Turk. J. Math. Comput. Sci.
16(2)(2024) 314–324
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DOI : 10.47000/tjmcs.1254943

Bipolar Fuzzy Soft Set Theory Applied to Medical Diagnosis

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Received: 22-02-2023 • Accepted: 03-09-2024

Abstract. The primary objective of this study is to advance Sanchez's method for medical diagnosis by incorporating fuzzy arithmetic operations. To achieve this, we generalize the existing approach through the application of bipolar fuzzy soft set theory, which enables the identification of two distinct types of medical knowledge within a unified framework. Additionally, we propose a novel decision-making algorithm tailored to this enhanced approach. The application of this algorithm in the medical field is illustrated through practical examples, demonstrating its potential to improve diagnostic processes and decision-making in medical practice.

2020 AMS Classification: 03E72, 91B06, 92C50

Keywords: Bipolar fuzzy soft sets, fuzzy numbers, defuzzification, decision making.

1. Introduction

Uncertainty is involved in most fields like engineering, economic and social disciplines, etc. All uncertainty problems that a person encounters in his life cannot be solved by using ancient mathematical skills. Because; information may be incomplete, not entirely reliable, vague, contradictory, or deficient in some other way. These various information deficiencies may result in fuzziness or vagueness. One of the first studies on the solution to uncertainty problems was the fuzzy set theory given by Zadeh [26] in 1965. In the following years, in 1994, Zhang [27] initiated the concept of bipolar fuzzy sets. In addition, many set theories such as rough sets [21] (this theory is based on equivalence relations), bipolar valued fuzzy sets [15], bipolar-valued hesitant fuzzy sets [16, 25], soft sets [20], bipolar fuzzy soft sets [1] have been proposed to solve the uncertainty problem in the most ideal way. Bipolar fuzzy sets, one of these cluster theories, are an extension of fuzzy sets whose membership degree range is $[-1, 1]$. In recent years bipolar fuzzy sets seem to have been studied and applied a bit enthusiastically and a bit increasingly. Abdullah et al. [1] introduced the notion of the bipolar fuzzy soft set which is a combination of bipolar fuzzy set and soft set. In addition, we can easily say that the studies for the solution to uncertainty problems are increasing day by day. Especially; works on soft set theory [20] are making progress rapidly [4–7, 10, 11, 14].

Although many studies have been conducted in the fuzzy medical diagnostic model, few of these studies have established a consistent relationship between symptoms and diseases. In one of these studies, Sanchez [24] formulated the diagnostic models involving fuzzy matrices representing the medical knowledge between the symptoms and diseases. This approach is grounded in fuzzy set theory, which was initially introduced by Zadeh [26] in 1965 to handle uncertainties and imprecision inherent in many real-world problems, including medical diagnosis. The Sanchez method

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leverages fuzzy relations to encapsulate the uncertain and imprecise nature of medical information. By constructing fuzzy matrices, Sanchez's models can depict the degrees of association between symptoms (inputs) and diseases (outputs). These degrees of association are represented by values in the interval $[0, 1]$, where 0 indicates no association and 1 indicates a full association. One of the beneficiaries of this method, De et al. [8] have studied Sanchez's [23, 24] method of medical diagnosis using an intuitionistic fuzzy set. Apart from that, the method developed by Sanchez [24] has been used in many studies [2, 19, 22].

Despite its innovative use of fuzzy logic, the Sanchez approach has several limitations:

• Limited Handling of Bipolar Information: The original Sanchez method does not adequately handle bipolar information, which consists of both positive (supporting) and negative (contradicting) evidence. In medical diagnosis, it is crucial to consider both aspects to make accurate decisions.

• Single-Type Knowledge Representation: The approach primarily deals with a single type of medical knowledge at a time. However, medical diagnoses often require integrating multiple types of information, such as symptoms, test results, and patient history.

• Complexity and Scalability: As the number of symptoms and diseases increases, the fuzzy relation matrices become larger and more complex to handle. This scalability issue can pose challenges in practical applications involving large datasets.

In this study, we applied bipolar fuzzy soft sets to the medical diagnostic approach developed by Sanchez [24], demonstrating its application with a hypothetical case. The existing algorithm by Çelik and Yamak [3] has been found inadequate in certain situations. To overcome these limitations, we have proposed a new algorithm. Specifically, we addressed these limitations by generalizing the Sanchez approach using bipolar fuzzy soft set theory. This enhancement allows us to:

• Integrate Bipolar Information: By using bipolar fuzzy soft sets, we can simultaneously handle positive and negative information, providing a more comprehensive diagnostic model.

• Multi-Type Knowledge Representation: Our approach enables the integration of various types of medical knowledge within a single framework, improving the robustness and accuracy of diagnoses.

• Improved Decision-Making Algorithm: We propose an advanced decision-making algorithm that efficiently processes the complex relationships captured by bipolar fuzzy soft sets, making it scalable and applicable to larger datasets.

By extending the Sanchez approach in these ways, our method offers a more powerful tool for medical diagnosis, capable of addressing the nuanced and multifaceted nature of medical information.

2. Preliminaries

In this section, we introduce the fundamental ideas behind fuzzy sets, fuzzy numbers, soft sets, bipolar fuzzy sets and bipolar fuzzy soft sets.

Throughout the paper, U is an initial universe, $P(U)$ is the power set of U and A, B, C are non-empty subsets of the parameter set E.

Definition 2.1 ([26]). Let U be a collection of objects denoted by u. Then, a fuzzy set X in U is defined as

$$X = \{\mu_X(u)/u : u \in U\},$$

where $\mu_X : U \to [0, 1]$ is called the membership function of X. The value $\mu_X(u)$ denotes the degree of membership of the element $u \in U$ into the set X.

Definition 2.2 ([13]). A bipolar fuzzy set A in a universe U is an object having the form, $A = \{(u, \mu_A^+(u), \mu_A^-(u)) : u \in U\}$, where $\mu_A^+ : U \to [0, 1]$, $\mu_A^- : U \to [-1, 0]$. So μ_A^+ denote for positive information and μ_A^- denote for negative information.

Definition 2.3 ([20]). Let U be an initial universe, E be the set of parameters, $A \subset E$ and $P(U)$ is the power set of U. Then, (F, A) is called a soft set, where $F : A \to P(U)$.

Definition 2.4 ([26]). A fuzzy subset μ of U is defined as a map from U to $[0, 1]$. The family of all fuzzy subsets of U is denoted by $F(U)$. Let $\mu, \nu \in F(U)$ and $u \in U$. Then, the union and intersection of μ and ν are defined in the following way:

$$(\mu \vee \nu)(x) = \mu(x) \vee \nu(x),$$
$$(\mu \wedge \nu)(x) = \mu(x) \wedge \nu(x),$$

$\mu \leq \nu$ if and only if $\mu(x) \leq \nu(x)$ for all $u \in U$.

Definition 2.5 ([1]). Let U be a universe, E a set of parameters and $A \subset E$. Define $F : A \to BF^U$, where BF^U is the collection of all bipolar fuzzy subsets of U. Then, (F, A) is said to be a bipolar fuzzy soft set over a universe U. It is defined by

$$(F, A) = \{(u, \mu_e^+(u), \mu_e^-(u)) : \forall u \in U, e \in A\}.$$

Example 2.6. Let $U = \{u_1, u_2, u_3\}$ be the set of three houses under consideration and $E = \{e_1 = scenic, e_2 = expensive, e_3 = large\ and\ comfortable, e_4 = garden, e_5 = traditional\}$ be the set of parameters and $A = e_1, e_3, e_4 \subset E$. Then,

$$(F, A) = \Bigg\{\Bigg(e_1, \Big(\big\{u_1, (0.45, -0, 30)\big\}, \big\{u_2, (0.75, -0, 23)\big\}, \big\{u_3, (0.61, -0, 45)\big\}\Big)\Bigg),$$
$$\Bigg(e_3, \Big(\big\{u_1, (0.25, -0, 68)\big\}, \big\{u_2, (0.36, -0, 13)\big\}, \big\{u_3, (0.24, -0, 67)\big\}\Big)\Bigg),$$
$$\Bigg(e_4, \Big(\big\{u_1, (0.95, -0, 18)\big\}, \big\{u_2, (0.65, -0, 32)\big\}, \big\{u_3, (0.58, -0, 15)\big\}\Big)\Bigg)\Bigg\}.$$

Definition 2.7 ([1]). Let U be a universe and E a set of attributes.Then, (U, E) is the collection of all bipolar fuzzy soft sets on U with attributes from E and is said to be bipolar fuzzy soft class.

Definition 2.8 ([1]). Let (F, A) and (G, B) be two bipolar fuzzy soft sets over a common universe U. We say that (F, A) is a bipolar fuzzy soft subset of (G, B), if
(i) $A \subseteq B$ and
(ii) For all $e \in A$, $F(e)$ is a bipolar fuzzy subset of $G(e)$. We write $(F, A)\widetilde{\subseteq}(G, B)$.

Moreover, we say that (F, A) and (G, B) are bipolar fuzzy soft equal sets if (F, A) is a bipolar fuzzy soft subset of (G, B) and (G, B) is a bipolar fuzzy soft subset of (F, A).

Definition 2.9 ([1]). (i) A bipolar fuzzy soft set (F, A) is said to be the absolute bipolar fuzzy soft set over U, if $F(e) = BF^U$ for all $e \in A$.
(ii) A bipolar fuzzy soft set (F, A) is said to be the null bipolar fuzzy soft set over U, if $F(e) = \emptyset$ for all $e \in A$.

Definition 2.10 ([1]). The complement of a bipolar fuzzy soft set (F, A) is denoted $(F, A)^c$ and is defined by $(F, A)^c = \{(u, 1 - \mu_A^+(u), -1 - \mu_A^-(u)) : u \in U\}$.

It should be noted that $1 - F(e)$ denotes the fuzzy complement of $F(e)$ for $e \in A$.

Definition 2.11 ([1]). Let (F, A) and (G, B) be two bipolar fuzzy soft sets over a common universe U. Then,
(i) The union of bipolar fuzzy soft sets (F, A) and (G, B) is defined as the bipolar fuzzy soft set $(H, C) = (F, A) \cup (G, B)$ over U, where $C = A \cup B$, $H : C \to BF^U$ and

$$H(e) = \begin{cases} F(e), & \text{if } e \in A \setminus B \\ G(e), & \text{if } e \in B \setminus A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

for all $e \in C$.
(ii) The restricted union of bipolar fuzzy soft sets (F, A) and (G, B) is defined as the bipolar fuzzy soft set $(H, C) = (F, A) \cup_{\mathcal{R}} (G, B)$ over U, where $C = A \cap B \neq \emptyset$, $H : C \to BF^U$ and $H(e) = F(e) \cup G(e)$ for all $e \in C$.
(iii) The extended intersection of bipolar fuzzy soft sets (F, A) and (G, B) is defined as the bipolar fuzzy soft set $(H, C) = (F, A) \cap (G, B)$ over U, where $C = A \cup B$, $H : C \to BF^U$ and

$$H(e) = \begin{cases} F(e), & \text{if } e \in A \setminus B \\ G(e), & \text{if } e \in B \setminus A \\ F(e) \cap G(e), & \text{if } e \in A \cap B \end{cases}$$

for all $e \in C$.
(iv) The restricted intersection of bipolar fuzzy soft sets (F, A) and (G, B) is defined as the bipolar fuzzy soft set $(H, C) = (F, A) \cap_{\mathcal{R}} (G, B)$ over U, where $C = A \cap B \neq \emptyset$, $H : C \to BF^U$ and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition 2.12 ([12]). (i) A fuzzy subset μ on the universe of discourse $\mathbb{R}$ (the set of all real numbers) is convex if and only if for $a, b \in U$ $\mu(\alpha a + \beta b) \geq \mu(a) \wedge \mu(b)$, where $\alpha + \beta = 1$.
(ii) A fuzzy subset μ on the universe of discourse U is called a normal fuzzy subset if there exist $a_i \in U$ such that $\mu(a_i) = 1$.
(iii) A fuzzy number is a fuzzy subset defined on the universe of discourse $\mathbb{R}$ which is both convex and normal.

A fuzzy number μ on the universe of discourse $\mathbb{R}$ may be characterized by a triangular distribution function parameterized by a triplet (a, b, c). The membership function of the fuzzy number μ is defined as

$$\mu(u) = \begin{cases} 0, & \text{if } u < a \\ \frac{u-a}{b-a}, & \text{if } a \leq u \leq b \\ \frac{c-u}{c-b}, & \text{if } b \leq u \leq c \\ 0, & \text{if } u > c. \end{cases}$$

If the membership function $\mu(u)$ is piecewise linear, then μ is said to be a trapezoidal fuzzy number.

Let μ and β be two triangular fuzzy numbers parameterized by the triplet $\widetilde{y_1} = (x_1, y_1, z_1)$ and $\widetilde{y_2} = (x_2, y_2, z_2)$ respectively. Then, addition and multiplication of μ and β as given in [12] are

$$\mu \oplus \beta = \widetilde{y_1} \oplus \widetilde{y_2} = (x_1, y_1, z_1) \oplus (x_2, y_2, z_2) = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$$

and

$$\mu \otimes \beta = \widetilde{y_1} \otimes \widetilde{y_2} = (x_1, y_1, z_1) \otimes (x_2, y_2, z_2) = (x_1 \times x_2, y_1 \times y_2, z_1 \times z_2).$$

Next, we give the defuzzification method of a trapezoidal fuzzy number. Take a trapezoidal fuzzy number parameterized by a quadruplet (p, q, r, s) as shown in Figure 1.

Then, the defuzzification value t of the fuzzy number is calculated from the figure as follows:

$$(t-q)(l) + \frac{1}{2}(q-p)(l) = (r-t)(l) + \frac{1}{2}(s-r)(l)$$
$$\Rightarrow (t-q) + \frac{1}{2}(q-p) = (r-t) + \frac{1}{2}(s-r)$$
$$\Rightarrow 2t = \frac{s-r-q+p}{2} + q + r$$
$$\Rightarrow t = \frac{p+q+r+s}{4}.$$

Similarly, the defuzzification value e of a triangular fuzzy number (a, b, c) is equal to

$$e = \frac{a+b+b+c}{4}$$

3. Technical Details of the Method We Developed

In this section, an application in the field of medicine given by Çelik and Yamak [3] is reconsidered for bipolar fuzzy soft sets. In addition, the algorithm given by [3] has been developed here and results are obtained that are closer to more ideal, a related study example is given in the next section. The data obtained for patients will be subjected to fuzzy arithmetic operations and an algorithm for the detection of the disease will be shown. Assume that, there is a set of m patients, $P = \{p_1, p_2, p_3, ..., p_m\}$ with a set of n symptoms $S = \{s_1, s_2, s_3, ..., s_n\}$ related to a set of k diseases $D = \{d_1, d_2, d_3, ..., d_k\}$.

We apply bipolar fuzzy soft set theory to develop a technique through Sanchez's method to diagnose which patient is suffering from what disease. For this, construct a bipolar fuzzy soft set (F, P) over S where F is a mapping $F : P \to BF^S$. This bipolar fuzzy soft set gives a relation matrix Q, called patient-symptom matrix, where the entries are fuzzy numbers p parameterized by a triplet $(p-1, p, p+1)$.

Then construct another bipolar fuzzy soft set (G, S) over D, where G is amapping $G : S \to BF^D$. This bipolar fuzzy soft set gives a relation matrix (weighted matrix) R, called symptom-disease matrix, where each element denotes the weight of the symptoms for a certain disease. These elements are also taken as triangular fuzzy numbers.

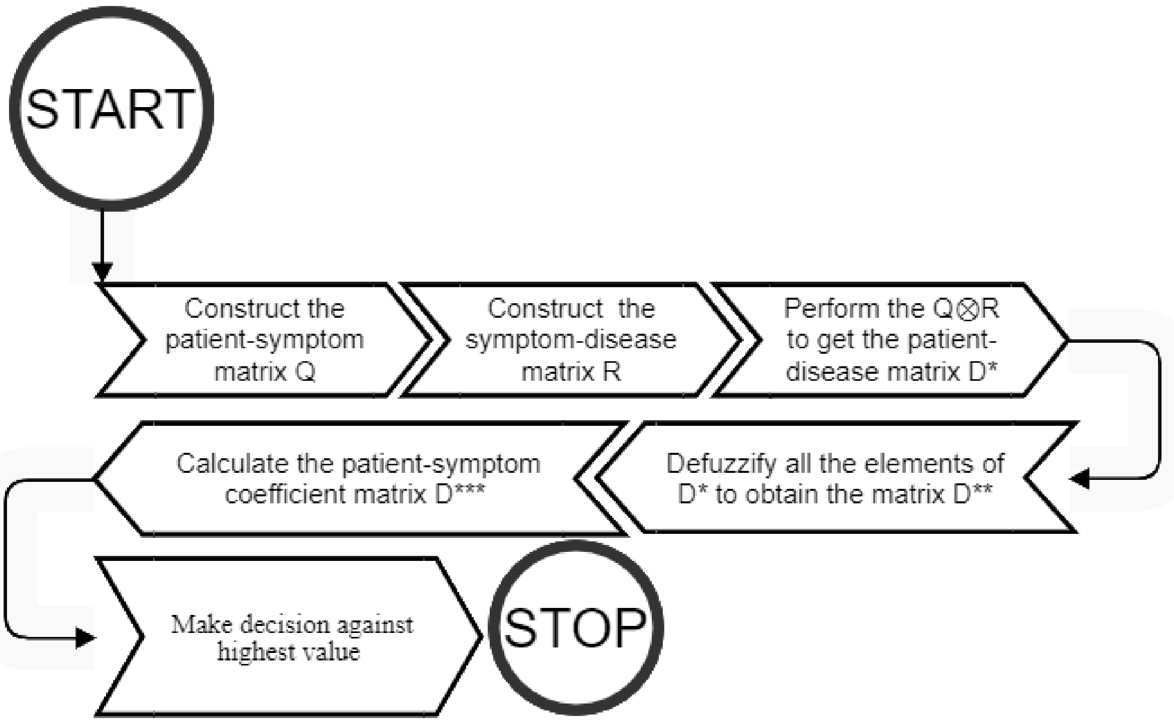

FIGURE 1. A trapezoidal fuzzy number parameterized by a quadruplet (p, q, r, s).

Thus, the general form of Q is

$$Q = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ \cdot \\ \cdot \\ \cdot \\ p_m \end{array} \begin{array}{c} \begin{array}{cccccc} s_1 & s_2 & s_3 & \cdots & s_n \end{array} \\ \left[\begin{array}{cccccc} (\widetilde{x^+_{11}}, \widetilde{x^-_{11}}) & (\widetilde{x^+_{12}}, \widetilde{x^-_{12}}) & (\widetilde{x^+_{13}}, \widetilde{x^-_{13}}) & \cdots & (\widetilde{x^+_{1n}}, \widetilde{x^-_{1n}}) \\ (\widetilde{x^+_{21}}, \widetilde{x^-_{21}}) & (\widetilde{x^+_{22}}, \widetilde{x^-_{22}}) & (\widetilde{x^+_{23}}, \widetilde{x^-_{23}}) & \cdots & (\widetilde{x^+_{2n}}, \widetilde{x^-_{2n}}) \\ (\widetilde{x^+_{31}}, \widetilde{x^-_{31}}) & (\widetilde{x^+_{32}}, \widetilde{x^-_{32}}) & (\widetilde{x^+_{33}}, \widetilde{x^-_{33}}) & \cdots & (\widetilde{a^+_{3n}}, \widetilde{a^-_{3n}}) \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ (\widetilde{x^+_{m1}}, \widetilde{x^-_{m1}}) & (\widetilde{x^+_{m2}}, \widetilde{x^-_{m2}}) & (\widetilde{x^+_{m3}}, \widetilde{x^-_{m3}}) & \cdots & (\widetilde{x^+_{mn}}, \widetilde{x^-_{mn}}) \end{array} \right] \end{array},$$

where $\left(\widetilde{\mu^+_{p_i}(s_j)}, \widetilde{\mu^-_{p_i}(s_j)}\right) = (\widetilde{x^+_{ij}}, \widetilde{x^-_{ij}})$ for all $1 \leq i \leq m$ and $1 \leq j \leq n$.

In addition, the general form of R is

$$R = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ \cdot \\ \cdot \\ \cdot \\ s_n \end{array} \begin{array}{c} \begin{array}{cccccc} d_1 & d_2 & d_3 & \cdots & d_k \end{array} \\ \left[\begin{array}{cccccc} (\widetilde{y^+_{11}}, \widetilde{y^-_{11}}) & (\widetilde{y^+_{12}}, \widetilde{y^-_{12}}) & (\widetilde{y^+_{13}}, \widetilde{y^-_{13}}) & \cdots & (\widetilde{y^+_{1k}}, \widetilde{y^-_{1k}}) \\ (\widetilde{y^+_{21}}, \widetilde{y^-_{21}}) & (\widetilde{y^+_{22}}, \widetilde{y^-_{22}}) & (\widetilde{y^+_{23}}, \widetilde{y^-_{23}}) & \cdots & (\widetilde{y^+_{2k}}, \widetilde{y^-_{2k}}) \\ (\widetilde{y^+_{31}}, \widetilde{y^-_{31}}) & (\widetilde{y^+_{32}}, \widetilde{y^-_{32}}) & (\widetilde{y^+_{33}}, \widetilde{y^-_{33}}) & \cdots & (\widetilde{y^+_{3k}}, \widetilde{y^-_{3k}}) \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ (\widetilde{y^+_{n1}}, \widetilde{y^-_{n1}}) & (\widetilde{y^+_{n2}}, \widetilde{y^-_{n2}}) & (\widetilde{y^+_{n3}}, \widetilde{y^-_{n3}}) & \cdots & (\widetilde{y^+_{nk}}, \widetilde{y^-_{nk}}) \end{array} \right] \end{array},$$

where $\left(\widetilde{\mu^{+}_{s_l}(d_t)}, \widetilde{\mu^{-}_{s_l}(d_t)}\right) = (\widetilde{y^{+}_{lt}}, \widetilde{y^{-}_{lt}})$ for all $1 \leq l \leq n$ and $1 \leq t \leq k$.

Now, performing the transformation operation $Q \otimes R$, we get the patient-diagnosis matrix D^* as follows:

$$D^* = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ \cdot \\ \cdot \\ \cdot \\ p_m \end{array} \begin{array}{c} \begin{array}{ccccccc} d_1 & d_2 & d_3 & \cdot & \cdot & \cdot & d_k \end{array} \\ \left[\begin{array}{ccccccc} (\widetilde{z^{+}_{11}}, \widetilde{z^{-}_{11}}) & (\widetilde{z^{+}_{12}}, \widetilde{z^{-}_{12}}) & (\widetilde{z^{+}_{13}}, \widetilde{z^{-}_{13}}) & \cdot & \cdot & \cdot & (\widetilde{z^{+}_{1k}}, \widetilde{z^{-}_{1k}}) \\ (\widetilde{z^{+}_{21}}, \widetilde{z^{-}_{21}}) & (\widetilde{z^{+}_{22}}, \widetilde{z^{-}_{22}}) & (\widetilde{z^{+}_{23}}, \widetilde{z^{-}_{23}}) & \cdot & \cdot & \cdot & (\widetilde{z^{+}_{2k}}, \widetilde{z^{-}_{2k}}) \\ (\widetilde{z^{+}_{31}}, \widetilde{z^{-}_{31}}) & (\widetilde{z^{+}_{32}}, \widetilde{z^{-}_{32}}) & (\widetilde{z^{+}_{33}}, \widetilde{z^{-}_{33}}) & \cdot & \cdot & \cdot & (\widetilde{z^{+}_{3k}}, \widetilde{z^{-}_{3k}}) \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ (\widetilde{z^{+}_{m1}}, \widetilde{z^{-}_{m1}}) & (\widetilde{z^{+}_{m2}}, \widetilde{z^{-}_{m2}}) & (\widetilde{z^{+}_{m3}}, \widetilde{z^{-}_{m3}}) & \cdot & \cdot & \cdot & (\widetilde{z^{+}_{mk}}, \widetilde{z^{-}_{mk}}) \end{array} \right] \end{array},$$

where

$$\widetilde{z^{+}_{il}} = \Big(\sum_{j=1}^{n} (x^{+}_{ij} - 1)(y^{+}_{jl} - 1), \sum_{j=1}^{n} x^{+}_{ij} y^{+}_{jl}, \sum_{j=1}^{n} (x^{+}_{ij} + 1)(y^{+}_{jl} + 1) \Big)$$

and

$$\widetilde{z^{-}_{il}} = \Big(\sum_{j=1}^{n} (x^{-}_{ij} - 1)(y^{-}_{jl} - 1), \sum_{j=1}^{n} x^{-}_{ij} y^{-}_{jl}, \sum_{j=1}^{n} (x^{-}_{ij} + 1)(y^{-}_{jl} + 1) \Big).$$

Then, defuzzifying each element of the above matrix by [26], we get the crisp diagnosis matrix as

$$D^{**} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ \cdot \\ \cdot \\ \cdot \\ p_m \end{array} \begin{array}{c} \begin{array}{ccccccc} d_1 & d_2 & d_3 & \cdot & \cdot & \cdot & d_k \end{array} \\ \left[\begin{array}{ccccccc} (\widetilde{v^{+}_{11}}, \widetilde{v^{-}_{11}}) & (\widetilde{v^{+}_{12}}, \widetilde{v^{-}_{12}}) & (\widetilde{v^{+}_{13}}, \widetilde{v^{-}_{13}}) & \cdot & \cdot & \cdot & (\widetilde{v^{+}_{1k}}, \widetilde{v^{-}_{1k}}) \\ (\widetilde{v^{+}_{21}}, \widetilde{v^{-}_{21}}) & (\widetilde{v^{+}_{22}}, \widetilde{v^{-}_{22}}) & (\widetilde{v^{+}_{23}}, \widetilde{v^{-}_{23}}) & \cdot & \cdot & \cdot & (\widetilde{v^{+}_{2k}}, \widetilde{v^{-}_{2k}}) \\ (\widetilde{v^{+}_{31}}, \widetilde{v^{-}_{31}}) & (\widetilde{v^{+}_{32}}, \widetilde{v^{-}_{32}}) & (\widetilde{v^{+}_{33}}, \widetilde{v^{-}_{33}}) & \cdot & \cdot & \cdot & (\widetilde{v^{+}_{3k}}, \widetilde{v^{-}_{3k}}) \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ (\widetilde{v^{+}_{m1}}, \widetilde{v^{-}_{m1}}) & (\widetilde{v^{+}_{m2}}, \widetilde{v^{-}_{m2}}) & (\widetilde{v^{+}_{m3}}, \widetilde{v^{-}_{m3}}) & \cdot & \cdot & \cdot & (\widetilde{v^{+}_{mk}}, \widetilde{v^{-}_{mk}}) \end{array} \right] \end{array}.$$

Now, let's calculate the coefficients to express what disease each patient suffering from. A coefficient is defined to determine the ranking order of all patients once the $(\widetilde{v^{+}_{ij}}, \widetilde{v^{-}_{ij}})$ of each patient p_i $(i = 1, 2, ..., m)$ which suffering from disease d_j $(j = 1, 2, ..., k)$ has been calculated. The coefficient of each patient suffering from any disease is calculated as:

$$\widetilde{c_{ij}} = -\frac{\widetilde{v^{-}_{ij}}}{\widetilde{v^{+}_{ij}} - \widetilde{v^{-}_{ij}}}, \quad i = 1, 2, ..., m, \quad j = 1, 2, ..., k$$

and, we get the patient-diagnosis coefficient matrix D^{***} as follows:

$$D^{***} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ \cdot \\ \cdot \\ \cdot \\ p_m \end{array} \begin{array}{c} \begin{array}{ccccccc} d_1 & d_2 & d_3 & \cdot & \cdot & \cdot & d_k \end{array} \\ \left[\begin{array}{ccccccc} \widetilde{c_{11}} & \widetilde{c_{12}} & \widetilde{c_{13}} & \cdot & \cdot & \cdot & \widetilde{c_{1k}} \\ \widetilde{c_{21}} & \widetilde{c_{22}} & \widetilde{c_{23}} & \cdot & \cdot & \cdot & \widetilde{c_{2k}} \\ \widetilde{c_{31}} & \widetilde{c_{32}} & \widetilde{c_{33}} & \cdot & \cdot & \cdot & \widetilde{c_{3k}} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \widetilde{c_{m1}} & \widetilde{c_{m2}} & \widetilde{c_{m3}} & \cdot & \cdot & \cdot & \widetilde{c_{mk}} \end{array} \right] \end{array}.$$

Obviously, the patient p_i is suffering from disease d_j as $\widetilde{c_{ij}}$ approaches to 1. In case $maxc_{ij}$ occurs for more than one value of l, $1 \leq j \leq k$, then we can reassess the symptoms.

Algorithm 1. .
Step 1*: Construct the bipolar soft set (F, P) to obtain the patient-symptom matrix Q.*
Step 2*: Construct the bipolar soft set (G, S) to obtain the symptom-disease matrix R.*
Step 3*: Perform the transformation operation $Q \otimes R$ to get the patient diagnosis matrix D^*.*
Step 4*: Defuzzify all the elements of the matrix D^* by [26] to obtain the matrix D^{**}.*
Step 5*: Calculate the coefficients $\widetilde{c_{ij}}$ of each patient p_i $(i = 1, 2, ..., m)$ which suffering from disease d_j $(j = 1, 2, ..., k)$ to obtain the patient-symptom coefficient matrix D^{***} using the data in the matrix D^{**}.*
Step 6*: Find s for which $c_{is} = maxc_{ij}$. In other words, make decision against highest value involved in the row. Then, we conclude that the patient p_i is suffering from disease d_s.*

4. An Application in the Field of Medicine

Suppose there are four patients Ahmet, Ali, Fatma, Ayşe in a hospital with symptoms suprapubic pain, sweating, chills, renal ultrasound findings and weakness. Let the possible diseases related to the above symptoms be urinary tract infection (UTI), acute pyelonephritis (AP), nonspecific urethritis (NU). In this case, take $P = \{p_1 = Ahmet,\ p_2 = Ali,\ p_3 = Fatma,\ p_4 = Ay\c{s}e\}$ as the set of patients, $D = \{d_1 = UTI,\ d_2 = AP,\ d_3 = NU\}$ as the set of diseases and $S = \{s_1 = suprapubic\ pain,\ s_2 = sweating,\ s_3 = chills, s_4 = renal\ ultrasound\ findings,\ s_5 = weakness\}$ as the set of symptoms.

Suppose

$$F(p_1) = \{s_1/[\widetilde{5}, \widetilde{-2}], s_2/[\widetilde{7}, \widetilde{-1}], s_3/[\widetilde{2}, \widetilde{-6}], s_4/[\widetilde{8}, \widetilde{-3}], s_5/[\widetilde{4}, \widetilde{-5}]\},$$
$$F(p_2) = \{s_1/[\widetilde{1}, \widetilde{-3}], s_2/[\widetilde{4}, \widetilde{-6}], s_3/[\widetilde{7}, \widetilde{-2}], s_4/[\widetilde{5}, \widetilde{-1}], s_5/[\widetilde{9}, \widetilde{-4}]\},$$
$$F(p_3) = \{s_1/[\widetilde{4}, \widetilde{-5}], s_2/[\widetilde{2}, \widetilde{-3}], s_3/[\widetilde{1}, \widetilde{-6}], s_4/[\widetilde{7}, \widetilde{-2}], s_5/[\widetilde{8}, \widetilde{-1}]\},$$
$$F(p_4) = \{s_1/[\widetilde{7}, \widetilde{-2}], s_2/[\widetilde{8}, \widetilde{-1}], s_3/[\widetilde{5}, \widetilde{-3}], s_4/[\widetilde{1}, \widetilde{-9}], s_5/[\widetilde{2}, \widetilde{-7}]\}.$$

Now, using the bipolar fuzzy soft set (F, P) given over S, the patient-symptom matrix Q is given by

$$Q = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccccc} s_1 & s_2 & s_3 & s_4 & s_5 \end{array} \\ \left[\begin{array}{ccccc} (\widetilde{5}, \widetilde{-2}) & (\widetilde{7}, \widetilde{-1}) & (\widetilde{2}, \widetilde{-6}) & (\widetilde{8}, \widetilde{-3}) & (\widetilde{4}, \widetilde{-5}) \\ (\widetilde{1}, \widetilde{-3}) & (\widetilde{4}, \widetilde{-6}) & (\widetilde{7}, \widetilde{-2}) & (\widetilde{5}, \widetilde{-1}) & (\widetilde{9}, \widetilde{-4}) \\ (\widetilde{4}, \widetilde{-5}) & (\widetilde{2}, \widetilde{-3}) & (\widetilde{1}, \widetilde{-6}) & (\widetilde{7}, \widetilde{-2}) & (\widetilde{8}, \widetilde{-1}) \\ (\widetilde{7}, \widetilde{-2}) & (\widetilde{8}, \widetilde{-1}) & (\widetilde{5}, \widetilde{-3}) & (\widetilde{1}, \widetilde{-9}) & (\widetilde{2}, \widetilde{-7}) \end{array} \right] \end{array}.$$

Then; suppose

$$G(s_1) = \{d_1/[\widetilde{1}, \widetilde{-2}], d_2/[\widetilde{2}, \widetilde{-3}], d_3/[\widetilde{8}, \widetilde{-1}]\},$$
$$G(s_2) = \{d_1/[\widetilde{4}, \widetilde{-8}], d_2/[\widetilde{9}, \widetilde{-5}], d_3/[\widetilde{5}, \widetilde{-6}]\},$$
$$G(s_3) = \{d_1/[\widetilde{6}, \widetilde{-6}], d_2/[\widetilde{8}, \widetilde{-4}], d_3/[\widetilde{7}, \widetilde{-4}]\},$$
$$G(s_4) = \{d_1/[\widetilde{9}, \widetilde{-4}], d_2/[\widetilde{5}, \widetilde{-3}], d_3/[\widetilde{2}, \widetilde{-2}]\},$$
$$G(s_5) = \{d_1/[\widetilde{3}, \widetilde{-1}], d_2/[\widetilde{1}, \widetilde{-5}], d_3/[\widetilde{1}, \widetilde{-1}]\}.$$

Now, let's express the bipolar fuzzy soft set (G, S) given above as matrix R:

$$R = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \left[\begin{array}{ccc} (\widetilde{1}, \widetilde{-2}) & (\widetilde{2}, \widetilde{-3}) & (\widetilde{8}, \widetilde{-1}) \\ (\widetilde{4}, \widetilde{-8}) & (\widetilde{9}, \widetilde{-5}) & (\widetilde{5}, \widetilde{-6}) \\ (\widetilde{6}, \widetilde{-6}) & (\widetilde{8}, \widetilde{-4}) & (\widetilde{7}, \widetilde{-4}) \\ (\widetilde{9}, \widetilde{-4}) & (\widetilde{5}, \widetilde{-3}) & (\widetilde{2}, \widetilde{-2}) \\ (\widetilde{3}, \widetilde{-1}) & (\widetilde{1}, \widetilde{-5}) & (\widetilde{1}, \widetilde{-1}) \end{array} \right] \end{array}.$$

Then, performing the transformation operation $Q \oplus R$, we get the patient-diagnosis matrix D^* as

$$D^* = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{bmatrix} (\widetilde{129}, \widetilde{-65}) & (\widetilde{133}, \widetilde{-69}) & (\widetilde{109}, \widetilde{-43}) \\ (\widetilde{131}, \widetilde{-74}) & (\widetilde{128}, \widetilde{-70}) & (\widetilde{96}, \widetilde{-53}) \\ (\widetilde{105}, \widetilde{-79}) & (\widetilde{77}, \widetilde{-65}) & (\widetilde{71}, \widetilde{-52}) \\ (\widetilde{84}, \widetilde{-73}) & (\widetilde{133}, \widetilde{-85}) & (\widetilde{135}, \widetilde{-45}) \end{bmatrix} \end{array},$$

where

$$(\widetilde{129}, \widetilde{-65}) = [(85, -108), (129, -65), (183, -32)], \quad (\widetilde{133}, \widetilde{-69}) = [(87, -111), (133, -69), (189, -37)],$$

$$(\widetilde{109}, \widetilde{-43}) = [(65, -79), (109, -43), (163, -17)], \quad (\widetilde{131}, \widetilde{-74}) = [(87, -116), (131, -74), (185, -42)],$$

$$(\widetilde{128}, \widetilde{-70}) = [(82, -111), (128, -70), (184, -39)], \quad (\widetilde{96}, \widetilde{-53}) = [(52, -88), (96, -53), (150, -28)],$$

$$(\widetilde{105}, \widetilde{-79}) = [(65, -122), (105, -79), (155, -46)], \quad (\widetilde{77}, \widetilde{-65}) = [(35, -107), (77, -65), (129, -33)],$$

$$(\widetilde{71}, \widetilde{-52}) = [(31, -88), (71, -52), (121, -26)], \quad (\widetilde{84}, \widetilde{-73}) = [(43, -121), (84, -73), (135, -35)],$$

$$(\widetilde{133}, \widetilde{-85}) = [(90, -132), (133, -85), (186, -48)], \quad (\widetilde{135}, \widetilde{-45}) = [(94, -86), (135, -45), (186, -14)].$$

Now, defuzzifying the above matrix, we get

$$D^{**} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{bmatrix} (131.5, -67.5) & (135.5, -71.5) & (111.5, -45.5) \\ (133.5, -76.5) & (130.5, -72.5) & (98.5, -55.5) \\ (107.5, -81.5) & (79.5, -67.5) & (73.5, -54.5) \\ (86.5, -75.5) & (135.5, -87.5) & (137.5, -47.5) \end{bmatrix} \end{array}.$$

Finally, we get the patient-diagnosis coefficient matrix D^{***} as:

$$D^{***} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{bmatrix} 0.3392 & 0.3454 & 0.2898 \\ 0.3643 & 0.3571 & 0.3604 \\ 0.4312 & 0.4592 & 0.4258 \\ 0.4660 & 0.3924 & 0.2568 \end{bmatrix} \end{array}.$$

The analysis of the results presented in the matrices reveals distinct patterns in the disease profiles of the patients. Specifically:

- **Patients Affected by Disease d_2 = AP:**
  - **Patient p_1 = Ahmet**: Ahmet exhibits high scores in the AP category. This suggests that Ahmet is significantly affected by disease AP, showing considerable impairment in this area compared to other patients.
  - **Patient p_3 = Fatma**: Similarly, Fatma's performance in the AP category is notably high in our study's normalized results, reflecting a substantial impact from disease AP.

- **Patients Affected by Disease d_1 = UTI:**
  - **Patient p_2 = Ali**: Ali shows elevated scores in the UTI category across the normalized results. His consistently high scores suggest that Ali is prominently suffering from UTI.
  - **Patient p_4 = Ayşe**: Similarly, Ayşe's results in the UTI category are significant in the normalized results. Despite some variation, the high normalized scores indicate a notable level of suffering from UTI.

Person	**UTI (Our Study)**	**AP (Our Study)**	**NU (Our Study)**
Ahmet	0.3392	0.3454	0.2898
Ali	0.3643	0.3571	0.3604
Fatma	0.4312	0.4592	0.4258
Ayşe	0.4660	0.3924	0.2568

Table 1. Results from This Study

Person	**UTI (Sanchez's Method)**	**AP (Sanchez's Method)**	**NU (Sanchez's Method)**
Ahmet	182.625	187.125	160.125
Ali	184.875	181.500	145.500
Fatma	155.625	124.125	117.375
Ayşe	132.000	187.125	189.375

Table 2. Results from Sanchez's Method

5. Discussion

The comparative analysis of the results derived from our study and those obtained using Sanchez's method provides insightful contrasts in the performance metrics evaluated. Our study's results, which are presented in terms of normalized scores for the variables UTI, AP, and NU, reveal notable differences when juxtaposed with the outcomes from Sanchez's methodology.

Our method yields varied scores across different individuals, with Fatma and Ayşe showing the highest and lowest normalized values respectively in the UTI metric, while Ali demonstrates the most balanced performance across all variables. In contrast, Sanchez's method presents absolute scores that are generally higher or more disparate across individuals. For instance, Fatma's scores in Sanchez's method are significantly lower compared to her normalized values, suggesting a discrepancy that might be attributable to methodological differences or scaling effects.

The results obtained from our study indicate that Fatma consistently performs better in the normalized metrics compared to other individuals. Her normalized UTI, AP, and NU scores (0.431216931, 0.459183673, and 0.42578125 respectively) reflect superior performance relative to the others. This trend is not mirrored in Sancezin's method, where Fatma's scores (155.625, 124.125, and 117.375) are notably lower, particularly in the AP and NU metrics. This discrepancy suggests that Sancezin's method may emphasize different performance aspects or introduce biases that affect the comparative results.

The observed differences highlight the impact of methodological choice on performance evaluation. Our normalized scores, being dimensionless, provide a comparative perspective that might be more reflective of relative performance. In contrast, Sanchez's method absolute scoring system may be more sensitive to the scale and distribution of data. The higher scores for individuals like Ahmet and Ali in Sancezin's method, compared to our study, could suggest a methodological bias or differing benchmarks for performance assessment.

The variance between the results from our study and Sanchez's method underscores the importance of methodological transparency and the potential influence of chosen metrics on outcome interpretation. Future work should explore these methodological differences further to understand their implications on performance evaluation and to potentially harmonize the results for more robust comparisons. Additionally, cross-validation with other methods could offer a more comprehensive view of the performance metrics assessed.

6. Conclusion and Future Studies

In this study, we have extended Sanchez's method of medical diagnosis by incorporating bipolar fuzzy soft sets, offering a refined approach to handling the complexity of medical information. Our proposed algorithm addresses some of the limitations found in previous methods, such as those described in [3], particularly in cases where the symptoms are closely related. The primary advantage of our proposed method is its ability to model both positive and negative aspects of medical information simultaneously, providing a more nuanced and accurate diagnosis. This dual approach enhances the decision-making process by offering a comprehensive analysis of complex and ambiguous data, potentially leading to improved diagnostic accuracy and better patient outcomes. This enhancement aims to aid in timely diagnosis and prompt treatment, which is crucial in emergency medical settings.

While our method presents advancements, it also has limitations. The computational complexity introduced by bipolar fuzzy soft sets may require more extensive processing power, especially for large datasets or real-time applications. Additionally, the algorithm's effectiveness may vary based on the quality and completeness of the input data.

The integration of bipolar fuzzy soft sets into our approach aligns with recent advancements in the field. Mahmood [17] provides foundational insights into bipolar soft sets, demonstrating their potential applications and enhancing our method's framework. Building on this, Mahmood and Ur Rehman [18] explore bipolar complex fuzzy sets and their use in generalized similarity measures, which supports the broader applicability of our method. Additionally, Jaleel [9] illustrates the practical use of bipolar complex fuzzy soft sets in agricultural robotics, showcasing how these concepts can be effectively applied in various complex systems. These references collectively suggest that our approach could be further developed and adapted for a range of complex decision-making scenarios beyond medical diagnosis.

We believe that further exploration of bipolar soft sets, bipolar complex fuzzy sets, and bipolar complex fuzzy soft sets could yield valuable insights and improvements in various fields, including medical diagnostics and beyond.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Authors Contribution Statement

The authors wrote, read and approved the final version of the manuscript.

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