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Research Article

Sensing Magnetic Field with Single-Spin Dynamical Probe State: Control over Sensing Precision via Quantum Fisher Information

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Abstract

Quantum sensors play an important role in many branches of modern science, and they occupy a huge segment of the growing market for quantum devices. Quantum sensors use qubits and their analogs as detecting and analyzing quantum elements. Some sensors can be based on a single qubit, which is often presented as a system making its evolution on the so-called Bloch sphere. Different criteria are used to evaluate the efficiency of the sensing process. One of the most popular is the Quantum Fisher Information Matrix (QFIM) based on Fisher information. The magnitudes of the QFIM elements are strongly related to the precision of the sensing. As an analog of the classical Cramér theorem, one can define the quantum Cramér-Rao bound for the variance V, which is equal to $V = 1/NF$ where F is the corresponding quantum Fisher information element, and N stands for the number of repeated sensory measurements. In this work, we develop our quantum Fisher information-based approach for a single feedback-driven qubit-type element for sensing external magnetic fields. We demonstrate the efficiency of our algorithm and discuss its further possible improvement. The approach developed here can be easily extended to other sensing schemes: collective spin systems and multi-qubit-based sensors. Alternative control algorithms can be applied to drive the probe state vector for maximization of the QFIM components. The particular choice of the control algorithm is defined by the specific experimental set-up.

Keywords: Quantum bit, Bloch sphere, Fisher information, Cramér-Rao bound, Quantum sensing, Feedforward control.

Tek Dönüşlü Dinamik Araştırma Durumu ile Manyetik Alanı Algılama: Kuantum Fisher Bilgileri Yoluyla Algılama Hassasiyeti Üzerinde Kontrol

Öz

Kuantum sensörleri, modern bilimin birçok dalında önemli bir rol oynar ve kuantum cihazları için büyüyen pazarın büyük bir bölümünü işgal eder. Kuantum sensörleri, kuantum öğelerini tespit etmek ve analiz etmek için kübitleri ve analoglarını kullanır. Bazı sensörler, genellikle evrimini sözde Bloch küresi üzerinde gerçekleştiren bir sistem olarak sunulan tek bir kübite dayalı olabilir. Algılama sürecinin etkinliğini değerlendirmek için farklı kriterler kullanılır. En popüler olanlardan biri, Fisher bilgilerine dayanan Kuantum Fisher Bilgi Matrisidir (KFBM). KFBM öğelerinin büyüklükleri, algılama hassasiyeti ile güçlü bir şekilde ilişkilidir. Klasik Cramér teoreminin bir benzeri olarak, $V = 1/NF$'ye eşit olan V varyansı için kuantum Cramér-Rao bağı tanımlanabilir; burada F, karşılık gelen kuantum Fisher bilgi öğesidir ve N, tekrarlanan duyusal ölçümlerin sayısını temsil etmektedir. Bu çalışmada, harici manyetik alanları algılamak için tek bir geri bildirim odaklı kübit tipi eleman için kuantum Fisher bilgi tabanlı yaklaşımımızı geliştirmekteyiz. Algoritmamızın verimliliğini göstermekte ve olası iyileştirmelerini tartışmaktayız. Burada geliştirilen yaklaşım, toplu döndürme sistemleri ve çoklu kübit tabanlı sensörler gibi diğer algılama şemalarına kolayca genişletilebilmektedir. KFBM bileşenlerinin maksimize edilmesi için araştırma durumu vektörünü sürmek üzere alternatif kontrol algoritmaları uygulanabilir. Kontrol algoritmasına özgü yapılacak seçim belirlenen deneysel düzenek tarafından tanımlanır.

Anahtar Kelimeler: Kuantum biti, Bloch küresi, Fisher bilgisi, Cramér-Rao sınırı, Kuantum algılama, İleri besleme kontrolü.

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1. Introduction: Quantum Fisher Information for Sensing

Quantum sensors play an important role in many branches of modern science: photonics, microscopy, gravitational wave detecting, and others (Laurenza et al., 2018; Koppenhöfer et al., 2022); they occupy a huge segment of the growing market for quantum devices (ReportLinker, 2022).

Quantum sensors use qubits and their analogs (Nielsen and Chuang, 2004) as detecting and analyzing quantum elements (Degen et al., 2017). Some sensors can be based on a single qubit, which is often presented as a system making its evolution on the so-called Bloch sphere (Bloch, 1946).

Different criteria are used to evaluate the efficiency of the sensing process. One of the most popular is the Quantum Fisher Information Matrix (QFIM) based on Fisher information (Fisher, 1922). In the quantum case, it can be defined via the Bures or, alternatively, the Hellinger distance between quantum states (Zhong et al., 2013). Here we use the Bures metric.

For the vector parameter **x** encoding the density matrix $\rho(\mathbf{x})$, the components of QFIM are defined as (Amari and Nagaoka, 2000):

$$F_{ab} = \mathrm{Tr}\left(L_b \frac{\partial \rho}{\partial x_a} \right) = -\mathrm{Tr}\left(\rho \frac{\partial L_b}{\partial x_a} \right); \quad (1)$$

and for the diagonal elements as:

$$F_{aa} = \mathrm{Tr}\left(\rho L_a^2 \right). \quad (2)$$

Here the sub-indices a,b numerate the elements of **x**, and L_a denotes the symmetric logarithmic derivative for the component x_a. This derivative is a Hermitian operator with the expected value $\mathrm{Tr}(\rho L_a) = 0$; and in general case it is defined for the density matrix ρ and an operator A as (Braunstein and Caves, 1994):

$$i[\rho, A] = \frac{1}{2}\{\rho, L(A)\} , \quad (3)$$

with the commutator $[X,Y] = XY - YX$, and anticommutator $\{X,Y\} = XY + YX$. For (1)-(2) one should take (Liu et al., 2019):

$$\frac{\partial \rho}{\partial x_a} = \frac{1}{2}(\rho L_a + L_a \rho) . \quad (4)$$

The magnitudes of the QFIM elements are strongly related to the precision of the sensing. As an analog of the classical Cramér theorem (Cramér, 1946), one can define the quantum Cramér-Rao bound (Nielsen, 2013) for the variance V, which is equal to:

$$V = \frac{1}{NF} , \quad (5)$$

where F is the corresponding quantum Fisher information matrix element, and N stands for the number of repeated sensory measurements.

In this work, we develop our quantum Fisher information-based approach (Borisenok, 2018) for a single feedback-driven qubit-type element for sensing external magnetic fields. We demonstrate the efficiency of our algorithm and discuss its further possible improvement.

2. Quantum Sensing of An External Magnetic Field

Let's consider a magnetic field vector **B** represented by the spherical coordinate set of one magnitude B and two angles θ and φ as ($B\cdot\cos\theta\cdot\cos\varphi$, $B\cdot\cos\theta\cdot\sin\varphi$, $B\cdot\sin\varphi$). Here we suppose also for simplicity that one angle, let's say φ, is known (Liu et al., 2019), such that our sensing deals with the estimation of B and θ.

The magnetic field detection in this case may be organized via alternative algorithms:

- A single-spin system (Pang and Brun, 2014; Liu et al., 2015);
- Collective spin system (Jing et al., 2015);
- Two-qubit system: a probe qubit serves as a sensing element, while the companion *ancilla* qubit does not interact with the magnetic field (Yuan, 2016).

Surely, the single-qubit sensing setup is the simplest from the point of its experimental realization. The interaction part of its Hamiltonian is given by $H_{\mathrm{int}} = -B\cdot\mathbf{n_0}\cdot\boldsymbol{\sigma}$, where

$$\mathbf{n}_0 = \left(\cos\theta, 0, \sin\theta\right) , \quad (6)$$

and the vector $\boldsymbol{\sigma}$ consists of the Pauli matrix components ($\sigma_x,\sigma_y,\sigma_z$). Then the QFIM can be represented via the Hamiltonian components (Pang and Brun, 2014; Liu et al., 2015):

$$H_B = t\mathbf{n_0}\cdot\boldsymbol{\sigma} ; \qquad H_\theta = -\frac{1}{2}\sin(Bt)\mathbf{n_1}\cdot\boldsymbol{\sigma} \quad (7)$$

in the form (Liu et al., 2019):

$$F_{BB} = 4t^2\left[1-(\mathbf{n_0}\cdot\mathbf{r_p})^2\right]; \qquad F_{\theta\theta} = \sin^2(Bt)\left[1-(\mathbf{n_1}\cdot\mathbf{r_p})^2\right]; \qquad F_{B\theta} = 2t\sin(Bt)(\mathbf{n_0}\cdot\mathbf{r_p})(\mathbf{n_1}\cdot\mathbf{r_p}) , \quad (8)$$

Here:

$$\mathbf{n_1} = \left(\cos(Bt)\sin\theta, \sin(Bt), -\cos(Bt)\cos\theta\right), \quad (9)$$

and $\mathbf{r_p}$ is the Bloch vector of the probe state.

Eqs (8) demonstrate the main handicap of the sensing with a single spin: the algorithm maximizes the Fisher matrix components F_{BB} and $F_{\theta\theta}$ when the probe state vector $\mathbf{r_p}$ is orthogonal to both vectors $\mathbf{n_0}$ and $\mathbf{n_1}$. But in this case, the component $F_{\theta\theta}$ is bounded by the sine. Another handicap is the periodical vanishing of the components $F_{\theta\theta}$ and $F_{B\theta}$, when $\sin(Bt)$ becomes equal to 0.

As we mentioned above, the two-qubit sensing could be an option for precision improvement, but here we focus on the alternative approach for a single sensing element: making the probe state vector to be controlled to maximize the Fisher information components. In other words, we describe here the case of dynamical $\mathbf{r_p}$ in the place of a static one.

3. Sensing Magnetic Field with a Single Spin by A Dynamical Probe State

For single-qubit-based quantum sensors, optimal and sub-optimal feedback (closed-loop) control has already been studied in (Borisenok, 2018; Poggiali et al., 2018).

Here we develop our approach (Borisenok, 2018), which also used the evaluation criteria based on the Fisher information, to adapt it to the efficient sensing of an external magnetic field. We apply here a feedforward (open-loop) form of the control taking the dynamical probe state vector $\mathbf{r_p}(t)$ as a control parameter.

To do it, let's express the probe state vector via the angle parameter $\theta_p(t)$ as:

$$\mathbf{r_p} = \left(\cos\theta_p(t), 0, \sin\theta_p(t)\right) \tag{10}$$

(the magnitude of the vector is equal to 1).

By (10) and (6),(9) one can present the vectors $\mathbf{n_0}$, $\mathbf{n_1}$ in the form:

$$\begin{aligned} \mathbf{n}_0 \cdot \mathbf{r_p} &= \cos(\theta - \theta_p(t)) \ ; \\ \mathbf{n_1} \cdot \mathbf{r_p} &= \cos(Bt)\sin(\theta - \theta_p(t)) \ . \end{aligned} \tag{11}$$

The system (8) becomes:

$$\begin{aligned} F_{BB} &= 4t^2\left[1 - \cos(\theta - \theta_p(t))\right]^2 \ ; \\ F_{\theta\theta} &= \sin^2(Bt)\left[1 - \cos(Bt)\sin(\theta - \theta_p(t)\right]^2 \ ; \\ F_{B\theta} &= 2t\sin(Bt)\cos(Bt)\sin(\theta - \theta_p(t))\cos(\theta - \theta_p(t)) \ . \end{aligned} \tag{12}$$

Now let's focus on F_{BB} and $F_{\theta\theta}$. From (12) one can learn easily that the maximization of both of them simultaneously has a conflict: different trigonometrical contributions from the variable θ. For this reason, we organize the controlled algorithm for θ_p in two stages: at the first stage we focus on the measurement of the amplitude B, and at the second one – on the angle variable θ.

At the first stage, we define the feedforward control as: $\theta_p = vt$, where the control frequency $v >> B$, such that it covers all minima and maxima of $\sin(Bt)$ (it will be important for the second stage).

From now we rescale the time variable t in the dimensionless units: vt, and the magnetic field amplitude is expressed in the dimensionless units B/v (which must be <1). The Fisher information for F_{BB} is limited by the growing magnitude $4t^2$, and in principle, it is increasing virtually for all ranges of θ_p, apart from the values close to the magnetic field angle θ. Having few measurements N, one can achieve good precision of the variance (5).

In the second stage, we suppose to know the amplitude B, and we start this stage from an integer moment $t = n$ (which starts close to $\sin(Bt) = 0$ due to the magnetic field scaled in the v units is: $B << 1$). We need to measure the angle θ close to the moment of the maximum for $F_{\theta\theta}$ for the given B.

The maximum magnitude of $\cos(Bt)$ corresponds to:

$$\cos(Bt) = \max\left\{\frac{1}{3\sin(\theta - \theta_p)}\left(1 \pm \sqrt{1 + 3\sin^2(\theta - \theta_p)}\right), 1\right\} \ . \tag{13}$$

For the numerical evaluation, we chose $\theta = \pi/4$ and $B = 0.2$ (in the dimensionless units of v). Then the plot for the Fisher information F_{BB} is given in Figure 1 (the first stage):

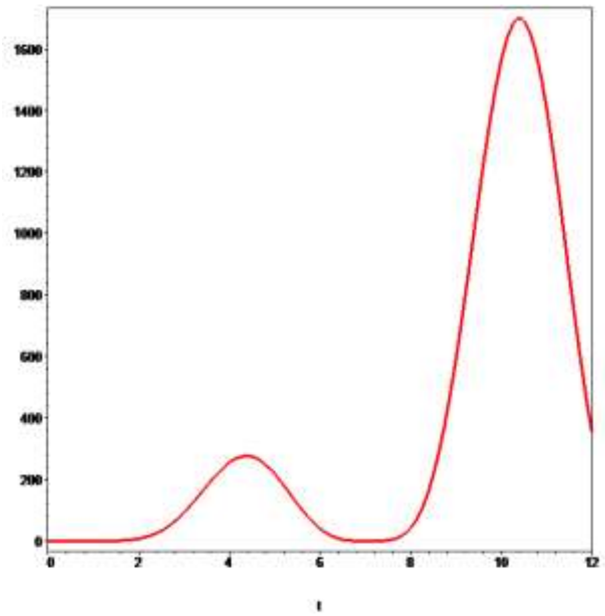

Figure 1. The Fisher information F_{BB} vs time t.

In Fig.1 one can easily observe the achievement of the maximum for the magnetic field amplitude Fisher information F_{BB} around $t = 10$. A similar investigation for the maximum we do for the magnetic field angle Fisher information $F_{\theta\theta}$.

For the second stage, we obtain Figure 2:

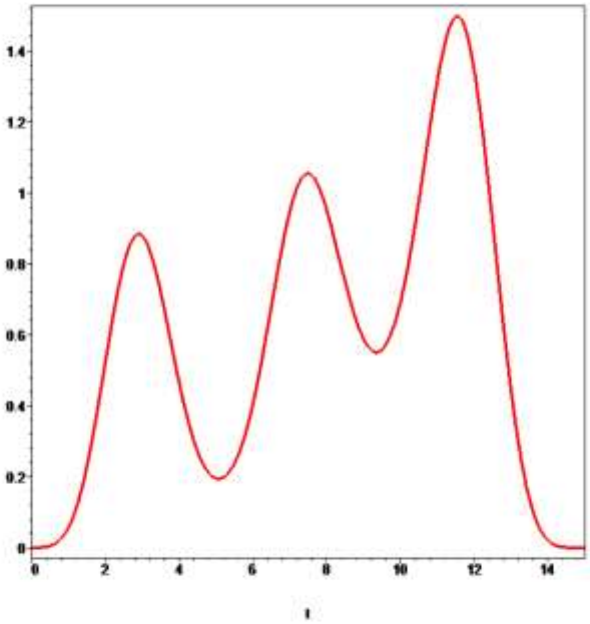

Figure 2. The Fisher information $F_{\theta\theta}$ vs time t.

Thus, for the first stage, the measurements are taken around the time moment t = 10, and for the second stage – around t = 11.

4. Results and Discussion

We develop a dynamical algorithm for the maximizing of the Fisher information in the process of detecting the amplitude and the angle of an external magnetic field. Our algorithm is numerically simple, open-loop and provides the best optimization of the variance parameters for single-spin measurements.

Alternative control algorithms can be applied to drive the vector $\mathbf{r_p}$ for maximization of the QFIM components. The particular choice of the control algorithm is defined by the specific of the experimental set-up and by the compromise between the precision of the control goal achievement and the optimization of the numerical complexity.

Algorithms for the quantum detecting and evaluation of the magnetic fields also work for other sensing applications: quantum photonics, renewable energy, nuclear and geothermal energy, and many others (Crawford et al., 2021).

5. Conclusions and Recommendations

Open-loop algorithm based on a single spin/qubit measurements is able to provide a good variance of the measurements for external fields due to the maximization of the quantum Fisher information.

Our approach can be extended for more advanced algorithms including different closed-loop realizations: optimal feedback, gradient methods, and forming target attractors in the dynamical system (Fradkov, 2007; Kolesnikov, 2014; Pechen et al., 2022). It also can be easily extended to other sensing schemes: collective spin systems and multi-qubit-based sensors.

The effects related to quantum sensing at finite temperatures (Wu and Shi, 2021) should be studied additionally.

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References

Amari, S., Nagaoka, H. (2000). *Methods of Information Geometry*, Providence, USA: American Mathematical Society.

Bloch, F. (1946). Nuclear induction, *Physical Review*, 70, 460-474.

Borisenok, S. (2018). Control over performance of qubit-based sensors, *Cybernetics and Physics*, 7(3), 93-95.

Braunstein, L. V., Caves, C. M. (1994). Statistical distance and the geometry of quantum states, *Physical Review Letters*, 72, 3439-3443.

Cramér, H. (1946). *Mathematical Methods of Statistics*, Princeton Mathematical Series, Princeton, USA: Princeton University Press.

Crawford, S. E., Shugayev, R. A., Paudel, H. P., Lu, P., Syamlal, M., Ohodnicki, P. R., Chorpening, B., Gentry, R., Duan, Y. (2021). Quantum sensing for energy applications: Review and perspective, *Advanced Quantum Technologies*, 4(8), 2100049.

Degen C. L., Reinhard, F., Cappellaro, P. (2017). Quantum sensing, *Review of Modern Physics*, 89, 035002.

Fisher, R. A. (1922). On the mathematical foundations of theoretical statistics, *Philosophical Transactions of the Royal Society of London, Series A*, 222, 309-368.

Fradkov, A. L. (2007). *Cybernetical Physics. From Control of Chaos to Quantum Control*, Berlin, Heidelberg, Germany: Springer.

Jing, X. X., Liu, J., Xiong, H. N., Wang, X. (2015). Maximal quantum Fisher information for general su(2) parametrization processes, *Physical Review A*, 92, 012312.

Kolesnikov, A. A. (2014). *Introduction of synergetic control*, 2014 American Control Conference, Portland, USA, 3013-3016.

Koppenhöfer, M., Groszkowski, P., Lau, H., Clerk, A. A. (2022). Dissipative superradiant spin amplifier for enhanced quantum sensing, *PRX Quantum*, 3, 030330.

Laurenza, R., Lupo, C., Spedalieri, G., Braunstein, S. L., Pirandola, S. (2018). Channel simulation in quantum metrology, *Quantum Measurements and Quantum Metrology*, 5, 1-12.

Liu, J., Jing, X., Wang, X. (2015). Quantum metrology with unitary parametrization processes, *Scientific Reports*, 5, 8565.

Liu, J., Yuan, H., Lu, X., Wang, X. (2019). Quantum Fisher information matrix and multiparameter estimation, *Journal of Physics A: Mathematical and Theoretical*, 53(2), 023001.

Nielsen, F. (2013). *Cramer-Rao lower bound and information geometry*, In: R. Bhatia, C. S. Rajan, A. I. Singh, Eds, Connected at Infinity II. Texts and Readings in Mathematics, vol 67. Gurgaon, India: Hindustan Book Agency.

Nielsen, M. A., Chuang, I. L. (2004). *Quantum Computation and Quantum Information*, Cambridge, UK: Cambridge University.

Pang, S., Brun, T. A. (2014). Quantum metrology for a general Hamiltonian parameter, *Physical Review A*, 90, 022117.

Pechen, A. N., Borisenok, S., Fradkov, A. L. (2022). Energy control in a quantum oscillator using coherent control and engineered environment, *Chaos, Solitons & Fractals*, 164, 112687.

Poggiali, F., Cappellaro, P., Fabbri, N. (2018). Optimal control for one-qubit quantum sensing, *Physical Review X*, 8, 021059.

ReportLinker. (2022). The Global Quantum Sensors Market size is expected to reach $619.8 million by 2028, rising at a market growth of 16.4% CAGR during the forecast period. [Online]. Available: https://www.globenewswire.com/news-release/2022/11/22/2561070/0/en/The-Global-Quantum-Sensors-Market-size-is-expected-to-reach-619-8-million-by-2028-rising-at-a-market-growth-of-16-4-CAGR-during-the-forecast-period.html

Wu, W., Shi, C. (2021). Criticality-enhanced quantum sensor at finite temperature, *Physical Review A*, 104, 022612.

Yuan, H. (2016). Sequential feedback scheme outperforms the parallel scheme for Hamiltonian parameter estimation, *Physical Review Letters*, 117, 160801.

Zhong, W., Sun, Z., Ma, J., Wang, X., Nori, F. (2013). Fisher information under decoherence in Bloch representation, *Physical Review A*, 87, 022337.