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Investigation of Secondary School Students' Processes of Constructing Area and Volume Relations of Rectangular Prisms

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Abstract. The purpose of this study is to examine the knowledge construction processes of 5th grade students, who do not have any knowledge about the surface area and volume of the rectangular prism, within the framework of the observable epistemic actions of the RBC (Recognizing, Building-with, and Constructing) abstraction model. This study was carried out according to the case study pattern, which is one of the qualitative research types. The study was carried out in the 2021-2022 academic year. For the research, six students were selected by the maximum diversity sampling method. Three homogeneous groups were formed and semi-structured interviews were conducted. An activity paper consisting of eight problems was used to obtain data. The obtained data were evaluated in the context of observable epistemic actions of RBC. Only a student with a low level of mathematics achievement couldn't construct knowledge of the surface area of the prism. All students have found the number of unit cubes that can fit inside the rectangular prism. However, three students with a high level of success were able to construct and use volume formulas. In the research, it has been seen that recognition and building-with actions are easier to perform than construction action. As the level of mathematics achievement decreased, the speed and success of abstraction decreased.

Keywords. Surface area, volume, abstraction, RBC+C.

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In the information society, which has a dynamic structure, the accumulation of knowledge is increasing every day and becoming complex. For this reason, education systems focus on individuals who can access information and use the obtained information in solving any problem, instead of raising individuals who memorize everything. In this context, the Ministry of National Education [MoNE] made a radical change and adopted the constructivist education approach (İlhan & Aslaner, 2018). The constructivist approach focuses on how knowledge is formed in the process of learning, which is a personal phenomenon (Delil & Güleş, 2007). The student should be actively involved in the learning process. He can learn by doing, experiencing, thinking about the concept and discussing his ideas about the concept with others (Baykul, 2014). In this process, it is aimed to abstract the operational and conceptual information by using together.

Mathematics, based on some characteristics of concepts and objects, provides the possibility to classify, establish certain relationships and make generalizations. Mathematics is an abstract subject and the abstraction of information requires a process. Dienes (1963) abstraction, classification of common aspects in different situations (cited in Altaylı Özgül & Kaplan); Hershkowitz et al. (2001) defines it as "the activity of vertically rearranging previously acquired mathematical knowledge to form a new mathematical structure". There are two basic approaches to abstraction. One of these approaches is the cognitive approach and the other is the socio-cultural approach. RBC is a socio-cultural theory that approaches abstraction. According to the socio-cultural approach, personal characteristics, use of tools, social interaction and environmental conditions are important in the abstraction process (Altun & Yılmaz, 2008; Hershkowiz, et al., 2001; Yeşildere & Türnüklü, 2008). Abstraction is related to the individual's past learning, the socio-cultural and physical environment in which learning activities are performed, and the structures obtained from the results of previous learning activities (Hershkowitz et al., 2001; Schwarz et al., 2009). Therefore, activities should be organized by considering the contextual factors affecting the abstraction process. Considering all these contexts, Hershkowitz et al. (2001) suggested that abstraction includes some observable actions. These actions are recognizing, building-with and constructing. Recognizing, recall of familiar structures (Bikner-Ashbash, 2004); building-with, the use of known knowledge in solving the problem (Tsamir & Dreyfus, 2002); constructing is the vertical restructuring of familiar constructions and the construct of new constructions (Bikner-Ahsbahs 2004). Constructing, includes recognizing and building-with and building-with includes recognizing (Hershkowitz, et al., 2001). The three epistemic actions are intertwined. This shows that actions do not always occur sequentially, but can occur simultaneously

(Dreyfus, 2007; Hershkowitz et al., 2001; Tsamir & Dreyfus, 2002). Based on these actions, the RBC theory has been developed. According to the RBC theory, abstraction occurs in three stages (Hershkowitz et al. 2001).

a) The need for a new construction.

b) The constructions existing in the mind are used for the construction of a new abstract entity in a process in which recognition and building-with are dialectically intertwined.

c) Consolidation of the new abstract entity for easy recognizing and building-with in subsequent abstractions.

Mathematics education consists of many interconnected subjects. In any matter, how abstraction processes take place can be demonstrated through epistemic actions of the RBC model. One of these issues is the surface area and volume of the rectangular prism, which is located in the geometry and measurement learning area. According to Bal (2012), geometry allows students to start their mental activities, produce various ideas, carry out problem-solving processes, compare the properties of objects, make generalizations and abstractions. However, studies on this issue show that students cannot perceive the properties of prisms and have misconceptions about measuring the surface area and volume of prisms (Avgören, 2011; Ben-Haim, Lappan, & Houang, 1985; Dağlı, 2010; Ergin, 2014; Gökdal, 2004; Okuyucu, 2019; Okuyucu & Kurtuluş, 2019; Olkun, 2003; Tan Şişman & Aksu, 2009; Voulgaris & Evangelidou, 2004). However, researches show that students try to reach the result without making sense of the concepts of area and volume, by memorizing the formula and only through procedural knowledge (Aydın Karaca, 2014; Çavuş Erdem & Gürbüz, 2018; Gürefe, 2018; Olkun, Çelebi, Fidan, Engin, & Gökgün, 2014; Tan Şişman & Aksu, 2009). One of the reasons leading to this situation is that focusing on the result rather than the process in mathematics teaching leaves the student out of the process (Altaylı Özgül, 2018). According to Okuyucu & Kurtuluş (2019, p. 1027), students find it difficult to make a connection between the two concepts. In the mathematics curriculum, the separation of these two concepts and their processing at different grade levels can be seen as another factor that prevents students from establishing the connection between the two. Considering all these, in this research, it is desired to focus on the formation process of the surface area and volume information of the rectangular prism, and to reveal the connection between them. In this research, a problem solving activity prepared on the surface area and volume of prisms, which is a geometry issue that students have difficulty with, will be presented. Throughout the activity process, the way students construct knowledge will be examined according to the RBC abstraction model. Difficulties and deficiencies encountered in constructing knowledge will be tried to be determined. In this context, it is aimed to examine how the secondary school 5th grade students, who have not seen the surface area and volume of the rectangular prism before, process the knowledge about these issues. It was found worth investigating how the students formed knowledge during the process of constructing the surface area and volume formulas of the rectangular prism. In the literature, there are studies examining the surface area and volume of the rectangular prism. These studies generally examine students' misconceptions, teachers' content knowledge and the effect of any teaching method or theory on teaching the issue. In the issue of surface area and volume of prisms, students' knowledge construction processes have not been examined within the theoretical framework of RBC. In this respect, it is thought that the research will contribute to the literature.

Method

In this section, the type of research, select of participants, data collection tools, data collection process and data analysis have been presented.

Research Model

The design of this research was determined as a case study, which is one of the qualitative research methods. Qualitative research offers the researcher the opportunity to look at the situation through the eyes of the participants (Coşkun, 2019). In qualitative research, it is aimed to obtain in-depth knowledge on the subject from a small number of participants and to evaluate the the data within certain criteria. One of these methods is case study. In case studies, the system, individual, group or phenomenon is described and analyzed in depth within certain limits (Merriam, 2013).

Study Group

Participants of the study were selected through maximum variation sampling, which is one of the purposive sampling methods. In order to observe the construction of a new knowledge, students must be seeing that subject for the first time. For this reason, fifth grade students who have not yet seen the attainment of the surface area and volume of the prism were included in the study. The study was carried out in the fall semester of the 2021-2022 academic year in a state secondary school affiliated to the Ministry of National Education in the Odunpazarı district of Eskişehir. The fact that the researcher works in this school has been effective in choosing the school. The study was carried out with six students at the fifth grade level. A code between S1 and S6 was assigned to the students who participated in the application. The researcher is coded with a "T". While coding, it was started from the student with the highest mathematics achievement level. In the selection of students, first mathematics written grade and 4th grade mathematics grade averages were taken into consideration. Accordingly, two students were selected from 100-85, two students from 84-70 and two students from 69-45 grades.

Data Collection Tools

In the study, an activity sheet consisting of eight problems was given to the students as a data collection tool. The problems were prepared within the framework of the attainments in mathematics curriculum and the explanations of these attainments. Two of these problems are about calculating the surface area of the rectangular prism, one problem is determining the number of cubes in the structures formed from unit cubes, two problems are calculating the amount of unit cubes that can fit inside the rectangular prism, two problems are using the volume formula of the rectangular prism, and the last problem is using the area and volume information. The reason for the problem-oriented preparation of the activity is that the observable epistemic actions of the RBC model, which are recognizing, building-with and constructing, can be observed more easily in problem solving activities. It is thought that the intertwined structure of epistemic actions, students' mathematics achievement and learning speed will be effective in this process.

Process

In this study, semi-structured interview technique, which is one of the interview types, was used. Because this technique gives the interviewee the opportunity to express himself and the analysis of the data is easy (Büyüköztürk, et al., 2018). Semi-structured interview technique consists of open-ended questions related to the main topic. The interviewee is directed to conduct an in-depth examination. In this way, it is ensured that he answers open-ended questions freely (Memnun, 2011). The interviews were conducted in two parts, one hour apart, during the day (See Table 1.). All interviews were video recorded.

Table 1.

Interview Duration

Group	**Group 1**		**Group 2**		**Group 3**	
Chapter	Chapter 1	Chapter 2	Chapter 1	Chapter 2	Chapter 1	Chapter 2
Duration (min.)	33	23	36	32	40	37

The students were asked to write the answers to the questions on the problem solving paper provided and hand them over to the researcher after the interview. In the interviews, guiding questions were asked according to the answers and reactions of the students. These questions were used to enable students to fully reveal their thoughts and to clear possible blockages in the process. The videos obtained as a result of the interviews were analyzed.

Data Analysis

The video recordings of the semi-structured interviews with each group were transcribed and written down. The data obtained were subjected to descriptive analysis within the scope of the epistemic actions of the RBC theory. In this context, the data obtained from the semi-structured interviews, observations and answer sheets of the students were evaluated according to the predetermined themes. During the analysis, student dialogues were directly quoted and images from student solutions were included.

While analyzing the data, some key statements were determined in order to observe and reveal the epistemic actions of the RBC theory. Key phrases were used to determine which actions were taken in the discourses and behaviours of the students during the solution process. Key phrases and the epistemic actions they refer to are given in Table 2. Thus, the interpretation of the findings has become easy. Some discourses do not always require observing the same epistemic action. This is due to the intertwined nature of actions.

Table 2.

Key Phrases Used to Identify Epistemic Actions (Altaylı Özgül, 2018)

Recognizing	Building-with	Construction
Remembering Exemplification Expressing the property of a geometric shape or object	Problem solving Make assumptions Defending a proposal Reasoning Explaining a situation Building relationships	Building relationships Building new structure Mathematical language development Reasoning

There are a number of ways to ensure credibility (internal validity) in a qualitative research. According to Lincoln and Guba (1985), these ways are the long-term interaction provided throughout the research, deep focused data collection, diversification, expert review and participant confirmation (Cited by Yıldırım & Şimşek, 2016). The problems prepared in this study were examined by field education experts. In addition, the credibility of the research was ensured by using different methods such as observation, interview and document review. Participants with different mathematical achievements were selected for the research and the data obtained were described in detail. According to Yıldırım & Şimşek (2016), purposive sample selection and descriptive direct citation of the data increase the level of transferability (external validity) of the research. According to Yıldırım & Şimşek (2016), purposive sample selection and descriptive direct citation of the data increase the level of transferability (external validity) of the research.

In order to ensure the consistency (internal reliability) of a research, the research should be evaluated by a different expert than the researcher (Yıldırım & Şimşek, 2016). For this reason, all the data obtained from the research were examined by the researcher and a field expert. The results of both reviews were compared and it was seen that the comments made were consistent. Finally, a qualitative research must be verifiable (external reliability). According to Guba & Lincoln (1989), verifiability means impartiality. This is possible by minimizing the subjective judgments of the researcher (Cited by Aydın Çınar, 2019). For this reason, the results of the researcher should be verified with the data he has obtained and researcher should provide logical explanations (Yıldırım & Şimşek, 2016). In this case, the chain of evidence formed increases the reliability of the research. In this research, the results of the researcher were constantly compared and confirmed with the findings obtained from the video transcripts and student papers.

Results

In this section, the results obtained from the application are given. Results were analyzed under a different heading for each question.

Results from the First Problem

In this problem, the surface area of an open rectangular prism is asked. Students are expected to perform the actions of *recognizing* and *bulding-with* the properties of the prism and the area of the rectangle. The answers given by the three groups and the results of the epistemic actions they have taken are given below.

19T: Since you are asking the area of the rectangle, how is it calculated?

20S1: By multiplying one side with the other. I mean the long side and the short side (Feature speaking - Recognition).

...

1T: What geometric shape do the black lines form when you cut this cardboard, S3?

2S3: Rectangle (Recognition).

3T: In this question, "how much cardboard do you need?" he asks, what does he mean?

4S4: How many cm will the rectangle consist of.

5T: cm or cm²?

6S4: Cm².

7T: If cm², what are we trying to calculate?

8S4: Field (Recognition).

12S3: 12S3: Multiply the short side and the long side of the rectangle (Recognition).

...

2S5: He was going to make a parcel out of rectangular cardboard. He asks us how many cm² of cardboard we need?

3T: OK (the student is considered to have understood the question).

4S5: There is 20 cm there. 20 cm in the middle? (It shows on the figure). 20 cm next to it. Opposite is 20 cm. There are 35 cm. It is also 35 cm above (Recognition).

In this problem, all groups realized that the given shape consists of rectangles and that the areas of these rectangles must be found (20S1, 2S3, 8S4, 2S5). The students of the first two groups recognized the area relation of the rectangle. The third group was reminded by the researcher.

5T: What do we call this object, which is formed as a result of folding, mathematically?

6S1: We say rectangular prism (Recognition - Recognition).

...

19T: What geometric object do we get when we fold this cardboard and form a parcel?

20S3: Is it a cube?

21S4: Cube (Incorrect recognition).

22T: If all the lengths were the same, we would have obtained the cube.

23S4: Square (Incorrect recognition).

Only one student recognized the information that the given figure is the expansion of a rectangular prism (6S1). While the students in the second group gave the answer of square or cube (20S3, 21S4), the students in the third group could not give any answer.

21T: How to find the area of the rectangles here?

22S1: One side of the rectangle I showed is 35 cm, and the other side is 20 cm. If we multiply 35 by 20, we can find the area of that rectangle (Building-with).

23T: How much?

24S2: 700.

25S1: The short side of the rectangle next to it is 8 cm. Since its long side is equal to the other side, it becomes 20 cm (Specification - Recognition).

26T: What is its area in cm²?

27S1: 20 × 8 becomes 160.

36T: Now that we have found the area of all the rectangles, can we figure out how much cardboard is needed?

37S1: If we add all of these, we can find out how much cardboard is needed. It becomes 2280 cm² (Reasoning – Building-with).

...

14S3: The area of the rectangle you showed is 700 cm² (Problem solving – Building-with).

15T: What would be the area of the small rectangle I showed next to that? How can we find it?

16S3: Teacher, the short side of the rectangle with an area of 700 cm² is 20 cm. The long side of the rectangle you show is also 20 cm. If we multiply 20 by 8, 160 cm² (Feature speaking - Recognition) (Reasoning - Problem solving – Building-with).

38T: What would be the area of this carton?

43S3: I found 2280 (Problem solving - Building-with).

44S4: I found 2120.

...

33S5: Sorry, one side is 35 cm. We're going to multiply 35 by eight. Subtract 280. The upper side is also 280 (Reasoning - Using).

34T: What would be the area of the whole carton?

35S5: We will add them all, 2270 (wrong account).

36T: What is the unit of 2270? (S5 thought but did not answer). How much do you think it will be S6?

37S6: 2280.

Students recognized the properties of the prism, the quality to be measured and the area relation of the rectangle. Then, they used this information to solve the problem. None of the groups had any problems while calculating the area (22S1, 24S2, 14S3, 16S3, 33S5). As seen in Figure 1, the students summed up the results after calculating the areas of the rectangles. But, they sometimes made a mistake (37S1, 43S3, 44S4, 35S5, 37S6).

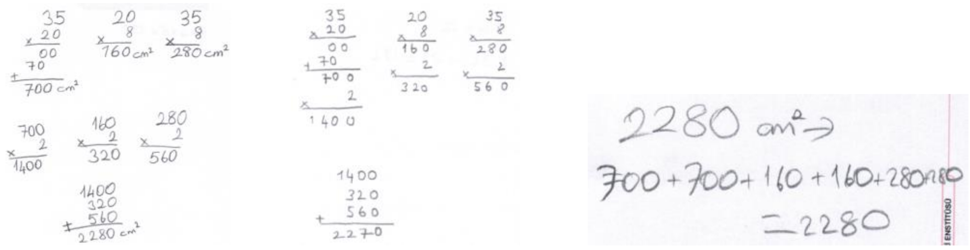

Figure 1. The Operations of the Students for the Solution of the First Problem.

In this problem, the students reached the aim by performing the actions of recognition and building-with. However, S1 developed a mathematical language by expressing that the required amount of cardboard is the sum of all the surface areas of the prism (39S1). This statement shows that the student has made progress in constructing knowledge.

Results from the Second Problem

In this problem, a question prepared for calculating the surface area of the container in the form of a rectangular prism, which is closed, was directed to the students. In addition to the recognition actions in the solution of the first problem, students were expected to construct surface area knowledge. The answers given by the three groups and the findings of the epistemic actions they have taken are given below.

1S1: If we calculate and add up the areas of the surfaces again, we can arrive at the result (Recognition – Recognizing).

2S2: I also think like S1.

3T: Calculate. Tell me the answers too. For example, what would be the area of the face I show?

4S1: 4 times 3 is 12. There are two of these 24 (Reflecting a process – Building-with).

5S2: The other is 2 times 3 out of 6. Above is 4 times 2 (Reflecting a process - Building-with).

6T: What about the whole field?

7S1: 52 m² (Problem solving - Building-with).

8S2: 52 (Solving the problem - Building-with).

9T: What have we found here?

10S1: The sum of the areas of the surfaces of the container.

...

2S4: We will add up the areas (Recognition – Recognition).

10S4: Multiply four by three, 12m² (Reflecting a process - Using).

13S3: It becomes 6 m² (Reflecting a process - Using).

14T: How long will the lower and upper face be?

15S4: We multiply four by two (Reflecting a process - Using).

19S4: Add 12 and 12 to get 24; rear and front 6-6 becomes 12 m; Add 8 to 8 to get 16. After that, the sum of 24 and 12 is 36. If we add 16, we get 52 (Problem Solving - Using).

20T: What attribute of the container did we find?

21S3: His area.

22S4: His area.

23A: S3, how much did you find the result?

24S3: I found 52 (Problem solving - Using).

...

7S5: We will multiply three by two, six. We are going to multiply four by three 16 (Problem solving - Using).

8S6: Makes 12 (Social learning).

11S5: Multiply 6 by 2, 12 by 2, and 8 by 2. 12, 24, this is 16. We will add up all of these (Problem solving - Using).

12T: Add up and tell me the result.

13S5: 52.

16S5: This is the same as the first question we did (Recognition - Recognition).

17T: What is the similarity with the first question?

18S5: In both, we found the sides and multiplied and found the area (Defend a proposal - Do not use).

19T: If we open the container, can we get the cardboard in the first question?

20S5: Yes. It would be exactly the same shape.

21T: How many did you find S6?

22S6: I found 56.

All students associated this problem with the previous one (1S1, 2S2, 2S4, 16S5, 20S5). They recognized the properties of the rectangular prism and the area knowledge of the rectangle. They quickly calculated the areas of the rectangles and solved the problem (4S1, 5S2, 10S4, 13S3, 15S4, 7S5, 8S6). As seen in Figure 2, all students added the measurements of the areas obtained and found the area to be painted (7S1, 8S2, 19S4, 24S3). However, only the first group used the area unit. S1 continued to defend the knowledge he obtained by stating that he found the area of all edges of the container, as in the previous problem (10S1). This situation supports the thoughts that the student has reached the constructing step. S3 and S4 stated that they found the area of the container (21S3, 22S4). It can be thought that the students construct the surface area calculation knowledge. S5 and S6 do not have any statements to construct the surface area knowledge of the prism.

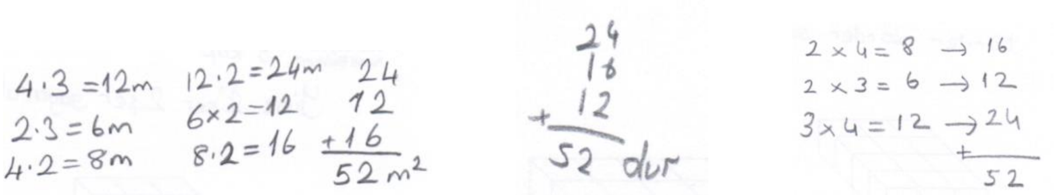

Figure 2. The Operations Performed by the Groups to Solve the Second Problem.

Results from the Third Problem

In this problem, constructions in the form of rectangular prisms construct from unit cubes are seen. The students were asked how to find the number of unit cubes contained in these constructions. It is aimed to discover the knowledge that the number of unit cubes that construct an object is the volume of that object. The answers given by the three groups and the findings of the epistemic actions they have taken are given below.

4T: How did you find it?

5S2: I counted (Reasoning - Using).

6S1: I found it by counting (Reasoning - Using).

7T: How many cubes are in the second figure? What do you think?

8S2: There are eight cubes.

9S1: Eight.

10S2: Counting again (Reasoning - Using).

11S1: We counted (Reasoning - Using).

12T: How did you count?

13S2: I counted twos.

14S1: Counting each cube since we know the shape of the cube. I counted by twos like S2.

15T: How else can we count?

16S1: Four by four.

...

11S3: I found it by counting again (Reasoning - Using).

12S4: Counting (Reasoning - Using).

13T: How did you count?

14S4: Four by four or one at a time.

...

13S5: I also counted four.

14S6: I multiplied four by two (Relating – Using).

In the first two constructions in this problem, the students found the number of unit cubes by counting (11S3, 12S4). They said that they counted the cubes as one, two, three and four (14S4). Only S6 used multiplicative reasoning in the second way (14S6).

19S1: A solution came to my mind. In the previous questions, we multiplied one side length by the other side length to find the area. Here, if we multiply the number of cubes on one edge by the number of cubes on the other edge, we can find out how many cubes there are (Associating – Using) (Reasoning – Creating).

...

16S5: There are 12 cubes. I got it by multiplying four by three.

In the last two constructions, students preferred multiplicative reasoning (16S5). In fact, S1 associated the number of cubes in a layer with the area calculation of the rectangle. Based on this, he multiplied the number of unit cubes arranged along two edges (19S1). S1, S4 and S5 adapted this method to the last figure. These three students first found the number of unit cubes in a layer in order to reach the number of unit cubes that make up the construction. Then he multiplied the result with the number of unit cubes along the third edge (See Figure 3.).

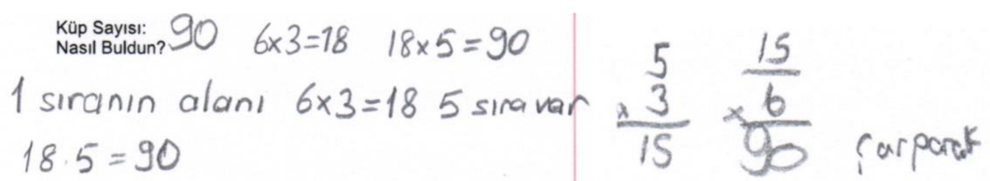

Figure 3. S1and S4's Answers for the Last Construction in the Third Problem.

All students remembered rhythmic counting and multiplicative reasoning. Recognizing and building-with actions have taken place. However, S1, S4 and S5 discovered a new way for them to find the number of unit cubes. Therefore, it is thought that three students constructed the knowledge to find the number of unit cubes that can fit inside the rectangular prism.

Results from the Fourth Problem

In this problem, a rectangular prism is given in the base layer and with a known number of unit cubes arranged along its height. Students are expected to use the formula *number of cubes = number of cubes in the base layer × height* and create the formula *volume = base area × height.*

1S1: I think like we just did. We know the height and width. If we multiply the width, length and height, we find both the area and how many unit cubes will fit inside (Recognition - Recognition).

2S2: I think so too.

3T: How many results can we find?

4S1: I found 192 (Reflecting a process – Using).

...

2S3: We will do it from width and height. Height is six units, four units are on the side, the bottom is the same as the top. (After showing six and four on the figure) Here is eight (showing the front of the object) (Reflecting a process – Using).

3T: What are we going to do then?

4S4: We will multiply.

5S3: I found 192.

...

8S5: Base 32.

9T: There are 32 cubes in the base. How many cubes would it be if we filled the entire prism?

10S5: If we count the bottom layer, there are six upwards. Multiplying six by 32 is 192 (Reflecting a process – Using).

In solving this problem, all groups tried to use the knowledge they gained from the previous problems (1S1, 2S3, 4S4). Four students found the cubic unit amount in the base layer by multiplicative reasoning. Then they thought that there would be six of these layers and multiplied the result by 6 (4S1, 5S3, 8S5, 10S5). For this reason, it is thought that the students proceed in the direction of constructing the *volume = base area × height* formula using the generalization of *cube unit number = unit cube number in the base layer × height* (See Figure 4.).

$$8 \times 4 \times 6 = 192$$

Figure 4. S1's Operations for the Solution of the Fourth Problem.

9S2: Thirty-two.

10T: How did you find it?

11S2: Multiplying four by eight. Here, I did not take six as one was given full. I took five and multiplied by 32. I found 160 (Reflecting a process – Using).

12T: Is the bottom part not included in this object?

13S2: I counted it.

14T: How many layers did you put on it?

15S2: Five floors. Hmm... then it would be 192 (Reasoning – Using).

...

11T: Do you think S6?

12S6: I counted the ones below 32. We multiply five by 32, then add 32.

S2 and S6 kept the lowest layer separate from the others. Therefore, he multiplied 32 by 5 (11S2). Then he added 32 to the result (160 br^{3}) and found the answer as 192 br^{3} (15S2, 12S6). The solution steps of S2 and S5 are shown in Figure 5.

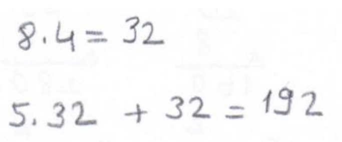

Figure 5. The Operations Performed by S5 for the Solution of the Fourth Problem.

Results from the Fifth Problem

In this problem, the students are asked how many cubes can fit inside a rectangular prism given the number of unit cubes arranged along its three sides. Students are expected to multiply the number of unit cubes arranged along the sides. Students are expected to form the generalisations *unit cube number = base area × height or unit cube number = width × length × height.*

2t: The number of unit cubes that can fit inside the prism shows which property of the object?

3S1: It can be area or volume. (He may have remembered the concept of volume from here because it is a concept in the 4th grade science lesson). But if we found the area, we would multiply the areas of their surfaces. So it can be volume (Recognition - Recognition).

4S2: We will use the same method as before. We multiply four by four. Then we multiply the height. 160 (Reflecting a process – Using).

...

1S3: Firstly, I multiplied four by four to find the cubes on the first floor and found 16. The length of this is 10 floors. Then I multiplied 16 by 10 and found 160 cubes (Reflecting a process - Using).

While solving the problem, S1, S2 and S3 established a relationship with the previous problem. As seen in Figure 6, three students found the number of unit cubes in a layer by multiplying the number of cubes arranged along two base edges. Then, they multiplied the result by the number of layers to find the number of unit cubes that could fit inside the object. It is seen that these three students constructed the aimed knowledge.

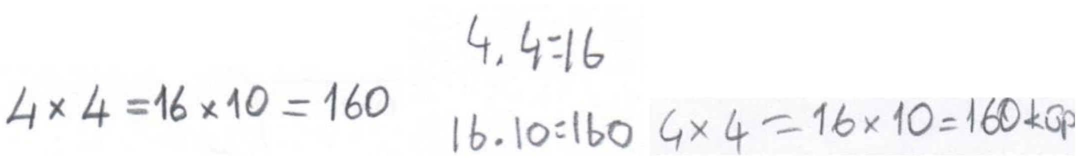

Figure 6. The Operations Performed by S1, S2 and S3 to Solve the Fifth Problem.

2S4: Everything will be reciprocal. Four cubes against four cubes. Two opposite him (he thinks a little). We will take the height as 10. (He starts over again) Four cubes there and four cubes there make 16 cubes. If we take the height as two, that's 20 cubes. If we multiply 16 by 10, we get 160 cubes. If we add them all together, we get 180 cubes.

...

1S5: There are 10 cubes upwards. At the bottom, on the left edge, there are four cubes. There are two cubes in the middle. To fill the gaps, I multiply four by 10, that's 40. I multiply 10 by two, that's 20. In the same way, I multiply four by 10 and two by 10. 40 + 40 = 80. 20 + 20 = 40. When I add it up, I get 100.

...

5S6: I do (after thinking for about two minutes). I found 92.

6T: How did you find 92?

7S6: It has four sides. I found it by multiplying.

S4, S5 and S6 determined the number of unit cubes arranged along the edges. However, he could not relate to the solution of the previous problem. Therefore, they could not give meaningful answers for the solution of the problem.

Results from the Sixth Problem

In this problem, the amount of load that a trailer with given edge lengths can take is asked. Students are asked to remember and use the knowledge they obtained from construction with unit cubes. They are expected to relate this information to a construction free of unit cubes and formulate the volume formula.

3S2: We multiply them all

4S1: Multiply the width, length and height (Recognition).

5T: How many?

6S2: We multiply three by two, six. We multiply six by eight, 48 (Reflecting on a process - Using).

...

12S3: Can we do it like this? Shall we multiply three by two and then multiply it by eight?

16S3: We can find the trailer of the truck by thinking like a rectangular prism. There are 48.

S1, S2 and S3 recognized the *unit cube number = number of unit cubes in the base layer × height* in order to find the number of cubes in construction with unit cubes. They adapted and used the knowledge they created from the previous problems (4S1, 6S2, 12S3, 16S3). They transferred the knowledge to a different structure by multiplying the width, length and height (3S2, 4S1). They rearranged the knowledge obtained from the constructions with unit cubes and adapted them to the prism given the edge lengths. The students restructured the existing knowledge in their minds and created a volume formula independent of unit cubes. It is seen in Figure 7 that S2 and S3 multiplied the width, length and height. S4 first made wrong inferences and performed wrong operation. Then, he did multiplication based on the statements of his group mate and was able to reach the result. It is thought that the student has realized the act of *building-with* but has not reached the stage of *constructing*.

Figure 7. The Operations S2 and S3 Do to Solve the Sixth Problem.

8S5: We can find the width × length × height (Reflecting on a process - Using it).

9T: How do we do it?

10S5: Width 2, length 3, sorry, length 8, height 3. We multiply them (Reflecting a process - Using).

11S6: I don't understand what we multiply.

12T: It will multiply the three lengths given here.

13S5: If we multiply 3 and 2, we get 6. If we multiply 6 and 8, we get 48 (Solving the problem - Using).

At first, S5 could not recognize the knowledge he had created in the previous process. For this reason, he could not find the amount of load the trailer would take. Then, with the guidance of the teacher, he was able to relate the previous problems to this problem. He reached the solution by multiplying the width, length and height. He found the result as 48 m^3 (12S5, 15S5, 17S5). S5 recognized and used the unit cube finding knowledge. However, since he did this with the guidance of the researcher, it cannot be said that he formed the general volume formula. S6 could not make any contribution to this question and could not reach the solution. Therefore, he could not formulate the general volume formula.

Results from the Seventh Problem

In this problem, it is requested to construct a rectangular prism with a definite volume with the help of unit cubes. It has been observed whether the epistemic actions are carried out in the process.

1S1: He was putting what he produced in a box and those boxes in another box. It's 27 boxes. When we find the area of a shape, we multiply its width and length, that is, its long side and its short side. If we multiply a number that will result in 27 by another number, we can find what we are going to design. Multiplying nine by three is 27 (confusing area with volume).

2T: How did you line up the boxes when multiplied nine by three? In a row? Side by side?

3S1: It goes like this (He shows by drawing. He draws three lines in a row and writes nine on them. He draws another line from the side and writes three on it). They do not overlap, in a single row. If it had a height, we would have found its volume. We can directly multiply nine by three to find the number of boxes it can contain.

4T: In this parcel you have designed, you have lined up the boxes nine at a time, one after the other. What would be the height if the length is nine and the width is three?

5S1: (Thinks for a bit) Then its height will be one (Notice – Recognition).

6T: Then just multiplying nine by three isn't enough. What should it be?

7S1: $9 \times 3 \times 1$ (Reasoning – Using).

...

1S3: The edge length is one unit. He wanted us to equate to 27. If its length was nine and its width was 3 units, we would have it equal to 27.

2T: You said the length is nine units and the width is three units. What about the height?

3S3: If we make two units....

4T: What is the height, the result of multiplication is 27?

5S3: One.

6T: Such a box can be designed. Is there any other alternative?

7S4: We swap nine for three.

S1 and S3 reasoned that two numbers with 27 multiplications could be the solution (1S1, 1S3). However, this reasoning is incomplete in that it contains two edges instead of three. Then they showed how they arranged the boxes by drawing models. S1 said that if he had a height in his drawing, he would have found the volume (3S1). After the researcher asked the question about height, they completed the missing information and concluded that the height was 1 br (5S1, 5S3). Two students showed that they abstracted the volume knowledge by saying "the multiplication of width, length and height is equal to volume" (See Figure 8.). S4 stated that the same result could be reached by taking the width as nine and the height as three (7S4). S4, on the other hand, stated that the same result can be achieved by taking the width as nine and the length as three (7S4). This situation is considered to be affected by S3's answer. This situation is considered to be affected by S3's answer. The deficiencies in *recognising* and *building-with* show that S4 could not perform the act of construction and therefore could not abstract the knowledge.

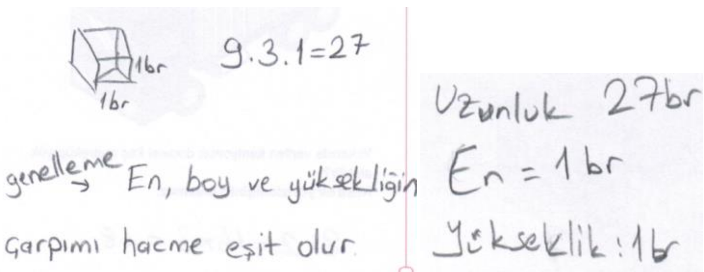

Figure 8. The Operations and Generalization of S1 and S3 to Solve the Seventh Problem.

S2, S5 and S6 could not find any idea for the solution. They could not reflect the experience they gained during the activity to this problem. Although S2 and S5 advanced in abstracting the information about finding volume in the previous problems, they could not find the edge lengths of a given volume. This situation indicates that the act of construction was not fully realised.

Results from the Eighth Problem

The students were expected to use the knowledge they had been trying to construct since the beginning of the activity in a problem in a different context. In this way, it was observed whether they could realise the act of construction. In this problem, a prism model is given whose volume and one of the segment lengths are certain. Students were expected to find other edge lengths and *surface area.*

1S2: There is a pool here. Ask how much water it takes. Only the length is given. We can divide.

2T: What do we divide?

3S2: Divide 480 by 20 (Reasoning – Using).

4T: 480 equals what?

5S1: width × length × height (Noticing - Recognising).

6T: What is 480 divided by 20?

7S2: 24.

8T: What does 24 tell us?

9S1: The multiplication of width and height (Relating - Using).

10T: If you wanted to design such a pool, what would you say about the width and height of this pool?

11S2: It can be three and eight (Solving the problem - Using).

12S1: It can be six and four (Solving the problem - Using).

...

9S3: We multiply the width and length and then multiply by the height.

10S4: Multiply the width, length and height and equal 480. We write 20 instead of width (Reasoning - Using).

12S4: We can say the length is 10. The height will be 200. But we can increase it. For example, as I said; 20 × 20 is 400. The rest will be 80.

18S3: For example, if we say the length is 3 metres. We multiply 20 by three, 60. The height is eight. If we multiply 60 by 8, we get 480.

23T: Is there another alternative?

24S4: We can choose from the divisors of 480.

25S3: If we make its height 6, we get 120 (multiplied by 20), and if we multiply it by 4, we get 480.

S1 and S3 are aware that the result of 480 is obtained from the formula *volume = width × length × height.* It is understood that they performed the act of recognition (5S1, 9S3). They divided 480 by 20 and said that the multiplication of width and height would be 24 (7S2, 9S1). Based on this, Ö1 found a width of 6 m and a height of 4 m (See Figure 9.). S2 and S3 stated that the edge lengths could be 8m and 3m (11S2, 18S3). It can be seen in Figure 9 that the students used the unit of meters for lengths, but did not use any units for area and volume while performing the operations. On the other hand, S4, S5, and S6 either could not make up their minds or could not reach a conclusion by reasoning incorrectly. As a result, it is seen that three students recognized and use the volume formula very easily. The self-confidence of the students while solving the question and the fact that they used the volume knowledge directly means that they have constructed the volume knowledge.

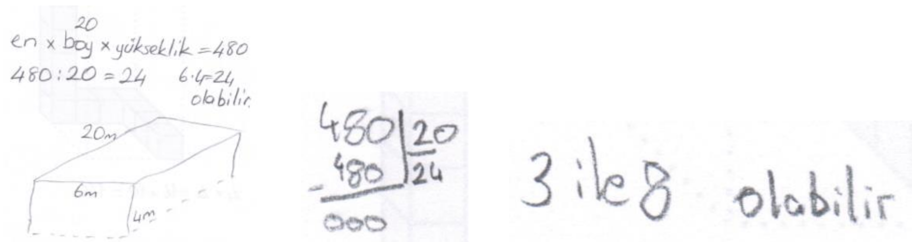

Figure 9. The Operations S1 and S2 Do to Solve the First Part of the Eighth Problem.

14S1: (After some reflection) He asks us not the width × length × height in this question, but the sum of the surface area of a shape, namely the pool, as in the first two questions, we did (Recognition – Recognition).

15T: Could you solve the question and tell me how you did it?

16S1: First, I multiplied six by four and got 24. Since there are two of these, I multiplied 24 by two and got 48. Then I multiplied 20 by four to get 80. Since there are two, I multiply by two to get 160. I multiplied 20 by six to get 120. There is one of these. It only has the bottom. The pool model has no upper face. We do not find its area. I added them all together and found 328 m² (Solving the problem - Using).

17T: What did you find Ö2?

18S2: I could not find it.

...

26S4: We multiply them all. Hmm... This question is similar to the first one we solved (Recognition – Recognition).

30S3: Height 3 m. If we tile on the long side, we multiply 20 by 3, if we tile on the short side, we multiply 8 by 3. Since they also have opposites (reciprocal congruent surfaces), we add both results with them (Reflecting a process - Using).

31T: How many surfaces will be tiled?

33S3: Isn't it five?

34T: So what should we do?

35S4: Just like we found the trailer of the truck (Relationship – Using).

36S3: Then we multiply 20 by eight, 160.

37T: What area of the pool (160m2) is this?

38S3: Base. We add this to the result.

In the second part of the question; S1, S3 and S4 made a connection between the question and the first two problems (14S1, 24S4, 35S4). They realized that they needed to calculate the surface area of the created pool. However, only S1 and S3 recognized the surface area knowledge that was constructed in the previous problems and used it in solving the problem (16S1, 30S3, 36S3, 38S3). In addition, S1 and S3 thought that the upper part of the pool was a void and did not include it in the calculation (See Figure 10.). Although S4 made a connection with the first two problems, he could not use the knowledge and could not reach a conclusion. As in the first stage of the problem, S1 and S3 easily realized the *recognition* and *building-with* actions at this stage. It is understood that both of them construct the surface area knowledge. Other students could not reach any solution in the second stage of the problem.

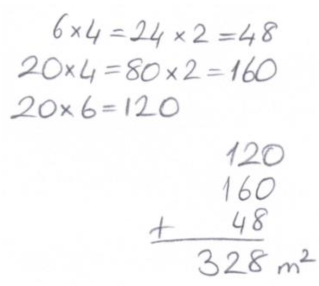

Figure 10. S1's Actions to Solve the Second Part of the Eighth Problem.

Discussion and Conclusion

In the first and second problems, two groups with high and medium achievement levels recognised the quality to be measured, the properties of the prism and the area relation of the rectangle. However, the group with low achievement level could only recognise the properties of the prism. They remembered the area relation of the rectangle with the guidance of the teacher. All students succeeded in using the area relation of the rectangle. In other words, the epistemic actions of *recognition* and *building-with* were realised. The use of the terms "bottom floor - top floor" instead of "base", "size" instead of "length", and "edge" instead of "side" can be given as examples of such misnaming. This situation is similar to the use of words such as "distance, line, length, perpendicular, line, vertical, edge" instead of the concept of "height" in the study of Gürefe & Gültekin (2016).

S1's statement that he found "the sum of the areas of all surfaces" and S3 and S4's statement that they calculated "the area of the container" can be considered as they progressed in constructing surface area knowledge. The fact that S2 and S5 calculated the surface area directly and without hesitation in the second problem can be interpreted as they constructed the surface area knowledge. S6, who had a low level of achievement, could not reach the construction stage. It is thought that S6 did not know the characteristic of area and had deficiencies about calculating the areas of geometric shapes. This situation prevented S6 from reaching the construction stage.

In the secondary school mathematics curriculum, "the number of unit cubes that can be placed in an object in such a way that there is no gap" is defined as the volume of that object. Students easily found the number of unit cubes in constructions consisting of unit cubes by using additive and multiplicative reasoning. In addition, it was not difficult to find the volume of the prism given the number of cubes in the base layer and along its height. They found the number of cubes in the base and multiplied by the number of cubes lined up along the height. This result agrees with the results obtained in the studies of Camci (2018) and Battista and Clements (1998). In this context, it can be said that the students started to construct the *base area × height* knowledge to find the volume. However, S4 with moderate mathematics achievement and two students with low achievement (S5 and S6) could not calculate the volume of a prism with a certain number of cubes arranged along its three edges. It has been observed that students with high success levels use the *base area × height* method faster.

According to the results obtained in the research, S4 with a medium level of mathematics achievement and S6 with a low level of mathematics could not establish a relationship between structures with and without a unit cube. Other students calculated the volume of the prism by multiplying the measurements of the width, length and height. It is seen that all students, except S4 and S6, made progress in constructing knowledge for calculating volume.

According to another result of the research, most of the students could not find the edge dimensions of a rectangular prism given the volume. Especially the group consisting of low-achieving students could not make any idea about this situation. However, S1 and S3 who reached the conclusion noticed the two edges of the prism and had difficulty in the third edge.

Another result is related to students' use of units. It has been observed that the measurement unit is not used in some cases. It has been observed that the measurement unit is not used in some cases. Students sometimes used the unit of length instead of the unit of area. It is understood that the students could not establish the connection between the quality to be measured and the unit used to measure that quality, or they did not care about using units. This result also agrees with the result of the study of Tan-Şişman & Aksu (2009). This result is also consistent with the result of Tan-Şişman & Aksu's (2009) study. The reason for this may be that the students had deficiencies in measurement in their previous term learning.

In order for the abstraction to be fully realised, it is expected that the constructed knowledge is directly used in problems with different contexts. S1 and S3 were able to use their surface area and volume knowledge in the last problem. S2 was able to reuse only the volume knowledge. It is seen that students with higher achievement level realise *recognition*, *building-with* and *construction* actions faster. It is understood that as the achievement level decreases, the acts of abstraction decrease. This result coincides with the results of the studies conducted by Ayanoğlu (2012), Aydın Çınar (2019), Hasar (2019), Memnun (2011), Temiz (2019), Ulaş & Yenilmez (2017), Yenilmez & Yüksel (2018) and Yeşildere & Türnüklü (2008).

Epistemic actions are intertwined, simultaneous and observable actions (Dreyfus, 2007; Dooley, 2007). According to the results of the research, the students performed the actions of *recognition*, *building-with* and *construction* sometimes sequentially and sometimes simultaneously. This result is consistent with the studies of Akkaya (2010), Altaylı Özgül & Kaplan (2016), Dinç (2018), Dreyfus (2007) and Eldekçi (2019).

Recommendations

It is seen that the recognized or created formula structures can be easily used by the students, but the connection between the operations and the concepts cannot be established. Activities for calculating surface area and volume without using formulas should be included. It is suggested to focus on the quality of the prism that the result obtained after performing the operation belongs to.

Mostly, it is seen that students have difficulties in applying the constructed knowledge to different problems. For this reason, it is recommended to include different activities at regular intervals after knowledge structures are established. In this way, permanent abstraction of the constructed knowledge can be ensured.

In the study, students working as a group showed that social learning environments were positive in terms of abstraction. Learning environments can be created to enable interaction within or between groups in the teaching of the issues.

One of the important concepts for measurement is the unit. In this study, it was observed that the students had problems in using units. They experience confusion in determining the unit of property to be measured. For this reason, examining the processes of constructing the concept of unit will help to eliminate such problems and help students to establish a connection between measurement and property.

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Conflict of Interest

It has been reported by the authors that there is no conflict of interest.

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Ethical Standards

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