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Embedding Hierarchical Fibonacci Cubes In Faulty Hierarchic Cubic Networks

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Abstract- Hierarchical Fibonacci Cubes HFC(n+2) can be obtained from the Hierarchic Cubic Network HCN(n,n) by removing certain nodes and edges. This problem is very simple when no faulty node exists in an HCN(n,n), however, it becomes very sophisticated if some faulty nodes appear in an HCN(n,n). In this paper, we tried to distinguish HFC(n+2) in faulty HCN(n,n), and it can also be considered as a fault-tolerant embedding in HCN(n,n). Then, we shall show how to dierctly embed a HFC(n+2) into a faulty HCN(n,n) and prove that if no more than two clusters which contain faulty nodes, then HFC(n+2) can be directly embedded. Another case is that if there are more than two clusters which contain faulty nodes, then the labels of faulty nodes must be (I_i,K) and (I_i,L) for each cluster.

Keywords. Hierarchical Cubic Network - HCN(n,n), Hierarchical Fibonacci Cube – HFC(n), Interconnection Network Embedding.

1.INTRODUCTION

When the number of nodes and links increase in the parallel systems, there is more chance to occur failure(s) in the system, especially because of higher failure rates to these bigger systems, there must be some methods to overcome this problem or make system continue to operate in the case of failures. As the number of components increases, so does the probability of a component failure. The number of nodes in HCN(n,n) increases exponentially, while dimension of HCN(n,n) increases linearly [1,3,6]. For a large system, the probability of faults arising is non-negligible, and over its task duration, the system might involve one or several faults.

In this paper, we consider the problem of embedding HFC(n+2) [2,4,5] in HCN(n,n) when there are node faults. While dimension of HCN(n,n) increases, the probability of fault arising in the system will increase as well. However, eventhough, faults occur in HCN(n,n), there is opportunity to derive special proper subgraphs of HCN(n,n) directly or by relabeling the nodes of HCN(n,n). The main idea of deriving HFC(n+2) from faulty HCN(n,n) is to translate labels of faulty nodes which involve more than one consecutive 1s. This case motivate us to consider the embedding of HFC(n+2) in the faulty HCN(n,n).

In this paper, we propose a method for embedding HFC(n+2) onto HCN(n,n) in case of faulty nodes. There are four cases

a) There is only single faulty node and let this node be v. Translating HCN(n,n) with respect to $v\oplus 1^{(n)}$.
b) If all faulty nodes are in the same cluster, and let this cluster be (I,*). Then translate faulty HCN(n,n) with respect to $I\oplus 1^{(n)}$.
c) If two faulty nodes are in two clusters and let these clusters be (I,J) and (K,L), then h=H(I,K) and permute HCN(n,n) with respect to $Pm(q,I\oplus K)=0^{(n-h)}|1^{(h)}$. Then translate resultant HCN(n,n) with respect to $Pm(q,I)\oplus 0^{(h)}|1^{(n-h)}$. Similar case can be seen in [7].

d) All faulty nodes are distributed into two clusters. Then the method applied in third case is also applicable in this case.

This paper is organized as follows. Section 2 describes HCN(n,n) and HFC(n) architectures and introduces the operations applied in embedding process. Section 3 contains the method proposed for embedding HFC(n+2) into HCN(n,n) in case of node faults. Then finally, last section concludes this paper.

2.PRELIMINERIES

In this section, we will introduce HCN(n,n), HFC(n) and some operations used in embedding process.

Node v has address $(v_{n-1}v_{n-2}\ldots v_0)$ and t has address $(t_{n-1}t_{n-2}\ldots t_0)$ for some integer n and $u=v\oplus t=(u_{n-1}u_{n-2}\ldots u_0)$ where $u_i=v_i\oplus t_i$, $0\le i<n$.

The **Hamming distance** between v and t is

$$H(v,t)=\sum_{i=0}^{n-1} v_i \oplus t_i \ .$$

HFC(n)

The HFC(n) interconnection network is a proper subgraph of HCN(n-2,n-2) whose definition can be given as follows [6,7]:

Definition 1. *[4] A HFC(n) is an interconnection network graph and it contains Fibonacci Cubes - FC(n)s as basic building blocks and the node label is (I,J) where I is the label of building block and J is the node number in I^{th} block. If HFC(n)=(V_H(n),E_H(n)), then $V_H(n)=\{(u,v) \mid u,v\in F$ and $F=\{0,1,\ldots, f(n)\}$ is a set of Fibonacci numbers $\}$*

and let (I,J), (K,L) be two nodes in HFC(n) and I, K are clusters' labels and J, L are nodes' labels, then

$$E_H(n)=\bigcup_{x,y\in V_H(n)} \{((I,J),(K,L)) \mid x=(I,J) \text{ and } y=(K,L)\wedge\big((I=K\wedge H(J,L)=1) \vee (I\neq K\wedge I=L\wedge J=K)\ \big)\}$$

In general, interconnection network HFC(n) can be derived from HCN(n-2,n-2) by removing all nodes, which are not included in Fibonacci cube, with incident edges from HCN(n-2,n-2). The edges ((I,I), $(\bar{I},\bar{I})$) are also removed from HCN(n-2,n-2) and remaining interconnection networks are interconnection network HFC(n)s. So, two nodes (I,J) and (K,L) in HFC(n) are connected if and only if one of the following conditions holds.

- I=L and J=K.
- I=K and H(J,L)=1.

The edges within a cluster are called horizontal edges and the edges between clusters are called diagonal edges.

The number of edges in HFC(n) is

$$|E_H(n))|=|V(n)|.|E(n)|+|V(n)|(|V(n)|-1)/2= f_n\left[\frac{2(n-1)f_n - nf_{n-1}}{5}\right]+\frac{f_n^2 - f_n}{2}$$

with initial condition $|E_H(3)|=3$, $|E_H(4)|=9$, and the number of nodes in HFC(n) is $(f_n)^2$. The node degree in HFC(n) is between $\left\lceil\frac{n+1}{3}\right\rceil-1$ and n-1.

Let us introduce some operations used in the embedding process and these operations are translation, and permutation.

Definition 2. *In [7], translation of a node v by a node t means a bitwise exclusive-OR of the address, $Tr(t,v)=t\oplus v$. With translation of graph G(V,E) with respect to a node t, it means a graph Tr(t,G(V,E))=G(Tr(t,V),Tr(t,E)), where $Tr(t,V)=\{Tr(t,v) \mid \forall v\in V\}$ and $Tr(t,E)=\{(Tr(t,u), Tr(t,v)) \mid \forall (u,v)\in E\}$.*

Definition 3. *In [7], permutation of a node v by a sequence q means a bitwise permutation of its address into a new sequence $q=(q_{n-1},q_{n-2},\ldots, q_0)$, $q_i\in D$, $\forall 0\le i\le n-1$, such that $Pm(q,v)=(v_{q_{n-1}}v_{q_{n-2}}\cdots v_{q_1}v_{q_0})$. With permutation of a graph G(V,E) with respect to a sequence q, it means a graph Pm(q,G(V,E))=G(Pm(q,V),Pm(q,E)), where $Pm(q,V)=\{Pm(q,v)|Pm(q,V)\}$, and $Pm(q,E)=\{(Pm(q,u),Pm(q,v))| \forall (u,v)\in E\}$.*

For example, let v=00100 and v is a faulty node label. If it is wanted that v has maximum number of 1s, then $v\oplus t=11111$. So, t=11011. Again if Pr(q,v)=00001, then q=(4,3,1,0,2).

3. EMBEDDING HFC(n+2) in FAULTY HCN(n,n)

HFC(n+2) is an interconnection network based on the FC(n)s, and it uses FC(n+2) as basic building blocks. It is known that FC(n+2) is a proper subgraph of n-cube and n-cube is used as basic building block while constructing HCN(n,n). Thus, HFC(n+2) will be a proper subgraph of HCN(n,n).

Therefore, while there is/are faulty node(s) in HCN(n,n), HFC(n+2) can be embedded onto this interconnection network. It is trivial to see the case where the faulty nodes are not included in the HFC(n+2). In this case, embedding of HFC(n+2) onto faulty HCN(n,n) is done by removing faulty nodes from HCN(n,n) directly. In the case of faulty edges like $((I,I),(\bar{I},\bar{I}))$, the solution is also trivial, since these edges of HCN(n,n) are not included in the HFC(n+2).

When the faulty nodes are included in the HFC(n+2), embedding of HFC(n+2) onto HCN(n,n) is not so simple. There are some cases due to faulty nodes distribution in HCN(n,n).

The terms or notations will be used in this paper can be briefly described as below, and these notations can be seen in [7] in partial.

- $1^{(n)}$ and $0^{(n)}$ represent the n consecutive 1 and n consecutive 0, respectively. Let S_1 and S_2 be binary string and $S_1|S_2$ denotes the concatination of S_1 and S_2.
- Let v be a node in a graph G and OP(x) denotes any operations defined in Definitions 2 and 3 on x. G_1=OP(G) always implies v_1=OP(v).
- The node label in HCN(n,n)/HFC(n+2) is (I,K). The cluster in this architecture can be denoted as (I,*). I is label of cluster and let the sets of labels of clusters in HCN(n,n), HFC(n+2) be C_{HCN} and C_{HFC}, respectively.

The method applied in this paper is inspired by [7]. F.-S. Jiang et al [7] applied this method in the case of embedding generalized Fibonacci Cubes onto hypercubes with faulty nodes. We applied this method in the case of embedding Hierarchical Fibonacci Cubes onto Hierarchic Cubic Networks with faulty nodes.

Case 1. One Faulty Node in HCN(n,n)

While there is a faulty node in HCN(n,n) whose label is (K,L), the translating pattern t is obtained by using L binary string, and $t=L\oplus 1^{(n)}$. After obtaining t, HCN(n,n) is translated with respet to t and let obtained architecture be HCN(n,n)'. It is easy to see that HFC(n+2) is a subgraph of HCN(n,n)'. Fig.1 illustrates the case of one faulty node in HCN(2,2) and obtained HCN(2,2)' inteconnection network which contains HFC(4). That is, HFC(n+2) is a subgraph of Tr(t,HCN(n,n)). The special case is that if each cluster contains only one faulty node and labels of all faulty nodes are same, the same procedure for single faulty node is also valid for this case.

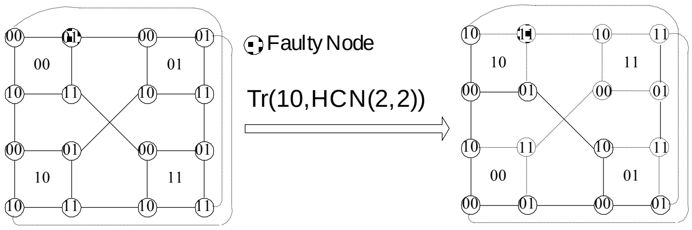

Figure 1. Obtaining HFC(n+2) from HCN(n,n) contains one faulty node.

Case 2. All Faulty Nodes in a Cluster

There are 2^n nodes in a cluster of HCN(n,n), and if all faulty nodes are in the same cluster, this case can be regarded as a node faulty in n-cube. Let all faulty nodes be in cluster (I,*) and if $I \notin C_{HFC}$, then HFC(n+2) is still embedable onto faulty HCN(n,n) without applying any operations on the labels of nodes in the HCN(n,n). If $I \in C_{HFC}$, then $t = I \oplus 1^{(n)}$ and HCN(n,n)'=Tr(t,C_{HCN}). After that, removing faulty nodes will remain HFC(n+2) in HCN(n,n)'. Fig. 2 illustrates the case of number of faulty nodes upto 2^3 in the cluster (010,*) and translating the labels of clusters with respect to 101.

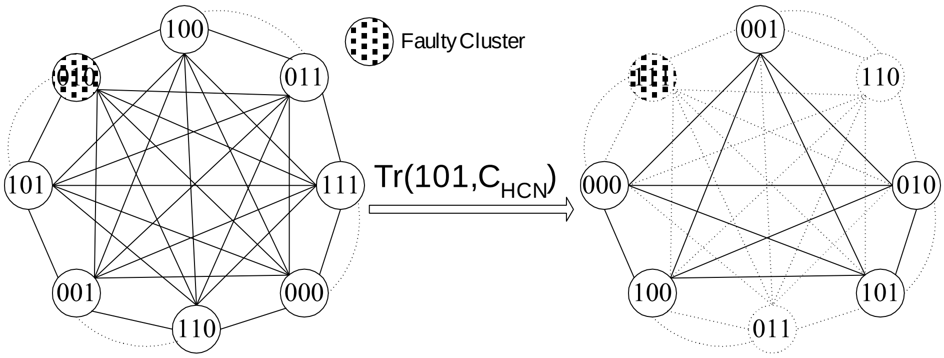

Figure 2. All faulty nodes are in cluster (010,*) and translating just labels of clusters will remain HFC(5) in resulting architecture.

If all faulty nodes are in cluster (I,*) and number of consecutive 1s in I is at most one, then translation process is applied to labels of clusters. Otherwise, the cluster contains faulty nodes is removed.

Case 3. Two Faulty Nodes in Different Cluster

Let faulty nodes be (I,J) and (K,L). If $I,K \notin C_{HFC}$, there is no need to apply translation or/and permutation operation(s) to faulty HCN(n,n) for embedding HFC(n+2) onto faulty HCN(n,n). Otherwise ($I,K \in C_{HFC}$ or $I \in C_{HFC}$ and $K \notin C_{HFC}$ or $I \notin C_{HFC}$ and $K \in C_{HFC}$ or $I,K \notin C_{HFC}$), we need convert labels I and K and they will not be in C_{HFC}.

In this case, only translation operation is not enough to convert labels of faulty clusters to labels which are not included in C_{HFC}. We must apply permutation and translation operations in consecutive manner. It is known that any label in HFC(n+2) does not have more than 1 consecutive 1s. The bit-positions which include 1 of $I \oplus K$ correspond to inconsistent dimension in HCN(n,n). So, these dimension must be located at leftmost/rightmost sides of labels of clusters. By this way, faulty clusters with converted labels will not be in HFC(n+2). Converting process is as follows.

h=H(I,K) and there is h inconsistent dimensions.

$M = I \oplus K$

The permutation sequence q must convert M to $M_1 = (0^{(n-h)} | 1^{(h)})$, That is, $M_1 = Pm(q,M) = (0^{(n-h)} | 1^{(h)})$.
After obtaining permutation sequence q,

$I_1 = Pm(q,I)$, $K_1 = Pm(q,K)$, and $C^1_{HCN} = Pm(q, C_{HCN})$.

The translation pattern t is $t=I_1\oplus(0^{(h)}|1^{(n-h)})$. Then C^1_{HCN} is translated by t, and $C^2_{HCN}=Tr(r, C^1_{HCN})$. After this point, the labels of faulty clusters have inconsistent dimensions as located at leftmost/rightmost side. The case of two faulty nodes in HCN(n,n) is depicted in Fig. 3. Special case is that if all clusters contain at most two faulty nodes, and let labels of faulty nodes be (I_i,K) and (J_i,L), $I_i\neq J_i$ and $I_i=\{0,1,\ldots,2^n-1\}$. Then permutation and translation operations are applied to node labels in the consecutive manner.

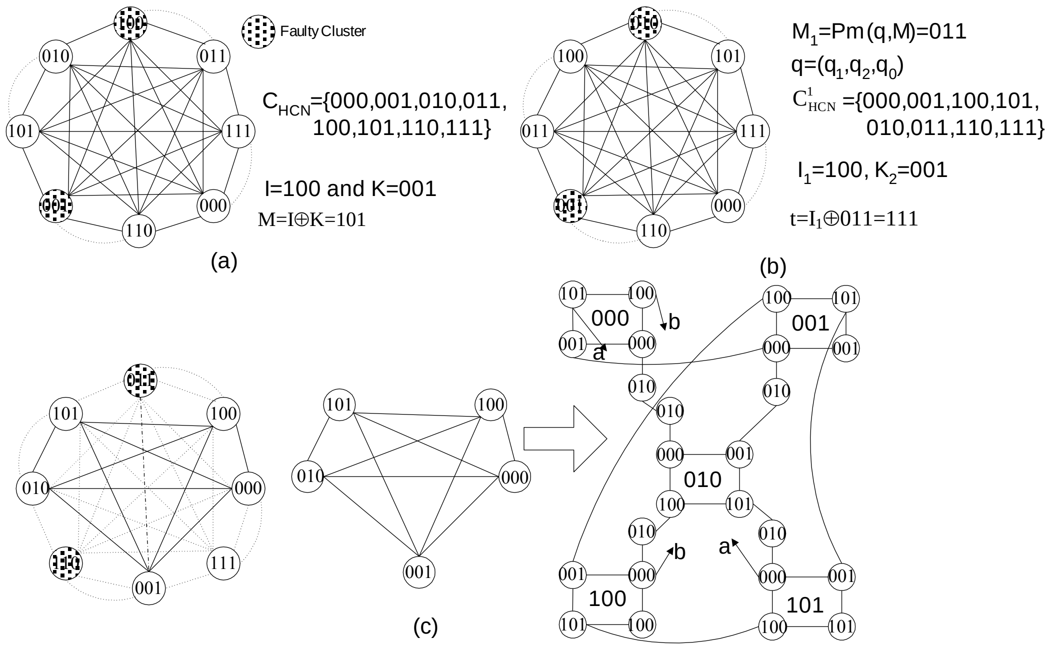

Figure 3. a) Two faulty nodes in HCN(3,3); b) Permuted faulty HCN(3,3) with respect to permutation sequence q=(1,2,0); c) Translated faulty HCN(3,3) with respect to t=111 and obtained result contains HFC(5) and its figure.

Case 4. Upto 2^{n+1} Faulty Nodes Distributed in Two Clusters.

This case is similar to Case 3. The labels of clusters are permuted and then translated with respect to obtained permutation and translation sequences, respectively. The clusters contain faulty nodes have labels contain more than 1 consecutive 1s, and so, these clusters can be removed from faulty HCN(n,n). Resulting architecture contains HFC(n+2).

4. CONCLUSION and DISCUSSION

HFC(n+2) is constructed by using FC(n+2) as building blocks and it is known that FC(n+2) is a proper subgraph of n-cube. HCN(n,n) is constructed by using n-cube as basic building blocks. So, it is obvious that HFC(n+2) can be derived from faulty HCN(n,n). The important point is that the number of faulty nodes and their distribution in HCN(n,n) limit us to derive HFC(n+2).

The proposed method in this paper can be considered as deriving HFC(n+2) from faulty HCN(n,n) in case of two faulty clusters. Since if all faulty nodes are distributed into two clusters, then the proposed method does work, or there are only at most two faulty nodes in each cluster have the same labels.

The processes applied to faulty HCN(n,n) for obtaining HFC(n+2) are not sophisticated and they are simple and easy to apply.

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