Professor Krishan L. Duggal: A Biographical Note

Ramesh Sharma* and Bayram Şahin

(Dedicated to the memory of Prof. Dr. Krishan Lal DUGGAL (1929 - 2022))

ABSTRACT

In this note, we present a biographical sketch of the life and academic contributions of late Professor Krishan L. Duggal. His contributions span from Riemannian and Lorentzian geometries of manifolds with various structural groups of the tangent bundle, Lightlike curves and submanifolds, Cauchy-Riemann geometry, Symmetries of semi-Riemannian manifolds, to Killing horizons. In particular, his approach to the study of lightlike submanifolds is remarkable and drawn considerable interest of many geometers.

1. Professor Krishan Lal Duggal

Figure 1. Professor Krishan L. Duggal

Professor Krishan L. Duggal was born on December 21, 1929 in India, and passed away on December 1, 2022 at 92 years of age. He served on the editorial board of the *International Electronic Journal of Geometry* for a long time. Respectfully, we present some information, to the best of our knowledge, about his life and work.

He was an avid musician in his free time singing spiritual music and playing the Indian instruments, the sitar and harmonium. His family and friends will always remember him saying " The sky is not too high" and "Work is worship".

Professor Duggal graduated with a B.A. degree (Science and Mathematics) from Punjab University (India) in 1951, subsequently got an M.A. degree (Mathematics) from Agra University (India) in 1954, followed by an M.Sc. degree (Mathematics) from University of Windsor (Canada) in 1967. Finally, he completed his Ph.D. degree in Mathematics from the University of Windsor (Canada) in 1969. His doctoral dissertation was titled "A generalization of differentiable structures on Riemannian manifolds" written under the guidance of Professor Hermes A. Eliopoulos. Professor Duggal had taught as a lecturer at Punjab university (India) during 1954-66, as Assistant, Associate and Full Professor at the University of Windsor (Canada) during 1969-72,

1972-79 and 1979-95 respectively. Finally, he was Professor Emeritus at the University of Windsor since 1995. His research was supported by National Science and Engineering Research Council of Canada Fellowship and grants (1967-2010), a travel grant under commonwealth exchange scheme (1980) and Bursary from French Government under research exchange (1970) . He also served on the editorial boards of (1) *Tensor Society of India*, (ii) *International Electronic Journal of Mathematics* (Turkiye) and (iii) *Bulletin of Mathematical Analysis and Applications* (Kosova). Professor Duggal was a prolific researcher having published over 150 journal articles, 5 research monographs and 2 edited conference proceedings with prestigious publishers, resulting in about 3050 citations.

Professor Duggal's contributions span from Riemannian and Lorentzian geometries of manifolds with various structural groups of the tangent bundle, Exact solutions of Einstein's field equations, Lightlike curves and submanifolds, Cauchy-Riemann manifolds and submanifolds, Symmetries of semi-Riemannian manifolds, to Killing and Black hole event horizons. In particular, his approach to the study of lightlike submanifolds is remarkable and drawn considerable interest of many geometers. The first author (R. Sharma) completed his second Ph.D. degree under Professor Duggal in 1986. The research involved in that dissertatation [22] was centered around Cauchy-Riemann submanifolds of Kaehler manifolds, Conformal vector fields and relativistic fluid dynamics. Professor Duggal possessed a remakable way of inspiring and persuading people to work harder. The first author co-authored a number of research articles (a selected list: [15], [16], [17], [18], [23]) published in refereed journals, a research monograph [19] and a co-edited conference proceeding [20].

It was D. N. Kupeli [21] who used the quotient of the tangent bundle by Radical distribution to describe singular hypersurfaces of a semi-Riemannian manifold The technique of using a non-degenerate screen distribution was first introduced by A. Bejancu [3] for null curves and then by Bejancu-Duggal [4] for hypersurfaces to study the induced geometry of lightlike submanifolds. Subsequently, this approach on lightlike submanifolds attracted the attention of several geometers. His last two projects were (1) Ricci almost solitons and physical applications [5], and (ii) Contact pseudo Framed manifolds with applications [6].

The second author, who came to the University of Windsor as a postdoctoral fellow in 2003, received postdoctoral education under the supervision of Professor Duggal. In the 1996 book by Professor Duggal and Professor Bejancu, CR-lightlike submanifolds of indefinite Kaehler manifolds were defined. However, this definition of CR-lightlike submanifold excluded invariant and anti-invariant lightlike submanifolds. For this reason, Professor Duggal and the second author have published their seminal paper [10] in which they defined and studied Screen Cauchy Riemann lightlike submanifolds containing invariant and anti-ivariant lightlike submanifolds as particular cases. This was the beginning of a productive collaboration. Their papers [9, 10, 11, 12, 13] were published in collaboration with Professor Duggal and the second author. In 2007, the second author was a visiting scholar at the University of Windsor at the invitation of Professor Duggal. Here, Professor Duggal and the second author wrote the monograph titled "Differential geometry of lightlike submanifolds" [14], which includes their and other related authors' recent work on the lightlike submanifolds, and this monograph was published by Springer in 2010. The friendship between Professor Duggal and the second author continued until Professor Duggal's death.

Following is a list of his graduate students: M.K. Kwong (Hong Kong), Shankar Prasad (India), R. Sudarshan (India), Sneh Lata (India). The researchers who visited and collaborated with Professor Duggal are: Dr. A. Bejancu (Romania, 1991.1994), Dr. G.G. Asgekar (India), Dr. D.H. Jin (Korea, 1998), Dr. B. Şahin (Türkiye, 2003), Dr. C.L. Bejan (Romania, 2003), Dr. A. Gimenez (Spain, 2004), Dr. C. Atindogbe (Benin, 2005) and Dr. S. Dragomir (Italy, 2005). Professor Duggal visited Indian Institute of Science (Bangalore, India, 1988), Oxford University (England, 1980) and University of Waterloo (Canada, 1979).

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Author's contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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Affiliations

RAMESH SHARMA
ADDRESS: University of New Haven, CT 06516, USA.
E-MAIL: rsharma@newhaven.edu
ORCID ID:0000-0001-5026-8564

BAYRAM ŞAHIN
ADDRESS: Ege University, İzmir,Türkiye
E-MAIL: bayram.sahin@ege.edu.tr
ORCID ID:0000-0002-9372-1151