DOI: 10.38061/idunas.1260138

IDUNAS	NATURAL & APPLIED SCIENCES JOURNAL	2023 Vol. 6 No. 1 (41-49)

Relations of Multiplicative Generalized $(\alpha,\beta)-$ Reverse Derivation and $\alpha-$ Commuting Maps

Research Article

Barış ALBAYRAK [1*]

[1]Department of Finance and Banking, Çanakkale Onsekiz Mart University, Turkey.

Author E-mails
balbayrak@comu.edu.tr

*Correspondence to: Barış ALBAYRAK, Department of Finance and Banking, Çanakkale Onsekiz Mart University, Turkey.

DOI: 10.38061/idunas.1260138

Received: 04.03.2023; Accepted: 03.07.2023

Abstract

In this paper, properties of the ideal I of semiprime ring R with multiplicative generalized $(\alpha,\beta)-$ reverse derivation with determined not necessarily additive map d is studied. We generalized previous studies for different derivations to multiplicative generalized $(\alpha,\beta)-$ reverse derivation F. We show that $[\beta(p),d(p)]I=0$ for all $p\in I$ or $[d(p)],\alpha(p)]I=0$ for all $p\in I$ under the given different conditions. Also, we give the relationship between map d and anti-automorphism α of semiprime ring R and automorphism β of semiprime ring R. Under the given different conditions, we examine whether d is $\alpha-$ commuting on ideal I or $\beta-$ commuting on ideal I and obtain new results.

Keywords: Reverse derivation, semiprime ring, commuting map

2010 Mathematics Subject Classification. Primary 16N60; Secondary 16U80, 16W25.

1. INTRODUCTION

The aim of our study is to investigate properties of the ideal I of semiprime ring R with multiplicative generalized $(\alpha,\beta)-$reverse derivation. How to generalize the work on semiprime rings involving derivation and how to obtain new results has been a long-studied topic in ring theory. The definition of derivation is given as additive map d that provides $d(rp)=d(r)p+rd(p)$ for $r,p\in R$. This definition has been generalized over time and studies have been generalized for different derivations. In order to contribute to these studies, we study multiplicative generalized $(\alpha,\beta)-$reverse derivation and we obtain new results. Before moving on to the main conclusions, let's give some previous studies and terms that we will use throughout this article.

Let $Z(R)$ denote the center of ring R. Assume that $pRp = (0)$ for any $p \in R$. R is said to be a semiprime ring, if $p = 0$. $[p, r]$ expression is used for commutator $pr - rp$ and (por) expression is used for anticommutator $pr + rp$. A subgroup I which is additive is said to be an ideal of R if IR and RI are included I.

The generalized derivation definition was first given by Bresar in [1]. F from R to R is said to be generalized derivation with determined derivation d if $F(rp) = F(r)p + rd(p)$ for $r, p \in R$. According to [2], generalized derivation F is called generalized $\alpha -$derivation with determined derivation d if $F(rp) = F(r)\alpha(p) + \alpha(r)d(p)$ for $r, p \in R$. On the other hand, Herstein introduced reverse derivation in [3]. If an additive map d provides $d(rp) = d(p)r + pd(r)$ for any $r, p \in R$, then d is a reverse derivation.

Later on, derivations with non-additive maps began to be studied. In [4,5,6], authors gave different definitions of the derivation when d is a non-additive map (not necessarily additive). A non-additive map d is said to be multiplicative derivation if it provides $d(rp) = d(r)p + rd(p)$ for $r, p \in R$. F from R in R is said to be multiplicative generalized derivation with determined non-additive map (not necessarily additive) d if $F(rp) = F(r)p + rd(p)$ for $r, p \in R$.

Next, in [7], authors gave the definitions of multiplicative generalized reverse derivation and multiplicative generalized $(\alpha, \beta) -$reverse derivation. F is said to be multiplicative generalized reverse derivation with determined a non-additive map (not necessarily additive) d if $F(rp) = F(p)r + pd(r)$. F is said to be multiplicative generalized $(\alpha, \beta) -$reverse derivation with determined a non-additive map (not necessarily additive) d if $F(rp) = F(p)\alpha(r) + \beta(p)d(r)$ for an automorphism β of R and anti-automorphism α of R.

On the other hand, different types of maps used in derivation studies were also defined. A map d from R to R that provides $[d(p), p] = 0$ for all $p \in R$, is said to be commuting on R. Also, for α automorphism of R, a map d from R to R that provides $[d(p), \alpha(p)] = 0$ for all $p \in R$, is said to be $\alpha -$ commuting on R. Similar definitions can be made for anti-automorphism. Authors introduced multiplicative left reverse $\alpha -$centralizer in [8]. A map d from R to R is called a multiplicative left reverse $\alpha -$centralizer satisfy $d(pr) = d(r)\alpha(p)$ holds for all $p, r \in R$. α is a mapping of R and d is a map such that not necessarily additive.

Let's take a brief look at the work we have done in this study. In [9] authors studied identities $F(por) + H(por) = 0$, $F(por) + H[p, r] = 0$, $F[p, r] + [\alpha(p), H(r)] = 0$, $F(por) + [\alpha(p), H(r)] = 0$, $F(rp) + [\alpha(p), H(r)] \in Z(R)$, $F(rp) + [H(p), H(r)] \in Z(R)$ for all $r, p \in I$ such that F is a multiplicative generalized derivation, I is an ideal semiprime ring R. We generalize their results to multiplicative generalized $(\alpha, \beta) -$ reverse derivation F for anti-automorphism α and automorphism β of semiprime ring R. Also, we examine the relationship between multiplicative generalized $(\alpha, \beta) -$ reverse derivations and $\alpha -$ commuting maps.

2. PRELIMINARIES

Let's first give the properties provided for the anticommutator and commutator for all $s, r, p \in R$. Next, we will give a lemma that we will use in our theorems.

- $[pr, s] = p[r, s] + [p, s]r$
- $[p, rs] = [p, r]s + r[p, s]$
- $(pr)os = p(ros) - [p, s]r = (pos)r + p[r, s]$
- $p \circ (rs) = (por)s - r[p, s] = r(pos) + [p, r]s$

Lemma 2.1 [10] *Let R be a $2-$torsion free semiprime ring and U a noncentral Lie ideal of R. If $pU=0$ for $p\in U$, then $p=0$.*

3. RESULTS

Let R be a semiprime ring, $0\neq I$ be an ideal of R, α be an anti-automorphism of R, β be an automorphism of R, $0\neq G: R\to R$ be a multiplicative left reverse $\alpha-$ centralizer and $0\neq F: R\to R$ be a multiplicative generalized $(\alpha,\beta)-$ reverse derivation determined with a map $0\neq d: R\to R$ such that it doesn't need to be an additive map. In the following theorems, we examine the conditions under which $[\beta(p),d(p)]I=0$ is provided for all $p\in I$.

Theorem 3.1: If $F(por)+G(por)=0$ for all $r,p\in I$, then $[\beta(p),d(p)]I=0$ for all $p\in I$.

Proof: Let $F(por)+G(por)=0$ for all $r,p\in I$. Replacing r by pr and using commutator properties, we have

$$\begin{aligned} 0 &= F\big(po(pr)\big)+G\big(po(pr)\big) \\ &= F\big(p(por)\big)+G\big(p(por)\big) \end{aligned}$$

$for\ all\ r,p\in I$. Using commutator properties and definitions of F and G, we get

$$F(por)\alpha(p)+\beta(por)d(p)+G(por)\alpha(p)=0$$

$for\ all\ r,p\in I$. Using hypothesis, we have

$$\beta(por)d(p)=0\ for\ all\ r,p\in I. \tag{1}$$

Replacing r by $\beta^{-1}(r)s$, $s\in I$ and using commutator properties, we have

$$r\beta(pos)d(p)+\beta[p,\beta^{-1}(r)]\beta(s)d(p)=0$$

$for\ all\ s,r,p\in I$. Using equation (1) in the above equation, we obtain

$$\beta\left[p,\beta^{-1}(r)\right]\beta(s)d(p)=0\ for\ all\ s,r,p\in I.$$

Since β is an automorphism of R, we write this relation as below relation.

$$[\beta(p),r]Vd(p)=0\ for\ all\ r,p\in I.$$

where $\beta(I)=V$ is a nonzero ideal of R. Replacing r by $d(p)r$, we have

$$[\beta(p),d(p)]rwd(p)=0\ \ for\ all\ r,p\in I, w\in V. \tag{2}$$

Replacing w by $w\beta(p)$, we have

$$[\beta(p),d(p)]rw\beta(p)d(p)=0 \tag{3}$$

$for\ all\ r,p\in I, w\in V$. Also, right multiplication of equation (2) by $\beta(p)$, we get

$$[\beta(p),d(p)]rwd(p)\beta(p)=0 \tag{4}$$

$for\ all\ r,p\in I, w\in V$. Comparing (3) and (4), we get

$$[\beta(p),d(p)]IV[\beta(p),d(p)]=0\ \ for\ all\ p\in I.$$

Since V is an ideal of R, we write

$$[\beta(p), d(p)]IVR[\beta(p), d(p)]IV = 0 \ \ for\ all\ p \in I.$$

Since R is a semiprime ring, we obtain

$$[\beta(p), d(p)]IV = 0 \ \ for\ all\ p \in I.$$

Specially, we write

$$[\beta(p), d(p)]I\beta(s) = 0\ for\ all\ s, p \in I. \tag{5}$$

Replacing s by $s\beta^{-1}(d(p))$, we have

$$[\beta(p), d(p)]I\beta(s)d(p) = 0 \tag{6}$$

for all $s, p \in I$. Also, since I is an ideal of R, from equation (5) we write

$$[\beta(p), d(p)]Id(p)\beta(s) = 0 \tag{7}$$

Theorem 3.2: If $F(por) + G[p, r] = 0$ for all $r, p \in I$, then $[\beta(p), d(p)]I = 0$ for all $p \in I$.

Proof: Let $F(por) + G[p, r] = 0$ for all $r, p \in I$. Replacing r by pr and using commutator properties, we have

$$0 = F\big(po(pr)\big) + G[p, pr] = F(p(por)) + G(p[p, r])$$

$for\ all\ r, p \in I$. Using commutator properties and definitions of F and G, we get

$$F(por)\alpha(p) + \beta(por)d(p) + G(por)\alpha(p) = 0$$

$for\ all\ r, p \in I$. Using hypothesis, we have

$$\beta(por)d(p) = 0\ for\ all\ r, p \in I.$$

This equation is the equation (1) in Theorem 3.1. If the proof is continued in a similar way,

$$[\beta(p), d(p)]I = 0$$

is obtained.

Theorem 3.3: If $F[p, r] + [\alpha(p), G(r)] = 0$ for all $r, p \in I$, then $[\beta(p), d(p)]I = 0$ for all $p \in I$.

Proof: Let $F[p, r] + [\alpha(p), G(r)] = 0$ for all $r, p \in I$. Replacing r by pr and using commutator properties, we have

$$F[p, pr] + [\alpha(p), G(pr)] = F\big[p[p, r]\big] + [\alpha(p), G(pr)] = 0$$

$for\ all\ r, p \in I$. Using commutator properties and definitions of F and G, we get

$$F[p, r]\alpha(p) + \beta[p, r]d(p) + [\alpha(p), G(r)]\alpha(p) + G(r)[\alpha(p), \alpha(p)] = 0\ for\ all\ r, p \in I$$

Using hypothesis, we have

$$\beta[p, r]d(p) = 0\ for\ all\ r, p \in I. \tag{8}$$

Replacing r by $\beta^{-1}(r)s$, $s \in I$ and using commutator properties, we have

$$\beta\left[p, \beta^{-1}(r)\right]\beta(s)d(p) + r\beta[p, s]d(p) = 0$$

$for\ all\ s, r, p \in I$. Using equation (8) in the above equation, we obtain

$$\beta\left[p,\beta^{-1}(r)\right]\beta(s)d(p)=0\ for\ all\ s,r,p\in I.$$

Since β is an automorphism of R, we write this relation as below relation.

$$[\beta(p),r]Vd(p)=0\ for\ all\ r,p\in I.$$

where $\beta(I)=V$ is a nonzero ideal of R. Replacing r by $d(p)r$, we have

$$[\beta(p),d(p)]rwd(p)=0\ \ for\ all\ r,p\in I,w\in V.$$

This equation is the equation (2) in Theorem 3.1. If the proof is continued in a similar way,

$$[\beta(p),d(p)]I=0$$

is obtained.

Theorem 3.4: If $F(por)+[\alpha(p),G(r)]=0$ for all $r,p\in I$, then $[\beta(p),d(p)]I=0$ for all $p\in I$.

Proof: Let $F(por)+[\alpha(p),G(r)]=0$ for all $r,p\in I$. Replacing r by pr and using commutator properties, we have

$$F\big(po(pr)\big)+[\alpha(p),G(pr)]=F(p(por))+[\alpha(p),G(pr)]=0$$

$for\ all\ r,p\in I.$ Using commutator properties and definitions of F and G, we get

$$F(por)\alpha(p)+\beta(por)d(p)+[\alpha(p),G(r)]\alpha(p)+G(r)[\alpha(p),\alpha(p)]=0\ for\ all\ r,p\in I.$$

Using hypothesis, we have

$$\beta(por)d(p)=0\ for\ all\ r,p\in I.$$

This equation is the equation (1) in Theorem 3.1. If the proof is continued in a similar way,

$$[\beta(p),d(p)]I=0$$

is obtained.

Now, using the Lemma 2.1, we can obtain the following result.

Corollary 3.5: Let R be a 2-torsion free semiprime ring, $I\not\subset Z(R)$ be an ideal of R, α be an anti-automorphism of R, β be an automorphism of R, $0\neq G:R\to R$ be a multiplicative left reverse $\alpha-$centralizer and $0\neq F:R\to R$ be a multiplicative generalized $(\alpha,\beta)-$ reverse derivation determined with a map $0\neq d:R\to R$ such that it doesn't need to be an additive map. If one of the following properties are provided $for\ all\ r,p\in I$, then d is $\beta-$ commuting on I.

1) $F(por)+G(por)=0$

2) $F(por)+G[p,r]=0$

3) $F[p,r]+[\alpha(p),G(r)]=0$

4) $F(por)+[\alpha(p),G(r)]=0$

Now, let's give the relationship between map d and anti-automorphism α.

Theorem 3.6: If $\beta(pr)=\alpha(rp)$ and $F(rp)+[\alpha(p),G(r)]\in Z(R)$ for all $r,p\in I$, then $[d(p),\alpha(p)]I=0$ for all $p\in I$.

Proof: Let $F(rp)+[\alpha(p),G(r)]\in Z(R)$for all $r,p\in I$. Replacing r by vr, $v\in I$ and using commutator properties, we have

$$F(v(rp))+[\alpha(p),G(vr)]\in Z(R) \tag{9}$$

$for\ all\ v,r,p\in I.$ Using commutator properties and definitions of F and G, we get

$$(F(rp)+[\alpha(p),G(r)])\alpha(v)+\beta(rp)d(v)+G(r)[\alpha(p),\alpha(v)]\in Z(R)$$

$for\ all\ v,r,p\in I.$ Since the element in the above equation is in the $Z(R)$, we write following equation for $\alpha(v)$.

$$[(F(rp)+[\alpha(p),G(r)])\alpha(v)+\beta(rp)d(v)+G(r)[\alpha(p),\alpha(v)],\alpha(v)]=0$$

$for\ all\ v,r,p\in I.$ Using hypothesis, we have

$$[\beta(rp)d(v),\alpha(v)]+[G(r)[\alpha(p),\alpha(v)],\alpha(v)]=0$$

$for\ all\ v,r,p\in I.$ Using the fact that $\beta(pr)=\alpha(rp)$ for all $r,p\in I$, we obtain

$$[\alpha(pr)d(v),\alpha(v)]+[G(r)[\alpha(p),\alpha(v)],\alpha(v)]=0 \tag{10}$$

$for\ all\ v,r,p\in I.$ Replacing p by vp and using commutator properties, we have

$$[\alpha(vpr)d(v),\alpha(v)]+[G(r)[\alpha(p),\alpha(v)]\alpha(v),\alpha(v)]=0 \tag{11}$$

$for\ all\ v,r,p\in I.$ Also, right multiplication of equation (10) by $\alpha(v)$, we get

$$[\alpha(pr)d(v)\ \alpha(v),\alpha(v)]+[G(r)[\alpha(p),\alpha(v)]\ \alpha(v),\alpha(v)]=0 \tag{12}$$

$for\ all\ v,r,p\in I.$ Comparing (11) and (12) and using properties of anti- automorphism for α, we get

$$[\alpha(r)\alpha(p)d(v)\ \alpha(v),\alpha(v)]-[\alpha(r)\alpha(p)\alpha(v)d(v),\alpha(v)]=0\ for\ all\ v,r,p\in I.$$

Arranging above equation, we have

$$[\alpha(r)\alpha(p)[d(v),\alpha(v)],\alpha(v)]=0 \tag{13}$$

$for\ all\ v,r,p\in I.$ Replacing r by rw, $w\in I$, we have

$$[\alpha(w)\alpha(r)\alpha(p)[d(v),\alpha(v)],\alpha(v)]=0$$

$for\ all\ w,v,r,p\in I.$ Using commutator properties, we get

$$\alpha(w)[\alpha(r)\alpha(p)[d(v),\alpha(v)],\alpha(v)]+[\alpha(w),\alpha(v)]\alpha(r)\alpha(p)[d(v),\alpha(v)]=0$$

$for\ all\ w,v,r,p\in I.$ Using equation (13) in the above equation, we obtain

$$[\alpha(w),\alpha(v)]\alpha(r)\alpha(p)[d(v),\alpha(v)]=0$$

$for\ all\ w,v,r,p\in I.$ Since α is an anti-automorphism of R, we write this relation as below relation.

$$[\alpha(w),\alpha(v)]VV[d(v),\alpha(v)]=0\ \ for\ all\ w,v\in I.$$

where $\alpha(I)=V$ is a nonzero ideal of R. Replacing w by v, we have

$$[\alpha(v),\alpha(v)]VV\ [d(v),\alpha(v)]=0\ \ for\ all\ v\in I.$$

Since V is an ideal of R, left and right multiplication of above equation by V , we get

$$V[\alpha(v),\alpha(v)]VRV\ [d(v),\alpha(v)]V=0\ \ for\ all\ v\in I.$$

Since R is a semiprime ring, we get

$$V[d(v), \alpha(v)]V = 0 \;\; for\; all\; v \in I.$$

Using properties of ideal V, we have

$$[d(v), \alpha(v)]VR[d(v), \alpha(v)]V = 0 \;\; for\; all\; v \in I.$$

Using the fact that R is a semiprime ring, we obtain

$$[d(v), \alpha(v)]V = 0\; for\; all\; v \in I.$$

Since V is an ideal of R, we get

$$[d(v), \alpha(v)]IV = 0\; for\; all\; v \in I.$$

Specially, we write

$$[d(v), \alpha(v)]I\alpha(s) = 0\; for\; all\; s, v \in I \tag{14}$$

Replacing s by $s\alpha^{-1}(d(v))$, we have

$$[d(v), \alpha(v)]I\alpha(s)d(v) = 0 \tag{15}$$

for all $s, v \in I$. Also, since I is an ideal of R, from equation (14), we write

$$[d(v), \alpha(v)]Id(v)\alpha(s) = 0 \tag{16}$$

for all $s, v \in I$. Comparing (15) and (16), we get

$$[d(v), \alpha(v)]I[d(v), \alpha(s)] = 0\; for\; all\; s, v \in I.$$

Replacing s by v we have

$$[d(v), \alpha(v)]I[d(v), \alpha(v)] = 0\; for\; all\; v \in I.$$

Since I is an ideal of R, we write

$$[d(v), \alpha(v))]IR[d(v), \alpha(v)]I = 0 \;\; for\; all\; v \in I$$

Since R is a semiprime ring, we obtain

$$[d(v), \alpha(v)]I = 0 \;\; for\; all\; v \in I.$$

Theorem 3.7: If $\beta(pr) = \alpha(rp)$ and $F(rp) + [G(p), G(r)] \in Z(R)$ for all $r, p \in I$, then $[d(p)], \alpha(p)]I = 0$ for all $p \in I$.

Proof: Let $F(rp) + [G(p), G(r)] \in Z(R)$ for all $r, p \in I$. Replacing r by vr, $v \in I$ and using commutator properties, we have

$$F(vrp) + [G(p), G(vr)] \in Z(R)$$

$for\; all\; v, r, p \in I$. Using commutator properties and definitions of F and G, we get

$$(F(rp) + [G(p), G(r)])\alpha(v) + \beta(rp)d(v) + G(r)[\alpha(p), \alpha(v)] \in Z(R)$$

$for\; all\; v, r, p \in I$. Since the element in the above equation is in the $Z(R)$, we write following equation for $\alpha(v)$.

$$[(F(rp) + [G(p), G(r)])\alpha(v) + \beta(rp)d(v) + G(r)[\alpha(p), \alpha(v)], \alpha(v)] = 0$$

$for\; all\; v, r, p \in I$. Using hypothesis in this relation, we have

$$[\beta(rp)d(v), \alpha(v)] + [G(r)[\alpha(p), \alpha(v)], \alpha(v)] = 0$$

$for\ all\ v, r, p \in I.$ Using the fact that $\beta(pr) = \alpha(rp)$ for all $r, p \in I$, we obtain

$$[\alpha(pr)d(v), \alpha(v)] + [G(r)[\alpha(p), \alpha(v)], \alpha(v)] = 0$$

$for\ all\ v, r, p \in I.$ This equation is the equation (10) in Theorem 3.6. If the proof is continued in a similar way,

$$[d(p)], \alpha(p)]I = 0$$

is obtained.

Now, using the Lemma 2.1, we can obtain the following result.

Corollary 3.8: Let R be a 2-torsion free semiprime ring, $I \not\subset Z(R)$be an ideal of R, α be an anti-automorphism of R, β be an automorphism of R such that $\beta(pr) = \alpha(rp)$ for all $r, p \in I$, $0 \neq G: R \to R$ be a multiplicative left reverse $\alpha -$ centralizer and $0 \neq F: R \to R$ be a multiplicative generalized $(\alpha, \beta) -$ reverse derivation determined with a map $0 \neq d: R \to R$ such that it doesn't need to be an additive map. If one of the following properties are provided $for\ all\ r, p \in I$, then d is $\alpha -$ commuting on I.

1) $F(rp) + [\alpha(p), G(r)] \in Z(R)$

2) $F(rp) + [G(p), G(r)] \in Z(R)$

5. CONCLUSIONS

In this paper, properties of the ideal I of semiprime ring R with multiplicative generalized $(\alpha, \beta) -$ reverse derivation with determined not necessarily additive map d is studied. Many studies have been done on the derivation and commutativity in the prime ring and the results have been reached. These studies and reached results is adapted for multiplicative generalized $(\alpha, \beta) -$ reverse derivation F in our study. Also, new results are given about the relationship between map d and anti-automorphism α. The studies and the results found can be used for different derivations and semiprime rings in the future and contribute to the ring theory.

6. REFERENCES

[1] Bresar, M. (1991). On the distance of the composition of two derivations to the generalized derivations. *Glaskow Math. J.,* 33,89-93.

[2] Chang, J. C. (2009). Right generalized $(\alpha, \beta) -$ derivations having power central values. *Taiwanese J. Math*., 13(4), 1111-1120.

[3] Herstein, I. N. (1957). Jordan Derivation of Prime Rings. *Proc. Amer. Math. Soc.* 8, 1104-1110.

[4] Dhara, B. and Ali, S. (2013). On Multiplicative (generalized) derivative in prime and semi-prime rings", *Aequat. Math.*, 86(1-2), 65-79.

[5] Daif, M.N. (1991) When is a multiplicative derivation additive. *Int. J. Math. Sci.,* 14(3), 615-618.

[6] Daif, M.N. and Tammam El Sayiad M.S. (2007). Multiplicative generalized Derivation which are additive. *East-west J. Math.* 9(1), 31-37.

[7] Tiwari, S.K., Sharma, R.K. and Dhara, B. (2008). Some theorems of commutatively on semiprime ring with mapping. *Southeast Asian Bull. Math.,* 42(2), 279-292

[8] Alhaidary, Z. S. M. and Majeed, A. H. (2021). Square closed Lie İdeals and Multiplicative (Generalized) $(\alpha,\beta)-$reverse derivation of Prime Rings. *Journal of Discrete Math. Sci. and Cryptography*, 24(7), 2037-2046.

[9] Malleswari, G. N., Sreenivasulu, S. and Shobhalatha. G. (2021). Semiprime rings with multiplicative (Generalized) – derivations involving left multipliers. *Create. Math. Inform*, 30(1), 61-68.

[10] Hongan, M., Rehman, N. and Al-Omary R. M. (2011). Lie ideals and Jordan triple derivations in rings. Rend. Sem. *Mat. Univ. Padova*, 125.