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Convergent result for linear conformable pseudo-parabolic equation

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Abstract

In this paper, we are interested to study conformable pseudo-parabolic equation. This equation has many applications in science and engineering. Our main goal is to show that the convergence result of the mild solution when the fractional order tends to 1^-. The main technique is to use evaluations in Hilbert scales spaces that incorporate some new inequalities.

1. Introduction

In recent years, fractional calculus has been the subject of active research. It accurately simulates some models where classical derivatives are constrained and describes numerous natural phenomena. The fractional derivatives must be used to model some viscous effects. Many fields, including physics, biology, and chemistry, have equations with fractional derivatives, see in [1,2].

Because to their numerous applications, including [3] circuits and chaotic systems in [4] dynamics, PDE with conformable operator have drawn interested mathematicians employing various methods. The link between these two derivative kinds has been studied by mathematicians using the following observation. A conformable fractional derivative of order α exists at s if f is a real function and $s > 0$, and only if it is (classically) differentiable at s.

$$\frac{{}^{C}\partial^{\alpha} f(s)}{\partial s^{\alpha}} = s^{1-\alpha}\frac{\partial f(s)}{\partial s}, \tag{1}$$

where $0 < \alpha \leq 1$. If f is defined in a general Banach space, then Equation (1) will not hold . This is also the main reason why the study of conformable PDEs in Hilbert spaces or Sobolev spaces is not as active as ODEs. Impressive work on diffusion equation with conformable derivative seems to be of Tuan and his colleagues [5], Hung and his coauthors [6].

Let $T > 0$, we consider the following problem

$$\begin{cases} \dfrac{{}^{C}\partial^{\alpha}}{\partial t^{\alpha}}\left(Z(x,t) - kZ_{xx}(x,t)\right) - Z_{xx}(x,t) = G(x,t), & (x,t) \in \Omega \times (0,T), \\ Z(0,t) = Z(\pi,t) = 0, & 0 < x < \pi, t \in (0,T), \\ Z(x,0) = f(x) & 0 < x < \pi, \end{cases} \tag{2}$$

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where f is the initial datum and G is the source function.

The pseudo-parabolic equation has been widely investigated and describes a number of significant physical phenomena. For further information, check the sources listed in [7–9]. The existence and blowup of solution, interested readers can find it in [10, 11].

Some papers on pseudo-parabolic equation with Caputo derivative or Riemann-Liouville can be found in [8, 12–14]. However, there are limited result on pseudo-parabolic equation with conformable derivative. The first work on conformable pseudo-parabolic equation seems to be of [15]. In [15], the authors studied the well-posedness of nonlinear conformable diffusion equation. However, there is an interesting question later about the limitation of the mild solution when $\alpha \to 1^-$ which has not been explored in [5, 15]. The main purpose of this paper is to answer this question.

2. Preliminary results

Definition 2.1. Let the function $g : [0, \infty) \to B$ where B is a Banach space. If the limitation exists in B

$$\frac{{}^{\mathscr{C}}\partial^\alpha f(t)}{\partial t^\alpha} := \lim_{h\to 0} \frac{g(t + ht^{1-\beta}) - g(t)}{h} \tag{3}$$

for each $t > 0$, then we call it the conformable dertivative. More information about conformable, we can provide some papers [16–19].

Definition 2.2. For any $s \geq 0$, the Hilbert scale space

$$\mathbb{H}^s(\Omega) = \left\{ \nu \in L^2(\Omega) \Big| \sum_{n=1}^{\infty} \lambda_n^{2s} \Big(\int_\Omega \nu(x)\psi_n(x)\mathrm{d}x \Big)^2 < \infty \right\},$$

with the norm

$$\|\nu\|_{\mathbb{H}^s(\Omega)} = \left(\sum_{n=1}^{\infty} \lambda_n^{2s} \Big(\int_\Omega \nu(x)\psi_n(x)\mathrm{d}x \Big)^2 \right)^{1/2}, \quad f \in \mathbb{H}^s(\Omega).$$

Theorem 2.3. *For $\varepsilon, \beta > 0$, we get*

$$\left| \exp\Big(-z\frac{t^\alpha}{\alpha} \Big) - e^{-zt} \right| \leq Ct^{\varepsilon(\alpha-\beta)} \Big((1-\alpha)^\beta + (1-\alpha) + |T^{1-\alpha} - 1| \Big)^\varepsilon z^\varepsilon, \; 0 < t \leq T, \tag{4}$$

where $\alpha \geq \alpha_0$ and $\beta \geq 0$. Here C depends on $\alpha_0, \varepsilon, \beta$.

Proof. The proof can be found in [20]. □

3. The mild solution and some lemmas

The mild solution which is given by

$$Z(x,t) = \sum_{n=1}^{\infty} \langle Z(.,t), \psi_n \rangle \psi_n(x), \quad \psi_n(x) = \sqrt{\frac{2}{\pi}} \sin(nx). \tag{5}$$

Problem (2) takes the inner product with ψ_n gives

$$\begin{cases} \frac{C\partial^\alpha}{\partial t^\alpha} \langle Z(.,t), \psi_n \rangle + n^2 \langle Z(.,t), \psi_n \rangle + kn^2 \frac{C\partial^\alpha}{\partial t^\alpha} \langle Z(.,t), \psi_n \rangle &= \langle G(.,t), \psi_n \rangle, \quad t \in (0,T), \\ \langle Z(.,0), \psi_n \rangle &= \langle f, \psi_n \rangle. \end{cases} \tag{6}$$

The first equation of (6) is a differential equation with a conformable derivative as follows

$$\frac{C\partial^\alpha}{\partial t^\alpha} \langle Z_\alpha(.,t), \psi_n \rangle + \frac{n^2}{1+kn^2} \langle Z_\alpha(.,t), \psi_n \rangle = \frac{1}{1+kn^2} \langle G(.,t), \psi_n \rangle.$$

Due to the result in Theorem 5, p.318, [18] we have

$$\langle Z_\alpha(.,t),\psi_n\rangle = \exp\left(-\frac{n^2}{1+kn^2}\frac{t^\alpha}{\alpha}\right) f_n + \frac{1}{1+kn^2}\int_0^t r^{\alpha-1}\exp\left(\frac{n^2}{1+kn^2}\frac{r^\alpha - t^\alpha}{\alpha}\right) G_n(r)dr.$$

Here we remind that

$$f_n = \int_0^\pi f(x)\psi_n(x)dx, \quad G_n(r) = \int_0^\pi G(x,r)\psi_n(x)dx.$$

This implies that

$$Z_\alpha(x,t) = \sum_{n=1}^\infty \exp\left(-\frac{n^2}{1+kn^2}\frac{t^\alpha}{\alpha}\right) f_n\psi_n(x) + \sum_{n=1}^\infty \left[\frac{1}{1+kn^2}\int_0^t r^{\alpha-1}\exp\left(\frac{n^2}{1+kn^2}\frac{r^\alpha - t^\alpha}{\alpha}\right) G_n(r)dr\right]\psi_n(x). \tag{7}$$

The main result of our paper is described as follows

Theorem 3.1. *Let Z^* be the mild solution to following problem*

$$\begin{cases} \frac{\partial Z^*}{\partial t}(Z^*(x,t) - kZ^*_{xx}(x,t)) - Z^*_{xx}(x,t) = G(x,t), & (x,t)\in\Omega\times\in(0,T), \\ Z^*(0,t) = Z^*(\pi,t) = 0, & 0<x<\pi, t\in(0,T), \\ Z^*(x,0) = f(x) & 0<x<\pi. \end{cases} \tag{8}$$

Let $\frac{1}{2}<\alpha<1$. Let $f\in\mathbb{H}^d(0,\pi)$ and $G\in L^\infty(0,T;\mathbb{H}^{d-2}(0,\pi))$. Then we have the following observation

$$\begin{aligned}\left\|Z_\alpha(.,t) - Z^*(.,t)\right\|_{\mathbb{H}^d(0,\pi)} &\lesssim k^{-\varepsilon}t^{\varepsilon(\alpha-\beta)}\Big((1-\alpha)^\beta + (1-\alpha) + |T^{1-\alpha}-1|\Big)^\varepsilon \|f\|_{\mathbb{H}^d(0,\pi)} \\ &+ \big((1-\alpha)^\beta + (1-\alpha) + |T^{1-\alpha}-1|\big)\|G\|_{L^\infty(0,T;\mathbb{H}^{d-2}(0,\pi))} \\ &+ k^{-1}\|G\|_{L^\infty(0,T;\mathbb{H}^{d-2}(0,\pi))}\left[\sqrt{(1-\alpha)^\beta + (1-\alpha) + |T^{1-\alpha}-1|} + 1 - \alpha\right].\end{aligned} \tag{9}$$

where $\varepsilon>0$ and $0<\beta<\min(\alpha, 2\alpha-1)$.

Proof. The mild solution to Problem (8)

$$Z^*(x,t) = \sum_{n=1}^\infty \exp\left(-\frac{n^2}{1+kn^2}t\right) f_n\psi_n(x) + \sum_{n=1}^\infty \left[\frac{1}{1+kn^2}\int_0^t r^{\alpha-1}\exp\left(\frac{n^2(r-t)}{1+kn^2}\right) G_n(r)dr\right]\psi_n(x). \tag{10}$$

From (7) and (10), we have that

$$\begin{aligned} &Z_\alpha(x,t) - Z^*(x,t) \\ &= \sum_{n=1}^\infty \left[\exp\left(-\frac{n^2}{1+kn^2}\frac{t^\alpha}{\alpha}\right) - \exp\left(-\frac{n^2}{1+kn^2}t\right)\right] f_n\psi_n(x) \\ &+ \sum_{n=1}^\infty \left[\frac{1}{1+kn^2}\int_0^t r^{\alpha-1}\left(\exp\left(\frac{n^2}{1+kn^2}\frac{r^\alpha-t^\alpha}{\alpha}\right) - \exp\left(\frac{n^2(r-t)}{1+kn^2}\right)\right)G_n(r)dr\right]\psi_n(x) \\ &+ \sum_{n=1}^\infty \left[\frac{1}{1+kn^2}\int_0^t \left(r^{\alpha-1}-1\right)\exp\left(\frac{n^2(r-t)}{1+kn^2}\right) G_n(r)dr\right]\psi_n(x) \\ &= B_1(x,t) + B_2(x,t) + B_3(x,t). \end{aligned} \tag{11}$$

Step 1. Estimate the term B_1.
By using Parseval's equality and in view of Theorem (2.3), we get that

$$\begin{aligned}\|B_1(.,t)\|^2_{\mathbb{H}^d(0,\pi)} &= \sum_{n=1}^{\infty} n^{2d}\Big[\exp\big(-\frac{n^2}{1+kn^2}\frac{t^\alpha}{\alpha}\big) - \exp\big(-\frac{n^2}{1+kn^2}t\big)\Big]^2 |f_n|^2 \\ &\lesssim \sum_{n=1}^{\infty} n^{2d}\big(\frac{n^2}{1+kn^2}\big)^{2\varepsilon} t^{2\varepsilon(\alpha-\beta)}\left((1-\alpha)^\beta + (1-\alpha) + |T^{1-\alpha}-1|\right)^{2\varepsilon}|f_n|^2. \end{aligned} \tag{12}$$

Step 2. Estimate the term B_2.
Using the inequality $|e^{-a}-e^{-b}| \le C|a-b|$ for any $\theta > 0$ and Theorem (2.3), we get

$$\begin{aligned}\Big|\exp\big(\frac{n^2}{1+kn^2}\frac{r^\alpha - t^\alpha}{\alpha}\big) - \exp\big(\frac{n^2(r-t)}{1+kn^2}\big)\Big| &\le C\big(\frac{n^2}{1+kn^2}\big)^2\Big[\Big|\frac{t^\alpha}{\alpha}-t\Big| + \Big|\frac{r^\alpha}{\alpha}-r\Big|\Big] \\ &\le Ck^{-2}\Big[\Big|\frac{t^\alpha}{\alpha}-t\Big| + \Big|\frac{r^\alpha}{\alpha}-r\Big|\Big]. \end{aligned} \tag{13}$$

In view of [20], we get the following bound for $0 \le r, t \le T$

$$\Big|\frac{t^\alpha}{\alpha}-t\Big| \le C(\alpha_0,\beta)t^{\alpha-\beta}\left((1-\alpha)^\beta + (1-\alpha) + |T^{1-\alpha}-1|\right), \tag{14}$$

and if we change the variable t to the variable r, we get an inequality equivalent to (14) Hence, using Parseval's equality, we derive that

$$\begin{aligned}&\|B_2(.,t)\|^2_{\mathbb{H}^d(0,\pi)} \\ &= \sum_{n=1}^{\infty} n^{2d}\left[\frac{1}{1+kn^2}\int_0^t r^{\alpha-1}\Big(\exp\Big(\frac{n^2}{1+kn^2}\frac{r^\alpha-t^\alpha}{\alpha}\Big) - \exp\Big(\frac{n^2(r-t)}{1+kn^2}\Big)\Big)G_n(r)dr\right]^2 \\ &\le \sum_{n=1}^{\infty}\frac{n^{2d}}{k^2n^4}\left(\int_0^t r^{\alpha-1}dr\right)\left[\int_0^t r^{\alpha-1}\Big(\exp\Big(\frac{n^2}{1+kn^2}\frac{r^\alpha-t^\alpha}{\alpha}\Big) - \exp\Big(\frac{n^2(r-t)}{1+kn^2}\Big)\Big)^2|G_n(r)|^2dr\right] \\ &\lesssim \left((1-\alpha)^\beta + (1-\alpha) + |T^{1-\alpha}-1|\right)^2 \\ &\qquad \times \sum_{n=1}^{\infty} n^{2d-4}\Big(\int_0^t r^{\alpha-1}t^{2\alpha-2\beta}|G_n(r)|^2dr + \int_0^t r^{\alpha-1}r^{2\alpha-2\beta}|G_n(r)|^2dr\Big). \end{aligned} \tag{15}$$

Thus, we can infer that

$$\begin{aligned}\|B_2(.,t)\|^2_{\mathbb{H}^d(0,\pi)} &\lesssim \left((1-\alpha)^\beta + (1-\alpha) + |T^{1-\alpha}-1|\right)^2 \\ &\quad \Big[t^{2\alpha-2\beta}\int_0^t r^{\alpha-1}\|G(.,r)\|^2_{\mathbb{H}^{d-2}(0,\pi)}dr + \int_0^t r^{3\alpha-2\beta-1}\|G(.,r)\|^2_{\mathbb{H}^{d-2}(0,\pi)}dr\Big]. \end{aligned} \tag{16}$$

It is obvious to see that

$$\int_0^t r^{\alpha-1}\|G(.,r)\|^2_{\mathbb{H}^{d-2}(0,\pi)}dr \le \|G\|^2_{L^\infty(0,T;\mathbb{H}^{d-2}(0,\pi))}\frac{t^\alpha}{\alpha}, \tag{17}$$

and

$$\int_0^t r^{3\alpha-2\beta-1}\|G(.,r)\|^2_{\mathbb{H}^{d-2}(0,\pi)}dr \le \|G\|^2_{L^\infty(0,T;\mathbb{H}^{d-2}(0,\pi))}\frac{t^{3\alpha-2\beta}}{3\alpha-2\beta}. \tag{18}$$

Some previous observations allows us to get that

$$\|B_2(.,t)\|^2_{\mathbb{H}^d(0,\pi)} \lesssim \left((1-\alpha)^\beta + (1-\alpha) + |T^{1-\alpha}-1|\right)^2 t^{3\alpha-2\beta}\|G\|^2_{L^\infty(0,T;\mathbb{H}^{d-2}(0,\pi))}. \tag{19}$$

Step 3. Estimate the term B_3.
Based on Parseval's equality, we get

$$\begin{aligned}\big\|B_3(.,t)\big\|^2_{\mathbb{H}^d(0,\pi)} &= \sum_{n=1}^{\infty} n^{2d}\left[\frac{1}{1+kn^2}\int_0^t \left(r^{\alpha-1}-1\right)\exp\left(\frac{n^2(r-t)}{1+kn^2}\right)G_n(r)dr\right]^2\\ &\le k^{-2}\sum_{n=1}^{\infty} n^{2d-4}\left[\int_0^t \left|r^{\alpha-1}-1\right|^2 \exp\left(2\frac{n^2(r-t)}{1+kn^2}\right)|G_n(r)|^2 dr\right]\\ &\le k^{-2}\big\|G\big\|^2_{L^\infty(0,T;\mathbb{H}^{d-2}(0,\pi))}\int_0^t \left(r^{2\alpha-2}+1-2r^{\alpha-1}\right)dr. \end{aligned} \tag{20}$$

Indeed, we get that

$$\int_0^t \left(r^{2\alpha-2}+1-2r^{\alpha-1}\right)dr = \frac{t^{2\alpha-1}}{2\alpha-1}-\frac{2t^\alpha}{\alpha}+t, \tag{21}$$

we have

$$\begin{aligned}\left|\frac{t^{2\alpha-1}}{2\alpha-1}-\frac{t^\alpha}{\alpha}\right| &= \frac{t^{\alpha-1}}{\alpha}\left|\frac{\alpha}{2\alpha-1}t^\alpha - t\right|\\ &\le \frac{t^{\alpha-1}}{\alpha}\frac{t^\alpha(\alpha-1)^2}{\alpha(2\alpha-1)}+\frac{t^{\alpha-1}}{\alpha}\left|\frac{t^\alpha}{\alpha}-t\right|. \end{aligned} \tag{22}$$

Hence, since the fact that $\alpha > \frac{1}{2}$, we derive that

$$\begin{aligned}\int_0^t \left(r^{2\alpha-2}+1-2r^{\alpha-1}\right)dr &= \frac{t^{2\alpha-1}}{2\alpha-1}-\frac{2t^\alpha}{\alpha}+t \le \frac{t^{\alpha-1}}{\alpha}\frac{t^\alpha(\alpha-1)^2}{\alpha(2\alpha-1)}+\frac{t^{\alpha-1}}{\alpha}\left|\frac{t^\alpha}{\alpha}-t\right|+\left|\frac{t^\alpha}{\alpha}-t\right|\\ &\lesssim t^{2\alpha-\beta-1}\Big((1-\alpha)^\beta+(1-\alpha)+|T^{1-\alpha}-1|\Big)+t^{2\alpha-1}(1-\alpha)^2. \end{aligned} \tag{23}$$

It follows from (20) that

$$\big\|B_3(.,t)\big\|_{\mathbb{H}^d(0,\pi)} \lesssim k^{-1}\big\|G\big\|_{L^\infty(0,T;\mathbb{H}^{d-2}(0,\pi))}\left[\sqrt{(1-\alpha)^\beta+(1-\alpha)+|T^{1-\alpha}-1|}+1-\alpha\right]. \tag{24}$$

Combining (11), (12), (19) and (24), we infer that

$$\begin{aligned}&\big\|Z_\alpha(.,t)-Z^*(.,t)\big\|_{\mathbb{H}^d(0,\pi)}\\ &\lesssim \big\|B_1(.,t)\big\|_{\mathbb{H}^d(0,\pi)}+\big\|B_2(.,t)\big\|_{\mathbb{H}^d(0,\pi)}+\big\|B_3(.,t)\big\|_{\mathbb{H}^d(0,\pi)}\\ &\lesssim k^{-\varepsilon}t^{\varepsilon(\alpha-\beta)}\Big((1-\alpha)^\beta+(1-\alpha)+|T^{1-\alpha}-1|\Big)^\varepsilon \big\|f\big\|_{\mathbb{H}^d(0,\pi)}\\ &+\Big((1-\alpha)^\beta+(1-\alpha)+|T^{1-\alpha}-1|\Big)\big\|G\big\|_{L^\infty(0,T;\mathbb{H}^{d-2}(0,\pi))}\\ &+k^{-1}\big\|G\big\|_{L^\infty(0,T;\mathbb{H}^{d-2}(0,\pi))}\left(\sqrt{(1-\alpha)^\beta+(1-\alpha)+|T^{1-\alpha}-1|}+1-\alpha\right). \end{aligned} \tag{25}$$

□

4. Conclusion

In this work, the conformable derivative is applied to pseudo-parabolic equation. The main target is to show that the convergence result of the mild solution when the fractional order tends to 1^-, with some new inequalities and using Hilbert scales spaces. In the future work, we will continue to study the convergence results for pseudo-parabolic equations with other derivatives such as: Caputo derivative, Atangana Baleanu Caputo derivative, Riemann-Liouville derivative, and some other non-integer order derivatives. .

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Declaration of Competing Interest

The author(s), declares that there is no competing financial interests or personal relationships that influence the work in this paper.

Authorship Contribution Statement

Anh Tuan Nguyen: Typesetting, Reviewing, Editing, Analysis.
Ngo Ngoc Hung: Reviewing, Model Formulation, Analysis, Supervision.
Nguyen Hoang Luc: Typesetting, Reviewing, Editing, Analysis.

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