sayı örüntüsüne dayalı olarak farklı yapıları kullanarak farklı şekil örüntüleri oluşturdukları ve bunu yaparken de farklı stratejiler kullandıkları belirlenmiştir. Bu stratejiler; sayma, şekil belirleme ve sayma, yinelemeli, çizme, belirgin ve sayıları parçalama stratejileridir. Ayrıca çalışmada bazı katılımcıların sorunlar yaşadıkları da saptanmıştır.

Anahtar Kelimeler: Sayı Örüntüsü, Şekil Örüntüsü, Lineer Olmayan Örüntü, Örüntü Oluşturma Stratejisi

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1. INTRODUCTION

Pattern is one of the important mathematical concepts involved in mathematics education. It is asserted that pattern is the heart and soul of mathematics (Zazkis & Liljedahl, 2002). There are many benefits of the pattern activities. Pattern activities contribute to the development of functional thinking (Souviney, 1994; Van De Walle, 2004; Warren & Cooper, 2006), are found helpful to make generalisations (Cathcart et al., 2003; Rivera & Becker, 2007; Threlfall, 1999), in seeing relationships and making connections (Cathcart et al., 2003), problem solving (Bassarear, 2008; Cathcart et al., 2003, Reys et al., 1998) counting (Bassarear, 2008; Frobisher & Threlfall, 1999), using number systems (Frobisher & Threlfall, 1999), and in arithmetical operations (Bassarear, 2008; Frobisher & Threlfall, 1999). Fox (2005) asserted that studying patterns is closely connected to mathematical content areas such as numbers, geometry, measurement, and data. Souviney (1994) indicated that patterns can be based on geometric attributes (shapes, regions, and angle), measurement attributes (colour, texture, length, weight, volume, number), relational attributes (proportion, sequence, functions), and affective attributes (values, likes, dislikes, familiarity, heritage, culture). Patterns can also be seen in spoken and written words, musical forms and video images, ornamental designs, natural geometry, traffic, and the objects that we create (Reys et al., 1998). Patterns are found in arithmetic and geometric sequences problem, as well as in various real-life situations.

There are many definitions of pattern in the literature. Souviney (1994), for example, defined a pattern as a systematic configuration of geometric figures, sounds, symbols, and actions. McGarvey (2012) defined a pattern as an act of perceiving or imposing structural regularities on physical, behavioural, visual, or symbolic phenomena. Mulligan & Mitchelmore (2009) asserted that a mathematical pattern may be described as any predictable regularity, usually involving numerical, spatial or logical relationships. Previous studies have shown many kinds of pattern types referring to the type of representation system within which the terms are expressed as numerical or figural/geometric forms. Stacey (1989) classified patterns as linear and quadratic patterns according to their nth terms expressed as $an+b$ and an^2+bn+c, respectively, while Orton and Orton (1999) indicated that for number patterns if constant differences first appear in the first difference row, then the rule is linear $an+b$, if it is the second differences where constants appear, the rule is quadratic an^2+bn+c. For instance, 2, 4, 6, 8, 10, 12, 14, …numbers represent a linear pattern, 1, 3, 6, 10, 15, ... numbers having a systematic configuration of numbers and 1, 1, 1, 1, ... constants being second differences represent a non-linear pattern. Smith (1997) indicated that patterns can be numerical (involving numbers) or non-numerical (involving shapes, sounds, or other attributes such as colour and position), while Lin et al. (2004) classified geometric patterns as linear and quadratic patterns. Bishop (2000, p. 110) indicated that "A number pattern is a sequence of numbers in which there is a well-defined rule for calculating each number from the previous numbers or from its position in the sequence. In a geometric number pattern, the numbers relate to a sequence of geometrical figures in which each figure is derived from the previous figure by some well-defined procedure."

In some studies, patterns are classified as repeating or growing (Cathcart et al., 2003; Mulligan & Mitchelmore, 2009; Reys et al., 1998; Van De Walle, 2004; Warren & Cooper, 2006). For example, Zazkis and Liljedahl (2002) classified patterns as numerical patterns, pictorial/geometric patterns, patterns in computational procedures, linear and quadratic patterns, and repeating patterns. Repeating patterns have a recognisable repeating cycle of elements, referred to as the 'unit of repeat' (Zazkis & Liljedahl, 2002). This kind of pattern can have one attribute such as the colour, size, shape, or orientation of objects (Threlfall, 1999). The followings are examples of repeating patterns: alphabetic letters such as A-B-A-B-A-B, and actions such as stand, sit, stand, sit, stand, sit, stand, sit (Warren & Cooper, 2006). Growing patterns change over time (Cathcart et al., 2003) and involve a progression from one step to the other step. Moreover, those patterns are called sequences (Van De Walle, 2004). Growing patterns may be linear such as Y B B Y B B B B Y B B B B B B (in this example, only B's are growing) (Reys et al., 1998) and 2, 4, 6, ... (Cathcart et al., 2003) or quadratic such as 1, 4, 9, 16, ... number pattern. Repeating patterns are particularly important, since they recur in measurement (which involves the iteration of identical spatial units) and multiplication (which involves the iteration of identical numerical units) (Mulligan & Mitchelmore, 2009). In Turkey, patterns have been incorporated into the Mathematics Curriculum in numbers and geometry learning areas since 2005. In the current New Middle School Mathematics Curriculum (5th grade to 8th grade), students are expected to extend number and figural patterns and find algebraic rules of patterns (Ministry of National Education, 2013).

In the literature, studies conducted with middle school students especially are focused on students' generalization of patterns (e.g., Akkan, 2013; Amit & Neria, 2008; Bishop, 2002; Jurdak & Mouhayar, 2014; Lannin, 2005; Rivera & Becker, 2008; Walkowiak, 2014) and representation types used in pattern activities (e.g. Akkan, 2013; Amit & Neria, 2008; Steele, 2008; Walkowiak, 2014). Bishop (2002), for example, focused on the capacity of students to generalize relationships and the strategies they use to answer questions about linear geometric number patterns. Lannin (2005) indicated that students used different strategies to generalize figural patterns. Rivera and Becker (2008) investigated tthe content and structure of middle school students' generalization of figural patterns. Amit and Neria (2008) focused on the generalization methods used by talented pre-algebra students in solving linear and non-linear pattern problems and the students showed mental flexibility, shifting smoothly between pictorial, verbal and numerical representations and abandoning additive solution approaches in favor of more effective multiplicative strategies. Steele (2008) demonstrated that students used different external representations such as drawing diagrams, creating tables, writing verbal generalizations, and constructing generalized symbolic expressions for pictorial growth and change problems. Akkan (2013) aimed to compare and determine 6th-8th graders'abilities, strategies and representations including students from 6th to 8th grades when dealing with problems related to linear and quadratic patterns. According to the results of their study when grade level increased, students' abilities of generalizing patterns improved in a positive way at all levels and the variety in pattern generalization strategies changed in all types of patterns. Jurdak and Mouhayar (2014) investigated students' (from grades 4 to 11) level of reasoning in pattern generalization tasks. Results showed that student level of reasoning exhibited an increasing trend across clusters of grade levels. Type of task (immediate, near, far) and function type (linear, non-linear) seem to mediate the development of level of reasoning across grade level. Walkowiak (2014) asked students to describe, extend, and generalize pictorial growth patterns and found that their generalizations included informal notation, descriptive words, and formal notation. Kılıç (2016) investigated middle school students' strategies while creating figural patterns based on 3, 5, 7, 9, 11,… linear number pattern. The results of the study indicated that participants produced different figural patterns and used different figural pattern generating strategies. The nature of the strategies that participants used was both visual and non-visual. Most of the participants preferred counting strategy while creating figural patterns. Moreover, in that study in addition to counting strategy, determining a figure and counting, recursive, drawing, explicit and chunking the numbers strategies were used and some of the participants had issues.

In summary, while a number of studies have investigated middle school students' generalization strategies and representation types of patterns, generating figural pattern strategies based on number patterns are not studied at all. However, given that generating figural patterns based on number patterns are important to develop students' pattern knowledge, algebraic thinking and problem solving and to understand their skills on geometry and spatial relationships, the present study aims to fill this gap in the literature by examining students' generating figural pattern strategies. Moreover, in current Turkish Middle School Mathematics Curriculum creating figural patterns based on non-linear number patterns are not given attention. As a result of the study some suggestions will be presented to the curriculum developers because of the mathematics curriculum scope regarding pattern activities. In this study, the following research questions were addressed:

- What kinds of figural patterns were produced by middle school students based on 2, 6, 12, 20, 30, ... non-linear number pattern?
- What kinds of figural pattern generating strategies were used by middle school students while creating figural patterns based on 2, 6, 12, 20, 30,... non-linear number pattern?

2. METHOD

2.1. Research design

In this study, case study was conducted. The data were collected by using a number pattern task. That task consisting of two questions was carried out by all participants. The task allowed the participants to produce different figural patterns and used different strategies related to a growing number pattern.

2.2. Participants

In this study 474 eight grade students attended from four different schools. The schools were selected from similar socio-economic neighborhoods. Of these participants, 254 were girls and 220 were boys and their ages ranged between 14 and 15 years. The reason for selection eight grades was that they studied how to recognise, identify, describe, and extend repeating and growing number and figural patterns and finding algebraic rules of patterns. It was assumed that all participants already had basic knowledge of patterns since they have started to learn about it from the first grade.

2.3. Data collection

Participants were asked to generate figural patterns individually considering 2, 6, 12, 20, 30, ... number pattern. Kılıç (2016) indicated that to determine middle school students' figural pattern generating strategies different pattern types such as quadratic number patterns should be asked to students. The pattern task is described as below:

Question 1: Could you generate a figural pattern for the first five stages regarding the 2, 6, 12, 20, 30, ... number pattern?

Question 2: Could you write how you created a figural pattern for the 2, 6, 12, 20, 30, ... number pattern?

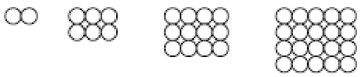
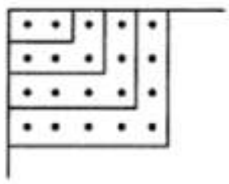
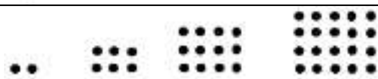
Those questions demand knowledge on pattern structure that grows according to the number of objects in each stage; thus, participants had the potential to generate several interesting figural patterns and could reflect their figural pattern generating strategies. To confirm the suitability of this task, the opinions of a mathematics educator were considered. The educator indicated that the task used in this study was suitable for middle school students. This task was chosen because it allowed us to assess how participants generated figural patterns based on number patterns and what kinds of strategies they use. After getting opinions from a mathematics educator, a pilot study was performed with a similar group consisting of 60 eight grade students. The pilot study aimed to assess the task's feasibility in terms of language, difficulty, time, and the number of questions. At the end of the pilot study, two questions were decided to be asked to the participants and 30 minutes were allowed for answering the task.

2.4. Data analysis

The data obtained from the study were analysed at two levels: (i) semantic analysis and (ii) descriptive analysis. In the semantic analysis, firstly the figural patterns or situations generated by participants and strategies they used were analysed in accordance with the non-linear number pattern. Secondly, the produced figural patterns or situations and the strategies that participants used while creating figural patterns were first listed and classified according to their semantic structures. Generated patterns or situations, strategies were then coded and issues were noted. After the semantic analysis of the generated patterns and generating figural pattern strategies, their frequencies were calculated. The descriptive analysis then provided descriptive information to offer an overall picture of the figural patterns generated and strategies used by participants.

Table 1 provide the figural pattern types related to the 2,6,12,20,30,.. number pattern in the literature. The generated figural patterns were analysed by using the Table 1 developed by the researcher based on previous studies of figural patterns related to the 2, 6, 12, 20, 30, ... number pattern (Orton et al., 1999; Van de Walle, 2004; Vogel, 2005). The figural representations of the 2, 6, 12, 20, 30, ... number pattern were coded as types (e.g., Type 1, Type 2, etc.) separately to analyse the data obtained from the study effectively. In the results section, besides the pattern types from the literature, some new pattern types created by the participants used for the linear number pattern are presented.

Table 1.
Examples of Figural Representations of 2, 6, 12, 20, 30, ... Number Pattern from the Literature

Types	Structures	Characteristics
Type 1	 (Van de Walle, 2004).	That pattern has been represented by circles reminding a rectangle.
Type 2	(Orton et al., 1999). 	In overlapping rectangles pattern, the first stage contains 2, second 6, third 12 dots, etc.
Type 3	 (Vogel, 2005).	In Type 3 sequence of rectangular numbers is presented as an array of dot patterns.

As seen in Table 1, the representation types of 2, 6, 12, 20, 30,.. seems to differ in terms of arrangement. Geometric figures such as lines, dots and circles are used to represent those types of patterns. Although the same geometric shapes are used for Type 2 and Type 3, the construction of those patterns differs because of the arrangement.

The figural pattern generating strategies that participants used were analysed based on categories was found in previous studies and categorized by Barbosa and Vale (2015). Although, those categories were for generalization strategies applied to visual patterns in this current study same categories were considered for generating figural patterns. The strategies are explained as below (Barbosa &Vale,2015, pp.59-60):

- *Counting -Visual;* drawing a figure and counting its elements.
- *Whole-object (no adjustment)-Non-visual;* considering a term of the sequence as unit and using multiples of that unit.
- *Whole-object w/visual adjustment-Visual;* considering a term of the sequence as unit and using multiples of that unit. A final adjustment is made based on the context of the problem.
- *Whole-object w/ numeric adjustment-Non-visual;* considering a term of the sequence as unit and using multiples of that unit. A final adjustment is made based on numeric properties.
- *Recursive-Non-visual;* extending the sequence using the common difference, building on previous terms (numeric relations).
- *Recursive-Visual;* extending the sequence using the common difference, building on previous terms (features of the figures).

- *Difference rate (no adjustment)-Non-visual;* using the common difference as a multiplying factor without proceeding to a final adjustment.

- *Difference rate w/adjustment-Visual;* using the common difference as a multiplying factor and proceeding to an adjustment of the result.

- *Explicit-Non-visual;* discovering a numerical rule that allows the immediate calculation of any output value given the correspondent input value.

- *Explicit-Visual;* discovering a rule, based on the context of the problem, that allows the immediate calculation of any output value given the correspondent input value.

- *Guess and Check- Non-visual;* guessing a rule by trying multiple input values to check its validity.

2.5. Validity and Reliability

In the study the researcher asked for the opinion and assessment of one colleague who was blinded to the data and unbiased regarding the code list and research findings. In order to examine inter-rater reliability and increase the reliability of the results, another colleague who has a mathematics education background independently classified the generated figural patterns. The formula of Miles and Huberman (1994) was used to calculate inter-rater reliability and this was determined to be 95% for question 1 and %94 for question 2. The pilot study also contributed to the validity and reliability of the number pattern task.

2.6. Ethical issues

In that study all the participants were informed about the aim of the research and asked to attend at the beginning of the study. They were informed that the results of the study will be used for academic studies only, and they will not have an effect on their grades.

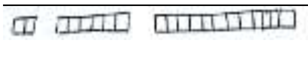
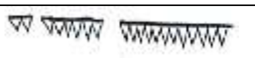
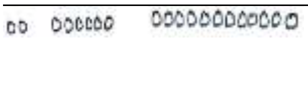
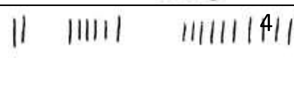
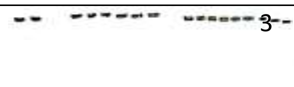
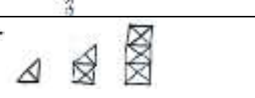
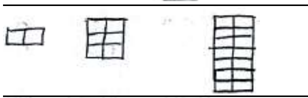
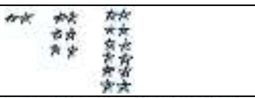
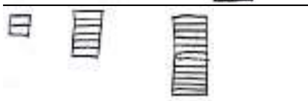
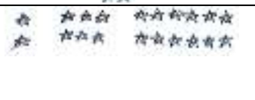
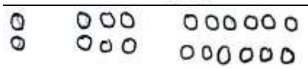
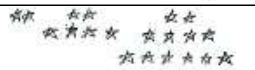
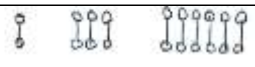
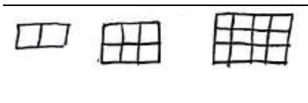
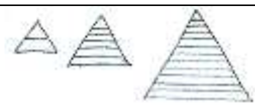
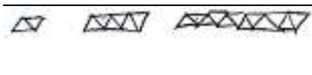
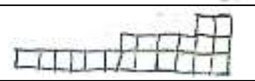
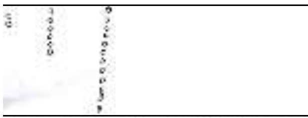
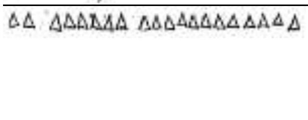
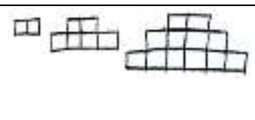
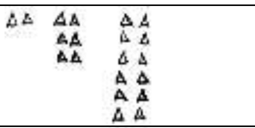
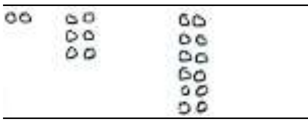
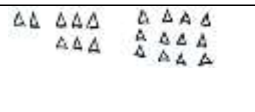
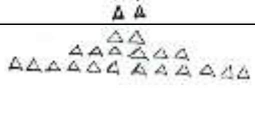
3. RESULTS

The figural patterns generated by middle school students based on a non-linear number pattern and strategies that they used while creating figural patterns are presented in the tables below. The produced figural patterns by participants for the 2, 6, 12, 20, 30, ... number pattern and their frequencies are displayed in Table 2 and the strategies they used while creating figural patterns and their frequencies are displayed in Table 3.

3.1. Structures of figural patterns generated by participants for 2, 6, 12, 20, 30, ….number pattern

Participants created 33 different figural patterns using geometrical forms such as squares, circles, lines, triangles, rectangles and stars and non-geometrical forms such as hearts for the 2, 6, 12, 20, 30, ... number pattern. Among those patterns, only one was created by using both circles and lines; the others were created by using only one type of shape.

Table 2.
Responses Related to the 2, 6, 12, 20, 30, ...Number Pattern

Patterns	f	Patterns	f	Patterns	f
	45		6		1
	30		4		1
	24		3		1
	16		3		1
	13		2		1
	13		2		1
	13		2		1
	12		2		1
	11		1		1
	10		1		1
	7		1		1
Total				232	

Fourty five of the participants used squares and 30 used circles to create a figural pattern. 24 participants used lines, 16 used squares and 13 used rectangles to create a figural pattern. 13 participants used circles or squares to create their figural patterns. 12 participants used triangles to create figural patterns in an incoherent format and another 10 participants used triangles to ensure that the shapes did not touch each other. 7 and other 11 participants used circles to create a vertical figural pattern and 6 created a Type 1 figural pattern.

Vertical lines were used by 4 participants and horizontal lines by 3 participants. 3 participants created a figural pattern as the bases of the triangles attached to one another.

2 participants used dots to create a figural pattern and 2 preferred stars. 2 participants used triangles or squares to create a star model figural pattern. The remaining 14 figural patterns were created by using lines, squares, triangles, hearts, stars and rectangles. Some participants found it difficult to produce figural patterns. Total, 232 participants out of 474 participants could produce correct figural patterns considering 2, 6, 12, 20, 30,.. number pattern.

3.2. Strategies that participants used while creating figural patterns following 2,6,12,20,30,.. number pattern

Different figural pattern generating strategies were identified falling into two categories, visual and non-visual based on students' exhibited strategies.

Table 3.
Strategies That Used by Participants While Creating Figural Patterns

Nature	Strategy	Description	f
Visual	Counting	Drawing figures in accordance with the numbers	116
	Drawing	Drawing figures correctly just mentioning drawing figures	36
	Determining a figure andcounting	Determining a figure and then drawing a figure in accordance with the numbers	18
	Recursive	Drawing a first figure and then creating the figural pattern using the common difference, building on previous terms (numeric relations)	7
Non-visual	Explicit	Discovering the number rule that pattern is increasing not constantly in every step and realizing that first differences of pattern are 4, 6, 8, 10,..	26
	Chunking the numbers	2, 2+4=6, 2+4+6=12, 2+4+6+8=20	1
Total			204
Issues	No explanation	Creating a figural pattern correctly but not giving any explanations	28
	Not creating any correct figural patterns		242
Total			270

In this section, six figural pattern generating strategies are identified, based on the explanations of the participants related to generating figural patterns. While four of the of these strategies like counting, determining a figure+counting, recursive, and drawing are visual strategies, two are such as explicit and chunking the numbers strategies are in non-visual nature. Considering each strategy, two categories emerged, being either visual or non-visual strategies. In non-visual strategies participants focused on the numbers and then created figural patterns. Barbosa and Vale (2015) indicated considering a term of the sequence as unit and using multiples of that unit, extending the sequence using the common difference, building on previous terms (numeric relations), using the common difference as a multiplying factor, discovering a numerical rule and guessing a rule represented non-visual strategies.

In some strategies the figures play an essential role in the discovery of the invariant and, in others, the work is developed in a numeric context.Among those strategies counting strategy was applied by most of the participants, that resorted to a drawing of the terms of the number pattern asked, in order to create a figural pattern. Hundred and sixteen participants preferred that strategy to create a figural pattern. One participant indicated that he drew rectangulars as many as the numbers in number pattern. The figural pattern that participants created and the strategy that he used is given in Figure 1.

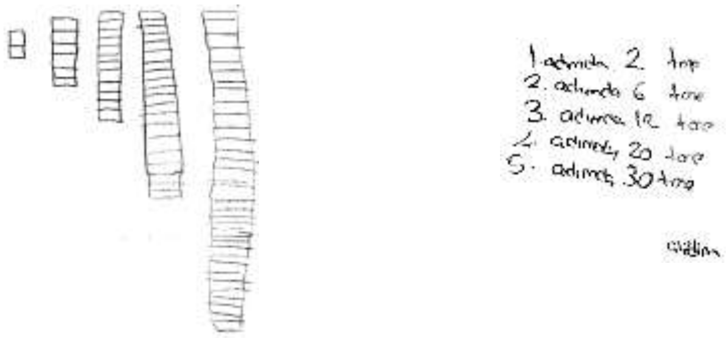


Figure 1

Thirty-six participants preferred drawing strategy being in a visual nature. They did not explain anything about how they drew the figures; they just mentioned that they drew the figures (see in Figure 2).

Figure 2

Eighteen participants used the strategy of determining a figureand counting strategy while producing a figural pattern. Participants mentioned that they determined a figure and then drew the figures in accordance with the numbers in number pattern. Figural pattern that participant produced and the strategy he used is given in Figure 3 as below;

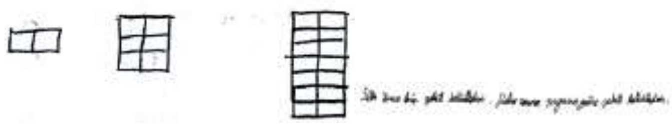


Figure 3

Recursive strategy was used by seven participants considering figures and differences between two consecutive terms (numeric relations). They first drew a figure and then created the figural patterns using the differences, building on previous terms (see in Figure 4).

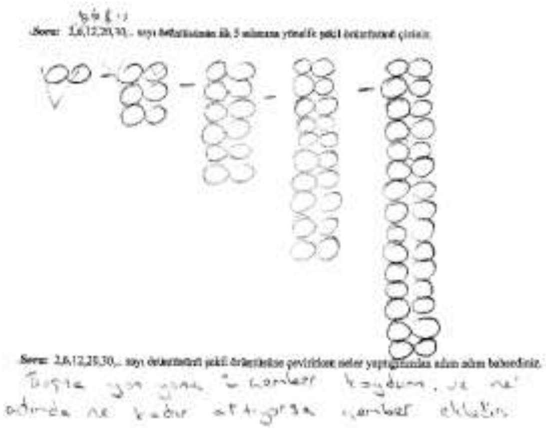


Figure 4

Explicit strategy being in non-visual nature was used by 26 participants. They mentioned that they discovered the number rule that pattern is increasing 4, 6, 8, 10, ... and they created figural patterns based on those differences (see in Figure 5).

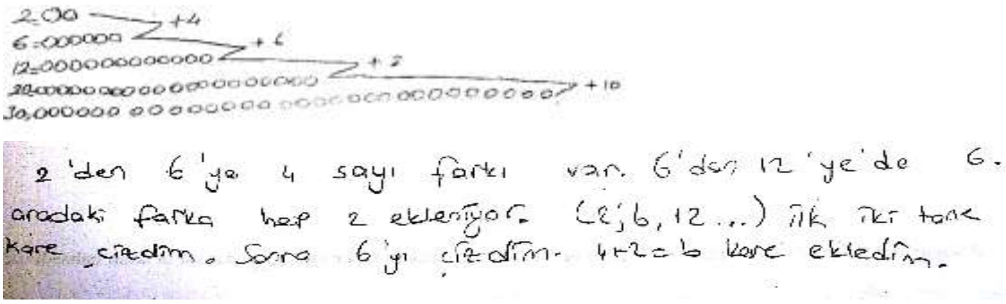


Figure 5

One participant preferred chunking the numbers strategy. He chunked the 2, 6, 12, 20, 30,.. number pattern as 2, 2+4=6, 2+4+6=12, 2+4+6+8=20,… and then created the figural pattern following those numbers. That example is given in Figure 6.

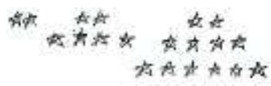


Figure 6

Although 28 participants created figural patterns correctly considering 2, 6, 12, 20, 30,.. non-linear number pattern, they did not explain how they created. Furthermore, 242 participants did not create any figural patterns representing 2,6,12,20,30,.. number pattern.

4. DISCUSSION AND CONCLUSION

Patterns were found to have many contributions to students. It contributes to the development of algebraic and functional thinking, problem solving, counting skills and arithmetical operations. In the literature, many pattern studies have focused on students' generalization of figural or number patterns (Akkan, 2013; Amit & Neria, 2008; Bishop, 2002; Jurdak & Mouhayar, 2014; Lannin, 2005; Rivera & Becker, 2008; Steele, 2008, Walkowiak, 2014), whereas studies of generating figural patterns following number patterns are not found so much in the literature. In recent studies Kılıç (2016) investigated middle school students' figural pattern generating strategies considering a lineer number pattern. Therefore, in this study it is aimed to fill this gap by examining the ability of middle school students to generate figural patterns and their strategies for generating figural pattern based on a non-linear number pattern. It is believed that producing figural patterns following number patterns have potential to contribute to students' problem solving skills and algebraic thinking. Moreover, generating figural patterns are an effective way to measure students' knowledge of geometry and spatial relationships related to pattern.

The data obtained from the current study indicated that different types of figural patterns were generated by participants according to 2, 6, 12, 20, 30,... non-linear number pattern. Most of the students generated different types of figural patterns that followed the given a non-linear number pattern and they used different figures (either geometric or non-geometric forms). Some of the figural patterns created by participants used geometric shapes like squares, circles, triangles and rectangles predominantly during the generating activity.A few participants preferred stars and hearts. Moreover, some new figural patterns representing the 2, 6, 12, 20, 30, ... non-linear number pattern emerged in the present study. In future studies, students should be motivated to produce different figural patterns based on the number patterns. Then, different figural patterns regarding the number patterns in the literature should be taught to extend their pattern generating performance.

Four of the strategies are primarily visual in nature and two of them are non-visual. Overall, students' strategies appeared to be predominantly visual. Those generating figural pattern strategies are counting, determining a figure andcounting, recursive, drawing strategies, which are in visual nature and explicit and chunking the numbers strategies that are in non-visual nature. In the current study, similar strategies emerged in the study of Kılıç (2016) and generalization strategies applied to figural patterns as categorized by Barbosa and Vale (2015) study. Among figural pattern generating strategies counting strategy was the most common strategy used by participants. Counting strategy seems easy to perform when compared to the other types of generating figural pattern strategies that emerged in the study. Recursive-visual and chunking the number-non visual strategies were used less. As indicated in the study of Barbosa and Vale (2015) recursive-visual strategy requires to extend the sequence building on previous terms (features of the figures), for that reason participants may not notice that connection between the previous and the next sequence. Many possible factors may have influenced middle school students' strategy selection; it can be highlighted that the type of the pattern task may have affected their selection. Task-based interviews should be conducted to investigate the other factors that could have influenced the participants' generating figural strategies.

In this study, participants encountered issues while producing figural patterns based on the number pattern. Similar results have also emerged in the study of Kılıç (2016). Some participants who failed to generate figural patterns correctly for 2, 6, 12, 20, 30,.. number pattern and describe the strategy tended to start out with creating figures in accordance with the numbers related to number pattern and chose the correct figures but they were not aware that figural patterns are formed of objects that convey positions in a structural relationship and resemble each other in some way. They did not recognize the regularities in the patterns produced. Because pattern activities do not include generating figural patterns following number pattern activities in the mathematics curriculum in Turkey, this lack of familiarity with the topic might have affected their pattern-generating performance. Furthermore, Fox (2005) found that patterns are closely connected to mathematical content areas such as geometry, which can be affected by participant performance**.** In addition, generating figural patterns are related to participants' ability to visualize pictures and shapes and thus this may have affected their performance. Moreover, other variables related to learners' performance such as learning style, spatial reasoning and mathematical thinking style might have influenced their performance. For that reason, to understand participants' performance more in depth, correlation studies and mixed method research may be conducted.

In the current study, the figural pattern generating strategies of middle school students requiring creating figural patterns based on a non-linear number pattern was investigated. Considering that generating patterns based on number patterns may improve students' algebraic thinking and problem solving in the future, repeating number patterns being other types of patterns can be given students and they can be asked to create figural patterns. Therefore, students should be encouraged to use different strategies while creating figural patterns based on number patterns. Most participants tried to use geometric shapes to create figural patterns. For that reason, future correlation studies should assess attitudes towards geometry and their pattern generating performance as well as the cognitive obstacles associated with creating figural patterns based on number patterns.

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GENİŞ ÖZET

1. Giriş

Örüntü konusu, aritmetik ve cebir konuları ile ilgili olması bakımından matematikte yer alan önemli konulardan biridir. Örüntünün önemli olmasının pek çok nedeni bulunmaktadır. Örüntü etkinlikleri sayesinde öğrenciler, cebirsel etkinliklere dahil olur ve cebiri öğrenirler, fonsiyonel düşünmeleri gelişir ve genelleme yapabilirler (Souviney, 1994; Van De Walle, 2004; Warren ve Cooper, 2006). Örüntülerle çalışmanın sayılar, geometri, ölçme ve veri konuları ile yakından ilişkili olduğu (Fox, 2005) ve aynı zamanda uzamsal yetenekler ile de ilgili olduğu belirtilmektedir. Örüntüler, aritmetik ve geometrik dizi problemlerinde ve günlük yaşam durumlarında, sözlü ve yazılı kelimelerde, müzik yapılarında ve video görsellerinde, doğal geometride, trafikte ve bizlerin oluşturduğu diğer nesnelerde karşımıza çıkmaktadırlar (Reys, Suydam, Lindquist ve Smith, 1998). Örüntü, geometrik şekiller, sesler, semboller ve eylemlerin sistematik bir yapılandırması olarak tanımlanırken Souviney (1994), matematiksel bir örüntünün desen genellikle, sayısal mekânsal ya da mantıksal ilişkileri içeren herhangi öngörülebilir düzen olarak tarif edilebileceği belirtilmektedir (Mulligan ve Mitchelmore, 2009).

Örüntü çeşitlerinin farklı biçimlerde sınıflandırıldığı görülmektedir. Stacey (1989) örüntüyü n.inci terimin ifade ediliş biçimine göre $an+b$ lineer ve an^2+bn+c kuadratik olarak sınıflamıştır. Smith(1997) ise örüntüyü sayısal ve sayısal olmayan biçiminde ikiye ayırmıştır. Bazı çalımalarda ise örüntüler tekrarlayan yada genişleyen biçiminde sınıflandırılmıştır(Cathcart, Pothier, Vance ve Bezuk, 2003;Mulligan ve Mitchelmore, 2009; Reys ve diğerleri, 1998; Van De Walle, 2004; Warren ve Cooper, 2006). Tekrarlayan örüntüye, A-B-A-B-A-B, yada örüntüleri (Warren ve Cooper, 2006) ve genişleyen örüntüye de Y B B Y B B B B Y B B B B B B (bu örnekte B genişliyor) (Reys ve diğerleri, 1998) ve 2,4,6,... (Cathcart ve diğerleri, 2003) örüntüleri örnek olarak verilebilinir. Ortaokul matematik dersi öğretim programında örüntüler konusuna 2005 yılında sayılar ve geometri öğrenme alanlarında yer verilmeye başlanmıştır. 2013 yılında yenilenen ortaokul matematik dersi öğretim programında ise örüntüler konusu ile ilgili olarak öğrencilerden, sayı ve şekil örüntülerini genişletme ve örüntülerin cebirsel kurallarını bulmaları beklenmektedir (MEB, 2013).

Örüntülerle ilgili yapılan çalışmaların çoğunun özellikle ortaokul öğrencilerinin sayı ve şekil örüntülerini genellemeleri üzerine olduğu görülmektedir. Ancak öğrencilerin örüntü bilgilerini ortaya çıkarmada önemli bir rolü olan ve öğrencilerin hem cebirsel düşünme, hem de problem çözme becerilerini geliştirmeye potansiyel katkısı olan sayı örüntülerine dayalı olarak şekil örüntüsü oluşturma çalışmalarına yer verilmediği göze çarpmaktadır. Bunu yanı sıra, sayı örüntülerine dayalı olarak şekil örüntüsü oluşturma etkinlikleri, bize öğrencilerin geometri ve uzamsal ilişkiler konusundaki yeterlilikleri ortaya çıkarma konusunda da yardımcı olacağı düşünülmektedir. Bu nedenle bu çalışma ile alandaki bu boşluk doldurulmak istenmektedir. Türkiye'de örüntüler konusuna matematik dersi öğretim programlarında 2005 yılında sayılar ve geometri öğrenme alanlarında yer verilmeye başlanmıştır. 2013 yılında yenilenen ortaokul matematik dersi öğretim programında, örüntülerle ilgili olarak öğrencilerden, sayı ve şekil örüntülerini genişletme ve örüntülerin cebirsel kurallarını bulmaları beklenilmektedir. Bu çalışmada aşağıdaki sorulara yanıtlar aranmıştır;

1- Öğrenciler lineer olmayan sayı örüntüsüne dayalı olarak ne tür şekil örüntüleri oluşturmuşlardır?

2- Öğrenciler lineer olmayan sayı örüntüsüne dayalı olarak şekil örüntüleri oluştururken hangi stratejileri kullanmışlardır?

2. Yöntem

Bu çalışmada durum çalışması benimsenmiş olup, çalışma 2014-2015 eğitim öğretim yılında gerçekleştirilmiştir. Araştırmaya, ortaokul 8. sınıfa devam eden toplam 474 öğrenci katılmıştır. Öğrencilerin yaşları 14 ve 15 yaşları arasında değişmektedir. Çalışmaya 8. sınıf öğrencilerinin seçilmesinin nedeni, bu öğrencilerin öğrenim yaşantıları boyunca tekrarlayan ve genişleyen sayı ve şekil örüntülerini tanıma, açıklama, devam ettirme ve örüntülerin cebirsel kurallarını bulma gibi çalışmalara yer vermiş olmalarıdır. Çalışmada öğrencilere lineer olmayan sayı örüntüsü olan 2, 6, 12, 20, 30, ...sayı örüntüsü verilmiş olup, bu örüntüye dayalı olarak şekil örüntüleri oluşturmaları ve bu örüntüyü şekil örüntüsüne çevirirken nasıl bir yol izlediklerini yazılı olarak belirtmeleri istenmiştir. Araştırmadan elde edilen veriler, ilk olarak semantik olarak analiz edilmiş ve daha sonra da bu veriler betimsel olarak analiz edilmiştir. Semantik analiz yapılırken, ilk önce ortaokul öğrencileri tarafından oluşturulan şekil örüntüleri analiz edilmiştir. İkinci olarak, katılımcılar tarafından oluşturulan şekil örüntüleri ve sayı örüntüsüne bağlı olarak şekil örüntüsü oluştururken kullanılan stratejiler listelenmiş ve semantik olarak sınıflandırılmış ve kodlanmıştır. Daha sonra oluşturulan şekil örüntülerinin ve stratejilerin frekans hesaplaması yapılmıştır.

2,6,12,20,30,..sayı örüntüsüne yönelik yer alan şekil örüntülerine Tablo 1'de yer verilmiştir. Tablo 1 oluşturulurken daha önceden bu konu üzerine yapılmış çalışmalardan yararlanılmıştır (Orton ve diğerleri, 1999; Van de Walle, 2004; Vogel, 2005). 2, 6, 12, 20, 30,..,.. sayı örüntüsüne yönelik şekilller Tip 1, Tip 2, … biçiminde kodlanmıştır. Katılımcıların sayı örüntüsünü şekil örüntüsüne çevirirken başvurdukları stratejilerin belirlenmesinde, Barbosa ve Vale (2015) tarafından daha önceden örüntü genelleme stratejilerinin ele alındığı araştırma sonuçlarına dayalı olarak ortaya koydukları stratejiler benimsenmiştir.

Bu çalışmada, bütün katılımcılara veri toplamadan önce araştırmanın genel amacından bahsedilmiş olup, çalışmanın başında böyle bir araştırmada yer alıp yer almak istemedikleri sorulmuştur. Çalışmaya katılımın gönüllülük esasına dayalı olduğu belirtilmiştir. Ayrıca çalışmadan elde edilecek olan sonuçların akademik çalışmalarda kullanılacağı ve bu araştırmada yapacakları çalışmaların herhangi bir biçimde notlandırılmayacağından da bahsedilmiştir.

3. Tartışma, sonuç, öneriler

Araştırmadan elde edilen bulgulara bakıldığında, öğrencilerin 2, 6, 12, 20, 30, ... lineer olmayan sayı örüntüsüne yönelik olarak dikdörtgen, kare, daire üçgen, çizgi ve yıldız gibi geometrik modelleri ve kalp gibi geometrik olmayan modelleri kullanarak farklı şekil örüntüsü oluşturdukları belirlenmiştir. Çalışmada 2, 6, 12, 20, 30, ... lineer olmayan sayı örüntüsünü şekil örüntüsüne çevirirken, görsel ve görsel olmayan yapıda stratejiler kullanmışlardır. Bu stratejiler sayma, şekil belirleme ve sayma, yinelemeli, çizme, belirgin ve sayıları parçalama stratejilerini kullandıkları belirlenmiştir. Çalışmada 242 öğrencinin şekil örüntüsünü oluşturamadıkları ve 28'inin ise oluşturdukları şekil örüntüsünü nasıl oluşturdukları konusunda açıklamada bulunamadıkları belirlenmiştir.

Araştırmadan elde edilen sonuçlara bakıldığında, öğrencilerin kendilerine verilen lineer olmayan sayı örüntüsüne yönelik farklı şekil örüntüleri oluşturdukları ve bunu yaparken de farklı stratejiler benimsedikleri saptanmıştır. 2, 6, 12, 20, 30, ... sayı örüntüsüne yönelik olarak 33 tane farklı şekil örüntüsü oluşturmuş olup, bu şekil örüntülerini oluştururken de altı farklı stratejinin kullanıldığı belirlenmiştir. Bu stratejiler arasında yer alan sayma stratejisi öğrenciler tarafından ağırlıklı olarak kullanılırken, yinelemeli ve belirgin stratejilerin ise pek kullanılmadığı görülmüştür. Öğrenciler lineer olmayan sayı örüntüsüne yönelik şekil örüntüleri oluşturulurken ağırlıklı olarak geometrik modelleri kullandıkları görülmektedir. Bu durum öğrencilerin geometriye olan yatkınlıkları ile açıklanabilir. Bunun yanı sıra, sayı örüntüsüne bağlı olarak şekil örüntüsü oluşturmada öğrencilerin bir takım sorunlar yaşadıkları da belirlenmiştir. Bu durum onların daha önceden böyle bir çalışma ile karşılaşmamış olmaları ile de açıklanabilir. Ileride yapılacak olan bu çalışmaya benzer çalışmalarda, öğrencilere farklı türden örneğin, lineer olan ya da tekrarlayan sayı örüntüleri verilip bu örüntüleri temsil edecek biçimde şekil örüntüleri oluşturmaları istenebilir ve bu şekil örüntülerini oluştururken ne tür stratejiler kullandıkları ve nasıl bir süreç yaşadıkları ele alınabilir. Ayrıca öğrencilerin sayı örüntüsünü şekil örüntüsüne çeviririken kullandıkları şekiller ve başvurdukları stratejileri seçme nedenlerini ortaya çıkarıcı çalışmalar yapılabilir.

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