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Research Article

Iterated Modified Tabu Search based Equitable Coloring for Scheduling Cricket World Cup Tournament

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Abstract : In this article, an Iterated Modified Tabu Search (IMTS) approach is presented by improving certain aspects of general Tabu Search to enhance the approximation of the Equitable coloring problem (ECP) problem for a real-world problem of scheduling the ICC Cricket World Cup tournament. The proposed IMTS introduces new point generation mechanisms and parameter updating rules to achieve this objective of the tournament schedule. The IMTS algorithm defines different k-ECP instances and utilizes the search process to determine the optimal solution for an instance of k-ECP by estimating the minimum k-coloring value. An illustration of resolving the Cricket World Cup tournament scheduling problem using the proposed IMTS algorithm is provided. Also, an assessment of the IMTS is also performed on a commonly used benchmark instance. Both the results illustrate that the IMTS provided comparatively better solutions with high quality and computational efficiency.

Keywords : Computational Efficiency, Cricket World Cup, Equitable Coloring Problem, Iterated Modified Tabu Search, Scheduling Problem

1 Introduction

The Equitable Coloring Problem (ECP), a specialized variant of the general graph coloring problem, incorporates an essential equity constraint: the disparity in size between any two random color classes must not exceed one unit. Real-world problems like scheduling can be modelled as ECP for finding equitable chromatic number and resolved efficiently by avoiding incompatible allocation of tasks or timings. However, the additional condition of ECP makes it NP-hard problem and difficult to solve especially for problems with large-sized instances. Hence for solving the NP-complete problem and finding equitable chromatic number of large-sized graph, the heuristics and search algorithms have been introduced. Tabu Search algorithm emerges as a prominent choice, applied extensively for solving ECP to automatically compute the k-chromatic number. However, studies indicate that, the Tabu Search algorithm increases time complexity particularly when confronted for larger graphs.

Graphs are discrete structures containing vertices connected via edges which are employed in all domains as abstract models for analysis and illustration of real-world processes and problems [1], [2]. The relations between the entities in most domains like bonds between atoms and elements in chemistry, work scheduling, bonds in DNA, etc. are more effectively illustrated in the graphs [3]. This leads to active graph analysis tasks that can be formulated into problems and resolved using strategic techniques and algorithms. Among all the challenges, scheduling quandaries encompass a universal classification involving allocation predicaments, transcending domain boundaries and necessitating adept solutions. Graph coloring problem (GCP) [4] is one such method to formulate the scheduling problems based on graph theory. The fundamental aim of GCP lies in the allocation of colors to nodes within a graph, ensuring that neighboring nodes connected by an edge remain distinct in coloration. Programming system tasks and the arrangement of objects often exhibit seamless compatibility with the principles of the Graph Coloring Problem (GCP) [5]. Nonetheless, this approach encounters constraints when addressing specific scheduling complexities, such as the equitable distribution of workloads among workers or the appropriate allocation of time to clients. In these instances, the challenge lies in avoiding scenarios where a biased distribution emerges one worker burdened with an excessive workload while another is assigned significantly fewer tasks, owing to an uneven allocation strategy. To overcome these issues, the ECP [6] is formulated as a variant of GCP. An equitable k-coloring of an undirected graph are partitions of its nodes into k disjoint independent sets where the colors of two independent sets vary maximum by one. In simple terms, an equitable coloring with k colors is a conflict-free coloring with k colors, if it fulfills the equitable coloring conditions [7]. As a variant of the GCP, the ECP apprehends defining a minimum k called as equitable chromatic number. Leveraging the framework of ECP, an abundance of challenges can be proficiently tackled using equitable scheduling strategies [8].

Round-robin sports tournament scheduling is one of the problems that can be resolved automatically using the ECP formulation. Round-robin sports timetables are pivotal in any multi-team tournament, including the World Cup Football tournaments and related domestic leagues, Olympic Games, Cricket World Cups and country-based cricket leagues especially the most popular Premier League Cricket tournaments and almost all multi-player and multi-team tournaments [9]. The round-robin scheduling includes n teams and constraint of all teams playing remaining teams precisely m times with a pre-determined number of rounds. Towards the end of the scheduled phases, the teams with higher points or most wins will advance to the next rounds. The two types of schedules in round robin are single round robins where two teams meet only once and double round robins where two teams meet twice before the end of the schedule. The major scheduling problem in these tournaments encompasses the traveling tournament problem and the availability of teams at specified times [10]. ECP conceptualizes round-robin scheduling as graph-based problems and tackles them using search-based algorithms.

However, ECP is a NP-complete problem (NP-hard and NP combined together) which renders its resolution more intricate. Many heuristic algorithms have been used to resolve ECP among which Tabu search is the most prevalent algorithm. In this paper, the ICC Cricket World Cup 2019 tournament scheduling problem is analysed and modeled as ECP. The Iterated Modified Tabu Search (IMTS) Algorithm is proposed, aiming to mitigate the time complexity associated with solving ECP. The proposed IMTS is an enhanced version of the Tabu search process whose new solution-generating process and updating of search parameters are improved to provide an efficient approximation. The subsequent sections are structured as follows: Section 2 provides a succinct overview of recent related research; Section 3 delineates the proposed approach for ECP resolution, while the Section 4 demonstrates its practical application in real-world scheduling scenarios. Section 5 presents the evaluation results, and finally, Section 6 encapsulates the conclusions drawn from the study.

2 Related Works

Due to its profound relevance across multiple studies, a multitude of research endeavours has been steadfastly directed towards the identification of efficient solutions for the ECP, with a parallel emphasis on their practical integration into real-world applications. Yan et al. [11] presented an innovative approach to equitable coloring of Cartesian products using balanced comprehensive manifold graphs. This approach utilizes the balancing factor to connect the graph based on the equity constraints of ECP. Bahiense et al. [12] offered a branch-and-cut approach for the ECP based on inter-programming representatives. This approach uses a primitive heuristic, splitting tactics and the first branch-and-cut strategy and enhances solutions for the ECP, exhibiting an improved average relative gap.

DSATUR, a pivotal graph colo‘ring algorithm devised by mathematician Daniel Brélaz and rooted in the principles of the greedy algorithm, stands as a cornerstone in this domain. Its extensive utilization for effectively addressing the challenges posed by the ECP underscores its significance. San Segundo [13] proposed a new DSATUR approach for precise vertex coloring problems by maximizing the saturation point to choose a new nominee vertex to color. Notably, it is one of the proficient solutions for any ECP approximation. Méndez-Díaz et al. [14] proposed a polyhedral approach associated with a 0,1-integer program design for ECP. Méndez-Díaz et al. [15] developed an exact DSatur-based algorithm with novel pruning procedures precisely developed from ECP constraints. Further expanding on this foundation, Méndez-Díaz et al. [16] also introduced an advanced pruning measure from equity constraints based on the popular DSATUR approach. This approach exploited arithmetical properties essential in equitable coloring and associated them with the methods of DSatur to provide an effective approximation. Though DSatur is widely recognized to provide better coloring than greedy algorithms, it falls short in comparison to the Recursive Largest First algorithm. This leads to the search for advanced DSatur and other algorithms for solving ECP.

Recent research has employed the heuristic algorithm for solving the NP-hard problem of the ECP. The profound and efficacious algorithm is the Tabu Search-based heuristic projected by Méndez-Díaz et al. [17] which uses a new local search criterion. This approach utilized the dynamic Tabu version of previous research to improve the ECP approximation. Wang et al. [18] proposed a hybrid Tabu search algorithm with feasible and infeasible searches for ECP. The process substitutes a possible local search where the search centers on the most applicable and practicable solutions and an infeasible local search where an organized exploration of solutions is acceptable by comforting the equity constraint. The hybrid Tabu search algorithm provided satisfactory performance on common benchmark instances.

Lai et al. [19] suggested a solution to the ECP using backtracking based iterated Tabu search in which the approximation of the ECP is performed with different fixed k values. The iterated Tabu search determines the k-coloring while the backtracking system adjusts k to an appropriate value and the binary search determines a good initial k value. The experimental analysis on common benchmark instances showed that this approach resulted in a better approximation of ECP than existing heuristic methods. Sun et al. [20] proposed a memetic search process that utilizes a backbone-based crossover operator, a 2-phase Tabu search strategy to resolve NP-hard ECP. Among the heuristic algorithms, the Tabu search based algorithms provide better colorings for ECP and also significantly reduce the complexity. However, as the scale of instances escalates to larger proportions, these algorithms tend to exhibit heightened time complexities. Hence, this study acknowledges these challenges as focal research issues and introduces IMTS algorithm for overcoming those limitations in resolving the ECP effectively.

3 Iterated Modified Tabu Search Algorithm for the ECP

The proposed IMTS operates in a solution space where the equity conditions is fulfilled and only the conditions of graph coloring may be ignored. The k-ECP can be resolved by determining the solutions even after k+1 and continuing with k-1, k-2,… until the solution is found [21]. The iterative process of IMTS results to obtain optimal k-coloring with k∈ k*-1,k*-2,…,k*-m where k* has receiving the minimum number of equitable k* colors and m is a back tracking depth parameter such that m>1. The algorithm of the complete solution finding approach using IMTS is given as follows:

Algorithm 1 Proposed approach using IMTS algorithm for ECP

Require: Graph $G = (V, E)$, the number of colors k, the perturbation parameter β, the depth of tabu search α
Ensure: The best number of colors k^* and an equitable k^*-coloring solution s^*

```
k^r, s^r are initial k and s values determined by binary search (BS)
(k^r, s^r) ← BS(V, E, α)
Update best results k* ← k^r, k ← k^r, s* ← s^r
repeat
    if k = k* − 1 or k = 2 then
        k ← k* − 1
    else
        k ← k − 1
    end if
    Resolving equivalent k-ECP using IMTS, s ← IMTS(k, G, α, β)
    if f(s) = 0 then
        k* ← k
        s* ← s
    end if
until Time T ≥ T_max
return k* and s*
```

Algorithm 1 consists of three main processes: binary search to set an appropriate initial k value (k^r), backtracking mechanism to set k for IMTS and finally using IMTS for solving the k-ECP and updating k^*. The proposed IMTS approach for ECP terminates once the time reaches maximum limit even if the solution is not found. At this stage, the smallest number k^* obtained in the final step is considered as the approximate solution for the ECP.

For a given k-ECP, the IMTS initially searches the solution space where all possible k-classes assuring the equity constraint. Ω_k is the search space which is formulated as

$$\Omega_k = \{C : ||V_i| - |V_j|| \leq 1; i \neq j\} \tag{1}$$

where C is the k-classes denoted as $C = \{V_1, V_2, \cdots, V_k\}$ and $i \leq 1, j \leq k$ are the instances. The whole search space Ω is exploited by th e IMTS as given by

$$\Omega = \bigcup_{k=1}^{n} \Omega_k \tag{2}$$

In this equation, the k-classes of the search space guarantee the equity constraint but the coloring constraint is not guaranteed and might result in adjacent vertices getting the same color [22], [23]. Hence it becomes important to estimate the quality of the k-class solution candidate. For achieving this objective, the IMTS familiarizes an evaluation function $f\ (s))$ by totalling the sum of all conflicting edges in a k-class solution. Assigning $s = \{V_1, V_2, \cdots, V_k\}$ as the k-class equity in Ω, the evaluation function is given as.

$$f\ (s) = \Sigma_{V_i, V_j \in E}\, \delta\ (i, j) \tag{3}$$

where,

$$\delta\ (i,j) = \begin{cases} 1 & if\ E \in \{1, 2, \cdots, k\} \\ 0 & otherwise \end{cases} \tag{4}$$

The selected k-class solution is considered ideally equitable k-coloring satisfying both the equity and coloring constraints only when the $s \in \Omega$ and $f(s) = 0$. This solution is considered ideal since the search process of IMTS filters between the available k-class solutions by attaining the optimal solution with evaluation $f(s) = 0$, thus resulting in effecting transitioning of solutions for k-ECP.

3.1 IMTS procedure for k-ECP

The proposed IMTS procedure includes the process of initialization and the process of applying the modified tabu search for obtaining the mandatory solution. This process is repeated for n iterations and the perturbation operator is used by IMTS to modify the mandatory solution to obtain new mandatory solution. This solution will be considered mandatory until the better solution is produced in other iterations. This iterated process of the modified search process will eliminate the conflicting solutions and results in optimal equitable k-coloring solution until β number of consecutive perturbations. Algorithm 2 shows the IMTS procedure for k-ECP. This algorithm determines the best mandatory solution based on the evaluation function $f(s)$.

Algorithm 2 IMTS procedure for k-ECP

Require: Graph $G = (V, E)$, k, β, α
Ensure: The best solution s

```
1: Initializing s ← Solution(V, E, k)
2: Applying s ← Modified Tabu Search(s, α)
3: Consecutive perturbation counter d ← 0 for unchanged s
4: repeat
5:     s′ ← Perturbation Operator(s)
6:     s″ ← Modified Tabu Search(s′, α)
7:     if f(s″) < f(s) then
8:         s ← s″
9:         d ← 0
10:    else
11:        d ← d + 1
12:    end if
13: until d = β or f(s) = 0
14: return s
```

Step 3 of Algorithm 2 presents the initialization process. The sole purpose of the solution initialization process will be to generate an initial solution with fewer conflicts. Algorithm 3 presents the solution initialization process. Let U be the list of unassigned vertex nodes and v is the randomly selected vertex. The set of neighbors of v in V_i is given by $\Gamma^i(v)$. This initialization process is performed by randomly selecting the vertices and assigning them with the suitable k color classes. After the completion of the assigning processes, the remaining vertices are categorized as unassigned vectors and are assigned to one of the existing k-color class based on the greedy approach.

Algorithm 3 Solution initialization of IMTS

Require: Graph $G = (V, E)$, k
Ensure: k-class candidate for ECP

```
for i ∈ [1, k] do
    V_i ← ∅
end for
Set of unassigned vertices U ← V
for i ∈ [1, k] do
    Select a vertex v randomly from V
    V_i ← V_i ∪ {v}, U ← U \ {v}
end for
i ← 1
while U ≠ φ do
    v ← arg min{|Γ^i(v)| : v ∈ U}
    V_i ← V_i ∪ {v}, U ← U \ {v}
    i ← 1 + (i mod k)
end while
```

The preliminary process of setting the solutions is performed by defining the initial value of i as 1 and i-th color class V_i as the initial class. This initial color class is assigned with the unassigned vertex v that has the smallest number of neighbors in V_i. This random assigning process is continued by setting $i \leftarrow 1 + i\ modk$, and repeating the steps again for all other vertices. After assigning color classes to all vertices, the newer solution is saved and the optimal solution is determined by the modified Tabu search process presented in steps 4 and 8 in Algorithm 2.

3.2 Modified Tabu Search Algorithm

The modified Tabu search is achieved by improving the new point generation mechanisms and parameter updating rules of the basic Tabu search [24]. The main purpose of modifying the basic Tabu search is to guarantee the best balance between exploration and exploitation searches and improve the convergence rate such that the algorithm does not end up in the local optimum. The entire search process of the Tabu search consists of the intensification, diversification and refinement phases. The intensification phase initializes the search to quickly obtain optimal point. The diversification phase exploits the unknown spaces for better optimal points while the refinement phase filters the obtained best points and selects the global optimum solution. As defined above, the intensification and refinement phases searches for optimal solution among the available points while diversification phase generates new solutions for the other phases. To improve this process, the Gaussian probability density function (pdf) is used in the modified Tabu search. The main constraint for generating new solutions is that the $68-95-99.7$ rule, which defines that 68% of the points obtained from a Gaussian distribution are inside one standard deviation from the mean value, while 95% of the points are within two and 99.7% are within three standard deviations. This constraint is satisfied by generating new points in a relatively small neighbor of the specific point. Similarly for diversification phase, better local optima must be obtained to avoid stagnating at a local optimal point. For this purpose, Cauchy pdf is used which produce new points that stay in unexplored spaces from the current optimal points without constraints like Gaussian. The Gaussian and Cauchy pdf are given by.

$$G(x_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} exp\left(-\frac{(x_i-\mu_i)^2}{2\sigma_i^2}\right) \tag{5}$$

$$C(x_i) = \frac{1}{\left(\pi\gamma_i\left[1+\left(\frac{x_i-a_i}{\gamma_i}\right)^2\right]\right)} \tag{6}$$

where x_i is the solution point, μ_i is the mean value, σ_i is the standard deviation, γ_i is the scaling parameter and a_i is the location of the optimal solution. Based on these pdf functions, the new solution points can be obtained as

$$x_i = \mu_i + \sigma_i P \tag{7}$$

$$x_i = a_i + \gamma_i\ tan\left[\pi\left(P-0.5\right)\right] \tag{8}$$

Here P is the accumulation function specified as the integral of either Gaussian or Cauchy pdf denoted as $F\ (u)$ in the below equation.

$$P = \int_{-\infty}^{\infty} F\left(u\right) du \tag{9}$$

Secondly, the parameter updating rule is improved based on the scalar parameter γ and standard deviation σ. Setting larger value of these parameters will yield in low convergence speed while a smaller value will reduce the global searching capability. Hence dynamic updating is used for automatically determining these parameters for obtaining optimal solution.

$$a_{i+1} = a_0 - a_0\left(1-\frac{1}{i}\right)^q \tag{10}$$

where $a_0 = \frac{x_u - x_l}{10}$ and $q = log_{29/30}(w)$ with w is the attenuation parameter and x_u and x_l are the higher and lower limits of the dynamic strategy.As the searches advances, a is reduced steadily and improves the converging speed of the algorithm. Based on this process, the search process is performed in IMTS which is given in Algorithm 4.

Specified a neighbourhood function N, the evaluation function $f(s)$ in equation (3), and a given initial solution s_0, the modified Tabu search provides the best solution to replace the mandatory solution. Here d is the parameter set to count the consecutive iterations where s_b is not updated i.e., s_b stays the best solution. By using the modified steps as described above, the optimal solution is obtained in less time and also the solution is globally optimal, thus averting the local optima situation. The utilization of Gaussian and Cauchy pdf in Tabu search has significantly improved the optimal solution determination as shown in Figure 1.

Algorithm 4 Modified Tabu Search process

Require: Input solution s_0, the neighbourhood N, and α
Ensure: Best solution s_b

```
Generate new solution using G(x_i) and C(x_i)
Current solution s ← s_0
Best solution obtained until now s_b ← s
Iteration counter d ← 0
repeat
    Select the best neighbourhood solution s' ∈ N(S)
    s ← s'
    Update parameters using Equations 8 and 9
    Update Tabu list
    if f(s) < f(s_b) then
        s_b ← s
        d ← 0
    else
        d ← d + 1
    end if
until f(s) = 0
return s_b
```

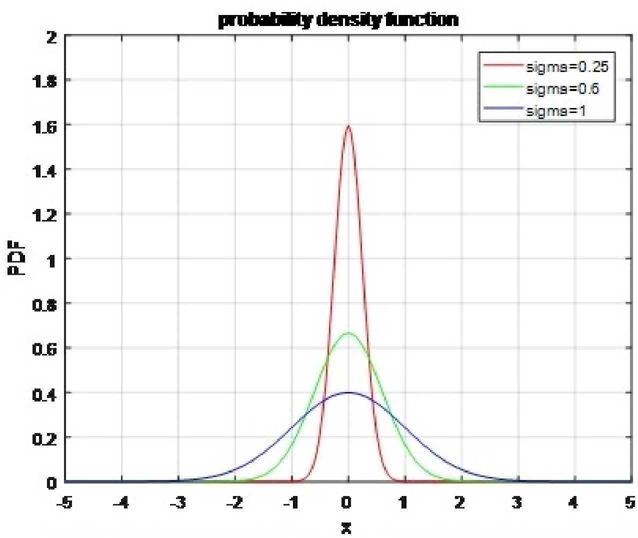

Figure 1: Gaussian and Cauchy pdf performance in IMTS

4 Application of IMTS for ECP in Scheduling ICC Cricket World Cup 2019

ICC Cricket World Cup is the global competition conducted by the International Cricket Council (ICC) every four years. The Men's World Cup tournament held in 2019 is modeled as ECP for scheduling the tournament. The 2019 event was contested by men's national teams of ten cricketing countries. It was from 30 May to 14 July, 2019 with the league matches taking place in round robin format. The semi-finals and final were played as knock-out format. The tournament was organized in seven cricket grounds across six cities in England and Wales. The matches were scheduled on the basis of flexibility of the TV audience with one match taking place each weekday and two matches taking place in weekends to attract the cricketing crowds. This strategy is mainly based on the profit model but it also considers the travel flexibility of the cricketing players and officials. In this work, the multi-nation tournament is considered for evaluating the effectiveness of ECP in solving the scheduling problems. For simplicity, only the main matches in the league stage is considered while knock-out matches are left out due to their easy scheduling.

The 10 teams participated in the tournament are sorted in alphabetical order for evaluation. Afghanistan, Australia, Bangladesh, England, India, New Zealand, Pakistan, South Africa, Srilanka and West Indies are the teams. They are numbered $1, 2, \cdots, 10$ for match ordering. By sorting the match combinations, the single round robin format will lead to 45 matches in 9 rounds which are shown in the Table 1

As the tournament is single round robin, the teams are ordered as the nodes of a graph $G = (V, E)$ where each vertex $v \in V$ as

Table 1: Round-robin schedule format

Round	Matches				
1	1,2	3,9	4,8	5,7	6,10
2	1,3	2,10	4,9	5,8	6,7
3	1,4	2,3	5,9	6,8	7,10
4	1,5	2.4	3,10	6,9	7,8
5	1,6	2.5	3,4	7,9	8,10
6	1,7	2,6	3,5	4,10	8,9
7	1,8	2,7	3,6	4,5	9,10
8	1,9	2,8	3,7	4,6	5,10
9	1,10	2,9	3,8	4,7	5,6

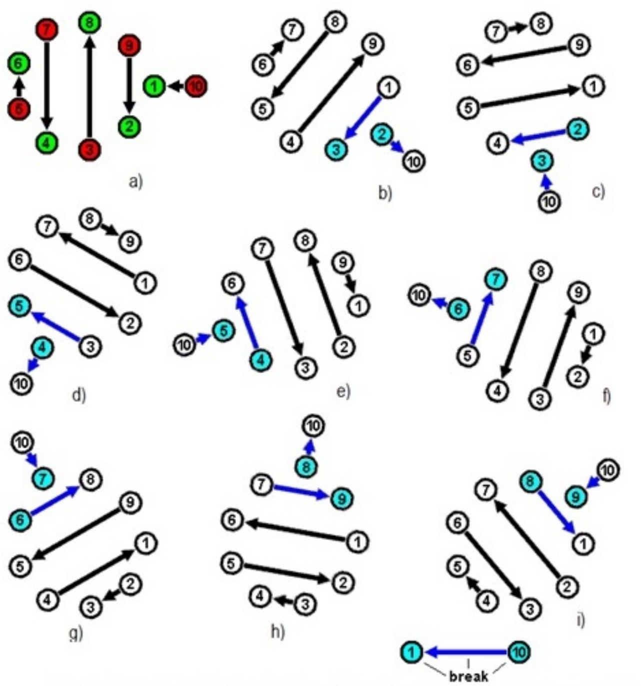

Figure 2: ICC World Cup Round robin schedule of nine rounds

an unordered pair $v = \{t_i, t_j\}$, representing a match amongst teams t_i and t_j. The vertices number $|V|$becomes $\frac{1}{2}n(n-1)$ and the number of possible colors $k = n - 1$ is determined for concrete scheduling. The edge dimensions are given by [25] as in equation 11

$$D = \frac{4(n-2)+1}{|V|-1} \tag{11}$$

Figure 2 shows the IMTS for World Cup tournament by splitting the scheduling process for each day. In this pattern, the tournament can be organized in 10 or 11 days including the knock-out matches. Each round can be contested in one day at 5 venues and thus the time can be minimized and also the expenditure.

Figure 3 shows the input graph plotted for evaluation for the tournament with the said 10 teams. The plot is obtained by the node plot structure used to construct the graph vertices. The edges are connected based on the matches assumed.

Figure 4 shows the Plot area of the graph for assigning the 10 teams as vertices of the graph G and utilizing it in scheduling the matches. Once scheduled, the edges of the graph denote the matches and coloring of the vertices illustrate the teams that can be scheduled in the same round. Scheduling of the matches on the same day reduces the overall timing of the schedule. On comparing with the original schedule of the tournament, few constraints are avoided. The main constraint is the scheduling based on TV audiences. The other constraint is the allocation of reserve days and possible extensions of playing time that might affect the travel schedule.

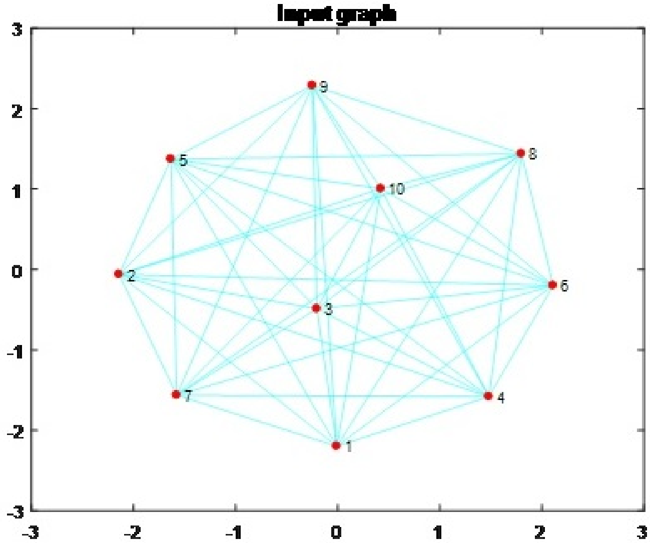

Figure 3: Input Graph of World Cup Tournament

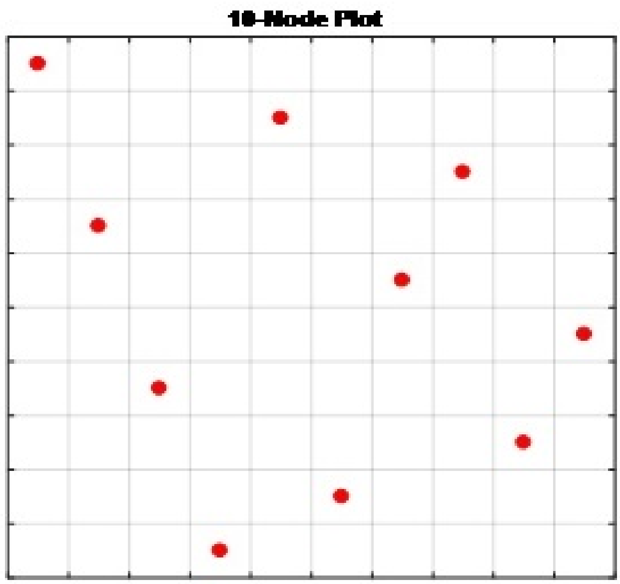

Figure 4: Plotting of the Scheduled Teams in Graph Vertices

Figure 5 shows the coloring output of the World Cup tournament scheduling problem. From the results, it can be seen that the coloring constraint of no adjacent vertices have the same colour and equity constraint are satisfied by the IMTS approach. This justifies the performance of the suggested IMTS algorithm for the ECP.

5 Performance Evaluation

The approximation of the proposed IMTS for ECP is performed in the previous section for the scheduling problem of ICC Cricket World Cup 2019. The results have shown effectiveness of the proposed approach. In addition to that evaluation, the proposed IMTS is applied on benchmark instances which are commonly in evaluating GCP and ECP problems. The experimental setup is given in the Table 2

The performance of the IMTS is provided in Table 3 along with a comparison of existing methods namely Tabu Search (TS) [17] and BITS [19]. The proposed IMTS is measured for the value of initial $k\left(\mathbf{k}_i\right)$, best $k\left(\mathbf{k}_b\right)$ and average values of $k\left(\mathbf{k_{avg}}\right)$ for 20 runs for estimating the success rate $(\mathbf{S}R)$. A total of 60 benchmark instances are utilized which weregenerated in DIMACS machine benchmark format with varying number of nodes available at the following link: https://turing.cs.hbg.psu.edu/txn131/graphcoloring.html. The evaluations are performed for the proposed IMTS for 20 runs on

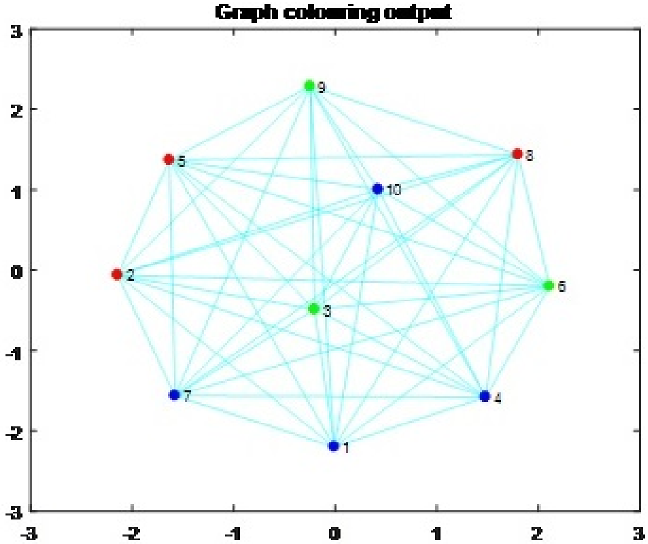

Figure 5: Equitable Coloring of the World Cup Scheduling Problem

Table 2: Experimental Setup

OS	Windows 7 and above, 32bit
Processor	Intel core i5 3470 3.2 GHz
Storage	500GB Intel SSD SC2CT060A3 ATA device
RAM	4GB DDR3
Network bandwidth	1 Gbps
Simulation tool	MATLAB v.2016b
Simulation time	600 seconds
Total Runs	20
Number of Instances	60

each instance to validate the success rates.

From the table 3, it can be found that the IMTS algorithm has better values of k_i, k_b and k_avg with significantly higher success rates than the existing schemes TS and BITs on most instances, thus suggesting that the proposed IMTS reduces the convergence rate and hence the final solutions are better. For a total of 20 runs of the proposed IMTS on each instance, the algorithm returned successful solutions on most runs, indicated by the SR rate. This shows the significance of the proposed approach in handling the ECP and its applications to real-world problems.

6 Conclusion

The introduced IMTS algorithm for tackling the equitable coloring problem demonstrates marked enhancements in performance, attributed to its integration of the modified Tabu search technique alongside a strategic backtracking approach. The algorithm's efficacy is substantiated through its application to address the scheduling conundrum of the ICC Cricket World Cup tournament, yielding favorable outcomes. Furthermore, assessments conducted on benchmark instances underscore the IMTS algorithm's superiority in delivering improved results and notably accelerated convergence rates for the ECP. The algorithm's effectiveness is a synergy of the combined efforts of its constituent processes. Notably, the IMTS algorithm's versatility extends to the successful resolution of other real-world NP-hard problems as well.

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Authors' Contributions

MV and KP have found new results and proved the same in this paper. D and INC have given the idea for the data and have drawn the graphs using MATLAB software. All four authors have read and approved the final manuscript.

Table 3: Computational results of IMTS algorithm on benchmark instances

Instance	Nodes	TS	BITS				IMTS			
		k_b	k_i	k_b	k_{avg}	SR	k_i	k_b	k_{avg}	SR
DSJC125.1	125	5	5	5	5	20/20	5	5	5	20/20
DSJC125.5	125	18	17	17	17.5	20-Oct	17	16	16.5	20-Dec
DSJC125.9	125	45	44	44	44	20/20	43	42	42.65	20/20
DSJC250.1	250	8	8	8	8	20/20	8	8	8	20/20
DSJC250.5	250	32	32	30	31.9	20-Jan	31	30	30.45	20-Aug
DSJC250.9	250	83	72	72	72	20/20	70	69	69.65	20/20
DSJC500.1	500	13	13	13	13	20/20	13	13	13	20/20
DSJC500.5	500	63	57	56	56.95	20-Jan	53	53	53.25	20-Jul
DSJC500.9	500	182	130	129	129.9	20-Feb	127	127	127.6	20-Nov
DSJR500.1	500	12	12	12	12	20/20	12	12	12	20/20
DSJR500.5	500	133	126	126	126.3	14/20	121	121	121.2	17/20
DSJC1000.1	1000	22	22	21	21.95	20-Jan	21	21	21.25	20-Jul
DSJC1000.5	1000	112	112	103	105.1	20-Mar	105	103	104.95	20-Dec
DSJC1000.9	1000	329	254	252	253.3	20-Jan	232	230	232.65	20-Nov
R125.1	125	-	5	5	5	20/20	5	5	5	20/20
R125.5	125	-	36	36	36	20/20	36	36	36	20/20
R250.1	250	-	8	8	8	20/20	8	8	8	20/20
R250.5	250	-	67	66	66.65	20-Jul	66	66	66	20-Nov
R1000.1	1000	-	20	20	20	20/20	20	20	20	20/20
R1000.5	1000	-	269	250	250.4	20-Dec	257	248	248.34	16/20
le450_5a	450	-	5	5	5	20/20	5	5	5	20/20
le450_5b	450	7	5	5	5	20/20	5	5	5	20/20
le450_5c	450	-	5	5	5	20/20	5	5	5	20/20
le450_5d	450	8	5	5	5	20/20	5	5	5	20/20
le450_15a	450	-	15	15	15	20/20	15	15	15	20/20
le450_15b	450	15	15	15	15	20/20	15	15	15	20/20
le450_15c	450	-	15	15	15.1	18/20	15	15	15	19/20
le450_15d	450	16	15	15	15.7	20-Jun	15	15	15	15/20
le450_25a	450	-	25	25	25	20/20	25	25	25	20/20
le450_25b	450	25	25	25	25	20/20	25	25	25	20/20
le450_25c	450	-	26	26	26	20/20	26	26	26	20/20
le450_25d	450	27	26	26	26	20/20	26	25	26.1	20/20
wap01a	2368	46	43	42	42.6	20-Aug	44	41	42.1	15/20
wap02a	2464	44	42	41	41.8	20-Apr	41	41	41	13/20
wap03a	4730	50	46	45	45.05	19/20	44	42	43.55	19/20
wap04a	5231	-	46	44	44.15	17/20	45	45	45.1	15/20
wap05a	905	-	50	50	50	20/20	50	50	50	20/20
wap06a	947	-	42	41	41.7	20-Jun	41	41	41	20-Nov
wap07a	1809	-	43	43	43.05	19/20	43	41	41.8	17/20
wap08a	1870	-	43	43	43.05	19/20	42	41	41.75	17/20
flat300_28_0	300	36	35	34	34.7	20-Jun	33	33	33.45	20-Sep
flat1000_50_0	1000	-	112	101	102.8	20-Jan	102	99	101.2	20-Mar
flat1000_60_0	1000	-	112	102	102.9	20-May	109	101	102.5	20-Aug
flat1000_76_0	1000	112	112	102	103.4	20-Mar	109	99	101.65	20-Jan
latin_square_10	900	130	129	115	120	20-Jan	121	113	117.17	20-Jun
C2000.5	2000	-	202	201	201.6	20-Jul	198	197	198.4	20-Aug
C2000.9	2000	-	504	502	502.4	20-Nov	502	493	498.6	14/20
mulsol.i.1	197	50	49	49	49	20/20	49	48	48.45	20/20
mulsol.i.2	188	48	36	36	36.35	13/20	33	31	32.8	18/20
fpsol2.i.1	496	78	65	65	65	20/20	61	61	61	19/20
fpsol2.i.2	451	60	47	47	47	20/20	43	43	43.25	20/20
fpsol2.i.3	425	79	55	55	55	20/20	53	51	52.5	20/20
inithx.i.1	864	66	54	54	54	20/20	49	49	50.35	20/20
inithx.i.2	645	93	36	36	36.35	13/20	35	31	33.75	15 /20
inithx.i.3	621	-	38	37	37.45	20-Nov	35	35	35	20-Oct
zeroin.i.1	211	51	49	49	49	20/20	45	45	45	19/20
zeroin.i.2	211	51	36	36	36	20/20	34	33	33.4	20/20
zeroin.i.3	206	49	36	36	36	20/20	33	33	33	20/20
myciel6	95	7	7	7	7	20/20	7	7	7	20/20
myciel7	191	8	8	8	8	20/20	8	8	8	20/20

Competing Interests

The authors declare that they have no competing interests.

References

[1] Jonathan L Gross and Jay Yellen. *Graph theory and its applications*. CRC Press, 2005.

[2] LR Foulds. *Graph theory applications*. Springer Science & Business Media, 2012.

[3] Narsingh Deo. *Graph theory with applications to engineering and computer science*. Courier Dover Publications, 2017.

[4] P. M. Pardalos, T. Mavridou, and J. Xue. The graph coloring problem: A bibliographic survey. In *Handbook of combinatorial optimization*, pages 1077–1141. Springer, 1998.

[5] T. Vredeveld and J. K. Lenstra. On local search for the generalized graph coloring problem. *Operations Research Letters*, 31(1):28–34, 2003.

[6] H. Furmanczyk and M. Kubale. Equitable coloring of graphs. *Contemporary mathematics*, 352:35–54, 2006.

[7] H. Furmańczyk. Equitable coloring of graph products. *Opuscula Mathematica*, 26(1):31–44, 2006.

[8] H. Furmańczyk and M. Kubale. Equitable and semi-equitable coloring of cubic graphs and its application in batch scheduling. *Archives of Control Sciences*, 25(1):109–116, 2015.

[9] A. Drexl and S. Knust. Sports league scheduling: graph-and resource-based models. *Omega*, 35(5):465–471, 2007.

[10] R. Lewis and J. Thompson. On the application of graph colouring techniques in round-robin sports scheduling. *Computers & Operations Research*, 38(1):190–204, 2011.

[11] Z. Yan, W. Wang, and X. Zhang. Equitable colorings of cartesian products with balanced complete multipartite graphs. *Discrete Applied Mathematics*, 180:200–203, 2015.

[12] L. Bahiense, Y. Frota, T. F. Noronha, and C. C. Ribeiro. A branch-and-cut algorithm for the equitable coloring problem using a formulation by representatives. *Discrete Applied Mathematics*, 164:34–46, 2014.

[13] P. San Segundo. A new dsatur-based algorithm for exact vertex coloring. *Computers & Operations Research*, 39(7): 1724–1733, 2012.

[14] I. Mendez-Diaz, G. Nasini, and D. Severin. A polyhedral approach for the equitable coloring problem. *Discrete Applied Mathematics*, 164:413–426, 2014.

[15] I. Mendez-Diaz, G. Nasini, and D. Severin. An exact dsatur based algorithm for the equitable coloring problem. *Electronic Notes in Discrete Mathematics*, 44:281–286, 2013.

[16] I. Mendez-Diaz, G. Nasini, and D. Severin. A dsatur based algorithm for the equitable coloring problem. *Computers & Operations Research*, 57:41–50, 2015.

[17] I. Mendez-Diaz, G. Nasini, and D. Severin. A tabu search heuristic for the equitable coloring problem. In *International Symposium on Combinatorial Optimization*. Springer, 2014.

[18] W. Wang, J. K. Hao, and Q. Wu. Tabu search with feasible and infeasible searches for equitable coloring. *Engineering Applications of Artificial Intelligence*, 71:1–14, 2018.

[19] X. Lai, J. K. Hao, and F. Glover. Backtracking based iterated tabu search for equitable coloring. *Engineering Applications of Artificial Intelligence*, 46:269–278, 2015.

[20] W. Sun, J. K. Hao, W. Wang, and Q. Wu. Memetic search for the equitable coloring problem. *Knowledge-Based Systems*, page 105000, 2019.

[21] P. Galinier and A. Hertz. A survey of local search methods for graph coloring. *Computers & Operations Research*, 33(9): 2547–2562, 2006.

[22] P. Galinier, Z. Boujbel, and MC. Fernandes. An efficient memetic algorithm for the graph partitioning problem. *Annals of Operations Research*, 191(1):1–22, 2011.

[23] P. Galinier, J. P. Hamiez, J. K. Hao, and D. Porumbel. Recent advances in graph vertex coloring. In *Handbook of optimization*, pages 505–528. Springer, 2013.

[24] S. An, S. Yang, S. L. Ho, T. Li, and W. Fu. A modified tabu search method applied to inverse problems. *A modified tabu search method applied to inverse problems*, 47(5):1234–1237, 2010.

[25] R. Lewis and J. Thompson. On the application of graph colouring techniques in round-robin sports scheduling. *Computers & Operations Research*, 38(1):190–204, 2011.