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High School Students' Achievement of Solving Quadratic Equations

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Abstract: The purpose of the present study was to investigate 10th grade students` achievement of solving quadratic equations, examine their tendency of using different solution ways such as completing square, factorization and quadratic formula and determine their errors. Fifty 10th grade students of a high school in northern part of Turkey constituted the sample of the study. In data collection, 9 open-ended questions related to quadratic equations were conducted to all students in the sample. Data were analyzed through content analysis and descriptive statistics such as percentage and frequency were also presented. Besides, the students' errors were showed through direct quotations from their answers. The findings revealed that most of the students had difficulty in solving quadratic equations and made various errors. The students' errors in solving quadratic equations were due to their weaknesses in mastering topics such as algebra, fractions, integers, the rules of quadratic equations` solution methods, calculation and algebraic simplification. In addition to this, the findings showed that students were usually in tendency to use factorization in order to solve quadratic equations.

Key Words: Algebra, quadratic equations, solution ways, high school students

Lise Öğrencilerinin İkinci Dereceden Denklemleri Çözme Başarıları

Özet: Bu çalışmanın amacı onuncu sınıf öğrencilerinin ikinci dereceden denklemleri çözme başarılarını incelemek, tam kareye tamamlama, çarpanlara ayırma, ikinci dereceden denklem formülü olmak üzere farklı çözüm yollarını kullanma eğilimlerini araştırmak ve öğrenciler tarafından yapılan hataları belirlemektir. Çalışmanın örneklemini Türkiye'nin kuzeyinde yer alan bir lisede öğrenim gören 50 onuncu sınıf öğrencisi oluşturmaktadır. Veriler ikinci dereceden denklemlere yönelik 9 sorunun örneklemdeki bütün öğrencilere uygulanmasıyla elde edilmiştir. İçerik analizi yapılarak veriler analiz edilmiş, yüzde ve frekans değerleri verilerek tanımlayıcı istatistiksel bilgiler sunulmuştur. Bunun yanı sıra, öğrencilerin cevaplarından alıntılar yapılarak hata türleri gösterilmiştir. Bulgular, öğrencilerin ikinci dereceden denklemleri çözmekte zorlandıklarını ve çözerken çeşitli hatalar yaptıklarını ortaya koymaktadır. Öğrencilerin ikinci dereceden denklemleri çözerken yaptıkları hatalar cebirsel ifadeler, kesirler, tam sayılar, ikinci dereceden denklem çözme kuralları, hesaplama ve cebirsel sadeleştirme gibi konuları tam öğrenememesine dayalı zayıflıklarından kaynaklanmaktadır. Ayrıca, öğrencilerin genellikle ikinci dereceden denklemleri çarpanlara ayırma yöntemiyle çözme eğiliminde oldukları sonucuna ulaşılmıştır.

Anahtar Kelimeler: Cebir, ikinci dereceden denklemler, çözüm yolları, lise öğrencileri

1. INTRODUCTION

Mathematical learning includes thinking, communicating and expressing mathematically (Hacısalihoglu, Mirasyedioğlu, & Akpınar, 2004). One of the mathematical expression types is algebra. Algebra is an important part of mathematics and a subject to be understood (Chazan, 1996). Although there are many different perspectives related to the definition of algebra, the common points of the definitions are solving equations, finding unknowns and using symbols. In general terms, algebra is considered as generalized arithmetic, a study of procedures for solving problems, the study of relationships among quantities and the study of structures (Usiskin, 1988). It is situated in every domain of life. Besides, learning of algebra is needed in terms of students. Due to the fact that algebra establishes a connection between subdomains of mathematics and the other branches of science in terms of theoretical and conceptual learning through its abstract thinking structure, algebra teaching is an important issue (Erbaş, Çetinkaya, & Ersoy, 2009).

Students have difficulty in understanding algebra and they usually tend to engage in algebra without realizing the real purposes and thinking context (Chazan, 1996; Sfard, 1991; Kieran, 1992). According to Kaput (1999) students do not like algebra since it is taught based on the rules and independent from the other domains in mathematics. In general, students feel the necessity of algebra in terms of accomplishing their objectives such as passing exam, entering a good high school or university (Usiskin, 1988). However, indeed they do not believe the importance of it due to their thought about uselessness of algebra in daily life. Thus, learning algebra must be made worthwhile (MacGregor, 2004). On the other hand, students learn arithmetic thinking and work with numbers at the beginning. As time goes by it replaces algebraic thinking since the development of algebra proceed from concrete to abstract (Katz, 1997). Therefore, this transition is not easy for students and algebra is considered abstract and meaningless. All these difficulties present the greatness of negative situation encountered in algebra teaching (Dede, Yalın, & Argün, 2002).

One of the essential and challenging subjects in algebra learning domain of secondary mathematics curriculum is quadratic equations (Kotsopoulos 2007; Vaiyavutjamai, Ellerton, & Clements, 2005). The acquirements related to quadratic equations entail “finding the roots of equations and solution set, showing the relationships between the roots of equation and coefficients, and posing the equation whose the roots are given” (MoNE, 2005). There are three methods that are normally taught in schools for solving quadratic equations (i) factorization, (ii) completing the square and (iii) the quadratic formula. Many researches related to mathematics education show that students have difficulty in quadratic equations and they comprehend quadratic equations as to make a calculation, focus on only symbols in order to solve equation and they are not aware of the essential concepts in quadratic equations (Didiş, Baş, & Erbaş, 2011; Lima, 2008; Makonye & Nhlanhla, 2014; Sarwadi & Shahrill; 2014; Vaiyavutjamai & Clements, 2006). Therefore, there are many errors performed by the students particularly in solving quadratic equations (Zakaria & Maat, 2010). Determining how students think, control their understanding of mathematical concepts and learn what kind of errors they made is important to remove the deficiencies.

Teaching of mathematics in schools is generally focused on the rules and formulas to show students how to get correct answers rather than teaching basic mathematical concepts and the logic behind the procedures (Sarwadi & Shahrill, 2014). The gap between new and previous knowledge in mathematics causes the various errors and misconceptions. According to Ashlock (2002) these errors and misconceptions develop due to overgeneralization of the rules and procedures while trying to give the meaning to new knowledge. Mathematics is seen as a union of rules by students (Tirosh, 1990). Focusing on students’ errors may help to learn how students understand mathematical concepts and how think mathematically. Hence, teachers may realize possible reasons of these errors and develop strategies to help the students (Sarwadi & Shahrill, 2014). The subject of quadratic equations is important since it enables to establish connection between various mathematical topics such as linear equations, functions and polynomials (Sağlam & Alacacı, 2012). Despite the importance of quadratic equations in secondary mathematics curriculum, the studies on teaching and learning quadratic equations in literature are limited (Didiş et al., 2011; Kieran, 2007; Vaiyavutjamai & Clements, 2006).

The purpose of this study was to investigate 10th grade students' achievement of solving quadratic equation, examine their tendency of using different solution ways such as completing the square, factorization and quadratic formula and determine the students' errors in solving quadratic equations. Thus, the research questions were as follows:

1. To what extent did the students have achievement in solving quadratic equations?
2. How was the students' tendency of using different solution types in solving quadratic equations?
3. What type of errors did the students make while solving quadratic equations?

Students can make lots of errors without being aware of them and if no precautions are taken to correct these mistakes, they may learn mathematics wrongly and make more errors (Pickthorne, 1983). Teachers should know students' mathematical thinking to shape their teaching approaches and correct students' wrong thinking ways to prevent errors (Sorensen, 2003). This study may help teachers to understand how students think mathematically and what kind of errors they make while solving quadratic equations and take precautions against them. It is also aimed to contribute to national literature regarding quadratic equations since a few studies were carried out in Turkey (Didiş et al., 2011; Didiş & Erbaş, 2015).

2. METHOD

In this research, case study was used in order to gain an in-depth understanding of participants' achievement, solution ways and errors regarding quadratic equations. In this technique, it is aimed to analyze and understand the related situation in detailed (Stake, 1994).

Sample of the Study

The sample of the study consisted of fifty students who were enrolled in 10th grade in a high school in northern part of Turkey. These students had the knowledge of quadratic equations in order to solve the questions asked in this study since the subject matter had been taught in this year. Due to the fact that the students were expected to solve these kinds of equations based on their previous knowledge, purposive sampling was preferred. In order to determine descriptive values and the students' errors and thinking ways, it was decided that fifty students were sufficient.

Data Collection

In accordance with the aim of the study, 9 open-ended questions related to solving quadratic equations were prepared to collect data. In the process of preparation of the questions, quadratic equations solution types (factorization, quadratic formula and completing the square) were analyzed in mathematics books and relevant questions were determined. The test was formed in accordance with two experts' opinions. 1-4 questions can be solved in three ways, 5th question has no real root and 6-9 questions cannot be solved by factorization but can be solved with quadratic formula or completing the square method. Besides, the quadratic equations were given in both standard and non-standard forms. The students were requested to find the roots and solution set of each quadratic equation. The students were given 45 minutes to solve the quadratic equations in the test.

Analysis of Data

The written documents including the solutions of students for the questions evaluated through content analysis. The main purpose in this analysis was to find the concepts and relationships to explain the meaning of obtained data. Therefore, the data should be conceptualized, organized judiciously and the themes should be determined (Yıldırım & Şimşek, 2004). The categorization such as true, false, no attempt and incomplete was used to examine the solutions of students and determine their achievement in solving quadratic equations in a way explained in Table 1. Also, the values of percentage and frequency of coding were calculated to facilitate interpretation and obtain a general view about students' achievement. It was aimed to provide descriptive analysis with this process.

Table 1.
Categories for the Solutions

Categories	**Description**
True	Correct solution and correct result
False	Incorrect solution and incorrect result
No attempt	No solution
Incomplete	Correct process of solution but no correct answer

In addition, qualitative data analysis was conducted to determine the nature of the students' errors regarding quadratic equations and the reasons behind these errors that influenced students' achievement negatively. The coding categories of error types that students made while solving quadratic equations were formed by the researcher through analysis of students' written solutions. The errors in the solutions were showed by quoting from the students' answers. In order to provide trustworthiness of the qualitative part of the present study, the investigator triangulation and member checking strategies were used.

3. FINDINGS

In this section, descriptive statistics regarding students' solutions and solution types in quadratic equations and also the examples of errors that students made were presented.

Descriptive Scores of the Students

The values of percentage and frequency of solution categories of students were calculated. The results of the analysis were given in Table 2.

Table 2.
The Values of Percentage and Frequency of Solutions

	True		**False**		**No Attempt**		**Incomplete**	
Questions	n	%	n	%	n	%	n	%
1	43	**86**	3	**6**	-	-	4	8
2	23	46	17	34	8	16	2	**4**
3	30	60	14	28	1	2	5	10
4	23	46	13	26	5	10	9	18
5	16	32	22	44	3	6	9	18

6	3	6	20	40	13	26	14	**28**
7	2	4	25	50	12	24	11	22
8	3	6	21	42	14	**28**	12	24
9	1	**2**	32	**64**	12	24	5	10

As it is seen in Table 2, the percentage of the students' true answers was between %86 and %2. The distinction between two values was obviously high. The students were relatively more successful in solving 1-4 questions that were factorable than in solving 6-9 questions that were non-factorable. It is seen that the percentages of true solutions in 6-9 questions were close to each other and students were notably unsuccessful in solving these type questions. The percentage of the students' false answers was between %6 and %64. It was noticed that the students obviously made incorrect solutions. Their achievement of solving quadratic equations were low. The percentage of no attempt varied at %0 and %28 whereas the rate of their incomplete solutions changed between %4 and %28. The findings revealed that students especially had difficulty in solving 6-9 questions. Thus, they gave incorrect answers, left empty or presented incomplete solutions for these quadratic equations. When false and incomplete answers of students were examined, these solutions showed that students

(i) factorized quadratic equations incorrectly
(ii) found incorrect or irrelevant roots by factorization although quadratic equation was not suitable for this method
(iii) did not complete solutions or think that there was no real root if they did not obtain roots by factorization at first
(iv) did not complete solutions after they understood there were real roots by finding discriminant
(v) made irrelevant operations.

The categories consisted of the solutions that included factorization, quadratic formula, completing the square method, both factorization and quadratic formula, the solutions that had no attempt or did not include these solution ways. Even though the solutions which were made by factorization, quadratic formula or completing the square method were not correct or finished, it was accepted in factorization, quadratic formula or completing the square category since it reflected the tendency of solution way of students in solving quadratic equations.

Table 3.
The Values of Percentage and Frequency of Solution Types in Solving Quadratic Equations

Questions	**Factorization**		**Quadratic Formula**		**Complete the Square**		**Factorization and Quadratic Formula**		**No Attempt**		**Other**	
	n	**%**	**n**	**%**	**n**	**%**	**n**	**%**	**n**	**%**	**n**	**%**
1	44	**88**	1	**2**	-	-	-	-	-	-	5	10
2	30	60	2	4	-	-	1	2	8	16	9	**18**
3	40	80	3	6	-	-	2	4	1	2	4	8
4	39	78	1	**2**	-	-	2	4	5	10	3	**6**
5	26	52	8	16	-	-	8	**16**	3	6	5	10
6	18	36	9	18	-	-	4	8	13	26	6	12
7	20	40	6	12	-	-	4	8	12	24	8	16
8	16	**32**	12	**24**	-	-	2	4	14	**28**	6	12
9	24	48	4	8	1	2	-	-	12	24	9	**18**

As it is seen in Table 3, commonly used solution way was factorization for every question. The percentage of using factorization was between %88 and %32. The percentage of preferring factorization was higher in 1-4 questions that could be solved by factorization than in 6-9 questions that could not be solved. When the use of quadratic formula was compared with completing the square, it was noticed that the students were more likely to apply quadratic formula. The percentage of using this method was between %2 and %24. It was observed that the rate of using quadratic formula to solve quadratic equations was not high as factorization as. Surprisingly, the percentage of applying the solution way of completing the square was %2. It was obviously low. It was seen that only one student preferred to use this method. The table shows that some students tried to solve quadratic equations by factorization at first and if they did not get the result in this way, they applied quadratic formula. The percentage of following this way was between %0 and %16. The reason of that the percentage of using the factorization and quadratic formula was more than the other strategies in 5. question might result from having no real root. Because of not finding the roots by factorization at first, students might think to control whether there were real roots or not through quadratic formula. It is seen that the percentage of no attempt was between %0 and %28 and it was higher in 6-9 questions that were non-factorable than in 1-4 questions that were factorable. Besides, the table shows that some students did not apply any of quadratic equations solution ways and tried to solve equations through different ways. The percentage of using other ways was between %6 and %18 for the questions. However, the examination of these solutions shows that students did not obtain correct answers using these ways.

According to the findings, %57 of the students, predominantly preferred using factorization. %10 of the students applied quadratic formula to solve the quadratic equations. %5 of the students first used factorization and then applied quadratic formula in some quadratic equations since they did not find the answer through first method. Moreover, only one of the students used completing the square method for solving only one question. In addition, %15 of the students generally had no attempt in the questions. It shows that they were not good at solving quadratic equations that could not be solved by factorization directly. Besides, %13 of the students was in tendency to solve quadratic equations differently from factorization, quadratic formula or completing the square method.

The Examples of Errors in the Solutions of Quadratic Equations

According to the findings, there were different types of errors made by the students related to signs, coefficients, rules and operations. The analysis of written solutions also showed that the students had misconceptions. In this part, the errors were illustrated by quoting from the students' solutions under the categories of errors.

Sign Errors

This kind of error was made in the process of making operations in order to reach the solution and find the possible values of the unknown. The students transferred the numbers or variables from one side of the equality to the other side wrongly. They failed to get the correct value of the unknown by forgetting to change sign of the term when transposing it. Sign errors might be made because of the students' lack of attention on the operations or having lack of knowledge about integers, linear equation and basic mathematical properties.

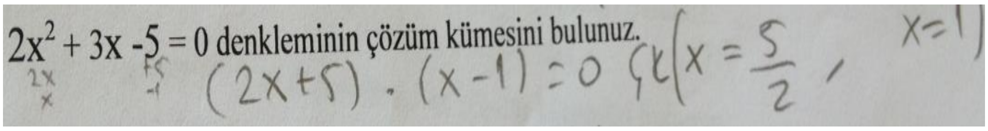

Figure 1. The representation of one student's solution for question 3

In this solution, although the quadratic equation was factorized correctly, the sign of one root was found as wrong. The student put the incorrect sign in front of the number of 5 while equating both parts of the equation by leaving x alone on one side, namely, s/he transferred the number to the right side of equation without changing the sign of it. Therefore, s/he found 5/2 instead of -5/2. When this solution process was examined carefully, it was realized that the student continued solution carelessly.

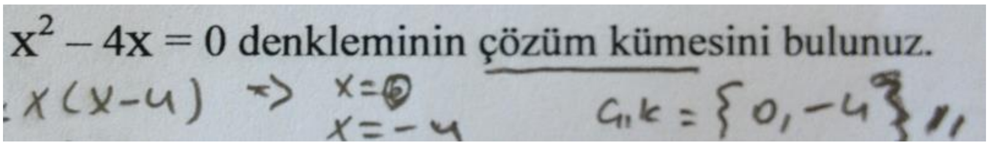

Figure 2. The representation of one student's solution for question 1

In a similar way, the student found the factors of the quadratic equation correctly but s/he equalized x to -4 directly without paying attention to change the sing. -4 was transferred to the right side but it kept the same sign in the left side. Thus, s/he obtained incorrect root.

Calculation Errors

While solving the quadratic equations, the students made errors related to operations such as addition, subtraction multiplication and division or the order of the operations. They applied the essential mathematical procedures or rules incorrectly. For example, they calculated the roots or discriminant incorrectly. Hence, they failed to produce correct solutions and get the correct answers. Errors in the students' calculations might result from carelessness or lack of previous knowledge.

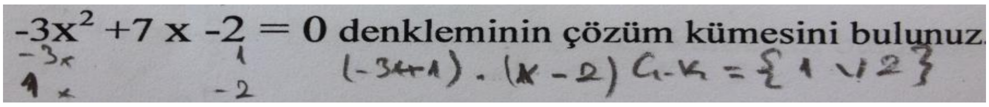

Figure 3. The representation of one student's solution for question 4

In this example, it is seen that the student factorized the quadratic equation correctly. However, while calculating the roots, s/he made mistake. In the linear equation of -3x+1= 0, s/he actually had to subtract 1 from both sides of equation. After obtaining -3x = -1, s/he had to multiply both sides by -1/3. Later, the students needed to find one of the root as 1/3 but s/he found 1 by solving the equation wrongly.

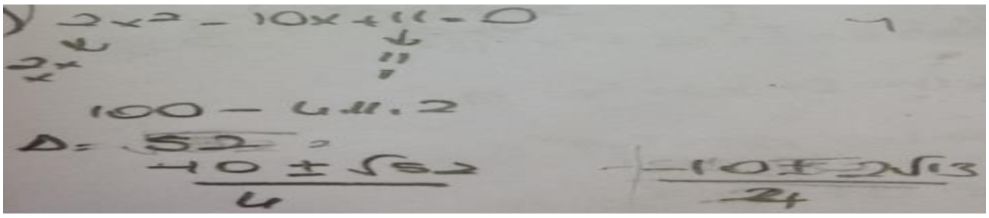

Figure 4. The representation of one student's solution for question 7

Here, the student tried to find the roots by factorization at first. Then, s/he understood that s/he would not be able to find in this way. S/he applied the quadratic formula and endeavored to calculate discriminant and the roots. However, s/he made calculation error and found discriminant as 52 instead of 12. Therefore, although s/he applied the correct quadratic formula, the roots obtained were wrong because of finding the value of delta faultily.

Simplification Errors

This type of error was particularly observed while the students were using the quadratic formula. The students tried to simplify numerator and denominator. However, they neglected one of the numbers in addition on the fraction bar. They should have separated the expression of $\frac{-b\pm\sqrt{\Delta}}{2a}$ as $\frac{-b}{2a} \pm \frac{\sqrt{\Delta}}{2a}$ or they should have taken the greatest common factor out of the parentheses in order to simplify the expression. However, the students worked on one number on the fraction bar and found incorrect roots by making simplification wrongly. These errors might be due to incorrect or incomplete knowledge of the students regarding fractions and mathematical procedures.

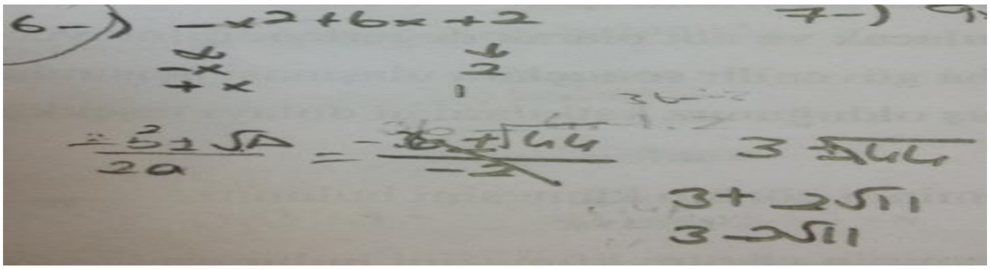

Figure 5. The representation of one student's solution for question 6

After not finding the roots by factorization, the student used the quadratic formula. S/he wrote the number value of a, b and delta correctly and then s/he simplified -2 and -6. However, the student forgot to simplify $\sqrt{44}$ so that s/he found the roots as 3±2$\sqrt{11}$ instead of 3±$\sqrt{11}$. Here, -2 was denominator of both -6 and $\sqrt{44}$ but the student might have thought that after simplifying -2 for once, there was no more -2. In addition to this, due to the fact that the fractions were not written separately as $\frac{-6}{-2} + \frac{\sqrt{44}}{-2}$, the student might be confused.

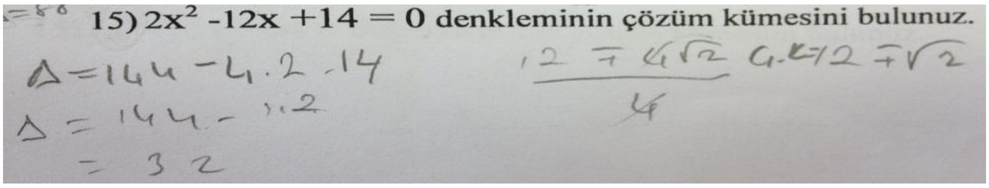

Figure 6. The representation of one student's solution for question 9

Similarly, the student tried to solve quadratic equation by using quadratic formula. While calculating the roots, s/he simplified 4 and 4$\sqrt{2}$ but she did not simplify 12. Thus, s/he found the roots as 12±$\sqrt{2}$ instead of 3±$\sqrt{2}$. Here, we see that the student thought that simplification could be made during addition and subtraction without taking the greatest common factor out of parentheses. Thus, s/he failed to simplify the expression and obtain correct roots.

Missing Root Errors

When the students performed all solution ways: factorization, quadratic formula or completing square, this type of error was observed. They focused on procedural process of the solution types without using the related conceptual understanding. Since procedural process that they used was failure, they made errors. The students found only one correct root and missed out the other root such as zero or negative root. It resulted from transforming the quadratic equation into linear equation through simplification, using the square root method by ignoring negative root or applying the quadratic formula incompetently.

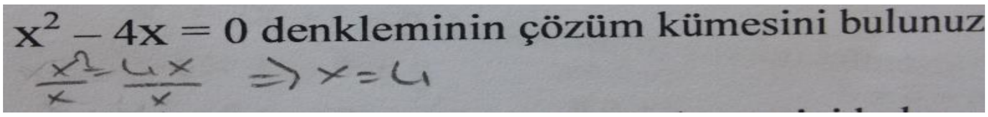

Figure 7. The representation of one student's solution for question 1

In this example, the student simplified the quadratic equation through dividing by x both part of equality and missed out the root came from the factor of x. After simplification, quadratic equation became linear equation and the student solved it. Thus, s/he neglected one of the roots of the equation, which is 0. The student canceled x from both sides but s/he did not recognize the root 0 was disappeared. Therefore, although s/he had to obtain two roots which were 0 and 4 from x.(x-4), the student reached only one root. It showed that the student was not aware of how many roots s/he had to obtain in solving quadratic equations.

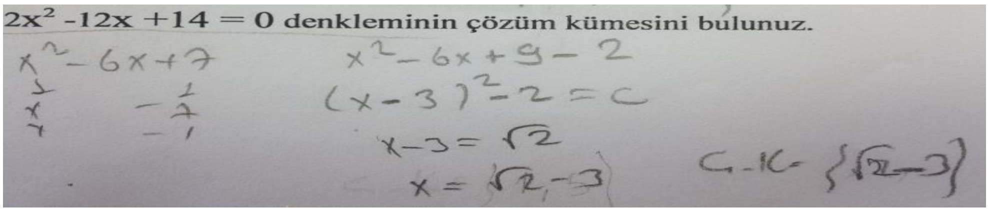

Figure 8. The representation of one student's solution for question 9

After not finding the roots by factorization, the student applied completing the square method and s/he tried to obtain $(x-3)^2$. At first, the student simplified the quadratic equation with 2 and obtained the quadratic equation of x^2-6x+7. Later, s/he wrote 7 as 9-2 and obtained $(x-3)^2$ from x^2-6x+9 and transferred -2 to the other side of the equations sign. While taking the square root of both part of equality, s/he ruled out that the square root of 2 could be $\sqrt{2}$ or -$\sqrt{2}$ and worked with only $\sqrt{2}$. In this situation, the student could find only one root and missed out the other root. The student had lack of conceptual understanding of that the square of a negative number could also be positive number. On the other hand, although the student applied the completing the square method correctly, s/he also made sign error and found $\sqrt{2}-3$ instead of $\sqrt{2}+3$.

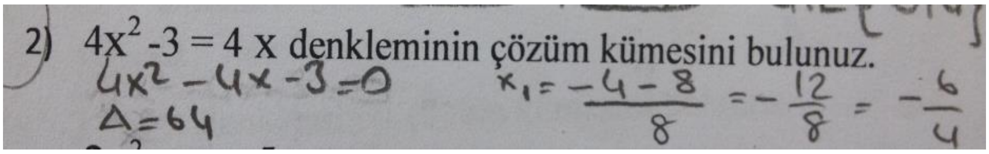

Figure 9. The representation of one student's solution for question 2

As it is seen in the solution, the student used quadratic formula to find the roots of quadratic equation. Although the roots were expressed as $\frac{-b\pm\sqrt{\Delta}}{2a}$ in quadratic formula, here the student found only one root as $-\frac{6}{4}$ and the other root $\frac{-4+8}{8}$ = $\frac{1}{2}$ was missed out. This failure might be caused by having difficulty in implementing the formula and not understanding the use of the it. In this solution, the student could not implement the quadratic formula accurately so that s/he could not solve the question completely. This quadratic equation was given in different form (e.g., ax^2+c=bx, where a, b, c ϵ R). Therefore, s/he might also have difficulty in understanding this equation and interpreting its roots.

Common Factor Errors

This type of error includes the use of the rules of algebra incorrectly. The students tried to change the quadratic equation to simpler form by taking the x out of the parentheses in order to make solution. However, they neglected basic mathematical properties and represented the quadratic equation mathematically in a wrong form. These errors might resulted from not having fluency in making mathematical operations or memorizing the procedural knowledge.

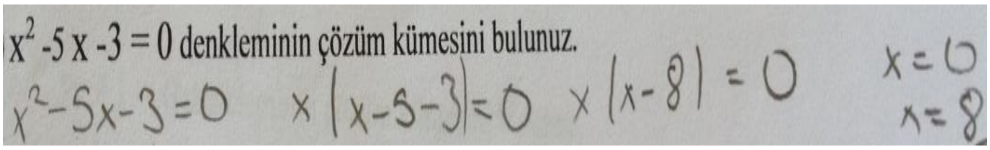

Figure 10. The representation of one student's solution for question 8

In this example, although -3 was constant and there was not the expression of -3x. The student considered it as -3x and took common factor parentheses incorrectly. Then, s/he made operations in parentheses and found failure roots as 0 and 8.

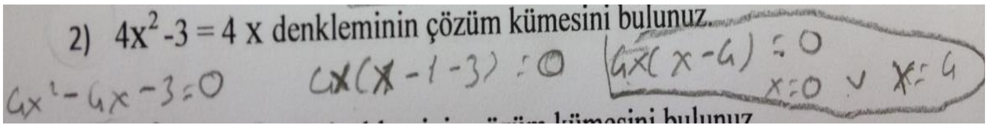

Figure 11. The representation of one student's solution for question 2

Same as above example, although -3 was the constant of quadratic equation, the student behaved the constant as if it was -3x. S/he tried to transform the equation into a form that could be easily manipulated. The student put the constant in parentheses and found the roots as 0 and 4 wrongly. The student would realize that if the terms in parentheses were multiplied by x, the expression would not be equal to the given quadratic equation.

One Method Errors

This type of error was observed because the students focused on only one solution type and insisted on using it. The students had lack of conceptual understanding about the solution types since they usually memorized them. Hence, the students could not make connections between the solution types or decide which of them was more suitable.

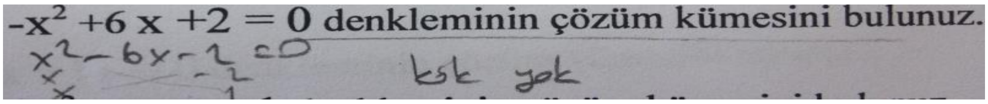

Figure 12. The representation of one student's solution for question 6

As it is seen in the solution, when the student realized that the quadratic equation could not be factorized, s/he believed that there was no real root. Besides, the student indicated that there was no real root without checking the accuracy of her/his claim by using different solution methods such as quadratic formula or completing the square.

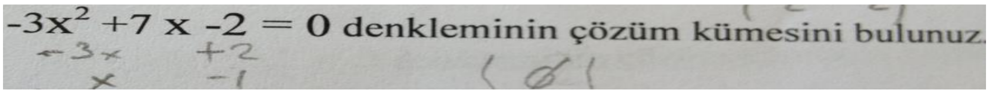

Figure 13. The representation of one student's solution for question 4

Similarly, after trying to factorize the quadratic equation, the student could not find the roots. Therefore, s/he directly thought that there were not suitable numbers to form the factors, although the quadratic equation was factorable. Furthermore, s/he thought that there was no real root. It showed that the student decided about the roots based on only one solution method. In addition, the student expressed her/his answer mathematically wrong. S/he sought to indicate that there was no real root so that the solution set was empty. However, the representation of her/his answer did not come to mean empty set mathematically.

Factorization Errors

Because of that the students could not find two correct linear factors, they could not find the roots accurately. This kind of error was also observed when the students tried to factorize the quadratic equations that were not factorable. Factorizing a quadratic equation requires to consider the first, middle and last term at the same time. For example, the addition of the factors of the first term and the last term must give the middle term in the equation (Makonye & Nhlanhla, 2014). However, the students neglected some of these terms or confused the rules such as multiplication or addition of the number pairs. This kind of error based on their incorrect guesses and no attempt to control the accuracy of linear factors while using the cross-multiplication method. Besides, this failure probably resulted from their lack of conceptual knowledge regarding factorization method.

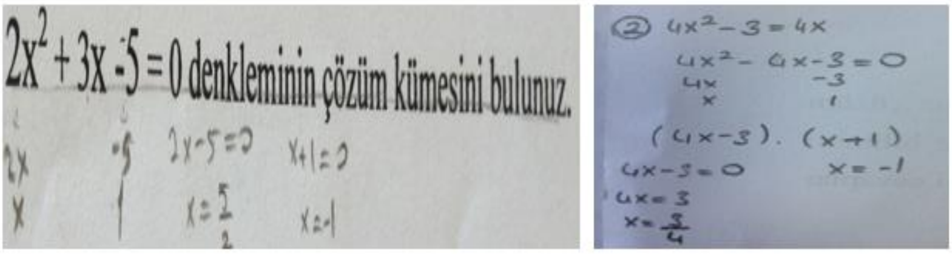

Figure 14. The representations of two students' solutions for questions 3 and 2

Although these two quadratic equations were factorable, the students factorized them incorrectly. They paid attention to write suitable numbers and signs for the coefficients of the first term and the last term of quadratic equations (e.g., $2x^2 \rightarrow 2x$ and x, -5 → -5 and 1 or $4x^2 \rightarrow$ 4x and x, -3 → -3 and 1) but they did not pay attention to the coefficient of the middle term and found wrong roots. The students confused the concept of factorizing. It is understood that they knew some rules but they could not apply correctly.

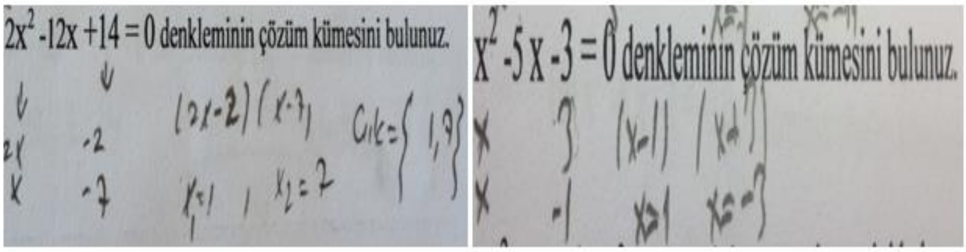

Figure 15. The representations of two students' solutions for questions 9 and 8

Although these two quadratic equations were non-factorable, the students tried to solve them by factorization and found wrong roots. Similar as above example, they paid attention to write suitable numbers and signs for the coefficients of the first and last terms of quadratic equations. If the students controlled the factors by multiplying each other, they could see that the obtained quadratic equation was not equal to the given quadratic equation at the beginning so that they would realize their own errors. However, they neglected the middle term. Although they considered the multiplication of the number combinations for the first and last terms, they did not control whether the sum of them were equal to the middle term.

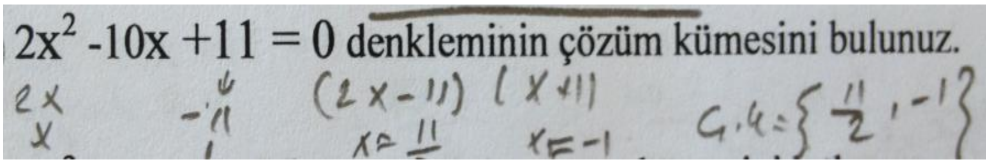

Figure 16. The representation of one student's solution for question 7

Although this quadratic equation could not be solved through factorization, the student sought to factorize it and found incorrect roots. S/he wrote suitable numbers for the coefficients of the first and last terms of quadratic equation but s/he made mistake in determining the sign. In order to obtain 11, 11 and 1 or -11 and -1 could be selected but the student wrote -11 and 1. Furthermore, s/he neglected the coefficient of the middle term and found incorrect linear factors and roots.

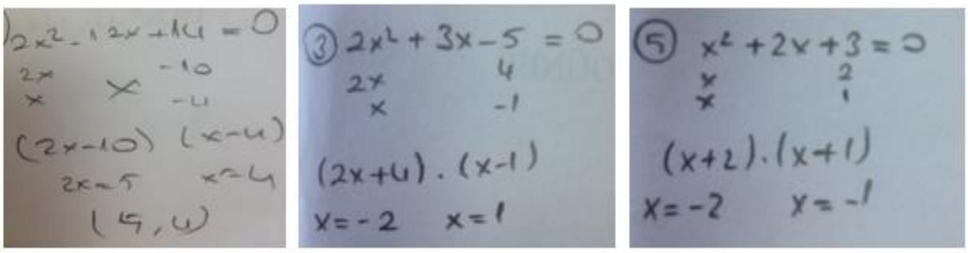

Figure 17. The representations of three students' solutions for questions 9, 3 and 5

As it is seen in the solution, the error that the students made was related to the constant of quadratic equation. The students wrote two numbers whose sums of them were equal to the constant without considering the signs (e.g., 14 → 10 and 4, 5 → 4 and 1 or 3 → 2 and 1), although they needed to write two numbers whose multiplications of them were equal to the constant. The students confused how to use the principle of factorization. It showed that the students had some knowledge about this method but it was not enough or not fully internalized.

Rule Errors

Students are tend to memorize the quadratic formula, when they cannot gain a procedural and conceptual understanding of quadratic equations. It was observed that they represented rule errors in their solutions since they remmembered and applied the quadratic formula incorrectly. Some students computed the discriminant incorrectly because of calculation errors and some students computed the discriminant correctly, but could not use the quadratic formula correctly since they had misremembered it. Besides, some of them remmembered the meaning of the discriminant incorrectly and made wrong interpretation about the roots of quadratic equations. In general, it was seen that they calculated the discriminant and use the quadratic formula incorrectly.

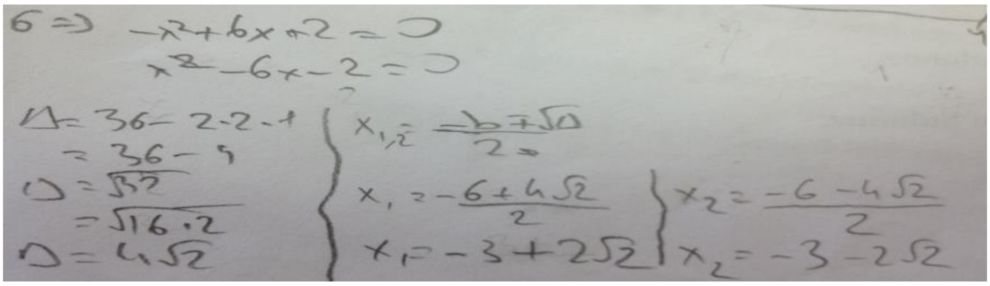

Figure 18. The representation of one student's solution for question 6

In this example, the student applied the correct quadratic formula to solve the quadratic equation but s/he remembered the formula of discriminant wrongly. Although the formula was b^2-4ac, the student calculated discriminant using the expression of b^2-2ac.

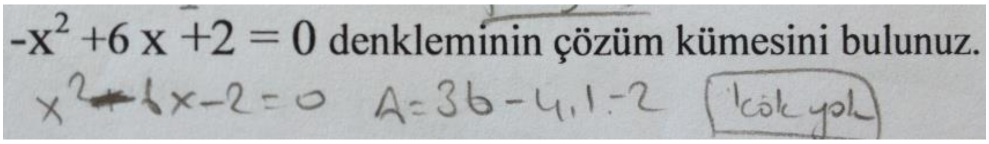

Figure 19. The representation of one student's solution for question 6

Here, the student used quadratic formula and calculated discriminant. Although delta was bigger than zero and came to mean of being two real roots of quadratic equations, the student interpreted this situation like that there was no real root. It probably resulted from remembering the meaning of the discriminant incorrectly. Therefore, s/he gave wrong answer.

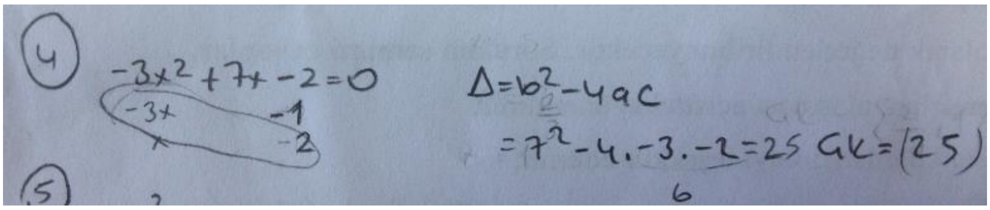

Figure 20. The representation of one student's solution for question 4

In this example, after not finding the roots by factorization, the student calculated delta by using the correct formula. Although s/he needed to continue solution by using $\frac{-b\pm\sqrt{\Delta}}{2a}$ in order to find the roots, the student accepted the result of delta as answer. Therefore, this situation showed that the student confused the concepts of discriminant and the roots.

Meaningless Solutions

Some of the students represented efforts which were not related to expected solution processes in order to solve the quadratic equations.

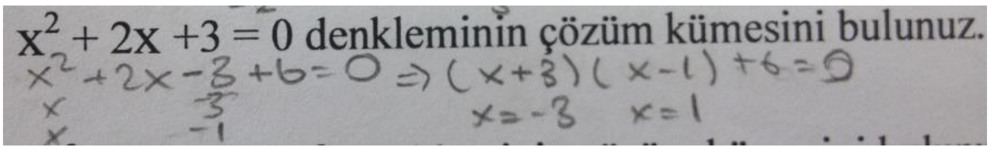

Figure 21. The representation of one student's solution for question 5

As it is seen in this solution, the student tried to transform the quadratic equation in a form that could be factorized since it was non-factorable. S/he wrote 3 as the addition of -3 and 6 so that s/he did not change the constant in the equation. Later, s/he factorized some part of quadratic equation (x^2+2x-3) and added on the remaining number. S/he neglected 6 and equalized (x+3).(x-1) to zero. Thus, the student found incorrect roots.

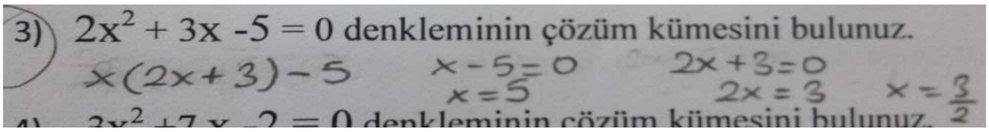

Figure 22. The representation of one student's solution for question 3

Here, the student determined the greatest common factor of the first and middle terms as x and wrote it in front of the parentheses correctly. Then, s/he put together the value of x and -5 and formed an expression of (x-5). After s/he equalized the expression of (2x+3) and the expression of (x-5) to zero, s/he found roots incorrectly. The student applied mathematically wrong and meaningless procedures.

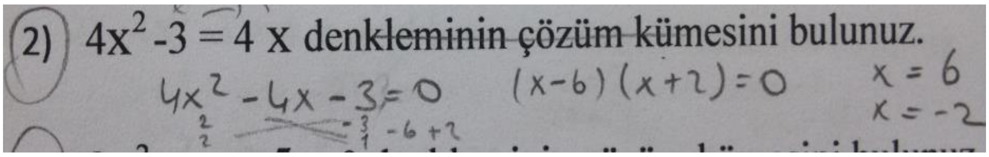

Figure 23. The representation of one student's solution for question 2

The student wrote -3 and 1 to obtain -3 and also 2 and 2 to obtain $4x^2$. Although s/he determined the correct number pairs for the coefficients of the first and last terms, the student continued the solution irrelevantly. S/he multiplied -3 and 1 by 2 and obtained -6 and 2. Later, s/he factorized the quadratic equation as (x-6).(x+2) and found incorrect roots.

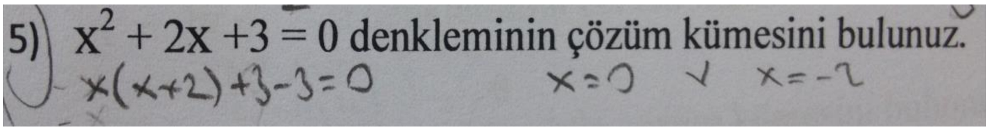

Figure 24. The representation of one student's solution for question 5

Here, the student determined the greatest common factor of the first and middle terms as x and put it out of the parentheses correctly. Then, s/he added -3 in order to remove constant but s/he did not pay attention adding the same number on the other part of equality to save the equation. This shows the student's the lack of understanding of the equals sign. In equality, the left side must be equal to the right side. In other words, if a change is made on one side, the same change must be done on the other side. In this example, s/he did not add -3 on the right side. Therefore, x.(x+2) remained and s/he found the roots as 0 and -2 wrongly.

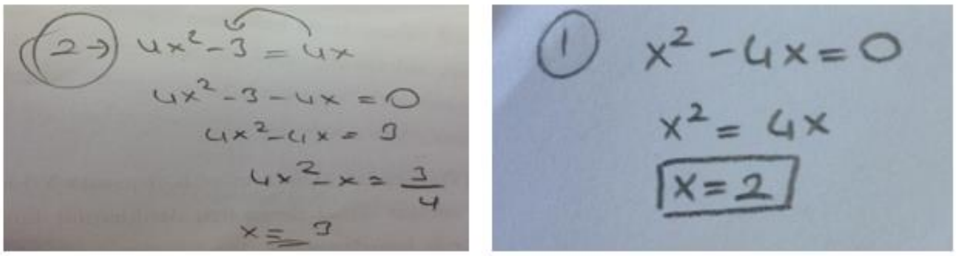

Figure 25. The representations of two students' solutions for questions 2 and 1

In these examples, the students made operations did not provide to reach correct solutions and they also made errors in calculations. Since the students did not write the equations in the standard form ($ax^2+bx+c=0$), they could not use a correct solution method. How the students obtained 3 or 2 was not clear and their solutions seemed meaningless.

4. DISCUSSION and CONCLUSION

When the findings are examined, it is seen that students` achievement of solving quadratic equations was notably low and they had some difficulties. The results of some studies also support this finding (Didiş et al., 2011; Lima, 2008; Makonye & Nhlankla, 2014; Sarwadi & Shahrill 2014; Vaiyavutjamai & Clements, 2006). Students were more successful in solving 1-4 questions that could be factorized than 6-9 questions that could not be factorized. It shows that non-factorable quadratic equations were challenging for the students to solve. If students could not factorize the quadratic equations at first, they did not usually try to find the roots by using different methods and left mostly empty or incomplete.

When the students` solution ways in solving quadratic equations are analyzed, it is seen that students were in tendency to use factorization as first method and more than quadratic formula or completing the square method. Besides, some students tended to use quadratic formula if they could not find the roots by factorization at first. Therefore, it is understood that the students firstly preferred factorization, secondly quadratic formula and least completing the square method. Because of deciding which expression can be completed the square and checking the accuracy of it requires thinking more mathematically than the other methods, students may avoid using this method and prefer the others. Similarly, in the study of Zakaria and Maat (2010), most of the students could not manage to perform the completing the square method as well. Students see this method more challenging than factorization and using quadratic formula (Makgakga, 2016). However, according to Snell (1958) since the integration of this method into lessons is essential for mathematics in higher levels, it should be learned in elementary course. The method of completing the square provides the algebraic manipulation of a quadratic equation for rendering it more suitable form. However, students do not prefer it since they have lack of knowledge regarding basic properties of square roots such as $\sqrt{x^2} = |x|$ and if $x^2 = a$ then $x = \pm\sqrt{a}$. It reveals the need for more time and attention of teaching these basic properties (López, Robles, & Martínez-Planell, 2016).

Similar to the findings of this study, various studies indicate that students mostly prefer factorization rather than the other methods when the quadratic equation is factorable (Bosse & Nandakumar, 2005; Didiş et al., 2011). However, this method does not allow for developing conceptual meaning of quadratic equations and students memorize the procedures and formulas to solve them (Taylor & Mittag, 2001). Students may see solving quadratic equations as calculations since they mainly made operations using symbols. Thus, it is likely that they do not recognize the meaning of the related concepts (Lima, 2008). Students make various errors because they confuse mathematical concepts, rules and procedures and have lack of knowledge about how and when to use them (Makonye & Nhlanhla, 2014). In teaching of quadratic equations, teacher should pay attention to tell the subject making connection between mathematical concepts and their meanings (Makonye & Nhlanhla, 2014; Stein, Smith, Henningsen, & Silver, 2000).

The solutions of students show that they made many errors in simplification, factorization (Norasiah, 2002; Roslina, 1997; Parish & Ludwig, 1994), calculation, remembering the rules (Zakaria & Maat, 2010) and made meaningless solutions. In this study, the errors made by the students were classified under 9 topics: sign errors, calculation errors, simplification errors, one method errors, factorization errors, rule errors, missing root errors, common factor errors and meaningless solutions. The students were failure to add, subtract, multiply and divide; transfer the variable or number considering the sing; remove or expand the parentheses; and simplify or group the terms. Similarly, Ersoy and Erbaş (2000) found that students were inadequate in making arithmetic operations and although they knew the rules, they could not remember mathematical knowledge. They also argued that the students made mathematical operations that were not correct by using the rules wrongly. These results show that students learn mathematical knowledge superficially and memorisingly (Schoenfeld, 1985; McCormick, 1997; Jinfa, 1998; Baki & Kartal, 2004). Furthermore, the students were unsuccessful in providing the equality of both sides since their lack of knowledge about the meaning of the equals sign. They also transmitted the letters or numbers to the other part of equality wrongly and obtained incorrect signs. Snell (1958) emphasize that the rule of changing the signs of numbers while replacing in equation can easily lead to mistakes. The researches of Norasiah (2002), Roslina (1997) and Parish and Ludwig (1994) support this finding as well. Besides, in some cases, the students' errors occurred due to carelessness. Moreover, while factoring the quadratic equations, they neglected the middle term and tried to find the factors of the first and last terms or they endeavored to determine the factors of last term considering addition instead of multiplication. The findings also revealed that students remembered the quadratic formula incorrectly, had difficulty in determining the factors through cross-multiplication method (Didiş & Erbaş, 2015) and avoided to apply the completing the square method. This showed the students' lack of conceptual understanding of the factorization, quadratic formula and the completing the square principles. In sum, the results revealed that students had lack of knowledge about fundamental mathematical concepts despite being in high school (Sarwadi & Shahrill, 2014).

In parallel with the results of Vaiyavutjamai et al. (2005), the results of this study revealed that the students were not aware of how many roots they should find after solving a quadratic equation. Therefore, they missed out some roots in their solutions. Besides, they did not know how to interpret the meaning of the roots such as two real roots, one real root or no real root. Furthermore, the missing roots (e.g., zero, negative root) showed that there were lacks in their previous knowledge such as linear equations, square root and numbers. As Lima (2008) and Vaiyavutjmai and Clements (2006) stated, this shows that students usually do not know the meaning of what they have found or done and they use rules without internalizing. Similarly, Didiş, Baş and Erbaş (2011) highlight that students use their knowledge of rules without considering why they used or whether they used correctly. The findings show that students endeavor to find the roots of quadratic equations without considering suitable solution method, the form and meaning of quadratic equation or the correctness of their own answers. Instead, they want to solve the question and get an answer quickly (Sönnerhed, 2009). Thus, introduction of mathematical concepts should prevent rote learning of the rules, procedures and formulas (Didiş et al., 2011).

It can be said that students' errors are resulted from memorization and misuse of rules, confusion of previous knowledge, the lack of ability to link arithmetic with algebra, the lack of knowledge about basic mathematical concepts and procedures. Wheatley (1995) argues that a number of errors arise because teachers focus on procedural aspect of the concepts rather than conceptual aspect and students are expected to perform a number of tasks with variables without taking into account the context of the subject (as cited in Kieran, 1992). Sarwadi and Shahrill (2014) identify the reasons of errors as incorrect knowledge and schema; the lack of interpreting symbols, operations and rules; and the tendency of overgeneralization regarding mathematical procedures and formulas. According to Zakaria and Maat (2010), the errors in quadratic equation solutions occur since students generally have problem in understanding and describing what is required by the questions and what the meanings of the terms used such as root, coefficient are. In addition, they defend that these weaknesses probably result from the lack of emphasis by the teachers in solution methods: factorization, quadratic formula and completing the square and the lack of emphasis by the teachers on understanding the language of mathematics and the needed skills. Using more than one solution way strengthens conceptual understanding so that learning becomes more permanent. In addition, solution methods' functionality changes depending on the type of questions. Whereas factorization is more suitable for one question, it may not be suitable for another. Therefore, teachers should teach different solution ways of quadratic equations and encourage students to apply more than one solution way since students are generally in tendency to prefer only one way and in order to make easy solving quadratic equations (Bossè & Nandakumar, 2005; Sönnerhed, 2009). According to Makgakga (2016), students are not provided enough time for discussing on the concepts and practicing them. It is recommended teachers to allow students for learning through discussion and practice. Vaiyavutjamai and Clements (2006) believe that when teachers focus on teaching the meaning of the symbols rather than the use of them, students can be more successful in solving quadratic equations. Although factorization is the most preferred method for solving quadratic equations, it can be very difficult when the coefficients and the constants include many pairs (Bossé & Nandakumar, 2005). Therefore, different types of solution ways should be taught and students should decide which to use.

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GENİŞ ÖZET

Öğrenciler cebir konusunu anlamada zorluk çekmekte ve genellikle cebirsel düşünme yapılarını kavramadan cebirle uğramaktadırlar (Chazan, 1996; Sfard, 1991; Kieran, 1992). Kaput (1999)'a göre, cebir konusu diğer matematik konularından ayrı olarak öğretildiği ve kurallara dayandığı için öğrenciler tarafından sevilmemektedir. Genel olarak öğrenciler cebiri sınavları geçmek, iyi bir liseye ya da üniversiteye yerleşmek gibi amaçları gerçekleştirmek için gereklilik olarak görmektedirler (Usiskin, 1988). Cebirin günlük yaşamda kullanılmayan bir konu olduğunu düşündükleri için bu alanın önemli olduğuna inanmamaktadırlar. Bu nedenle cebir öğrenimi üzerinde durulması gereken ve önemi kavratılması gereken bir konudur. Diğer bir yandan, öğrenciler başta aritmetik düşünmeyi öğrenmekte ve sayılar üzerinde çalışmaktadır. Fakat daha sonra bu durum yerini cebirsel düşünmeye bırakmaktadır (Katz, 1997). Cebirin gelişimi somuttan soyuta gerçekleştiğinden bu geçiş öğrenciler açısından kolay değildir. Bu nedenle öğrenciler cebirin soyut ve anlamsız olduğunu düşünmektedir. Bütün bu zorluklar cebir öğreniminde ve öğretiminde karşılaşılan negatif durumları ortaya koymaktadır (Dede, Yalın & Argün, 2002).

Cebir öğrenme alanındaki temel konulardan biri ikinci dereceden denklemlerdir. Bu konuyla ilgili müfredatta yer alan kazanımlar "denklemin köklerini ve çözüm kümesini bulma, denklemin kökleri ile katsayıları arasındaki ilişkiyi gösterme ve kökleri verilen denklemi oluşturma" hedeflerini içermektedir. İkinci dereceden denklemlerin çözümü için okullarda öğretilen üç tane yöntem bulunmaktadır: (i) çarpanlara ayırma, (ii) kareye tamamlama ve (iii) ikinci dereceden denklem formülü kullanma. Pek çok araştırma öğrencilerin ikinci dereceden denklemlerde zorluklar yaşadığını ve denklemleri çözerken çeşitli hatalar yaptığını ortaya koymaktadır (Zakaria & Maat, 2010). Öğrencilerin nasıl düşündüğünü belirlemek, matematiksel kavramları anlamlandırma şekillerini tespit etmek ve ne tür hatalar yaptıklarını tespit etmek öğrencilerdeki eksiklikleri gidermek açısından önemlidir. Bu doğrultuda çalışmanın amacı 10. sınıf öğrencilerinin ikinci dereceden denklem çözme başarılarını belirlemek, çarpanlara ayırma, kareye tamamlama ve ikinci dereceden denklem formülü uygulama gibi farklı çözüm yolları kullanma eğilimlerini incelemek ve öğrencilerin ikinci derece denklem çözümlerindeki hataları tespit etmektir.

Çalışmada nitel araştırma yöntemlerinden durum çalışması kullanılmıştır. Araştırmanın örnekleminin 10. sınıfta öğrenim gören 50 öğrenci oluşturmaktadır. Bu öğrencilere ikinci dereceden denklemlere yönelik 9 açık uçlu soru sorulmuştur. Öğrencilerin yaptığı çözümler detaylı bir şekilde incelenmiş ve çeşitli kategoriler altında yorumlanmıştır. Bulgular öğrencilerin ikinci dereceden denklemleri çözme başarılarının düşük olduğunu ve bu konuda zorlandıklarını göstermektedir (Didiş, Baş, & Erbaş, 2011; Lima, 2008; Makonye & Nhlankla, 2014; Sarwadi & Shahrill 2014; Vaiyavutjamai & Clements, 2006). Öğrencilerin çarpanlara ayrılabilen denklem türlerinde çarpanlara ayrılamayan denklem türlerine göre daha başarılı olduğu sonucuna ulaşılmıştır. Öğrenciler ilk başta çarpanlara ayırma yöntemiyle ikinci dereceden denklemi çözemezlerse eğer, genellikle denklemin köklerini bulmak için başka çözüm yöntemi denememekte, bu durumda soruyu ya boş bırakmakta ya da çözümünü tamamlayamamaktadır.

Öğrencilerin kullandıkları çözüm yolları incelendiğinde, çarpanlara ayrıma yöntemini kareye tamamlama ve ikinci dereceden denklem formülü uygulama yöntemlerine göre daha fazla kullanma eğiliminde oldukları görülmüştür. Bunun yanı sıra öğrenciler ilk başta çarpanlara ayırma yöntemiyle çözmedikleri denklemlerde ikinci dereceden denklem formülünü kullanarak çözüm yapmaya çalışmaktadırlar. Dolayısıyla, öncelikli olarak çarpanlara ayırma, ardından formül kullanma ve en son kareye tamamlama yöntemlerini tercih ettikleri anlaşılmaktadır. Hangi denklemin tam kareye tamamlandığını belirlemek ve bunun doğruluğunu kontrol etmek zor olduğundan kareye tamamlama yöntemi diğerlerine göre daha fazla matematiksel düşünmeyi gerektirmektedir. Benzer şekilde, Zakaria ve Maatt (2010) da çalışmalarında öğrencilerin bu yöntemi başarılı bir şekilde kullanamadıkları sonucuna ulaşmışlardır.

Bunların yanı sıra, bulgular öğrencilerin ikinci dereceden denklemleri çözerken çeşitli hatalar yaptıklarını ve bazı kavram yanılgıları olduğunu ortaya koymaktadır. Bu çalışmada, öğrenciler tarafından yapılan hatalar 9 başlık altında toplanmıştır. Bunlar; işaret hataları, hesaplama hataları, sadeleştirme hataları, ortak çarpan hataları, tek çözüm yolu kullanma hataları, çarpanlara ayırma hataları, kural hataları, eksik kök bulma hataları ve anlamsız çözümler şeklinde isimlendirilmiştir. Öğrenciler harfleri ya da sayıları eşitliğin diğer tarafına taşırken hatalar yapmakta ve işaretleri yanlış bulmaktadır. Snell (1958) sayıların denklemde yerini değiştirirken işaret değiştirme kuralının kolayca hatalara neden olabileceğini ifade etmiştir. Öğrencilerin ifadeleri sadeleştirmede, uygun çarpanları bulmada, hesap yapmada ve kuralları hatırlamada çeşitli hatalar yaptıkları görülmüştür (Norasiah, 2002; Roslina, 1997; Parish & Ludwig, 1994). Bu hataların öğrencilerin dikkatsizliklerinden, eksik bilgilerinden ve kavram yanılgılarından kaynaklanması muhtemeldir. Ersoy ve Erbaş (2000) öğrencilerin aritmetik işlem yapmada yetersiz olduklarını, kuralları bilmelerine rağmen doğru hatırlayamadıklarını ve kuralları yanlış uygulamalarına bağlı olarak hatalı matematiksel işlemler yaptıklarını belirtmişlerdir. Zakaria ve Maat (2010)'a göre öğrenciler bu tür hataları verilen soruda ne istenildiğini anlayamadıkları ve kök, katsayı gibi terimleri anlamlandıramadıkları için yapmaktadırlar. Bunun yanı sıra, öğretmenlerin çarpanlara ayırma, kareye tamamlama ve formül kullanma gibi farklı ikinci dereceden denklem çözüm yollarını ve uygun matematiksel dili kullanmamalarının da bu duruma neden olduğunu düşünmektedirler. Birden fazla çözüm yolu kullanmak kavramsal anlamayı güçlendirdiğinden ve her soruya uygun olan çözüm yolu farklı olabileceğinden, bu noktada öğretmenlere farklı çözüm yollarını öğretmenleri ve öğrencileri bunları kullanmaları yönünde teşvik etmeleri önerilmektedir.