

# Analysis of fractional random ordinary differential equations by Adomian Decomposition Method

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## Abstract

*In this study, random ordinary differential equations obtained by randomly choosing the coefficients or initial conditions of the ordinary differential equations will be analyzed by the Adomian Decomposition Method. The initial conditions or coefficients of the equations will be converted to random variables with normal and exponential distribution. Probability characteristics such as expected value, variance and confidence interval of the obtained random ordinary differential equations will be calculated. Obtained results will be drawn with the help of MATLAB (2013a) package program and random results will be interpreted.*

**Keywords:** Adomian decomposition method, fractional derivative, nonlinear ordinary differential equation

## Kesir mertebeden rastgele adi diferansiyel denklemlerin Adomian Ayrıştırma Yöntemi ile analizi

## Öz

*Bu çalışmada, adi diferansiyel denklemlerin katsayılarının veya başlangıç koşullarının rasgele seçilmesiyle elde edilen rasgele adi diferansiyel denklemler, Adomian Ayrıştırma Yöntemi ile analiz edilecektir. Denklemlerin başlangıç koşulları veya katsayıları, normal ve üstel dağılıma sahip rasgele değişkenlere dönüştürülecektir. Elde edilen rastgele adi diferansiyel denklemlerin beklenen değeri, varyansı ve güven aralığı gibi olasılık*

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özellikleri hesaplanacaktır. Elde edilen sonuçlar MATLAB (2013a) paket programı yardımıyla çizilecek ve rastgele sonuçlar yorumlanacaktır.

**Anahtar Kelimeler:** Adomian ayrıştırma yöntemi, kesirli türev, lineer olmayan adi diferansiyel denklem

## 1. Introduction

Application of nonlinear differential equations in physics, engineering etc. It has many applications in fields. Such equations often do not have analytical solutions. Many problems, such as physics and engineering, can be modeled mathematically. Since these models are expressed with the help of nonlinear differential equations, there are methods developed in the literature to obtain approximate analytical solutions of these equations. One of these methods is the Adomian Decomposition Method.

ADM was first introduced by Adomian in the early 1980s. Adomian applied this method to find approximate solutions of deterministic, linear and nonlinear problems with boundary and initial conditions. The method is constructed by decomposing nonlinear

$Ny$  terms.  $Ny = \sum_{n=0}^{\infty} A_n$  is defined as. Here,  $A_n$  are Adomian polynomials. Each

$A_n$  depends on the arguments  $y_0, y_1, y_2, \dots, y_n$  for  $n > 0$ . The formulas to obtain these polynomials were developed by Adomian [1-4]. In recent years, more and more researchers have applied this method to solving nonlinear systems [5–22]. We firstly study the algorithm and convergence analysis of ADM, and then apply ADM to constructing approximate solutions for nonlinear equations with initial data, including algebraic equations, fractional ordinary differential equations and fractional partial differential equations.

In recent years [27-33], many studies have been carried out on random differential equations, random partial differential equations and random integral equations. The deterministic model assumes that the parameters are fixed quantities, while the disease dynamics represented by the parameters may be random in nature. Therefore, we will use a random system of differential equations to represent this randomness. In our study, we transformed the parameters of the deterministic model into random variables to analyze the coefficients or initial conditions random transmission dynamics. The deterministic model assumes that the parameters are constant quantities, while the disease dynamics represented by the parameters can be random in nature. So we will use a random differential equation to represent this randomness.

The motivation for this study is the previous literature on random modeling of various diseases. Exponential and Normal (Gaussian) distributions will be used for the distributions of random parameters. In addition to expected values, random models and equations enable analysis of other numerical properties of the results. Results for expected values, variations, standard deviations, coefficients of variation, and confidence intervals provide analysis of changes in deterministic results. This analysis cannot be done using the deterministic model. Therefore, the random model offers much more than the random model. In recent years, the relationship of the mentioned method with the literature and the comparison results of the proposed articles are included in the [literature](#)[34-40].

The rest of this present paper is arranged as follows: Basic idea of Adomian Decomposition Method are introduced in Section 2. An attractive applications with graphical results to verify the effectiveness and reliability of our technique, along with the methodology of the proposed method, tables and discussions are given in Chapter 3. Finally, Chapter 4 concludes the output of the entire article.

**1.1.Preliminaries**

**Definition 1.** We define the Caputo fractional order derivative of the function  $f(t)$  [41-42]

$${}^C_0D_t^\alpha [f(t)] = \frac{1}{\Gamma(\beta - \alpha)} \int_0^t (t - \eta)^{\beta - \alpha - 1} f(\eta)^{(\beta)} d\eta$$

Where,  $\beta = [\alpha] + 1$  and  $[\alpha]$  represents the integral parts of  $\alpha$ .

**Definition 2.** The Riemann-Liouville fractional derivative of order  $0 < \alpha < 1$  of a function is defined as [43]

$${}^{RL}_0D_x^\alpha f(x) = \frac{1}{\Gamma(\alpha - 1)} \frac{d}{dx} \int_0^x (x - t)^{-\alpha} f(t) dt$$

where  $\Gamma(\cdot)$  denotes the Gamma function.

**2.Adomian decomposition method**

Let  $L(y) + R(y) + N(y) = f(t)$  be the ordinary differential equation. Here  $L$  is the highest order inverse derivative operator,  $R$  is linear differential operator,  $N$  is nonlinear operator and  $f$  is an independent term from  $y$ .  $y$  is the solution of the equation. The main purpose of the Adomian Decomposition Method is to implement the inverse operator. If  $L^{-1}$  is applied to both sides of the expression

$$Ly(t) = f(t) - Ry(t) - N(y(t))$$

giving the solution, then

$$y(t) = \psi_0(t) + L^{-1}f(t) - L^{-1}Ry(t) - L^{-1}N(y(t))$$

is obtained and  $\psi_0$  contains terms resulting from the use of initial conditions.

$$\psi_0(x) = \begin{cases} y(0), & L = \frac{d}{dt} \\ y(0) + ty'(0), & L^2 = \frac{d^2}{dt^2} \\ y(0) + ty'(0) + \frac{1}{2}t^2y''(0), & L^3 = \frac{d^3}{dt^3} \\ y(0) + ty'(0) + \frac{1}{2}t^2y''(0) + \frac{1}{3!}t^3y'''(0), & L^4 = \frac{d^4}{dt^4} \end{cases}$$

or

$$\sum_{n=0}^\infty y_n(t) = \psi_0(t) + L^{-1}f(t) - L^{-1}R \sum_{n=0}^\infty y_n(t) - L^{-1} \sum_{n=0}^\infty A_n(t) \tag{1}$$

and  $A_n(t)$  in (1) are the Adomian polynomials. If  $y_0(t)$  is defined as  $\psi_0(t) + L^{-1}f(t)$  the remaining components are determined by the following equations [6]:

$$y_0(t) = \psi_0(t) + L^{-1}f(t),$$

$$y_k(t) = -L^{-1}Ry_{k-1}(t) - L^{-1}A_{k-1}(t), \quad k = 1, 2, \dots$$

Adomian polynomials can be obtained as follows [1-4]:

$$A_0 = f(y_0),$$

$$A_1 = y_1 \frac{df(y_0)}{dy_0},$$

$$A_2 = y_2 \frac{df(y_0)}{dy_0} + \frac{y_1^2}{2!} \frac{d^2f(y_0)}{dy_0^2},$$

$$A_3 = y_3 \frac{df(y_0)}{dy_0} + y_1 y_2 \frac{d^2f(y_0)}{dy_0^2} + \frac{y_1^3}{3!} \frac{d^3f(y_0)}{dy_0^3},$$

$$A_4 = y_4 \frac{df(y_0)}{dy_0} + \left( \frac{1}{2!} y_2^2 + y_1 y_3 \right) \frac{d^2f(y_0)}{dy_0^2} + \frac{1}{2!} y_1^2 y_2 \frac{d^3f(y_0)}{dy_0^3} + \frac{y_1^4}{4!} \frac{d^4f(y_0)}{dy_0^4}, \dots$$

The authors of [44] also worked on ADM convergence analysis using the Cauchy-Kowalevskaya theorem formalism, which guarantees that the solutions of initial value problems for systems of ordinary differential equations with analytical vector fields are analytical over time for small time intervals.

### 3.Application

In this Chapter, we provide attractive and interesting examples with graphical results to demonstrate the effectiveness and simplicity of the method we proposed in Chapter 2.

#### Example 3. 1.

Consider the following random ordinary fractional differential equation

$$\frac{dy}{dt} = -\frac{d^{\frac{1}{2}}(-y)}{dt^{\frac{1}{2}}} - y, \tag{2}$$

subject to the initial conditions

$$y(0) = B$$

where  $B \sim N(\mu, \sigma^2)$  is parameter with Normal distribution. If  $L^{-1}$  is applied to both sides of the equation in equation (2), where  $L = \frac{d}{dt}$ , equation (3) is obtained.

$$L^{-1} \frac{dy}{dt} = -L^{-1} \frac{d^{\frac{1}{2}}(-y)}{dt^{\frac{1}{2}}} - L^{-1}y$$

$$y_{n+1} = -\frac{d^{-\frac{1}{2}}}{dt^{-\frac{1}{2}}} A_n - L^{-1}y_n \tag{3}$$

As Adomian Polynomials,  $f(y) = -y, f'(y) = -1, f''(y) = 0$ .

$$A_0 = B,$$

$$A_1 = -y_1 = \frac{2B}{\sqrt{\pi}} t^{\frac{1}{2}} + Bt,$$

$$A_2 = -y_2 = Bt - \frac{Bt^2}{2},$$

$$y_0 = B,$$

$$y_1 = -\frac{d^{-\frac{1}{2}}}{dt^{-\frac{1}{2}}} A_0 - L^{-1} y_0 = -\frac{d^{-\frac{1}{2}}}{dt^{-\frac{1}{2}}} B - L^{-1} B = -\frac{2B}{\sqrt{\pi}} t^{\frac{1}{2}} - Bt,$$

$$y_2 = -\frac{d^{-\frac{1}{2}}}{dt^{-\frac{1}{2}}} A_1 - L^{-1} y_1 = -\frac{d^{-\frac{1}{2}}}{dt^{-\frac{1}{2}}} \left[ \frac{2B}{\sqrt{\pi}} t^{\frac{1}{2}} + Bt \right] + L^{-1} \left[ \frac{2B}{\sqrt{\pi}} t^{\frac{1}{2}} + Bt \right] = -Bt + \frac{Bt^2}{2},$$

$$\begin{aligned} y_3 &= -\frac{d^{-\frac{1}{2}}}{dt^{-\frac{1}{2}}} A_2 - L^{-1} y_2 = -\frac{d^{-\frac{1}{2}}}{dt^{-\frac{1}{2}}} \left( Bt - \frac{Bt^2}{2} \right) + L^{-1} \left( Bt - \frac{Bt^2}{2} \right) \\ &= -\frac{4B}{3\sqrt{\pi}} t^{\frac{3}{2}} + \frac{8B}{15\sqrt{\pi}} t^{\frac{5}{2}} + \frac{Bt^2}{2} - \frac{Bt^3}{6}, \end{aligned}$$

In the equation  $y(t) = y_0(t) + y_1(t) + y_2(t) + y_3(t) + \dots$  instead of  $y_0(t)$ ,  $y_1(t)$ ,  $y_2(t)$ ,  $y_3(t)$  values written and edited, the following (4) solution will be found.

$$\begin{aligned} y(t) &= B - \frac{2B}{\sqrt{\pi}} t^{\frac{1}{2}} - Bt - Bt + \frac{Bt^2}{2} - \frac{4B}{3\sqrt{\pi}} t^{\frac{3}{2}} + \frac{8B}{15\sqrt{\pi}} t^{\frac{5}{2}} + \frac{Bt^2}{2} - \frac{Bt^3}{6} + \dots \\ y(t) &= \left( 1 - \frac{2}{\sqrt{\pi}} t^{\frac{1}{2}} + t^2 - 2t - \frac{4}{3\sqrt{\pi}} t^{\frac{3}{2}} + \frac{8}{15\sqrt{\pi}} t^{\frac{5}{2}} - \frac{t^3}{6} + \dots \right) B \end{aligned} \quad (4)$$

The moment generating function of the Normal distribution is

$$M_X(t) = E[e^{tX}] = e^{\frac{1}{2}\sigma^2 t^2 + \mu t}$$

The moments of the random variable  $B \sim N(\mu, \sigma^2)$  are calculated as

$$E[B] = \mu, \quad E[B^2] = \sigma^2 + \mu^2$$

Using the basic properties of the expected value for the random variable, the expected value of equation (4) is found as (5). If  $B \sim N(\mu = 2, \sigma^2 = 5)$  special values are selected, equation (6) is obtained.

$$\begin{aligned} E[y(t)] &= \left( 1 - \frac{2}{\sqrt{\pi}} t^{\frac{1}{2}} + t^2 - 2t - \frac{4}{3\sqrt{\pi}} t^{\frac{3}{2}} + \frac{8}{15\sqrt{\pi}} t^{\frac{5}{2}} - \frac{t^3}{6} + \dots \right) E[B] \\ E[y(t)] &= \left( 1 - \frac{2}{\sqrt{\pi}} t^{\frac{1}{2}} + t^2 - 2t - \frac{4}{3\sqrt{\pi}} t^{\frac{3}{2}} + \frac{8}{15\sqrt{\pi}} t^{\frac{5}{2}} - \frac{t^3}{6} + \dots \right) \mu \end{aligned} \quad (5)$$

$$E[y(t)] = 2 \left( 1 - \frac{2}{\sqrt{\pi}} t^{\frac{1}{2}} + t^2 - 2t - \frac{4}{3\sqrt{\pi}} t^{\frac{3}{2}} + \frac{8}{15\sqrt{\pi}} t^{\frac{5}{2}} - \frac{t^3}{6} + \dots \right) \quad (6)$$

If the expected value is plotted with MATLAB (2013a) for the given parameter values, the graph in Figure 1. is obtained.

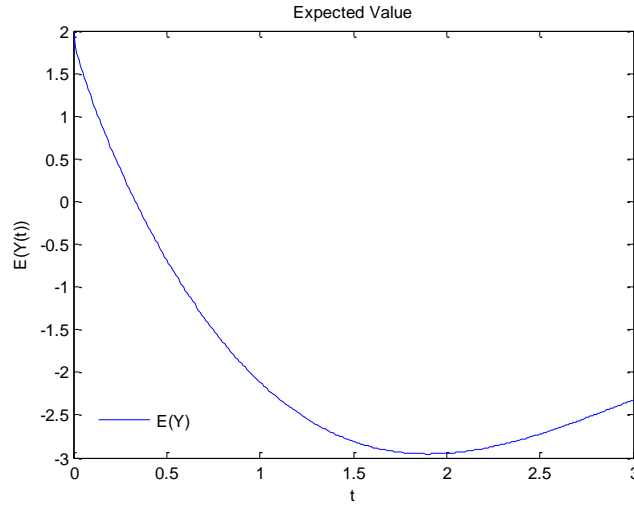


Figure 1. The solution behavior of the expected value of equation (4) for special values  $B \sim N(\mu = 2, \sigma^2 = 5)$ .

Using the basic properties of the variance, the variance of equation (4) is calculated as follows:

$$\begin{aligned} \text{Var}(B) &= E(B^2) - [E(B)]^2 = \sigma^2 + \mu^2 - \mu^2 = \sigma^2 \\ \text{Var}[y(t)] &= \left(1 - \frac{2}{\sqrt{\pi}}t^{\frac{1}{2}} + t^2 - 2t - \frac{4}{3\sqrt{\pi}}t^{\frac{3}{2}} + \frac{8}{15\sqrt{\pi}}t^{\frac{5}{2}} - \frac{t^3}{6} + \dots\right)^2 \text{Var}[B] \\ \text{Var}[y(t)] &= \left(1 - \frac{2}{\sqrt{\pi}}t^{\frac{1}{2}} + t^2 - 2t - \frac{4}{3\sqrt{\pi}}t^{\frac{3}{2}} + \frac{8}{15\sqrt{\pi}}t^{\frac{5}{2}} - \frac{t^3}{6} + \dots\right)^2 \sigma^2 \end{aligned}$$

Specifically, if  $B \sim N(\mu = 2, \sigma^2 = 5)$  is selected,

$$\text{Var}[y(t)] = 5 \left(1 - \frac{2}{\sqrt{\pi}}t^{\frac{1}{2}} + t^2 - 2t - \frac{4}{3\sqrt{\pi}}t^{\frac{3}{2}} + \frac{8}{15\sqrt{\pi}}t^{\frac{5}{2}} - \frac{t^3}{6} + \dots\right)^2$$

variance value. If the variance is plotted with MATLAB (2013a) for the given parameter values, the graph in Figure 2. is obtained.

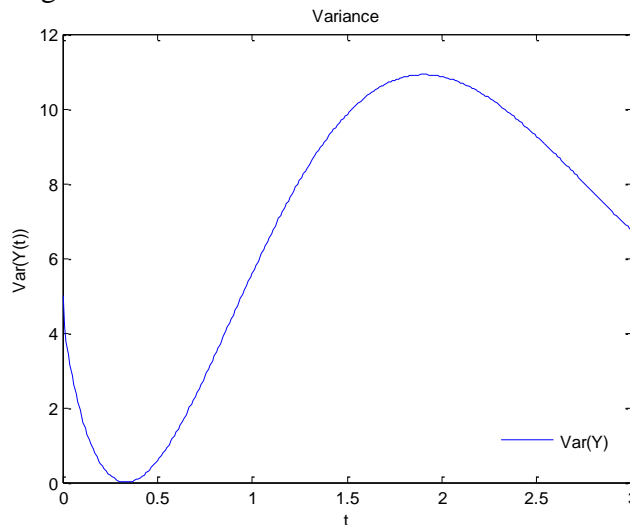


Figure 2. The solution behavior of the variance of equation (4) for special values  $B \sim N(\mu = 2, \sigma^2 = 5)$ .

Confidence intervals for expected values of random variables,

$$(E(y(t)) - K \cdot std(y(t)), E(y(t)) + K \cdot std(y(t)))$$

is equal to and this can be obtained through standard deviations. For  $K = 3$ , this formula gives approximately 99% confidence interval for the approximate expected value of the normally distributed random variable [23]. If the 99% confidence interval is plotted with MATLAB (2013a), the graph in Figure 3. is obtained.

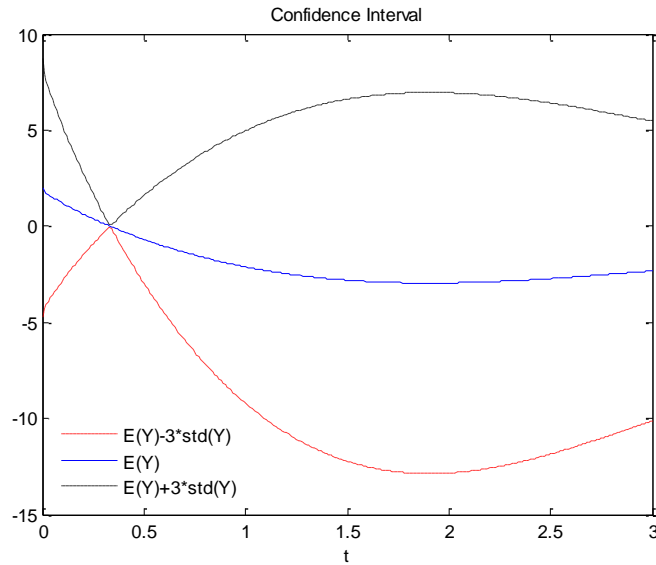


Figure 3. The solution behavior of the confidence interval of equation (4) for special values  $B \sim N(\mu = 2, \sigma^2 = 5)$ .

Table 1. Table for the expectation value, variance and 99% confidence interval

<b>t</b>	<b><math>E(y(t))</math></b>	<b><math>Var(y(t))</math></b>	<b><math>E(y(t)) - 3std(y)</math></b>	<b><math>E(y(t)) + 3std(y)</math></b>
<b>0.0</b>	2	5	-4.708203931	8.708203931
<b>0.1</b>	0.8603434431	0.9252385486	-2.025336189	3.746023075
<b>0.2</b>	0.1442786213	0.02602040111	-0.3396465896	0.6282038322
<b>0.3</b>	-0.4826270769	0.2911611188	-2.101407503	1.136153349
<b>0.4</b>	-1.048347668	1.373791039	-4.564612639	2.467917304
<b>0.5</b>	-1.562974221	3.053610517	-6.805349128	3.679400685
<b>0.6</b>	-2.031493046	5.158704992	-8.845327860	4.782341770
<b>0.7</b>	-2.456887690	7.545371395	-10.69753951	5.783764134
<b>0.8</b>	-2.841217533	10.09064634	-12.37095084	6.688515773
<b>0.9</b>	-3.186073332	12.68882909	-13.87248816	7.500341494
<b>1.0</b>	-3.492795001	15.24952115	-15.20798558	8.222395574

The expected value, variance and confidence interval for  $K = 3$  values are given in Table 1. It can be seen that the results for  $t \in [0,1]$  values are obtained similarly to the results for the case with Normal parameters.

**Example 3. 2.**

We will then consider the following random ordinary fractional differential equation

$$y^{\frac{1}{2}} = 1 - y^2 \tag{7}$$

Let equation (7) have initial conditions  $y(0) = A$ . Where  $A \sim exponential(\lambda)$  is parameter with exponential distribution.

$$\frac{d^{\frac{1}{2}}y}{dt^{\frac{1}{2}}} = 1 - y^2 \tag{8}$$

As in the previous example if  $L^{-\frac{1}{2}}$  is applied to both sides of the equation in equation (7), where  $L = \frac{d^{\frac{1}{2}}}{dt^{\frac{1}{2}}}$ , equation (8) is obtained.

$$L^{-\frac{1}{2}}\left(\frac{d^{\frac{1}{2}}y}{dt^{\frac{1}{2}}}\right) = L^{-\frac{1}{2}}(1) - L^{-\frac{1}{2}}(y^2)$$

$$y_{n+1} = -\frac{d^{-\frac{1}{2}}}{dt^{-\frac{1}{2}}}A_n + L^{-\frac{1}{2}}(1)$$

The terms  $A_n$  and  $y_n$  are obtained as  $f(y) = y^2$ ,  $f'(y) = 2y$  and  $f''(y) = 2$ .

$$A_0 = A^2,$$

$$A_1 = \frac{4}{\sqrt{\pi}}A^2(1 - A^2)t^{\frac{1}{2}},$$

$$A_2 = \left(-4A^4 + \frac{4}{\pi}(1 - A^2)\right)(1 - A^2)t + \frac{4A^2}{\sqrt{\pi}}t^{\frac{1}{2}},$$

$$y_0 = A,$$

$$y_1 = -\frac{d^{-\frac{1}{2}}}{dt^{-\frac{1}{2}}}A_0 + L^{-\frac{1}{2}}(1) = \frac{2}{\sqrt{\pi}}(1 - A^2)t^{\frac{1}{2}},$$

$$y_2 = -\frac{d^{-\frac{1}{2}}}{dt^{-\frac{1}{2}}}A_1 + L^{-\frac{1}{2}}(1) = -2A^2(1 - A^2)t + \frac{2}{\sqrt{\pi}}t^{\frac{1}{2}},$$

$$y_3 = -\frac{d^{-\frac{1}{2}}}{dt^{-\frac{1}{2}}}A_2 + L^{-\frac{1}{2}}(1) = \left(4A^4 - \frac{4}{\pi}(1 - A^2)\right)(1 - A^2)\frac{4}{3\sqrt{\pi}}t^{\frac{3}{2}} - 2A^2t + \frac{2}{\sqrt{\pi}}t^{\frac{1}{2}},$$

As  $y(t) = y_0(t) + y_1(t) + y_2(t) + y_3(t) + \dots$  the values  $y_0(t)$ ,  $y_1(t)$ ,  $y_2(t)$ ,  $y_3(t)$  are written instead of and if edited,

$$y(t) = A + \left(-\frac{2}{\sqrt{\pi}}t^{\frac{1}{2}} - 4t + \frac{32}{3\pi\sqrt{\pi}}t^{\frac{3}{2}}\right)A^2 + \left(2t + \frac{16}{3\sqrt{\pi}}\left(1 - \frac{1}{\pi}\right)t^{\frac{3}{2}}\right)A^4 - \frac{16}{3\sqrt{\pi}}t^{\frac{3}{2}}A^6 + \frac{6}{\sqrt{\pi}}t^{\frac{1}{2}} - \frac{16}{3\pi\sqrt{\pi}}t^{\frac{3}{2}} + \dots \tag{9}$$



solution is obtained.

$$M_X(t) = E[e^{tX}] = \frac{\lambda}{\lambda - t}$$

The moments of the random variable  $A \sim \text{exponential}(\lambda)$  can be calculated using the moment generating function of the exponential distribution.

If  $A \sim \text{exponential}(\lambda = 5)$  special values are selected, the expected value is obtained as equation (10).

$$\begin{aligned} E[y(t)] &= E[A] + \left(-\frac{2}{\sqrt{\pi}}t^{\frac{1}{2}} - 4t + \frac{32}{3\pi\sqrt{\pi}}t^{\frac{3}{2}}\right)E[A^2] + \left(2t + \frac{16}{3\sqrt{\pi}}\left(1 - \frac{1}{\pi}\right)t^{\frac{3}{2}}\right)E[A^4] - \\ &\quad \frac{16}{3\sqrt{\pi}}t^{\frac{3}{2}}E[A^6] + E\left(\frac{6}{\sqrt{\pi}}t^{\frac{1}{2}}\right) - E\left(\frac{16}{3\pi\sqrt{\pi}}t^{\frac{3}{2}}\right) + \dots \\ E[y(t)] &= \frac{1}{\lambda} + \left(-\frac{2}{\sqrt{\pi}}t^{\frac{1}{2}} - 4t + \frac{32}{3\pi\sqrt{\pi}}t^{\frac{3}{2}}\right)\frac{2!}{\lambda^2} + \left(2t + \frac{16}{3\sqrt{\pi}}\left(1 - \frac{1}{\pi}\right)t^{\frac{3}{2}}\right)\frac{4!}{\lambda^4} - \frac{16}{3\sqrt{\pi}}t^{\frac{3}{2}}\frac{6!}{\lambda^6} + \\ &\quad \frac{6}{\sqrt{\pi}}t^{\frac{1}{2}} - \frac{16}{3\pi\sqrt{\pi}}t^{\frac{3}{2}} + \dots \\ E[y(t)] &= \frac{1}{5} + \left(-\frac{2}{\sqrt{\pi}}t^{\frac{1}{2}} - 4t + \frac{32}{3\pi\sqrt{\pi}}t^{\frac{3}{2}}\right)\frac{2!}{5^2} + \left(2t + \frac{16}{3\sqrt{\pi}}\left(1 - \frac{1}{\pi}\right)t^{\frac{3}{2}}\right)\frac{4!}{5^4} - \\ &\quad \frac{16}{3\sqrt{\pi}}t^{\frac{3}{2}}\frac{6!}{5^6} + \frac{6}{\sqrt{\pi}}t^{\frac{1}{2}} - \frac{16}{3\pi\sqrt{\pi}}t^{\frac{3}{2}} + \dots \end{aligned} \quad (10)$$

If the expected value is plotted with MATLAB (2013a) for the given parameter values, the graph in Figure 4. is obtained.

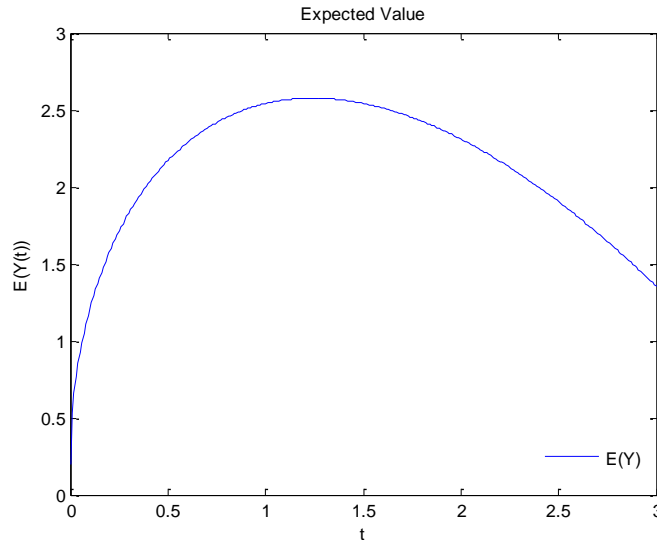


Figure 4. The solution behavior of the expected value of equation (9) for the special value  $A \sim \text{exponential}(\lambda = 5)$ .

The variance of random variable  $A$  is found as (10). If  $A \sim \text{exponential}(\lambda = 5)$  is chosen specifically, the variance is equal to (11).

$$\begin{aligned} \text{Var}[y(t)] &= \text{Var}[A] + \left(-\frac{2}{\sqrt{\pi}}t^{\frac{1}{2}} - 4t + \frac{32}{3\pi\sqrt{\pi}}t^{\frac{3}{2}}\right)^2 \text{Var}[A^2] + \\ &\quad \left(2t + \frac{16}{3\sqrt{\pi}}\left(1 - \frac{1}{\pi}\right)t^{\frac{3}{2}}\right)^2 \text{Var}[A^4] + \left(\frac{16}{3\sqrt{\pi}}t^{\frac{3}{2}}\right)^2 \text{Var}[A^6] + \dots \end{aligned}$$

$$\begin{aligned}
 \text{Var}[y(t)] &= \frac{1}{\lambda^2} + \left(-\frac{2}{\sqrt{\pi}}t^{\frac{1}{2}} - 4t + \frac{32}{3\pi\sqrt{\pi}}t^{\frac{3}{2}}\right)^2 \frac{20}{\lambda^4} + \left(2t + \frac{16}{3\sqrt{\pi}}\left(1 - \frac{1}{\pi}\right)t^{\frac{3}{2}}\right)^2 \frac{39744}{\lambda^8} + \\
 &\quad \left(\frac{16}{3\sqrt{\pi}}t^{\frac{3}{2}}\right)^2 \frac{478483200}{\lambda^{12}} + \dots \\
 \text{Var}[y(t)] &= \frac{1}{5^2} + \left(-\frac{2}{\sqrt{\pi}}t^{\frac{1}{2}} - 4t + \frac{32}{3\pi\sqrt{\pi}}t^{\frac{3}{2}}\right)^2 \frac{20}{5^4} + \left(2t + \frac{16}{3\sqrt{\pi}}\left(1 - \frac{1}{\pi}\right)t^{\frac{3}{2}}\right)^2 \frac{39744}{5^8} + \\
 &\quad \left(\frac{16}{3\sqrt{\pi}}t^{\frac{3}{2}}\right)^2 \frac{478483200}{5^{12}} + \dots \tag{11}
 \end{aligned}$$

The graph of the variance value of the random variable  $A$  is obtained as Figure 5. for the special value  $A \sim \text{exponential}(\lambda = 5)$ .

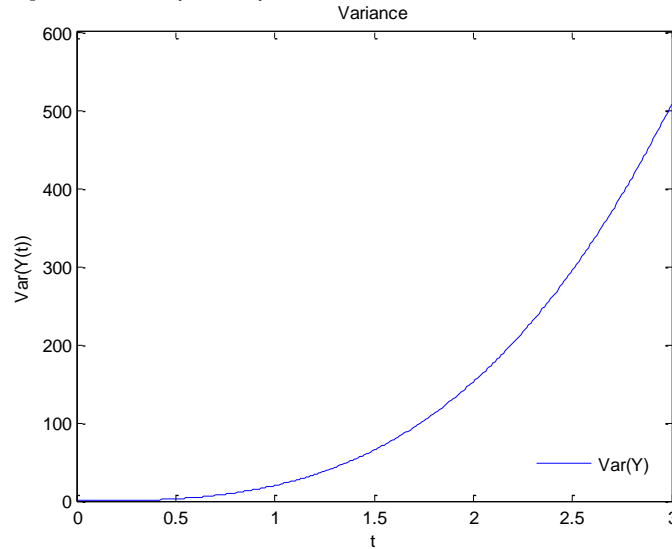


Figure 5. The solution behavior of the variance of equation (9) for the special value  $A \sim \text{exponential}(\lambda = 5)$ .

As in the previous application, the solution behavior of the approximately 98% confidence interval of the expected value of the random variable with exponential distribution for  $K = 3$  is Figure 6.

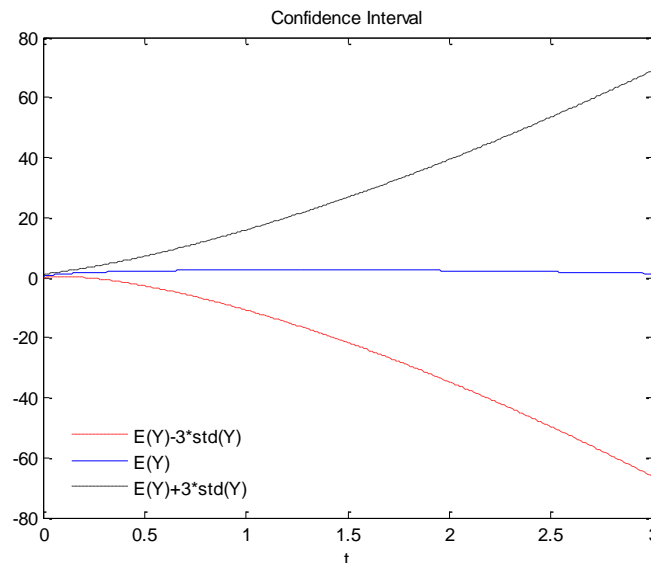


Figure 6. The solution behavior of the confidence interval of equation (8) for the special value  $A \sim \text{exponential}(\lambda = 5)$ .

Table 2. Table for the expectation value, variance and 98% confidence interval

$t$	$E(y(t))$	$Var(y(t))$	$E(y(t)) - 3std(y)$	$E(y(t)) + 3std(y)$
0.0	1/5	1/25	-0.4000000000	0.8000000000
0.1	1.193926410	0.08039504659	0.3433058014	2.044547018
0.2	1.557886374	0.2576957253	0.03497418607	3.080798560
0.3	1.808657407	0.6807656305	-0.6665982720	4.283913085
0.4	1.997120377	1.459717504	-1.627442705	5.621683459
0.5	2.143449284	2.704849520	-2.790478734	7.077377300
0.6	2.258219152	4.526504006	-4.124455526	8.640893829
0.7	2.347845506	7.035027282	-5.609242245	10.30493326
0.8	2.416594627	10.34075571	-7.230518996	12.06370825
0.9	2.467489460	14.55401064	-8.977426262	13.91240518
1.0	2.502774240	19.78509694	-10.84135827	15.84690675

The expected value, variance and confidence interval for  $K = 3$  values are given in Table 2. It can be seen that the results for  $t \in [0,1]$  values are obtained similarly to the results for the case with exponential parameters.

**Example 3. 3.**

As a third example,

$$\frac{d^{3/2}}{dt^{3/2}} = -y^2 + 3y + 1 \tag{12}$$

let's take the equation. Let the initial conditions of equation (12) be  $y(0) = A$  and  $y'(0) = B$ . Where  $A, B \sim N(\mu, \sigma^2)$ ,  $A$  and  $B$  being the random variable with normal distribution.

As in the previous example if  $L^{-\frac{3}{2}}$  is applied to both sides of the equation in equation (12), where  $L = \frac{d^{\frac{3}{2}}}{dt^{\frac{3}{2}}}$ , equation (13) is obtained.

$$\begin{aligned}
 y_{n+1} &= \frac{4}{3\sqrt{\pi}} t^{\frac{3}{2}} + 3L^{-\frac{3}{2}}(y_n) - L^{-\frac{3}{2}}(y^2) \\
 A_0 &= A^2 + 2ABt + B^2t^2, \\
 A_1 &= 2B(A + Bt) \left[ \left( \frac{4}{3\sqrt{\pi}} + \frac{4A}{\sqrt{\pi}} - \frac{4A^2}{3\sqrt{\pi}} \right) t^{\frac{3}{2}} + \left( \frac{8}{5\sqrt{\pi}} - \frac{16AB}{15\sqrt{\pi}} \right) t^{\frac{5}{2}} - \frac{32B^2}{105\sqrt{\pi}} t^{\frac{7}{2}} \right], \\
 y_1 &= \left( \frac{4}{3\sqrt{\pi}} + \frac{4A}{\sqrt{\pi}} - \frac{4A^2}{3\sqrt{\pi}} \right) t^{\frac{3}{2}} + \left( \frac{8}{5\sqrt{\pi}} - \frac{16AB}{15\sqrt{\pi}} \right) t^{\frac{5}{2}} - \frac{32B^2}{105\sqrt{\pi}} t^{\frac{7}{2}},
 \end{aligned}
 \tag{13}$$

$$y_2 = \frac{4}{3\sqrt{\pi}}t^{\frac{3}{2}} + \frac{9\sqrt{\pi}}{24}\left(\frac{4}{3\sqrt{\pi}} + \frac{4A}{\sqrt{\pi}} - \frac{4A^2}{3\sqrt{\pi}}\right)t^3 + \frac{15\sqrt{\pi}}{64}\left(\frac{8}{5\sqrt{\pi}} - \frac{16AB}{15\sqrt{\pi}}\right)t^4 - \frac{B^2}{20}t^5$$

$$- \frac{1}{4}AB\sqrt{\pi}\left(\frac{4}{3\sqrt{\pi}} + \frac{4A}{\sqrt{\pi}} - \frac{4A^2}{3\sqrt{\pi}}\right)t^3 - \frac{5B^2\sqrt{\pi}}{32}\left(\frac{4}{3\sqrt{\pi}} + \frac{4A}{\sqrt{\pi}} - \frac{4A^2}{3\sqrt{\pi}}\right)t^4$$

$$- \frac{5AB\sqrt{\pi}}{32}\left(\frac{8}{5\sqrt{\pi}} - \frac{16AB}{15\sqrt{\pi}}\right)t^4 + \frac{AB^2}{20}t^5 + \frac{AB^3}{30}t^5 - \frac{3B^4}{20}t^6,$$

As  $y(t) = y_0(t) + y_1(t) + y_2(t) \dots$  the values  $y_0(t)$ ,  $y_1(t)$ ,  $y_2(t)$  are written instead of and if edited,

$$y(t) = \frac{8}{3\sqrt{\pi}}t^{\frac{3}{2}} + \frac{8}{5\sqrt{\pi}}t^{\frac{5}{2}} + \frac{t^3}{2} + \frac{3}{8}t^4 + \left(1 + \frac{4}{\sqrt{\pi}}t^{\frac{3}{2}} + \frac{3}{2}t^3\right)A + Bt - \left(\frac{16}{15\sqrt{\pi}}t^{\frac{5}{2}} + \frac{t^4}{2} + \frac{t^3}{3}\right)AB - \left(\frac{4}{3\sqrt{\pi}}t^{\frac{3}{2}} + \frac{t^3}{2}\right)A^2 - \left(\frac{32}{105\sqrt{\pi}}t^{\frac{3}{2}} + \frac{t^5}{20} - \frac{5}{8}t^4\right)B^2 + \left(-t^3 - \frac{5}{8}t^4 + \frac{t^5}{20}\right)A^2B + \frac{9t^4}{24}A^2B^2 + \frac{t^3}{3}A^3B + \frac{t^5}{30}AB^3 - \frac{3t^6}{20}B^4 + \dots \tag{14}$$

solution is obtained. The parameters of the normally distributed random variable  $X$  are  $A, B \sim N(\mu, \sigma^2)$ . The moments of the random variable  $X$  with normal distribution are found with the help of the moment generating function.

$$E(y(t)) = \frac{8}{3\sqrt{\pi}}t^{\frac{3}{2}} + \frac{8}{5\sqrt{\pi}}t^{\frac{5}{2}} + \frac{t^3}{2} + \frac{3}{8}t^4 + \left(1 + \frac{4}{\sqrt{\pi}}t^{\frac{3}{2}} + \frac{3}{2}t^3\right)E[A] + E[B]t$$

$$- \left(\frac{16}{15\sqrt{\pi}}t^{\frac{5}{2}} + \frac{t^4}{2} + \frac{t^3}{3}\right)E[A]E[B] - \left(\frac{4}{3\sqrt{\pi}}t^{\frac{3}{2}} + \frac{t^3}{2}\right)E[A^2]$$

$$- \left(\frac{32}{105\sqrt{\pi}}t^{\frac{3}{2}} + \frac{t^5}{20} - \frac{5}{8}t^4\right)E[B^2] + \left(-t^3 - \frac{5}{8}t^4 + \frac{t^5}{20}\right)E[A^2]E[B]$$

$$+ \frac{9t^4}{24}E[A^2]E[B^2] + \frac{t^3}{3}E[A^3]E[B] + \frac{t^5}{30}E[A]E[B^3] - \frac{3t^6}{20}E[B^4] + \dots$$

$$E(y(t)) = \frac{8}{3\sqrt{\pi}}t^{\frac{3}{2}} + \frac{8}{5\sqrt{\pi}}t^{\frac{5}{2}} + \frac{t^3}{2} + \frac{3}{8}t^4 + \mu + \mu t - \left(\frac{16}{15\sqrt{\pi}}t^{\frac{5}{2}} + \frac{t^4}{2} + \frac{t^3}{3}\right)\mu^2$$

$$- \left(\frac{4}{3\sqrt{\pi}}t^{\frac{3}{2}} + \frac{t^3}{2}\right)(\sigma^2 + \mu^2) - \left(\frac{32}{105\sqrt{\pi}}t^{\frac{3}{2}} + \frac{t^5}{20} - \frac{5}{8}t^4\right)(\sigma^2 + \mu^2)$$

$$+ \left(-t^3 - \frac{5}{8}t^4 + \frac{t^5}{20}\right)(\sigma^2 + \mu^2)\mu + \frac{9t^4}{24}(\sigma^2 + \mu^2)^2 + \frac{t^3}{3}(3\sigma^2\mu + \mu^3)\mu$$

$$+ \frac{t^5}{30}\mu(3\sigma^2\mu + \mu^3) - \frac{3t^6}{20}(3\sigma^4 + 6\sigma^2\mu^2 + \mu^4) + \dots$$

$$E(y(t)) = \frac{8}{3\sqrt{\pi}}t^{\frac{3}{2}} + \frac{8}{5\sqrt{\pi}}t^{\frac{5}{2}} + \frac{t^3}{2} + \frac{3}{8}t^4 + 2 + 2t - 4\left(\frac{16}{15\sqrt{\pi}}t^{\frac{5}{2}} + \frac{t^4}{2} + \frac{t^3}{3}\right)$$

$$- 5\left(\frac{4}{3\sqrt{\pi}}t^{\frac{3}{2}} + \frac{t^3}{2}\right) - 5\left(\frac{32}{105\sqrt{\pi}}t^{\frac{3}{2}} + \frac{t^5}{20} - \frac{5}{8}t^4\right)$$

$$+ 10\left(-t^3 - \frac{5}{8}t^4 + \frac{t^5}{20}\right) + \frac{225t^4}{24} + \frac{28t^3}{3} + \frac{14t^5}{15} - \frac{129t^6}{20} + \dots$$

If the expected value is plotted with MATLAB (2013a) for the given parameter values, the graph in Figure 7. is obtained.

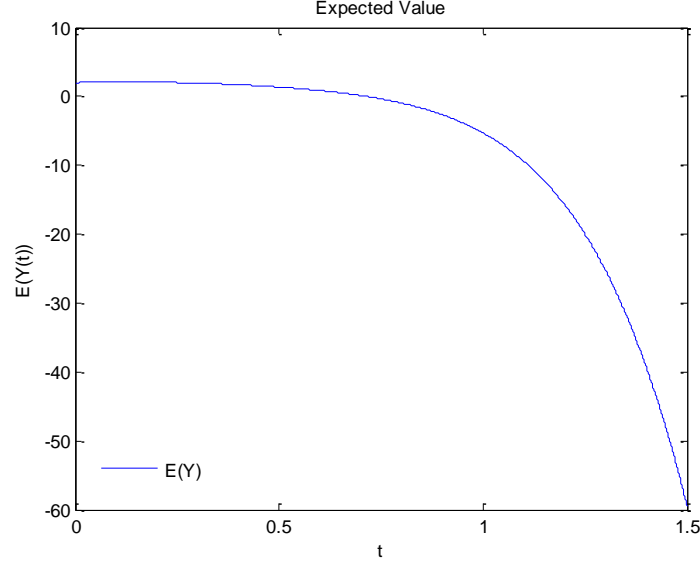


Figure 7. The solution behavior of the expected value of equation (14) for the special value  $A, B \sim N(\mu = 2, \sigma^2 = 1)$ .

$$\begin{aligned}
 Var(y(t)) &= \left(1 + \frac{4}{\sqrt{\pi}}t^{\frac{3}{2}} + \frac{3}{2}t^3\right)^2 Var[A] + Var[B]t^2 \\
 &+ \left(\frac{16}{15\sqrt{\pi}}t^{\frac{5}{2}} + \frac{t^4}{2} + \frac{t^3}{3}\right)^2 Var[A]Var[B] - \left(\frac{4}{3\sqrt{\pi}}t^{\frac{3}{2}} + \frac{t^3}{2}\right)^2 Var[A^2] \\
 &+ \left(\frac{32}{105\sqrt{\pi}}t^{\frac{3}{2}} + \frac{t^5}{20} - \frac{5}{8}t^4\right)^2 Var[B^2] \\
 &+ \left(-t^3 - \frac{5}{8}t^4 + \frac{t^5}{20}\right)^2 Var[A^2]Var[B] + \frac{81t^8}{576}Var[A^2]Var[B^2] \\
 &+ \frac{t^6}{9}Var[A^3]Var[B] + \frac{t^{10}}{900}Var[A]Var[B^3] + \frac{9t^{12}}{400}Var[B^4] + \dots \\
 Var(y(t)) &= \left(1 + \frac{4}{\sqrt{\pi}}t^{\frac{3}{2}} + \frac{3}{2}t^3\right)^2 \sigma^2 + \sigma^2 t^2 + \left(\frac{16}{15\sqrt{\pi}}t^{\frac{5}{2}} + \frac{t^4}{2} + \frac{t^3}{3}\right)^2 \sigma^4 \\
 &- \left(\frac{4}{3\sqrt{\pi}}t^{\frac{3}{2}} + \frac{t^3}{2}\right)^2 (2\sigma^4 + 4\sigma^2\mu^2) \\
 &+ \left(\frac{32}{105\sqrt{\pi}}t^{\frac{3}{2}} + \frac{t^5}{20} - \frac{5}{8}t^4\right)^2 (2\sigma^4 + 4\sigma^2\mu^2) \\
 &+ \left(-t^3 - \frac{5}{8}t^4 + \frac{t^5}{20}\right)^2 (2\sigma^4 + 4\sigma^2\mu^2)\sigma^2 + \frac{81t^8}{576}(2\sigma^4 + 4\sigma^2\mu^2)^2 \\
 &+ \frac{t^6}{9}(15\sigma^6 + 36\sigma^4\mu^2 + 9\sigma^2\mu^4)\sigma^2 \\
 &+ \frac{t^{10}}{900}\sigma^2(15\sigma^6 + 36\sigma^4\mu^2 + 9\sigma^2\mu^4) \\
 &+ \frac{9t^{12}}{400}[(105\sigma^8 + 420\sigma^6\mu^2 + 210\sigma^4\mu^4 + 28\sigma^2\mu^6 + \mu^8) \\
 &- (3\sigma^4 + 6\sigma^2\mu^2 + \mu^4)^2] + \dots
 \end{aligned}$$

$$\begin{aligned} \text{Var}(y(t)) = & \left(1 + \frac{4}{\sqrt{\pi}}t^{\frac{3}{2}} + \frac{3}{2}t^3\right)^2 + t^2 + \left(\frac{16}{15\sqrt{\pi}}t^{\frac{5}{2}} + \frac{t^4}{2} + \frac{t^3}{3}\right)^2 \\ & - 16\left(\frac{4}{3\sqrt{\pi}}t^{\frac{3}{2}} + \frac{t^3}{2}\right)^2 + 16\left(\frac{32}{105\sqrt{\pi}}t^{\frac{3}{2}} + \frac{t^5}{20} - \frac{5}{8}t^4\right)^2 \\ & + 18\left(-t^3 - \frac{5}{8}t^4 + \frac{t^5}{20}\right)^2 + \frac{729t^8}{16} + \frac{101t^6}{3} + \frac{101t^{10}}{300} + \frac{3006t^{12}}{25} + \dots \end{aligned}$$

The graph of the variance value of the random variable  $A$  is obtained as Figure 8. for the special value  $A, B \sim N(\mu = 2, \sigma^2 = 1)$ .

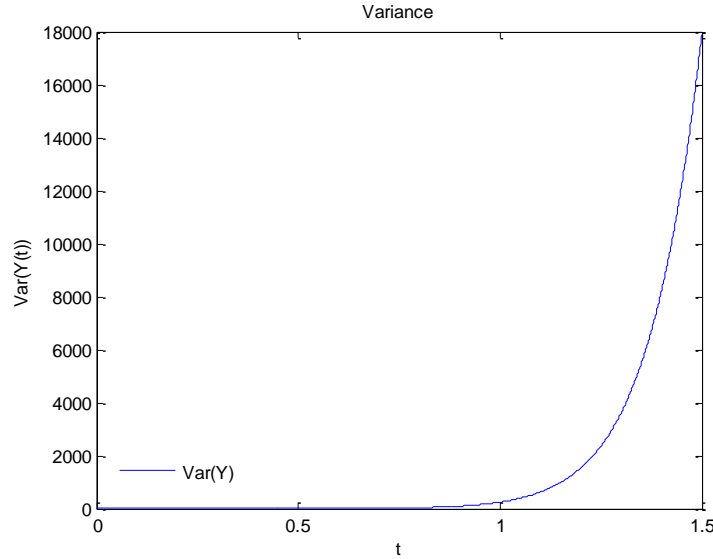


Figure 8. The solution behavior of the variance of equation (14) for the special value  $A, B \sim N(\mu = 2, \sigma^2 = 1)$ .

The solution behavior of the 99% confidence interval of the expected value of the random variable with normal distribution:

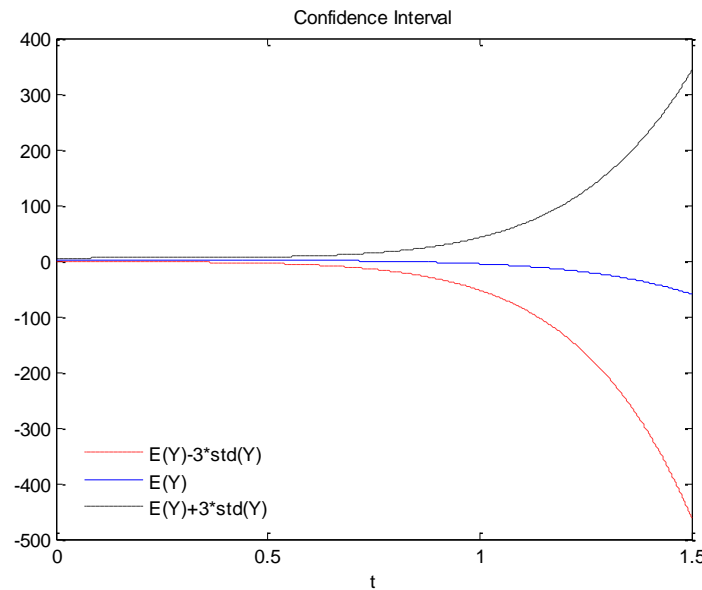


Figure 9. The solution behavior of the confidence interval of equation (14) for the special value  $A, B \sim N(\mu = 2, \sigma^2 = 1)$ .

Table 3. Table for the expectation value, variance and 99% confidence interval.

$t$	$E(y(t))$	$Var(y(t))$	$E(y(t)) - 3std(y)$	$E(y(t)) + 3std(y)$
0.0	2	1	-1.	5.
0.1	2.093158601	1.152122430	-1.126950383	5.313267585
0.2	2.069706384	1.439374512	-1.529511671	5.668924437
0.3	1.941382076	1.837539054	-2.125293652	6.008057803
0.4	1.707439933	2.453023722	-2.991199606	6.406079469
0.5	1.357458313	3.715405887	-4.425158132	7.140074757
0.6	0.8585391406	6.878988042	-7.009808358	8.726886638
0.7	0.1365155453	15.28764316	-11.59330919	11.86634028
0.8	-0.9478719200	37.66267347	-19.35884856	17.46310474
0.9	-2.627591560	96.43865557	-32.08854708	26.83336396
1.0	-5.262648018	247.6525278	-52.47358690	41.94829086

The expected value, variance and confidence interval for  $K = 3$  values are given in Table 3. It can be seen that the results for  $t \in [0,1]$  values are obtained similarly to the results for the case with Normal parameters.

#### 4. Conclusions

In this study, the solutions of randomly selected ordinary differential equations were found with the help of normal and exponential distributions using the Adomian Decomposition Method. The initial conditions or coefficients of random ordinary differential equations are chosen from the normal and exponential distribution, and the expected value, variance and confidence intervals, which are probability properties for the analysis of random effects, are obtained.

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