Review Article

Energy Structure Theory: A General Unified Thermodynamics Theory

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Abstract

This study, while reviewing some of the established unified equations and fundamentals of the energy structure and providing a detailed interpretation of their physical concepts, expands the relevant equations for new topics and applications, and in fact, establishes novel results and equations from the energy structure analysis. In fact, this paper establishes an energy components-based-general model inspired by the first and second laws of thermodynamics as well as using a new division to the total energy of the system. The established model is completed by extracting the physical direction for the feasible processes based on the energy components of the system. As two of the most important achievements of the energy components approach, using a new quasi-statistical approach as well as a novel energy conservation principle, an entropy equation is gained that has a common basis as the Boltzmann entropy equation as well as a general solution to the different formulations of the second law of thermodynamics is established. The established equations are gained without any limiting assumptions, and are governed to any physical system. Several basic examples have been studied, and matching the obtained results with expected ones is shown.

Keywords: Unified thermodynamics; high particle number systems; dependent energy components; energy structure equation; feasible processes; compatibility conditions; Boltzmann entropy equation.

1. Introduction

While the principles of motion determine the relationship between forces and accelerations, the principle of energy conservation, as a unifying principle in physics, with respect to kinetic energies, different potential energies, etc., examines the general behavior of the system due to exchange energy with the surrounding [1-2]. Of course, this issue can also be raised in thermodynamics. [1]. Two thermodynamic laws were developed by Clausius in 1865[3]. The first law of thermodynamics is a result of the principle of energy conservation for thermodynamic cycles and states that the sum of work done and heat exchange in a closed cycle is equal to zero. While the second law of classical thermodynamics examines the feasibility of thermodynamic processes. Clausius [3] using the concept of entropy, stated the second law as a prediction of the increase of entropy in physical processes, and it is based on the fact that based on the second law, it is necessary that the entropy of the universe is always increasing. In fact, the second law of thermodynamics is also referred to as a summary of all laws governing nature [3]. The use of the above expression in scientific theories is not easy as we know that Isaac Newton does not pay attention to entropy in compiling the principles of mechanics, in other words, classical mechanics are written reversibly [4]. Certainly, there are ideas for applying the second law on the mechanic and thermodynamic coupling problems on a macroscopic scale, based on thermodynamic entropy [5]. The second law has attracted many scholars and has not lost its credibility even with the advent of advanced scientific theories [6,7]. Various mathematical models are proposed for the second law of thermodynamics [8,9]. It has

also been proven in various ways [10]. Some of these methods are based on statistical mechanics and some on the basis of quantum mechanics. However, in the quantum mechanical science, the law entitled "the generalization of the second law of thermodynamics" is formulated and presented, as well as the thermodynamic concept for negative entropy [11]. Today, the concept of entropy is wider than that defined by Clausius which is referred to as thermodynamic entropy, while some researchers have also defined entropy in addition to thermodynamic entropy that is used in relevant scientific fields to investigate problems [12,13]. These definitions are based on the energies that are involved in the relevant issues and the entropy changes depend on these energy changes. Processes in which entropy production is present are referred to as irreversible processes. The reasons of process irreversibility can be seen microscopically in theories that are used to prove the second law [8, 11]. In the macroscopic view, the existence of friction, the heating of a system with a finite rate, the presence of a temperature gradient, and etc., are considered as factors that cause the irreversibility of a process as well as entropy production [14-16]. Change in the structure and properties of the system can be considered as the main factors that cause the irreversibility in the microscopic view. These factors are the base of analysis in some branches of science such as Tribo-fatigue, unified mechanics theory, and also mechanothermodynamics [17-19]. In mechanothermodynamics, as a new branch of physics, an entropy is defined which is used as the base of MTD system analysis [20]. Also, energy and entropy are two main quantities to understand nanoparticle behavior [21]. In

addition, other quantities are including of chemical potential, strongly affecting the solubility, nucleation, and stability of the nanoparticle. Also, these quantities can be coupled with Gibbs free energy [21-23]. To investigate nanoparticle behavior from the perspective of the energy application, chosen element structure can has a very important role [24-25]. Moreover, in statistical physics, entropy is considered as a function of attainable states of the system at the corresponding energy level [26]. Achievable states can be calculated at the corresponding energy level using quantum concepts as well as probability theory [26]. Considering the assumptions of statistical physics, Boltzmann calculated her entropy equation in terms of attainable states of the system at the corresponding energy level [27]. Boltzmann equation can be used in general in particle systems as well as continuous media. This equation can be generally used to calculate entropy changes in reversible and irreversible processes and is not directly dependent on energy losses in the performed process. According to the basis of statistical physics, in order to study the energy losses in particle systems, the dynamics of system particles can be investigated [29-31]. Various cases have been observed both in the scientific position and in the applied position of scientific principles in nature, which require the simultaneous mastery of the first and second laws of thermodynamics as governing principles. As an example, the formation of two-layer oxide films in the material structure can be mentioned [32-40]. Double layer oxide films with their outstanding features are still in the stage of being known. However, according to what is known so far about these films and the conditions of their formation, they are formed in a highly irreversible process, and in fact, in their analysis, the results related to the energy conservation principle cannot be considered for these issues, and in fact, reaches obvious, and of course, inapplicable results in the study of these films. Therefore, in this situation, the practical role of the second law of thermodynamics, and of course, the role of coupled equations and the possibility of their applicability become very important. In this situation, there are various challenges that researchers are working on. For example, the unequal form of the second law generally cannot be used as an equation that can be applied in problems, in order to obtain the behavior of the system in a specific energy exchange with the surroundings. In fact, the relevant form is in the position of checking the feasibility of a physical solution for the behavior of the studied system. Also, the existence of different expressions and formulations for the second law of thermodynamics brings many practical and scientific challenges [41-55]. Attempts have been made to find a general solution for various expressions of the second law of thermodynamics in terms of system energy components for physical processes [56]. Also, the energy components of the system have been used to provide a quasistatistical equation for entropy, such that the corresponding equation has the same basis as Boltzmann's entropy equation [57]. In fact, we can apply some of the effects of the second law on the energy structure of the system using its energy components [50,56,58-59]. Borchers presents his equations by considering mechanical and thermal variables in order to determine the behavior of the system in the representation of its internal energy, and also, using the relevant mathematical structure resulting from the definition of entropy in the second law of thermodynamics [60]. In this respect, the energy structure approach can be similar to Borchers approach. In fact, according to the achievements that have

been obtained from the perspective of the energy structure, the issue of its expansion and completion can be very valuable. References [50,56-59] provide the equation of energy structure with the assumptions of irreversibility in a physical process, and are used to provide the quasi-statistical equation for entropy as well as the different expressions of the second law of thermodynamics. In this study, the energy structure equation is extracted with the relevant mathematical analysis process, and used to define a new energy space. Feasible processes are investigated, and system compatibility conditions are derived. Also, the dynamic energy definition of the system is provided based on the extracted energy structure, from which valuable results can be obtained on the behavior of physical systems from the point of view of its energy components. The relevant perspective provides a simultaneous view of the first and second laws of thermodynamics, and is used in the energy coordinates of the system in order to express the processes that can be performed as a result of energy exchange with the surrounding. Another issue that results from this point of view is determining the structure of irreversibility, and also, defining the components of irreversibility. The analysis of irreversibility in a physical process using the energy structure equation is available in reference [59]. In this paper, according to the mathematical process established for deriving the equation of the energy structure as well as the definition of the new dynamic energy component and system compatibility conditions, a more comprehensive approach to the results presented for the analysis of irreversibilities is obtained. Finally, it must be noted that one of the goals of the energy structure perspective is to try to improve the relevant forms to express the energy of the system by providing its equations on the basis of the activated energy components and their structure and apply the governed principles that have recently become one of the challenges of scientists regarding the correct forms of the first and second laws of thermodynamics are converted, as well as the mathematical errors corresponding to these laws.

2. Notations and Definitions

In this part, the most important used concepts and notations are defined. At the first step, we need to provide a standard formulation to the first law of thermodynamics.

2.1 N.D.1. Energy Space

Energy space is combined of the all activated and nonactivated energy components of the system in the performed process. During a physical process, one of the sub-spaces of the energy space participates in the performed process. All analysis are done based on the energy space of the system, and relations are written based on the independent and dependent energy components of the system. Notation u_i is used for energy components of system.

2.2 N.D.2. Independent Energy Components

For a performed process, independent energy components are that components that are activated independently of that how energy is applied to the system. In fact, these components are the basis of the performed process, and will be activated in all conditions of energy applying for the considered process.

2.3 N.D.3. Dependent Energy Components

Dependent energy components can be activated dependently to the independent energy components. In fact, these components are dependence that how energy is applied to the system, and also can be ranged as value-dependence ($g_j = g_j(u_1, u_2, ..., u_m)$) and rate-dependence ($h_p = h_p(\dot{u}_1, ..., \dot{u}_m)$) to the independent energy components activated in the performed processes. Where *m* is number of the activated independent energy components.

2.4 N.D.4. Energy Structure Equation

Energy structure equation is established and developed as an equation that can formulate the performed processes using activated, independent and dependent, energy components of the system, and also shows their dependence. Therefore, energy structure equation is used to study the physical processes from the perspective of the energy space of the system.

2.5 N.D.5. Quasi-Static Path

In a quasi-static path, energy is applied to the system with a zero rate. In fact, for independent components, it can be considered that in a quasi-static path $\dot{u}_i \cong 0$. This path is used as a reference path, and other paths are studied using comparison with the quasi-static path. In this path, only independent and value-dependence energy components can be activated.

2.6 N.D.6. General Paths

In a general path, energy is applied to the system with a non-zero rate (Or $\dot{u}_i \neq 0$).

2.7 N.D.7. Particular Process

All processes that have the same active independent energy components.

2.8 N.D.8. Various Conditions

All conditions that same energy amount is applied to the system.

3. Energy In A Physical System

When the principle of energy conservation is used for a physical problem, at first, it is necessary to express the proper term for the system energy, which includes those participating in the processes that are possible for the system and will change through. The principle of energy conservation relates these changes to each other as well as the exchange of energy with the environment. For example, when the principle of mechanical energy conservation is to be applied, the total mechanical energy is considered as a sum of potential energies and kinetic energy [61].

$$E_M = U_{sys} + K_{sys} \tag{1}$$

In that E_M is the total mechanical energy, U_{sys} and K_{sys} are respectively potential energy and kinetic energy. In this case, the principle of energy conservation states that the total energy change rate is equal to the exchange rate of energy with the environment.

If the desired problem has thermal effects too, then in writing the principle of energy conservation, it is necessary to consider the terms related to heat exchange. For a unit mass of the body, if dH is the variation in the heat collected, dE is the internal energy variation, dW of the work performed by the external forces, dV is the change of the

kinetic energy stored and dR is the heat changes transformed to the environment, then the principle of energy conservation is thus written [1]:

$$dH = dE + dW + dV + dR \tag{2}$$

Equation (2) identifies the relationship between the intrinsic, thermal and kinetic energy changes of the system to the work done and changes in the heat exchanged with the surrounding environment.

As another example, it can be argued that a vulnerable solid object can exchange heat with the surrounding environment. Sosnovsky [12] named these systems as mechanothemodynamic systems. He also considered the energy of the whole to analyze the mechanothermodynamic systems as follows:

$$E = E_T + E_M \tag{3}$$

Where *E* is the total energy, E_T and E_M are respectively thermal and mechanical energy. He used this expression to determine the mechanothermodynamic entropy.

In this paper, it is assumed that the total energy for the system can be considered as follows, as shown in figure 1:

$$U_T = U_{ND} + U_D \tag{4}$$

That U_T is the total energy, U_D is the dynamic energy and U_{ND} is the sum of all other types of energy. Fig 1 takes a scheme of this division:



Figure 1. A new division of the total energy.

Dynamic energy is the sum of energies that are directly related to the inertia of the system. Non-dynamic energy is also the sum of energies that are not directly related to the inertia of the system and are related to the deformation of the system and its displacement. Energies such as the potential energy of the massless spring, the energy associated with the massless damper, the energy associated with changing the distance between the plates of a massless capacitor, etc. are the examples of non-dynamic energy. A component of the kinetic energy such as kinetic energy of a moving rigid body could be an example of dynamic energy, because it is directly related to the inertia of the system. This division enables us to apply the effects of the surrounding environment on the equation governing the behavior of the system, thus representing each physical process as a term appropriate for dynamic and non-dynamic energies. This manner of presentation allows us to incorporate the internal structure of the system and the processes that can be implemented for it in terms of non-dynamic energies.

Eq. (4) introduces a new approach for investigating the total energy of the system which its base is the direct dependence on the inertia of the system. Each energy type in this equation whether dynamic or non-dynamic could be divided into different components e.g. in studying damage of mechanothermodynamic systems, irreversible effective

energy in damage will be one of them, therefore this equation is more general than any other relation considering only a phenomena such as tribo-fatigue.

Using this assumption, a study on the energy structure of physical systems can be performed.

4. Non-Dynamic Energy Structure

In physical systems, the concepts of reversibility and irreversibility are an important issue for the performed process. It is in this situation that paying special attention to irreversibility factors has a special place [28-31]. In fact, physical processes are generally irreversible and this issue needs to be considered especially in the basis of the governing equations. Figure 2 shows a physical system in energy exchange with the surrounding environment and also according to its energy components:



Figure 2. Scheme of a physical system with its energy components.

In Figure 2, a physical system with its non-dynamic energy components is shown. This system receives some energy from its surrounding. The total non-dynamic energy for the system can be expressed as follows:

$$U_{ND} = \sum_{i} u_i \tag{5}$$

That u_i is a non-dynamic energy component.

In a process performed by the system, some of the energy components remain constant, some others change independently in the process, and remaining will also change as a function of independent components. Therefore, assuming that the first m components are independently participating in the process, k components are the dependent ones, and the other components remain unchanged, the non-dynamic variation can be written as follows:

$$\delta U_{ND} = [\delta u_1 + \delta u_2 + \dots + \delta u_m] + [\delta u_{m+1} + \dots + \delta u_{m+k}] + 0$$
(6)

Assuming that the process is quasi-static ($\dot{u}_i \cong 0$), then the dependent components can be rewritten as follows, as shown in figure 3.

$$\delta g_i \equiv \delta u_{m+i} \tag{7}$$

That:

 $g_j = g_j(u_1, u_2, \dots, u_m) \tag{8}$

So the Eq. (6) can be rewritten as follows:

$$\delta U_{ND} = [\delta u_1 + \delta u_2 + \dots + \delta u_m] + [\delta g_1 + \dots + \delta g_k] \quad (9)$$

By calculating the non-dynamic energy of the Eq. (9):

$$U_{ND} = (u_1 + u_2 + \dots + u_m) + [g_1 + \dots + g_k] + U_{T_0} \quad (10)$$

That U_{T_0} is the sum of all the energies that have not participated in the process:

$$U_{T_0} = \sum_i u_{m+k+i} \tag{11}$$



Figure 3. Non-dynamics energy change in the quasi-static process.

Path 1-2 is an ideal path on which the variations of the independent components occur in a quasi-equilibrium state. The quasi-static path occurs with remaining the system in internal equilibrium, even for an ideal process to be performed it could be supposed that infinite time is needed. As expected, in this case the relationship between the energy components is independent of time (because the functions g_j do not depend on time).

And in general case ($\dot{u}_i \neq 0$), as shown in figure 4, the following statement is used:



Figure 4. Non-dynamics energy change in general case.

$$U_{ND} = (u_1 + u_2 + \dots + u_m) + [g_1 + \dots + g_k] + [h_1 + \dots + h_n] + U_{T_0}$$
(12)

$$h_p = h_p(\dot{u}_1, \dots, \dot{u}_m) \tag{13}$$

Based on the Eq. (12), can be named *Energy Structure Equation*, in a physical process, the energy components that are activated can depend on the rate of energy applied to the system by the environment, and also on how it is applied, as well as the internal structure of the system. Eq. (12) states that in a non-static state, the energy given to the system is divided into more components than the static state.

The changes in Eq. (12) can be expressed as follows:

$$\delta U_{ND} = \sum_{i=1}^{m} \delta u_i + \sum_{j=1}^{k} \left(\frac{\partial g_j}{\partial u_1} \delta u_1 + \dots + \frac{\partial g_j}{\partial u_m} \delta u_m \right) + \sum_{p=1}^{n} \left(\frac{\partial h_p}{\partial \dot{u}_1} \delta \dot{u}_1 + \dots + \frac{\partial h_p}{\partial \dot{u}_m} \delta \dot{u}_m \right)$$
(14)

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As δU_T is exchanged between system and surrounding, activated energy components will change based on how energy is applied to the system. So, considering this process, Eq. (15) can take the variations of the activated independent energy components (δU_T is not necessarily heat or work, and in general it can be a combination of them or the cause of a variation in the independent components of energy):

$$\delta u_1 = \alpha_1 \delta U_T$$

$$\vdots$$

$$\delta u_m = \alpha_m \delta U_T$$
(15)

The coefficients α_i can be named as loading coefficients that depend on how energy is applied to the system. Therefore, by assuming that:

$$\alpha = \left[\sum_{i=1}^{m} \alpha_i \left(1 + \sum_{j=1}^{k} \frac{\partial g_j}{\partial u_i}\right)\right] + \left[\sum_{i=1}^{m} \dot{\alpha}_i \left(\sum_{p=1}^{n} \frac{\partial h_p}{\partial \dot{u}_i}\right)\right]$$
(16)

$$\beta = \sum_{i=1}^{m} \alpha_i \left(\sum_{p=1}^{n} \left(\frac{\partial h_p}{\partial u_i} \right) \right) \tag{17}$$

Equation (14) can be rewritten as follows:

$$\delta U_{ND} = \alpha \delta U_T + \beta \delta \dot{U}_T \tag{18}$$

The coefficients α and β depend on the amount and rate of energy applied to the system, as well as the inherent properties of the system. δU_T is not necessarily heat or work, and in general it can be a combination of them or the cause of a variation in the independent components of energy.

Equation (18) can also be expressed as:

$$\dot{U}_{ND} = \alpha \dot{U}_T + \beta \ddot{U}_T \tag{19}$$

Equation (19) depends on the variation of non-dynamic energies to the amount and variation of the rate of the energy that applied from the environment to the system as well as the coefficients α and β .

As an example, in one degree of freedom vibrational systems [30], the potential energy depends on the displacement of the mass while the dissipated energy depends on the velocity. Therefore, by assuming the potential energy as the independent energy component, the dissipated energy could be considered as the dependent one. It is also notable that some of these energy components could be considered as non-dynamic energy of the system.

The energy structure equation is based on the energy components of the system as well as the way of energy exchange between the system and the surrounding environment. In fact, according to the energy structure equation, in a physical process, some components are activated independently, and some participate in the physical process depending on the independent components, and the rest of the components remain inactive. Dependent components can be a function of the amount or rate of independent components. The components dependent on the rate of change of the independent components, in fact, show the irreversibility aspects of the performed physical process.

The energy structure perspective has no contradiction with the second law of thermodynamics. In fact, based on sub-structural approaches to the second law of thermodynamics [62-64], the perspective of energy structure is in complete agreement with this law.

5. Feasible Processes

In order to study the performed processes, classical thermodynamics considers their initial and final states, and heat exchange and work done in different paths are studied. Therefore, for thermodynamic processes with transferred heat and work done, as well as considering constant initial and final states, in paths that have similar initial and final states, the internal energy changes are equal, although the heat exchanged and also the work done in these paths are different, as shown in figure 5.



Figure 5. System behavior in different paths with fixed first and final states.

In this approach, as shown in figure 6, the energy exchanged between the system and the surrounding environment is considered equal in different paths, and the changes of the energy components in these paths are studied.



Figure 6. System behavior by applying the same energy in different paths.

To investigate the feasible processes, same amounts of energy are applied to a system in two different paths and thus the variation of the energy components will be compared. In fact, it is assumed that the variations of non-dynamic energies are equal in these two different paths and then the variations of independent components will be studied. In other words, the perspective of the energy structure studies the paths that have equal energy exchange and derives the governing condition for the feasible processes based on the energy components of the system.

As shown in figure 6, if Eq. (9) and Eq. (14) are equaled when the component u_j is the only independent component of the process, then the following relation can be concluded (That δu_j takes the variation of component u_j in general path ($\dot{u}_j \neq 0$) for the case that δU_T is exchanged between system and surrounding. Also, $\delta u'_j$ takes the variation of component u_j for the quasi-static path ($\dot{u}_j \approx 0$) for the case that δU_T is exchanged between system and surrounding.):

$$\delta u_{j} + \sum_{i=1}^{k} \left(\frac{\partial g_{i}}{\partial u_{j}} \delta u_{j} \right) + \sum_{p=1}^{n} \left(\frac{\partial h_{p}}{\partial \dot{u}_{j}} \delta \dot{u}_{j} \right) = \delta u'_{j} + \sum_{i=1}^{k} \left(\frac{\partial g_{i}}{\partial u_{j}} \delta u'_{j} \right)$$
(20)

Therefore:

$$\left(1 + \sum_{i=1}^{k} \left(\frac{\partial g_i}{\partial u_j}\right)\right) \left(\delta u_j - \delta u'_j\right) = -\left(\sum_{p=1}^{n} \left(\frac{\partial h_p}{\partial \dot{u}_j}\right)\right) \delta \dot{u}_j$$
(21)

$$-\frac{\left(\sum_{p=1}^{n} \left(\frac{\partial h_p}{\partial \dot{u}_j}\right)\right)}{\left(1+\sum_{l=1}^{k} \left(\frac{\partial g_l}{\partial u_j}\right)\right)} = \frac{\left(\delta u_j - \delta u'_j\right)}{\delta \dot{u}_j}$$
(22)

$$-\frac{\left(\sum_{p=1}^{n}\left(\frac{\delta h_{p}}{\partial u_{j}}\right)\right)}{\left(\left(1+\sum_{i=1}^{k}\left(\frac{\partial g_{i}}{\partial u_{j}}\right)\right)\right)}\left(\frac{\delta u_{j}-\delta u'_{j}}{\dot{u}_{j}}\right) = \frac{\left(\delta u_{j}-\delta u'_{j}\right)}{\delta \dot{u}_{j}}\left(\frac{\delta u_{j}-\delta u'_{j}}{\dot{u}_{j}}\right)$$
(23)

$$\left[\frac{\left(\Sigma_{p=1}^{n}\left(\frac{\partial h_{p}}{\partial \dot{u}_{j}}\right)\right)}{\left(1+\Sigma_{i=1}^{k}\left(\frac{\partial g_{i}}{\partial u_{j}}\right)\right)}\right]\left(\frac{\delta u_{j}-\delta u'_{j}}{\dot{u}_{j}}\right) = -\frac{\left(\delta u_{j}-\delta u'_{j}\right)^{2}}{\dot{u}_{j}\delta \dot{u}_{j}} = -2\frac{\left(\delta u_{j}-\delta u'_{j}\right)^{2}}{\delta (\dot{u}_{j})^{2}} \tag{24}$$

Therefore, since a quasi-static path is used as reference path:

$$\left[\frac{\left(\sum_{p=1}^{n} \left(\frac{\partial h_{p}}{\partial \dot{u}_{j}}\right)\right)}{\left(1+\sum_{i=1}^{k} \left(\frac{\partial g_{i}}{\partial u_{j}}\right)\right)}\right] \left(\frac{\delta u_{j}-\delta u'_{j}}{\dot{u}_{j}}\right) \leq 0$$
(25)

The terms $\left(\sum_{p=1}^{n} \frac{\partial h_p}{\partial \dot{u}_j}\right)$ and $\left(\frac{\delta u_j - \delta u'_j}{\dot{u}_j}\right)$ are actually a description of the more energy distribution in applying energy to the system at a non-zero rate. The term $(1 + \sum_{i=1}^{k} \frac{\partial g_i}{\partial u_j})$ also depends on the internal structure of the system and does not depend on the rate of energy applying.

Relationship (25) is established for any process that the system experiences. This inequality is expressed in the following Eq. (18) and shows that a physical system cannot experience any arbitrary process and inequality (25) must be established. This condition is very similar to the second law of thermodynamics, which states that in real processes, some entropy will always be generated.

Relationship (25) can also be considered as a constraint on processes performed by the system.

6. A Novel Energy Conservation Principle

Borchers considered the law of conservation of energy in terms of mechanical and thermal variables [60]. The perspective of energy structure actually uses energy components by considering the related dependence structure in order to express the energy of system. In fact, one of the achievements of the energy structure perspective mentioned in the references is providing a new expression for the energy conservation principle using a standard formulation to the first law of thermodynamics [50,57,65-66]. In this part, a brief overview of this statement is provided. Considering figure 7.

$$\delta U_j = \delta u_j - \delta u'_j \tag{26}$$

So, relation (25) takes:

$$\left[\frac{\left(\sum_{p=1}^{n} \left(\frac{\partial h_{p}}{\partial \dot{U}_{j}}\right)\right)}{\left(1+\sum_{i=1}^{k} \left(\frac{\partial g_{i}}{\partial \dot{U}_{j}}\right)\right)}\right] \left(\frac{\delta U_{j}}{\dot{U}_{j}}\right) \leq 0$$
(27)

Or:

$$\frac{\left[\left(1+\sum_{i=1}^{k} \left(\frac{\partial g_{i}}{\partial U_{j}}\right)\right)\right]}{\left(\sum_{p=1}^{n} \left(\frac{\partial h_{p}}{\partial U_{j}}\right)\right)}\right]\left(\frac{\delta U_{j}}{\dot{U}_{j}}\right) \leq 0$$
(28)

Eq. (28), like Borchers equations [60], in addition to energy exchange, depends on the internal structure of the system as well as the way of energy exchange. Eq. (28) is general in system energy coordinates and according to the energy structure equation, and it is obtained without any limiting assumption. These conditions are provided in Borchers unified view in terms of mechanical and thermal variables that can provide a complete description of the system.



Figure 7. Difference between two different paths [65].

7. The Physical Significance of Entropy

Borchers considered the law of conservation of energy in terms of mechanical and thermal variables [60]. The perspective of energy structure

The second law can be expressed as follows [67]:

$$\oint \frac{\delta \varrho}{T} \le 0 \tag{29}$$

Where δQ is the heat exchanged at temperature *T*. Also, for a continuous distribution of matter:

$$\frac{d}{dt} \int_{p} \rho s \, dV \ge \int_{p} \rho e \, dV - \int_{\partial p} \frac{q_{i} n_{i}}{T} \, da \tag{30}$$

So, in the differential form:

$$\frac{ds}{dt} - e - \frac{1}{\rho} \left(\frac{q_i}{T}\right)_{,i} \ge 0 \tag{31}$$

Where p is a material particle, and ∂p is the surface containing the volume of the particle p. Also, e is the rate of entropy entry from surrounding surface by external sources per unit mass of the material, q_i is the heat flux vector passing through the unit surface, ρ is density, n_i is the normal vector of surface, T is temperature, and s is entropy of the considered material point.

The equality state is for reversible processes, and the inequality state is for irreversible processes. Among the most important equations of classical thermodynamics in order to study irreversibility in performed process, Eq. 32 can be considered: [68]:

$$\vec{l_k} = \sum_l L_{kl} \vec{F_l} \tag{32}$$

Where $\overrightarrow{l_k}$ is a set of general currents and $\overrightarrow{F_l}$ is thermodynamic forces. Also, L_{kl} is a matrix of kinematic coefficients. For matrix L_{kl} it is required to satisfy the

symmetry equations in the OSAGER equations [69]. In this perspective, the irreversibility is studied using investigating the sub-structures of the system.

However, when there is no irreversibility in the performed process, Eq. (33) takes the entropy change in the process:

$$Tds = dQ \tag{33}$$

Based on the second law of thermodynamics, $ds \ge \frac{dQ}{T}$ or $ds_{gen} \ge 0$, that s_{gen} is the entropy generated, and must be always positive in all feasible process. The entropy generated is the basis for studying the amount of available work lost. In fact, irreversibility in a physical process is directly related to the amount of entropy production. The generated entropy increases the amount of thermal energy of the system, and this can be studied in the increase of changes related to the dependent components h_p activated in the performed process. The amount of irreversibility is considered as $T_0 \delta S_{gen}$ that is defined based on entropy generation and the environment temperature [67].

From the perspective of the statistical physics, Planck's theorem, entropy is considered as a function of the probability [70]. So:

$$S = f(W) \tag{34}$$

Where S is entropy, and W is the probability. Based on the considered properties for entropy, function f must satisfies the following conditions [70]:

$$S = S_1 + S_2 \tag{35}$$

$$f((W_1, W_2) = f(W_1) + f(W_2)$$
(36)

That W_1 and W_2 are probabilities of sub-systems 1 and 2. So:

$$S = kln(W) + constant$$
(37)

That *k* is a constant. Plank equation for entropy is [26]:

$$S = k[(N+P)\ln(N+P) - N\ln(N) - P\ln(P)]$$
(38)

That N is resonators, and P is an integer. The equation of probability has been provided with different approaches [27]. As an example, Fermi-Dirac provide the following equation:

$$W = \prod_{j} \frac{g_{j}!}{(g_{j}-1)!N_{j}!}$$
(39)

That N_j is the number of particles with the weighting factor of energy level g_j . Statistical mechanics determines the absolute value of entropy according to the energy levels of its constituent elements. In this view, the energy level of each particle is considered as a variable, and according to the set of all energy levels, statistical mechanics will be able to calculate the absolute value of entropy.

8. Energy Component's Approach to the Boltzmann Entropy Equation

Borchers considered the law of conservation of energy in terms of mechanical and thermal variables [60]. The perspective of energy structure

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As one of the achievements of the energy structure perspective in the references, it is the presentation of the quasi-statistical equation of entropy, which has a common basis as Boltzmann's entropy equation [57]. In this section, a brief overview of this equation is provided. Figure 8 shows a scheme of the energy components approach:



Figure 8. Energy Component's approach for particles. system [57].

Considering Eq. (26) and rewriting Eq. (21):

$$\left(1 + \sum_{i=1}^{k} \left(\frac{\partial g_i}{\partial U_j}\right)\right) \left(\delta U_j\right) = -\left(\sum_{p=1}^{n} \left(\frac{\partial h_p}{\partial \dot{U}_j}\right)\right) \left(\delta \dot{U}_j\right) \quad (40)$$

So, if Eq. (40) is replaced in the relation (28):

$$\left(-\frac{\delta \dot{v}_j}{\dot{v}_j}\right) \le 0 \tag{41}$$

Or:

$$\delta[\ln(\dot{U}_j)] \ge 0 \tag{42}$$

Because it is necessary that the relation (42) be established for each independent activated component, so relation (43) can be derived:

$$\delta\left[\ln(\dot{U}_{1})\right] + \delta\left[\ln(\dot{U}_{2})\right] + \dots + \delta\left[\ln(\dot{U}_{m})\right] \ge 0 \tag{43}$$

Therefore:

$$\delta[\ln(\prod_{j=1}^{m} \dot{U}_j)] \ge 0 \tag{44}$$

Considering $w_{u_j} = \dot{U}_j$, and also $W_u = \prod_{j=1}^m w_{u_j}$, therefore:

$$\delta[\ln(W_u)] = \delta\left[\ln\left(\prod_{j=1}^m w_{u_j}\right)\right] = \sum_{j=1}^m \delta \ln\left(w_{u_j}\right) \ge 0$$
(45)

Finally, as the Boltzmann equation, the quasi-statistical definition of entropy can be provided as follows:

$$\delta s = K_{MS} \,\delta[\ln(W_{\mu})] \tag{46}$$

Where K_{MS} is a universal constant. The quasi-statistical equation of entropy relates entropy changes to the rate of energy components that are measured relative to the quasi-static path. It also obtains entropy changes depending on the amount and manner of energy exchange. Also, applicability for systems with a continuous distribution of matter as well

as the amount of acceptable calculations can be mentioned as two features of this equation [57].

9. Dynamic Energy and Compatibility Conditions of System

Using Eq. (4), the variation of the dynamic energy in a process can be written as follows:

$$\delta U_D = \delta U_T - \delta U_{ND} \tag{47}$$

By placing the Eq. (18) in (47):

$$\delta U_D = (1 - \alpha) \delta U_T - \beta \delta \dot{U}_T \tag{48}$$

Eq. (48) shows the dependence of the dynamic energy variation to the amount and rate of energy applied from the environment to the system as well as the coefficients α and β . By using Eq. (15) dynamic energy can also be written in terms of independent components:

$$\delta U_T = \frac{\delta u_j}{\alpha_j} , 1 \le j \le m , \alpha_j \ne 0$$
⁽⁴⁹⁾

$$\delta \dot{U}_T = \left(\frac{1}{\alpha_j}\right) \delta \dot{u}_j - \left(\frac{\dot{\alpha}_j}{\alpha_j^2}\right) \delta u_j \tag{50}$$

Therefore:

$$\delta U_D = \left[\left(\frac{1 - \alpha}{\alpha_j} \right) + \left(\frac{\beta \dot{\alpha}_j}{\alpha_j^2} \right) \right] \delta u_j - \left(\frac{\beta}{\alpha_j} \right) \delta \dot{u}_j \equiv \gamma_j \delta u_j + \gamma_j' \delta \dot{u}_j$$
(51)

Eq. (51) gives the variation of dynamic energy in terms of the independent component of u_j and its changing rate. γ_j and γ_j' are the coefficients of influence of δu_j and $\delta \dot{u}_j$ on the variation of the dynamic energy of the system. The dependence of dynamic energy on the rate of the independent energy components indicates the possibility of more energy distribution by increasing the rate of applied energy to the system. The following relationship will also exist between the independent energy components:

$$\gamma_j \delta u_j + \gamma_j' \delta \dot{u}_j = \gamma_i \delta u_i + \gamma_i' \delta \dot{u}_i , 1 \le i, j \le m$$
(52)

The Eq. (52) indicates possible paths for independent energy components. This equation can be rewritten as follows:

$$\left(\gamma_{j}\delta u_{j}-\gamma_{i}\delta u_{i}\right)=\left(\gamma_{i}'\delta\dot{u}_{i}-\gamma_{j}'\delta\dot{u}_{j}\right)=\mu_{ij}$$
(53)

Where μ_{ij} is a scalar that depends on the performed process as well as δu_i and δu_j components. For a process performed by the system, each of the *m* independent components should be analyzed with (m-1) other components by using the Eq. (52). Therefore, the number of $\frac{m(m-1)}{2}$ values could be calculated for μ_{ij} . These $\frac{m(m-1)}{2}$ equations state the compatibility conditions of the system that must be satisfied for the performed process.

If $\mu_{ii} = 0$ and $\gamma_i, \delta u_i, \delta \dot{u}_i, \gamma_i' \neq 0$:

$$\frac{\delta u_j}{\delta u_i} = \frac{\gamma_i}{\gamma_j} \tag{54}$$

And also:

$$\frac{\delta \dot{u}_j}{\delta \dot{u}_i} = \frac{\gamma_i{}'}{\gamma_j{}'} \tag{55}$$

Eq.s (54) and (55) must be established together. Therefore if $\mu_{ij} = 0$, m(m-1) equations should be satisfied for the performed process, which in fact they specify the performed process as well as system compatibility conditions.

By using Eq. (19), the dynamic energy rate can also be expressed as:

$$\dot{U}_D = (1 - \alpha)\dot{U}_T - \beta \ddot{U}_T \tag{56}$$

That takes the rate of dynamic energy.

10. Energy State Equation

Using Eq. (4), the variation of the dynamic energy in a process can be written as follows:

In the case of $\beta = 0$, then dynamic and non-dynamic energy changes are independent of the exchange rate of energy with the environment and will only depend on the amount of energy exchanged. In this case, it is expected that for each value of the energy given to the system, the dynamic and non-dynamic energies will establish in a given relationship, which is dependent on the internal structure of the system.

If we consider this relation as follows:

$$G(U_{ND}, U_D) = 0 \tag{57}$$

In this case, by calculating the variation of Eq. (57) and using Eq.s (18) and (56), the following differential equation is obtained:

$$\alpha \frac{\partial G}{\partial U_{ND}} + (1 - \alpha) \frac{\partial G}{\partial U_D} = 0$$
(58)

If $\alpha = 0$ or $\alpha = 1$, we obtain from the differential equation (58) that G = c, where c is a constant value. The value of $\alpha = 0$ represents the process by which all energy exchanged with the environment is converted to dynamic energy and in the case of $\alpha = 1$, all energy exchanged also increases non-dynamic energy and does not result in a change in the dynamic energy. If $\alpha = 1/2$, then from the differential Eq. (58), $U_{ND} - U_D = c$ is obtained, where c is a constant value.

In fact for a physical system, if $\beta = 0$ then every function G in the form of Eq. (58) represents a feasible process in the system.

11. A Few Energy-Related Basic Examples

In this part, the presented model, novel concepts, and established relations are developed for some energy-related basis examples. The main purpose is an applicable validation for the established concepts as well as relations. In the following examples, based on the established model, the distribution of the whole energy given to the system is shown. In some of them, a part of energy may be wasted and the remaining may be stored in the components of the system. No contradiction will be found in the relations. These basis problems can be combined with others to create a variety of systems, ranging from classical to modern domains, and are used to show the performance ability of the established energy structure approach. Also, the established model is used to investigating the linear viscoelasticity models.

11.1 First Basic Example: Newton's Particle System

For Newton's particle shown in figure 9 with $F_{i.ext}$ as external force, and $F_{i,int}$ as internal force applied on the particle *i*:

$$\alpha = \beta = 0 \tag{59}$$



Figure 9. Dynamic and non-dynamic energy of Newton's particle system.

In this case, we obtain of the Eq. (60):

$$\dot{U}_D = \dot{U}_T \tag{60}$$

This equation states that the energy given to the system is completely transformed into dynamic energy.

In this case, the relation (28) is established in the equality state and the function G is also G = 0.

11.2 Second Basic Example: Elasticity Properties

An elastic rod with hardness k under the axial force p at the end is supposed, as shown in figure 10.



Figure 10. Non- dynamic energy related to the elasticity properties.

The elastic potential energy of the spring is considered as an independent component of the process:

$$U_{ND} = u_e \tag{61}$$

That u_e is the elastic potential energy. Therefore, the corresponding coefficient in Eq. (15) can be considered as follows:

$$\alpha_e = \frac{\sqrt{2ku_e}}{p} \tag{62}$$

From the Eq. (62), it can be concluded that the coefficient is a function of the current system condition and also the amount of energy applied to it.

Given that $\gamma_e = \left(\frac{1-\alpha_e}{\alpha_e}\right)$ and $\gamma_e' = 0$, Eq. (63) also obtains the dynamic energy as follows:

$$\delta U_D = \left(\frac{1 - \alpha_e}{\alpha_e}\right) \delta u_e \tag{63}$$

Therefore:

$$\delta U_D = \left(\frac{1 - \left(\frac{\sqrt{2ku_e}}{p}\right)}{\left(\frac{\sqrt{2ku_e}}{p}\right)}\right) \delta u_e \equiv (p - kv) \dot{v} \delta t \tag{64}$$

Where v is the displacement of the force location p. Using this equation, we will be able to calculate the dynamic energy available in the system. This energy belongs to all elements of the system and based on Eq. (64) is a function of the elastic potential energy of the system, $U_D = U_D(u_e)$.

As can be seen, Eq. (64) is completely equivalent to the result that expected, which determines the total entire system's dynamic energy.

Given that an independent energy component participates in the process, there is no need to apply a specific compatibility condition to the changes in the independent energy components ($\frac{m(m-1)}{2} = 0$ for m = 1).

The Eq. (42) indicates that the dynamic energy does not dependent on the rate of change of the component u_e and therefore dissipation does not occur in the system due to non-zero energy applied rate and the more energy distribution is not occurred, in the other words, given that for this problem $(\sum_{p=1}^{n} \frac{\partial h_p}{\partial u_j}) = 0$, the relation (28) will be in the state of equality.

The differential equation for the function G is obtained as follows:

$$\left(\frac{\sqrt{2kU_{TV}}}{p}\right)\frac{\partial G}{\partial U_{ND}} + \left(1 - \frac{\sqrt{2kU_{TV}}}{p}\right)\frac{\partial G}{\partial U_D} = 0$$
(65)

In fact, the above equation shows the energy states for the problem of figure 10.

11.3 Third Basic Example: System with Mass and Elastic Bounding

This example represents a sub-system of two point masses with elastic bonding so that the point masses can be moved along the line of their connection, as shown in figure 11:



Figure 11. Non-dynamic energy for a system with mass and elastic bounding.

If the gravitational potential energy is ignored, and also considering u_e as elastic energy component, the nondynamic energy for this system can be expressed as follows:

$$U_{ND} = u_e \tag{66}$$

Given that $\sum_{p=1}^{n} \frac{\partial h_p}{\partial \dot{u}_j} = 0$, the relation (28) is established for this problem in its equality state.

11.4 Fourth Basic Example: System with Electric Charges and Elastic Bonding

The figure 12 is shown a system including two point electric charges with elastic bonding so that the point electric charge can be moved along the line of their connection:



Figure 12. Non-dynamic energy for a system with electric charges and elastic bonding.

Considering u_e as elastic energy component, and u_q as electrical energy component, the following equation can be expressed for non-dynamic energy of this system:

$$U_{ND} = u_e + \frac{c_1}{c_2 \sqrt{u_e} + c_3} \tag{67}$$

Where c_1 , c_2 and c_3 are constants depend on the structure of the system and the initial and final conditions of the process. Eq. (67) can explain the processes that electric charges move along the lines of each other.

Given $\sum_{p=1}^{n} \frac{\partial h_p}{\partial u_j} = 0$, relation (28) is established for this problem in its equality state.

11.5 Fifth Basic Example: Combining Third and Fourth Examples

Figure 13 illustrates two examples by combining the third and fourth examples. These examples are samples of multi-component systems. For models shown in figure 13, the components of energy that are important are: the potential energy of springs and electrical charge, and also the compatibility conditions will be important for the investigation of these systems.



Figure 13. Two different schemes of combining two points electric charge with elastic bonding.

For figure (13-a) with considering $u_e' = 2u_e$, nondynamic energy equation can be written:

$$U_{ND} = u_e' + \frac{c_1}{c_2 \sqrt{u_e'} + c_3} \tag{68}$$

And for figure (13-b) it can be written:

$$U_{ND} = u_{e1} + u_{e2} + \frac{c_1}{c_2 \sqrt{u_{e1}} + c_3}$$
(69)

That u_{e1} and u_{e2} are the elastic potential energy of k_1 and k_2 . Respectively based on Eq. (30) dynamic energy is:

$$\delta U_D = \gamma_j \delta u_{ej} + \gamma_j' \delta \dot{u}_{ej} , \ j = 1 \text{ or } 2$$
(70)

And compatibility condition for this system is:

$$\gamma_1 \delta u_{e1} + \gamma_1' \delta \dot{u}_{e1} = \gamma_2 \delta u_{e2} + \gamma_2' \delta \dot{u}_{e2}$$
(71)

Because for this problem β_1 and β_2 are zero, therefore γ_1' and γ_2' are zero too, so Eq.s (70) and (71) can be rewritten as follows:

$$\gamma_1 \delta u_{e1} = \gamma_2 \delta u_{e2} \tag{72}$$

$$\delta U_D = \gamma_j \delta u_{ej} , \ j = 1 \text{ or } 2$$
(73)

And by using Eq. (51):

$$\left(\frac{1-\alpha}{\alpha_{e1}}\right)\delta u_{e1} = \left(\frac{1-\alpha}{\alpha_{e2}}\right)\delta u_{e2} \tag{74}$$

$$\delta U_D = \left(\frac{1-\alpha}{\alpha_{ej}}\right) \delta u_{ej} , \ j = 1 \text{ or } 2$$
(75)

Also by using the Eq.s (16) and (17):

$$\alpha = \alpha_{e1} + \alpha_{e2} \left(1 - \frac{c_1 c_2}{\left(2\sqrt{u_{e1}}\right)\left(c_2\sqrt{u_{e1}} + c_3\right)^2}\right)$$
(76)

$$\beta = 0 \tag{77}$$

Therefore Eq.s (74) and (75) can be rewritten as follow:

$$\left(\frac{\delta u_{e1}}{\alpha_{e1}}\right) = \left(\frac{\delta u_{e2}}{\alpha_{e2}}\right) \tag{78}$$

$$\delta U_{D} = \left(\frac{1 - (\alpha_{e1} + \alpha_{e2} \left(1 - \frac{c_{1}c_{2}}{(2\sqrt{u_{e1}})(c_{2}\sqrt{u_{e1}} + c_{3})^{2}}\right))}{\alpha_{ej}}\right) \delta u_{ej}, \ j = 1 \text{ or } 2$$
(79)

Given the Eq. (15), the Eq. (79) has the same result as we expected. The Eq. (58) gives the dynamic energy of the system in all possible processes. By using Eq. (76), Eq. (79) can be rewritten as following too:

$$U_D = D\delta U_T \tag{80}$$

Where:

δ

$$D = \left(1 - \left(\alpha_{e1} + \alpha_{e2} \left(1 - \frac{c_1 c_2}{(2\sqrt{u_{e1}})(c_2\sqrt{u_{e1}} + c_3)^2}\right)\right)\right)$$
(81)

The coefficient D indicates how much of the input energy into the system will be converted to dynamic energy. This coefficient can provide a description of the internal structure of the system, its inertia as well as the processes that can be performed. Also, according to Eq. (47), energy conservation principle, the rest of the energy will be converted to non-dynamic energy of the system, therefore:

$$\delta U_{ND} = \delta U_T - \delta U_D \tag{82}$$

$$\delta U_{ND} = (1 - D)\delta U_T \equiv D^* \delta U_T \tag{83}$$

The coefficient D^* denotes the share of non-dynamic energies. This coefficient can provide a description of the possibility of distributing non-dynamic energies in the system. If the variation of the Eq. (69) is calculated, the result will be the Eq. (82).

11.6 Sixth Basic Example: Generalized Maxwell and Kelvin's Viscoelasticity Models

From the point of view of energy behavior, viscoelastic materials have components of stored energy and wasted energy. The way of activation and dependence of the relevant components depends on the relevant viscoelasticity model, and also the amount and manner of energy applying is included in determining the loading coefficients of the components. Therefore, the investigation of viscoelastic materials and their energy behavior in the perspective of energy structure can provide important results about them, and also be a suitable topic to express the applications of the perspective of energy structure. So, In this section, the presented approach is used to linear viscoelasticity problems. For example, generalized Kelvin and Maxwell's models are studied. Eq. (84) shows the general linear form of U_{ND} with two components: one independent component and one dependent component.

$$U_{ND} = u_1 + (c_1 \dot{u}_1 + c_2) \tag{84}$$

Eq. (84) can be used for investigating the linear viscoelasticity models. Since, if the component of stored energy is shown by u_e and the component of dissipated energy shown by u_d , for the kelvin model [35], as shown in figure 14:



Figure 14. Energy structure equation for viscoelasticity kelvin model.

$$\frac{du_e}{dx} = Ex \tag{85}$$

$$\frac{du_d}{dx} = \eta \dot{x} \tag{86}$$

Therefore:

$$\frac{d^2 u_e}{dxdt} = \left(\frac{E}{\eta}\right) \frac{du_d}{dx}$$
(87)

$$u_d = \left(\frac{E}{\eta}\right) \dot{u}_e + constant \tag{88}$$

And also for the Maxwell model [35], as shown in figure 15:



Figure 15. Energy structure equation for viscoelasticity Maxwell model.

$$\dot{\varepsilon}^{OV} = \frac{E}{r} \varepsilon^e \tag{89}$$

Therefore:

$$\frac{du_d}{d\varepsilon^{OV}} = \eta \dot{\varepsilon}^{OV} = E\varepsilon^e = \frac{du_e}{d\varepsilon^e}$$
(90)

$$\frac{du_d}{\varepsilon^{OV}dt} = \frac{du_e}{\varepsilon^e dt} \tag{91}$$

$$u_e = \frac{\eta}{2E} \dot{u}_d \tag{92}$$

In fact, if the independent component is assumed the component of stored energy, that means $u_1 = u_e$ and $c_1 = \frac{E}{\eta}$, equation (84) can be used for kelvin model and if the independent component is assumed the component of dissipated energy, that means $u_1 = u_d$ and $c_1 = \frac{\eta}{2E}$, Eq. (84) can be used to Maxwell model. And also c_1 is determined with Eq.s (88) and (92).

And also by using of Eq.s (16) and (17), for example, for the kelvin model:

$$\alpha = \alpha_e + c_1 \dot{\alpha}_e \tag{93}$$

$$\beta = c_1 \alpha_e \tag{94}$$

And about determining the α_e , it can be written based on the energy conservation principle:

$$\delta U_T = \delta u_e + \delta u_d \tag{95}$$

And also:

$$\delta u_d = \left(\frac{E}{\eta}\right) \delta \dot{u}_e \tag{96}$$

$$\delta u_d = \left(\frac{\eta}{E}\right) \left[\dot{\alpha}_e \delta U_T + \alpha_e \delta \dot{U}_T\right] \tag{97}$$

Therefore:

$$\delta U_T = (\alpha_e) \delta U_T + \left(\frac{E}{\eta}\right) \left[\dot{\alpha}_e \delta U_T + \alpha_e \delta \dot{U}_T\right]$$
(98)

Or:

$$\dot{U}_T = (\alpha_e)\dot{U}_T + \left(\frac{E}{\eta}\right)[\dot{\alpha}_e\dot{U}_T + \alpha_e\ddot{U}_T]$$
(99)

$$\dot{\alpha}_e + \left[\frac{\dot{\nu}_T + \left(\frac{E}{\eta}\right)\dot{\nu}_T}{\left(\frac{E}{\eta}\right)\dot{\nu}_T}\right]\alpha_e = \left(\frac{\eta}{E}\right) \tag{100}$$

Therefore, α_e is calculated from Eq. (100) in functional of the how energy is applied to the system.

The obtained energy structure can be easily generalized for different combinations of the corresponding elements. For example, figure 16 can be considered:



Figure 16. Energy structure equation for generalized Kelvin elements.

$$U_V = u_{e1} + u_{e2} + \dots + u_{eN} + c_{e1}\dot{u}_{e1} + c_{e2}\dot{u}_{e2} + \dots + c_{eN}\dot{u}_{eN} + constant$$
(101)

Derived structure of Eq. (101) also can be used for generalized Maxwell elements:



Figure 17. Energy structure equation for generalized Maxwell element.

$$U_V = u_{d1} + u_{d2} + \dots + u_{dN} + c_{d1}\dot{u}_{d1} + c_{d2}\dot{u}_{d2} + \dots + c_{dN}\dot{u}_{dN} + constant$$
(102)

In addition to obtain the structure of non-inertial energies, it actually obtains the feasible processes and the way energy components change as a result of energy exchange between the system and the surrounding environment.

12. Irreversibility Analysis

Analysis of irreversibility in the perspective of energy structure has been done using a novel definition in references [58-59,71]. In this section, a brief overview is provided. Considering figures 18 and 19.

Considering Eq.s (103-104):

$$\delta H_i = \left(\sum_{p=1}^n \left(\frac{\partial h_p}{\partial \dot{\nu}_i}\right)\right) \delta \dot{U}_i \tag{103}$$

$$\delta \Psi = \sum_{i=1}^{m} \delta H_i$$

So, irreversibility is derived as follows:

$$\varphi = \frac{\delta\Psi}{\delta U_T} = \sum_{i=1}^m \frac{\delta H_i}{\delta U_T} = \sum_{i=1}^m \varphi_i \tag{105}$$

(104)

That always has positive sign (as well as $\varphi_i \ge 0$) [58-59,71].

Eq. (105) gives irreversibility from the point of view of the energy structure. Also, another important feature of this equation is obtaining irreversible components that determine the amount and manner of the effect of each component in the total irreversibility. Therefore, irreversibilities are clearly dependent on the internal structure of the system, the amount and manner of energy exchange between the system and the surrounding, which are presented in Eq. (105).



Figure 18. Different paths for the performed process [71].



Figure 19. Deviation from the quasi-static path to the general [71].

13. Conclusions

The perspective of energy structure suggests the use of activated energy components and their structure to represent energy in physical systems. In fact, due to the role of the energy conservation principle as a unifying principle in physics, energy components are able to expand the range of responses and processes that can be performed by physical systems. Valuable achievements have also been presented in this direction, which have been gathered in this paper, as well as the perspective of the energy structure has been developed and the relevant equations for the development of a closed space of a scientific theory have been established and generalized in order to explain and analyze the physical processes of physical systems. The effort has been to find the simplest form of equations in order to close the solution spaces. In fact, given the energy components of the system as quantities that can describe the process performed, new space is created for the set of governing equations. In the resulting space, the effects of the second law of thermodynamics can be directly applied to the energy structure of the system and in fact, the sensitivity of the system to the second law can be examined. Also, one of the advantages of this new approach is that a particular process in various conditions is studied instead of studying a particle of the system. In fact, this approach gives a general view regarding the studied process as well as the internal structure of the system in response to the energy exchange with the environment. This paper established an energy componentsbased-general model inspired by the first and second laws of thermodynamics as well as using a new division to the total energy of the system. The established model is completed by extracting the physical direction for the feasible physical processes based on the energy components of the system. Also, the presented model has developed for the high particle number systems as well as some energy-related basis examples. As three cases of the other achievements of the energy structure approach is the providing a quasi-statistical equation to entropy, a general solution to the different formulations of the second law of thermodynamics as well as irreversibility analysis using a novel definition. The problems studied are the energy-related basis systems that can be combined with others to create a variety of systems. Eq. (52) can also derive dynamic energy in terms of independent energy components. These relations can also describe the dynamics of the system with respect to independent component changes. The energy coordinates of the system by relying on the energy conservation principle obtain a standard and error-free mathematical method for the first and second laws in such a way that unification and a wide range of applications are provided, and applicability in classical and modern problems is created.

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Nomenclature

- g_j: Value-dependent energy component with SI (metric) dimension (*Joule*).
- *h_n*: Rate-dependent energy component with SI (metric) dimension (*Joule*).
- K_{MS} : Universal constant of quasi-statistical entropy definition without SI (metric) dimension.
- *u_i*: Energy component (Dependent and Independent) of the system with SI (metric) dimension (*Joule*).
- U_T: Total energy of the system with SI (metric) dimension (*Joule*).
- U_{ND} : Non-Dynamic energy component of the system with SI (metric) dimension (*Joule*).
- **U**_D: Dynamic energy component of the system with SI (metric) dimension (*Joule*).
- **s** : Entropy of the system with SI (metric) dimension $(\frac{Joule}{\kappa^{\circ}})$.

Greek Letters

- *φ*: Irreversibility component with SI (metric) dimension (*Joule*).
- α : Change energy coefficient without SI (metric) dimension.
- β : Change energy-rate coefficient without SI (metric) dimension.

- **γ:** Change energy component coefficient without SI (metric) dimension.
- Ψ: Irreversibility with SI (metric) dimension (Joule).
- ε: Strain without SI (metric) dimension. σ: Stress with SI (metric) dimension $(\frac{N}{m^2})$.
- η: Viscosity coefficient with SI (metric) dimension $(\frac{m^2}{s})$.

References:

- [1] A. Bejan, *Advanced engineering thermodynamics*, New Jersey: John Wiley & Sons, 2016.
- [2] P. G. Tait, Sketch of thermodynamics, Edinburgh: D. Douglas, 1877.
- [3] R. Clausius, "The second law of thermodynamics", *The World of Physics*, 1, 1865.
- [4] J. M. Heath, XXVI. "On the principles of thermodynamics", The London, Edinburgh, and Dublin *Philosophical Magazine and Journal of Science*, Vol. 40, no. 266, pp. 218-220, 1870, doi.org/10.1080/14786447008640386.
- [5] C. Maes, and H. Tasaki, "Second law of thermodynamics for macroscopic mechanics coupled to thermodynamic degrees of freedom", *Letters in Mathematical Physics*, Vol. 79, pp. 251-261, 2007, doi.org/10.1007/s11005-006-0136-9.
- [6] S. F. Wu, Wang, B. Yang, G.H. and P. M. Zhang, "The generalized second law of thermodynamics in generalized gravity theories", *Classical and Quantum Gravity*, Vol. 25, no. 23, p.235018, 2008, doi.org/10.1088/0264-9381/25/23/235018.
- J. Xuereb, S. Campbell, J. Goold, and A. Xuereb, "DQC1 as an Open Quantum System", arXiv preprint arXiv:2209.03947, 2022, doi.org/10.48550/arXiv.2209.03947.
- [8] B. C. Eu, "Kinetic theory and irreversible thermodynamics", Accounts of Chemical Research, Vol. 19, no. 5, pp. 153-160, 1986, doi.org/10.1021/ar00125a005.
- [9] E. H. Lieb, and J. Yngvason, "The physics and mathematics of the second law of thermodynamics", *Physics Reports*, Vol. 310, no. 1, pp. 1-96, 1999, doi.org/10.1016/S0370-1573(98)00082-9.
- [10] R. C. Nichols, XLIV. "On the proof of the second law of thermodynamics", *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, Vol. 1, no. 5, pp. 369-373, 1876, doi.org/10.1080/14786447608639054.
- [11] L. D. Rio, J. Åberg, R. Renner, O. Dahlsten, and V. Vedral, "The thermodynamic meaning of negative entropy", *Nature*, Vol. 474, no. 7349, pp. 61-63, 2011.
- [12] L. A. Sosnovskiy, and S. S. Sherbakov, "Mechanothermodynamic entropy and analysis of damage state of complex systems", *Entropy*, Vol. 18, no. 7, p.268, 2016, doi.org/10.3390/e18070268.
- [13] L. A. Sosnovskiy, and S. S. Sherbakov, "A model of mechanothermodynamic entropy in tribology", *Entropy*, Vol. 19, no. 3, p.115, 2017, doi.org/10.3390/e19030115.

- [14] P. V. Coveney, "The second law of thermodynamics: entropy, irreversibility and dynamics", *Nature*, Vol. 333, no. 6172, pp. 409-415, 1988, doi.org/10.1038/333409a0.
- [15] W. j. Macquorn Rankine, C. E. LL. D. F. SS. L, "On the second law of thermodynamics", *The London , Edinburg,* and Dublin Philosophical Magazine and journal of science, Vol. 30, no. 203, 1965.
- [16] C. M. Dafermos, "The second law of thermodynamics and stability", *Archive for Rational Mechanics and Analysis*, Vol. 70, no. 2, pp. 167-179, 1979.
- [17] C. Basaran, and C. Y. Yan, "A thermodynamic framework for damage mechanics of solder joints" *J. Electron. Packaging*, 120, 4, 1998, doi.org/10.1115/1.2792650.
- [18] C. S. Desai, C. Basaran, T. Dishongh, and J. L. Prince, "Thermomechanical analysis in electronic packaging with unified constitutive model for materials and joints", *IEEE Transactions on Components, Packaging, and Manufacturing Technology: Part B*, Vol. 21, no. 1, pp. 87-97, 1998, doi.org/10.1109/96.659511.
- [19] L. A. Sosnovskiy, and S. S. Sherbakov, "On the development of mechanothermodynamics as a new branch of physics", *Entropy*, Vol. 21, no. 12, p. 1188, 2019, doi.org/10.3390/e21121188.
- [20] R. Bendikiene, A. Bahdanovich, R. Cesnavicius, A. Ciuplys, V. Grigas, A. Jutas, D. Marmysh, A. Nasan, L. Shemet, S. Sherbakov, and L. Sosnovskiy, "Tribofatigue behavior of austempered ductile iron MoNiCa as new structural material for rail-wheel system", *Materials Science*, Vol. 26, no. 4, pp. 432-437, 2020, doi.org/10.5755/j01.ms.26.4.25384.
- [21] T. Hiemstra, "Formation, stability, and solubility of metal oxide nanoparticles: Surface entropy, enthalpy, and free energy of ferrihydrite", *Geochimica et Cosmochimica Acta*, 158, pp. 179-198, 2015, doi.org/10.1016/j.gca.2015.02.032.
- [22] M. Sheikholeslami, M. Jafaryar, E. Abohamzeh, A. Shafee, and H. Babazadeh, "Energy and entropy evaluation and two-phase simulation of nanoparticles within a solar unit with impose of new turbulator", *Sustainable Energy Technologies and Assessments*, 39, p.100727, 2020, doi.org/10.1016/j.seta.2020.100727.
- [23] M. Sheikholeslami, M. Jafaryar, E. Abohamzeh, A. Shafee, Z. Li, and R. U. Haq, "Heat transfer of nanoparticles employing innovative turbulator considering entropy generation", *International Journal* of Heat and Mass Transfer, 136, pp. 1233-1240, 2019, doi.org/10.1016/j.ijheatmasstransfer.2019.03.091.
- [24] M. W. Glasscott, A. D. Pendergast, S. Goines, A. R. Bishop, A. T. Hoang, C. Renault, and J. E. Dick, "Electrosynthesis of high-entropy metallic glass nanoparticles for designer, multi-functional electrocatalysis", *Nature Communications*, Vol. 10, no. 1, p.2650, 2019, doi.org/10.1038/s41467-019-10303-z.
- [25] C. Zhang, Z. Peng, C. Huang, B. Zhang, C. Xing, H. Chen, H. Cheng, J. Wang, and S. Tang, "High-energy allin-one stretchable micro-supercapacitor arrays based on 3D laser-induced graphene foams decorated with

mesoporous ZnP nanosheets for self-powered stretchable systems", *Nano Energy*, 81, p.105609, 2021, doi.org/10.1016/j.nanoen.2020.105609.

- [26] J. F. Klein, *Physical significance of entropy or of the second law*, New York: D. Van Nostrand Company, 1910, doi.org/10.5962/bhl.title.19423.
- [27] E. G. D. Cohen, "Boltzmann and Einstein: Statistics and dynamics—An unsolved problem", *Pramana*, 64, pp. 635-643, 2005, doi.org/10.1007/BF02704573.
- [28] W. Thomson, "The kinetic theory of the dissipation of energy", *Proceedings of the Royal Society of Edinburgh*, 8, pp. 325-334, 1875, 9. doi.org/10.1017/S0370164600029680.
- [29] S. H. Burbury, "A theorem on the dissipation of energy", *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, Vol. 13, no. 83, pp. 417-419, 1882, LII. doi.org/10.1080/14786448208627206.
- [30] R. Lord, "On the dissipation of energy", Van Nostrand's Eclectic Engineering Magazine, Vol. 12, no. 78, pp. 1869-1879.
- [31] W. Thomson, "On a Universal Tendency in Nature to the Dissipation of Mechanical Energy", *Proceedings of the Royal Society of Edinburgh*, 3, pp.139-142, 1857, 2. doi.org/10.1017/S0370164600027541.
- [32] H. Lee, N. Campbell, J. Lee, T. J. Asel, T. R. Paudel, H. Zhou, J. W. Lee, B. Noesges, J. Seo, B. Park, and L. J. Brillson, "Direct observation of a two-dimensional hole gas at oxide interfaces", *Nature materials*, Vol. 17, no. 3, pp. 231-236, 2018, doi.org/10.1038/s41563-017-0002-4.
- [33] M. Akbarifar, M. Divandari, S. M. A. Boutorabi, S. H. Ha, Y. O. Yoon, and S. K. Kim, "Short-time oxidation of Al–Mg in dynamic conditions", *Oxidation of Metals*, 94, pp. 409-429, 2020,doi.org/10.1007/s11085-020-09999y.
- [34] A. Dehhaghi, and S. M. A. Boutorabi, "Thickness estimation (folds and wrinkles) on short time dynamically formed double oxide film in Al-7Si-Mg alloyed with beryllium", *Materials Research Express*, Vol. 6, no. 10, p.106574, 2019,doi.org/10.1088/2053-1591/ab35ed.
- [35] J. Campbell, "The consolidation of metals: The origin of bifilms", *Journal of materials science*, 51, pp. 96-106, 2016, doi.org/10.1007/s10853-015-9399-9.
- [36] G. Gyarmati, G. Fegyverneki, T. Mende, and M. Tokár, "Characterization of the double oxide film content of liquid aluminum alloys by computed tomography", *Materials Characterization*, 157, p.109925, 2019, doi.org/10.1016/j.matchar.2019.109925.
- [37] H. Lee, N. Campbell, J. Lee, T. J. Asel, T. R. Paudel, H. Zhou, J. W. Lee, B. Noesges, J. Seo, B. Park, and L. J. Brillson, "Direct observation of a two-dimensional hole gas at oxide interfaces", *Nature materials*, Vol. 17, no. 3, pp. 231-236, 2018,doi.org/10.1038/s41563-017-0002-4.
- [38] A. Dehhaghi, and S. M. A. Boutorabi, "Nano Double Oxide Film (NDOF) Characteristics of A356 Alloy Casting Protected With Beryllium in Turbulence Conditions", *Founding Research Journal*, Vol. 2, no. 2,

pp. 71-78, 2018.

- [39] S. M. A. Boutorabi, P. Torkaman, J. Campbell, and A. Zolfaghari, "Structure and properties of carbon steel cast by the ablation process", *International Journal of Metalcasting*, 15, pp. 306-318, 2021, doi.org/10.1007/s40962-020-00466-7.
- [40] A. H. Fazeli, H. Saghafian, S. M. A. Boutorabi, and J. Campbell, "The fluidity of aluminium ductile irons", *International Journal of Metalcasting*, pp. 1-10, 2021, doi.org/10.1007/s40962-021-00581-z.
- [41] Y. Mishin, "Thermodynamic theory of equilibrium fluctuations", *Annals of Physics*, 363, pp. 48-97, 2015, doi.org/10.1016/j.aop.2015.09.015.
- [42] A. Bravetti, C. S. Lopez-Monsalvo, and F. Nettel, "Contact symmetries and Hamiltonian thermodynamics", *Annals of Physics*, 361, pp. 377-400, 2015, doi.org/10.1016/j.aop.2015.07.010.
- [43] R. Dann, and R. Kosloff, "Quantum signatures in the quantum Carnot cycle", *New Journal of Physics*, Vol. 22, no. 1, p.013055, 2020, doi.org/10.1088/1367-2630/ab6876.
- [44] G. Bulnes Cuetara, A. Engels, and M. Esposito, "Stochastic thermodynamics of rapidly driven quantum systems", *New Journal of Physics*, 17, p.055002, 2015, doi.org/10.1088/1367-2630/17/5/055002.
- [45] R. Gallego, J. Eisert, and H. Wilming, "Thermodynamic work from operational principles", *New Journal of Physics*, Vol. 18, no. 10, p. 103017, 2016, doi.org/10.1088/1367-2630/18/10/103017.
- [46] G. Gour, M. P. Müller, V. Narasimhachar, R. W. Spekkens, and N. Y. Halpern, "The resource theory of informational nonequilibrium in thermodynamics", *Physics Reports*, 583, pp. 1-58, 2015, doi.org/10.1016/j.physrep.2015.04.003.
- [47] A. Puglisi, A. Sarracino, and A. Vulpiani, "Temperature in and out of equilibrium: A review of concepts, tools and attempts", *Physics Reports*, 709, pp. 1-60, 2017, doi.org/10.1016/j.physrep.2017.09.001.
- [48] Z. Said, L. S. Sundar, A. K. Tiwari, H. M. Ali, M. Sheikholeslami, E. Bellos, and H. Babar, "Recent advances on the fundamental physical phenomena behind stability, dynamic motion, thermophysical properties, heat transport, applications, and challenges of nanofluids", *Physics Reports*, 946, pp. 1-94, 2022, doi.org/10.1016/j.physrep.2021.07.002.
- [49] M. N. Bera, A. Riera, M. Lewenstein, and A. Winter, "Generalized laws of thermodynamics in the presence of correlations", *Nature communications*, Vol. 8, no. 1, p.2180, 2017, doi.org/10.1038/s41467-017-02370-x.
- [50] S. Shahsavari, "Possiblity unique combination for first and second law of thermodynamics", *Quantum Journal of Engineering, Science and Technology*, Vol. 2, no. 5, pp. 51-52, 2021,.
- [51] L. Qiu, N. Zhu, Y. Feng, E. E. Michaelides, G. Żyła, D. Jing, X. Zhang, P. M. Norris, C.N. Markides, and O. Mahian, "A review of recent advances in thermophysical properties at the nanoscale: From solid state to colloids", *Physics Reports*, 843, pp. 1-81, 2020,

doi.org/10.1016/j.physrep.2019.12.001.

- [52] Á. Rivas, "Strong coupling thermodynamics of open quantum systems", *Physical review letters*, Vol. 124, no. 16, p.160601, 2020, doi.org/10.1103/PhysRevLett.124.160601.
- [53] I. Peshkov, M. Pavelka, E. Romenski, and M. Grmela, , "Continuum mechanics and thermodynamics in the Hamilton and the Godunov-type formulations", *Continuum Mechanics and Thermodynamics*, 30, pp.1343-1378, 2018, doi.org/10.1007/s00161-018-0621-2.
- [54] K. Ptaszyński, and M. Esposito, "Thermodynamics of quantum information flows", *Physical review letters*, Vol. 122, no. 15, p.150603, 2019, doi.org/10.1103/PhysRevLett.122.150603.
- [55] L. Masanes, and J. Oppenheim, "A general derivation and quantification of the third law of thermodynamics", *Nature communications*, Vol. 8, no. 1, pp. 1-7, 2017, doi.org/10.1038/ncomms14538.
- [56] S. Shahsavari, and M. Moradi, "A General Solution to the Different Formulations of the Second Law of Thermodynamics", *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, Vol. 82, no. 2, pp. 61-71, 2021, doi.org/10.37934/arfmts.82.2.6171.
- [57] S. Shahsavari, M. Moradi, and P. Torkaman, "A Quasi-Statistical Approach to the Boltzmann Entropy Equation Based on a Novel Energy Conservation Principle", *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, Vol. 101, no. 2, pp. 99-110, 2023, doi.org/10.37934/arfmts.101.2.99110.
- [58] S. Shahsavari, and M. Moradi, "A New Approach to the Energy Conservation Principle and Physical Systems", Ph.D. dissertation, Department of Mechanical Engineering, Isfahan University of Technology, IR, 2022.
- [59] S. Shahsavari, and M. Moradi, "A Study of the Entropy Production in Physical Processes from a New Perspective of the Energy Structure", *Asian Journal of Applied Sciences* (ISSN: 2321-0893), 8, 2020, doi.org/10.24203/ajas.v8i6.6472.
- [60] H. J. Borchers, "Some remarks on the second law of thermodynamics", *Reports on mathematical physics*, Vol. 22, no. 1, pp. 29-48, 1985, doi.org/10.1016/0034-4877(85)90004-7.
- [61] B. C. Eu, "Kinetic theory and irreversible thermodynamics", *Accounts of Chemical Research*, Vol. 19, no. 5, pp. 153-160, 1986, doi.org/10.1021/ar00125a005.
- [62] R. D. Gregory, *Classical mechanics*, Cambridge University Press, 2006.
- [63] É. Roldán, Irreversibility and dissipation in microscopic systems, Springer, 2014, doi.org/10.1007/978-3-319-07079-7.
- [64] D. R. Bland, *The theory of linear viscoelasticity*, New York: Courier Dover Publications, 2016.
- [65] S. Shahsavari, and P. Torkaman, "Energy Conservation Principle from the Perspective of the Energy Structure Theory", *Asian Journal of Applied Sciences* (ISSN:

2321–0893) Vol. 10, no. 5, 2022, doi.org/10.24203/ajas.v10i5.6950.

- [66] T. Z. Kalanov, "On the correct formulation of the first law of thermodynamics", *In APS April Meeting Abstracts*, pp. D1-055, 2006.
- [67] S. Carnot, R. Clausius, and W. T. B. Kelvin, *The Second Law of Thermodynamics*, American Book Company, 1899.
- [68] R. Luzzi, Á. R. Vasconcellos, and J. G. Ramos, "On the statistical foundations of irreversible thermodynamics", *Fortschritte der Physik: Progress of Physics*, Vol. 47, no. 4, pp. 401-432, 1999.
- [69] L. Onsager, "Reciprocal relations in irreversible processes", *I. Physical review*, Vol. 37, no. 4, p. 405, 1931, doi.org/10.1103/PhysRev.37.405.
- [70] M. Planck, "On the law of the energy distribution in the normal spectrum", *Ann. Phys*, Vol. 4, no. 553, pp. 1-11, 1901, doi.org/10.1016/B978-0-08-012102-4.50013-9.
- [71] S. Shahsavari, M. Moradi, and M. Esmaeilpour, "On the Irreversibility in Mechanical Systems Using a New Macroscopic Energy Structure Modeling", *Asian Journal of Applied Sciences* (ISSN: 2321–0893), Vol. 8, no. 6, 2020, doi.org/10.24203/ajas.v8i6.6473.
- [72] M. R. von Spakovsky, "The second law: A unified approach to thermodynamics applicable to all systems and all states", *In AIP Conference Proceedings*, Vol. 1033, no. 1, pp. 302-308, American Institute of Physics, 2008, doi.org/10.1063/1.2979049.
- [73] V. N. Pokrovskii, "A derivation of the main relations of

nonequilibrium thermodynamics", *International Scholarly Research Notices*, 2013, doi.org/10.1155/2013/906136.

- [74] M. Esposito, and C. Van den Broeck, "Three faces of the second law. I. Master equation formulation", *Physical Review E*, Vol. 82, no. 1, p.011143, 2010, doi.org/10.1103/PhysRevE.82.011143.
- [75] M. S. Ma, and R. Zhao, "Corrected form of the first law of thermodynamics for regular black holes", *Classical* and Quantum Gravity, Vol. 31, no. 24, p.245014, 2014, doi.org/10.1088/0264-9381/31/24/245014.
- [76] T. W. Xue, and Z. Y. Guo, "What is the real Clausius statement of the second law of thermodynamics", *Entropy*, Vol. 21, no. 10, p.926, 2019.
- [77] V. N. Pokrovski, "Extended thermodynamics in a discrete-system approach", *European journal of physics*, Vol. 26, no. 5, p.769, 2005, doi.org/10.1088/0143-0807/26/5/009.
- [78] V. A. Etkin, "Methodological principles of modern thermodynamics", arXiv preprint arXiv:1401.0550, 2014.
- [79] J. Serrin, "Conceptual analysis of the classical second laws of thermodynamics", *Archive for Rational Mechanics and Analysis*, Vol. 70, no. 4, pp. 355-371, 1979, doi.org/10.1007/BF00281160.
- [80] P. Podio-Guidugli, "A virtual power format for thermomechanics", *Continuum Mechanics and Thermodynamics*, 20, pp. 479-487, 2009, doi.org/10.1007/s00161-009-0093-5.