*Turkish Journal of Civil Engineering, 2023, 81-98, Paper 752, **Research Article***

On a New Method of Quasi-static and Dynamic Analysis of Viscoelastic Plate on Elastic Foundation

Gülçin TEKİN[1*]
Fethi KADIOĞLU[2]

ABSTRACT

An alternative solution technique based on mixed finite element (MFE) formulation in the Laplace-Carson domain is proposed for quasi-static and dynamic analyses of viscoelastic plate (VEP) resting on an elastic foundation (EF). This work contributes a numerical solution to the problem of a viscoelastic Kirchhoff plate supported on a Winkler foundation. VEP-EF interaction problems are taken into account under different wave-type loadings. The viscoelastic material behavior of the plate is modeled by the Zener rheological solid model. A four-nodded linear isoparametric element with sixteen degrees of freedom is used to model the VEP. The functional developed in the Laplace-Carson domain based on the Gâteaux differential method is transformed to the real time domain by utilizing the Dubner and Abate (D&A) inverse Laplace transform technique (ILTT). To evaluate the applicability of the results, five numerical samples are considered. Further analyzes are performed on different wave type loadings to offer a new perspective on the time-dependent behavior of VEP on EF.

Keywords: Viscoelastic plate, plate-foundation interaction, wave-type loadings, Gâteaux differential, Laplace-Carson transform, Dubner & Abate.

1. INTRODUCTION

Analysis of interaction of different structural elements with the supporting foundation is a typical problem in diverse fields of modern engineering disciplines such as structural, pavement and foundation. Numerous authors have employed various approximate numerical methods to investigate structural element - foundation interaction problems in recent years.

Note:
- This paper was received on March 21, 2023 and accepted for publication by the Editorial Board on September 4, 2023.
- Discussions on this paper will be accepted by January 31, 2024.

• https://doi.org/10.18400/tjce.1268992

1 Department of Civil Engineering, Yıldız Technical University, Istanbul, Türkiye
gulcint@yildiz.edu.tr - https://orcid.org/0000-0003-0207-4305

2 Department of Civil Engineering, Istanbul Technical University, Istanbul, Türkiye
fkadioglu@itu.edu.tr - https://orcid.org/0000-0001-7049-1704

* Corresponding author

Plate-foundation interaction problems are very common in civil engineering. In many practical applications, elastic and viscoelastic plates are in contact with the soil. Elastic ground effects are among the many factors that can have significant and unexpected effects on the behavior of plates. Therefore, there is a need to understand the behavior of composite plates resting on the elastic foundation. There are Winkler and Pasternak models, elastic half-space models, and other approximate models describing elastic foundations [1-2]. The Pasternak model has been widely adopted to describe the mechanical behavior of elastic soils, and the well-known Winkler model is a special case of this model. The Winkler elastic foundation model provides satisfying results in many practical engineering applications. A concise review of elastic foundation models is considered by Wang et al. [3] and Kerr [4]. In many research papers, elastic plate-Winkler elastic foundation interaction problems are investigated using various techniques such as boundary element, finite difference and finite element [5-14], which assumes that the plate material is linear elastic. When viscoelastic materials have been incorporated as structural elements in many engineering situations, the need for studies on viscoelastic problems has increased. Viscoelastic constitutive relations are more realistic than elastic constitutive relations to reflect the material behavior. Basic concepts regarding the mechanical behavior of viscoelastic structural components are presented in the studies [15-21]. The investigation of ground effects on the behavior of elastic/viscoelastic plates has been the subject of many studies [22-35]. Based on our literature review, many research activities have been devoted to the development of theoretical and computational methods for elastic/viscoelastic plate - elastic/viscoelastic foundation interaction problems. However, to the best of our knowledge, this study is unique in that it proposed MFE formulation to analyze the response of thin VEPs, which is modeled by the Zener rheological solid model, subjected to wave-type loadings and resting on EF. In the present study, different quasi-static and dynamic example problems are considered for quantifying the effect of EF on the behavior of VEPs. This study has contributions to scientific knowledge as follows: this study presents an improved MFE formulation that is simple, reliable and efficient in computations. By mixed formulation moment and shear force values (if any) are obtained independently from displacements without any back substitution process, which is unavoidable in displacement models. All field equations can be enforced to the functional systematically. For instance, the Gâteaux differential method can be applied to any field equations for which stationary functional and boundary condition terms of the problem under consideration are not known beforehand. Geometric and dynamic boundary conditions can be obtained more easily. For the Gâteaux differential method, boundary conditions are wholly dealt with through mathematical manipulations. The field equations can be verified via potential testing, which is defined by [36]. The Gâteaux differential method is applicable if the operator form of the field equations is potential (positive definite and self-adjoint). A numerical methodology presented in this study was applied for the analysis of viscoelastic structural members by Aköz and Kadıoğlu [36] and Kadıoğlu and Aköz [37-39]. In these studies, MFE formulations of viscoelastic parabolic and circular beams were derived based on the Gâteaux Differential method. Moreover, the proposed solution procedure was successfully applied to the analysis of viscoelastic Kirchhoff plates constituted by different rheological models by Aköz et al. [40] and to the analysis of Mindlin-Reissner plates made of viscoelastic materials based on the shear deformation theory by Tekin and Kadıoğlu [41].

2. METHOD

2.1. Winkler Elastic Foundation

In the Winkler foundation model, the relevance between the external pressure q_0 and the deflection of the foundation surface parallel to the z-axis w (Fig. 1) is given by:

$$q_k = k\,w \tag{1}$$

Here, k is the modulus of subsoil response of foundation. When applied to laterally loaded plates, the mathematical model for the plate-elastic foundation interaction system yields a biharmonic differential equation as:

$$D\,\nabla^4 w + kw = q_0 \tag{2}$$

Here, D indicates the bending stiffness of the plate and ∇^4 indicates a biharmonic operator:

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \tag{3}$$

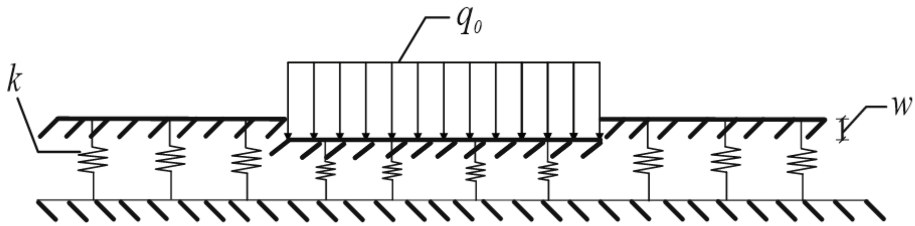


Fig. 1 - Winkler foundation model

2.2. Viscoelastic Plate - Winkler Elastic Foundation Coupling

The governing equations of plates made of viscoelastic materials were derived in Laplace-Carson space by [40] using the Kirchhoff plate theory with internal forces of which their positive directions are depicted in Fig. 2.

The Laplace-Carson transform $\bar{g}(r)$ of a function $g(t)$ with respect to time is defined by

$$\bar{g}(r) = r\int_0^\infty e^{-rt}\, g(t)\, dt \tag{4}$$

Here, r is the parameter of the Laplace transform, which is in general a complex number. Implementation of the Laplace-Carson transform with regard to time in field equations to remove time derivatives yields the following field equations in terms of four variables in the Laplace-Carson space, including the bending and twisting moments ($\bar{M}_x, \bar{M}_y, \bar{M}_{xy}$) and a component of displacement ($\bar{w}$) shown as follows:

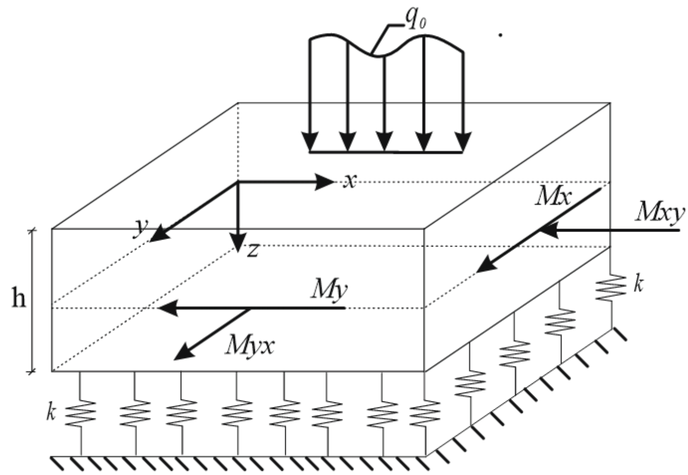


Fig. 2 - Internal forces of laterally loaded plate structure

$$
\begin{aligned}
-\frac{\partial^2 \bar{M}_x}{\partial x^2} - \frac{\partial^2 \bar{M}_y}{\partial y^2} - 2\,\frac{\partial^2 \bar{M}_{xy}}{\partial x \partial y} &= \bar{q}_0 - k\bar{w} \\
-\bar{M}_x - \bar{D}^*\left(\frac{\partial^2 \bar{w}}{\partial x^2} + \nu\,\frac{\partial^2 \bar{w}}{\partial y^2}\right) &= 0 \\
-\bar{M}_y - \bar{D}^*\left(\frac{\partial^2 \bar{w}}{\partial y^2} + \nu\,\frac{\partial^2 \bar{w}}{\partial x^2}\right) &= 0 \\
-\bar{M}_{xy} - (1-\nu)\,\bar{D}^*\,\frac{\partial^2 \bar{w}}{\partial x \partial y} &= 0
\end{aligned}
\tag{5}
$$

Here, q_0 is the wave-type load distribution, ν is Poisson's ratio of the plate and D^* is the operator form of the plate's flexural rigidity, which is defined for plates in the hereditary integral form. More information can be found from [40 and 42].

Field equations of VEPs - Winkler foundation interaction problem with the boundary conditions in the Laplace-Carson domain can be shown in operator form as:

$$\bar{Q} = \bar{L}\bar{\mathrm{v}} - \bar{f} \tag{6}$$

$$
\begin{bmatrix}
k & \bar{P}_{12} & \bar{P}_{13} & \bar{P}_{14} & 0 & 0 & 0 & 0 \\
\bar{P}_{21} & \bar{P}_{22} & \bar{P}_{23} & 0 & 0 & 0 & 0 & 0 \\
\bar{P}_{31} & \bar{P}_{32} & \bar{P}_{33} & 0 & 0 & 0 & 0 & 0 \\
\bar{P}_{41} & 0 & 0 & \bar{P}_{44} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\bar{w} \\ \bar{u}_2 \\ \bar{u}_3 \\ \bar{u}_4 \\ \bar{w}_0 \\ \bar{w}_0{}' \\ \bar{M} \\ \bar{T}
\end{bmatrix}
=
\begin{bmatrix}
\bar{q}_0 \\ 0 \\ 0 \\ 0 \\ \hat{\bar{T}} \\ -\hat{\bar{M}} \\ \hat{\bar{w}}' \\ -\hat{\bar{w}}
\end{bmatrix}
\tag{7}
$$

where

$$
\begin{aligned}
\bar{P}_{12} &= -\bar{D}\left(\frac{\partial^2}{\partial x^2} + \nu\frac{\partial^2}{\partial y^2}\right)\\
\bar{P}_{13} &= -\bar{D}\left(\frac{\partial^2}{\partial y^2} + \nu\frac{\partial^2}{\partial x^2}\right)\\
\bar{P}_{14} &= -\bar{D}(1-\nu)\frac{\partial^2}{\partial x\,\partial y}\\
\bar{P}_{22} &= \bar{P}_{33} = -\bar{D}\\
\bar{P}_{23} &= \bar{P}_{32} = -\nu\bar{D}\\
\bar{P}_{44} &= -\frac{1}{2}\bar{D}(1-\nu)\\
\bar{u}_2 &= \frac{\bar{M}_x - \nu\bar{M}_y}{\bar{D}(1-\nu^2)}\\
\bar{u}_3 &= \frac{\bar{M}_y - \nu\bar{M}_x}{\bar{D}(1-\nu^2)}\\
\bar{u}_4 &= \frac{2\bar{M}_{xy}}{\bar{D}(1-\nu)}
\end{aligned}
\tag{8}
$$

These are the matrix shape of the operators. Here $\bar{L}$ is the derivative operator, $\bar{\text{v}}$ is the variables and $\bar{f}$ is external loads in the Laplace-Carson space. $\bar{Q}$ is a potential and these inner products

$$< d\bar{Q}\,(\bar{v}\,,\bar{v}\,'),\bar{v}* >= < d\bar{Q}\,(\bar{v},\,\bar{v}*),\bar{v}\,' > \tag{9}$$

must be equal [43] in which $d\,\bar{Q}\,(\bar{v}\,,\bar{v}\,')$ is the Gâteaux derivative of the operator $\bar{Q}$.

$$
d\,\bar{Q}\,(\bar{v}\,,\,\bar{v}*) = \begin{bmatrix}
k\bar{w}^* - \frac{\partial^2\bar{M}^*{}_x}{\partial x^2} - \frac{\partial^2\bar{M}^*{}_y}{\partial y^2} - 2\frac{\partial^2\bar{M}^*{}_{xy}}{\partial x\partial y}\\
-\bar{D}\left(\frac{\partial^2\bar{w}^*}{\partial x^2} + \nu\frac{\partial^2\bar{w}^*}{\partial y^2}\right) - \bar{M}^*{}_x\\
-\bar{D}\left(\frac{\partial^2\bar{w}^*}{\partial y^2} + \nu\frac{\partial^2\bar{w}^*}{\partial x^2}\right) - \bar{M}^*{}_y\\
-(1-\nu)\bar{D}\frac{\partial^2\bar{w}^*}{\partial x\partial y} - \bar{M}^*{}_{xy}\\
\bar{T}^*\\
-\bar{M}^*\\
\bar{w}^{*\prime}\\
-\bar{w}^*
\end{bmatrix}
\tag{10}
$$

Thus, the functionals stand for the field equations which form the basis of finite element formulation can be found as (see, Oden and Reddy [43]):

$$I(\bar{v}) = \int_0^1 < \bar{Q}\,\left(\varphi\,\bar{v},\bar{f}\right),\bar{v} > d\varphi \tag{11}$$

φ being a scalar. Finally, from Eq. (11), the explicit expression of the functional of the thin VEP-EF interaction problem in the Laplace-Carson space takes the form:

$$I(\bar{v}) = \frac{1}{2}k\,[\bar{w},\bar{w}] + \left[\frac{\partial\bar{w}}{\partial x},\frac{\partial\bar{M}_{xy}}{\partial y}\right] + \left[\frac{\partial\bar{w}}{\partial y},\frac{\partial\bar{M}_{xy}}{\partial x}\right] - \frac{1}{2\bar{D}\,(1-\nu^{2})}\left\{[\bar{M}_x,\bar{M}_x] + [\bar{M}_y,\bar{M}_y]\right\}$$
$$-[\bar{q}_0,\bar{w}] + \left[\frac{\partial\bar{M}_x}{\partial x},\frac{\partial\bar{w}}{\partial x}\right] + \left[\frac{\partial\bar{M}_y}{\partial y},\frac{\partial\bar{w}}{\partial y}\right] + \frac{\nu}{\bar{D}\,(1-\nu^{2})}[\bar{M}_x,\bar{M}_y] - \frac{1}{\bar{D}\,(1-\nu)}[\bar{M}_{xy},\bar{M}_{xy}] \quad (12)$$
$$-\left[\hat{\bar{T}},\bar{w}\right]_{\sigma} - \left[\bar{w}',\left(\bar{M}-\hat{\bar{M}}\right)\right]_{\sigma} - [\hat{\bar{w}}',\bar{M}]_{\varepsilon} - [\bar{T},(\bar{w}-\hat{\bar{w}})]_{\varepsilon}$$

The expressions in the square brackets with the subscripts σ and ε represent the boundary conditions which are classified into two types: dynamic and geometric boundary conditions, respectively. The explicit form of the boundary conditions of the problem can be written in the form:

$$[\bar{T},\bar{w}] = \left[\left(\frac{\partial\bar{M}_x}{\partial x} + \frac{\partial\bar{M}_{xy}}{\partial y}\right)n_x + \left(\frac{\partial\bar{M}_y}{\partial y} + \frac{\partial\bar{M}_{xy}}{\partial x}\right)n_y,\bar{w}\right]$$
$$[\bar{M},\bar{w}'] = \left[\bar{M}_x,\frac{\partial\bar{w}}{\partial x}n_x\right] + \left[\bar{M}_y,\frac{\partial\bar{w}}{\partial y}n_y\right] + \left[\bar{M}_{xy},\left(\frac{\partial\bar{w}}{\partial x}n_y + \frac{\partial\bar{w}}{\partial y}n_x\right)\right] \quad (13)$$

The expression $[\bar{q}_0,\bar{w}]$ in Eq. (12) corresponds to:

$$[\bar{q}_0,\bar{w}] = \frac{1}{2}\rho\omega[\bar{w},\bar{w}]$$

for dynamic analysis, where ω is the angular frequency.

The linear rectangular element that has a number of four nodes is constructed for the finite element analysis of the VEP-elastic foundation interaction problem. The interpolation functions of this element (Fig. 3), expressed in the (ξ,η) coordinates that are mere translations of (x, y), is given by [44]:

$$\Psi_i = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i) \qquad i = 1,2,3,4 \quad (14)$$

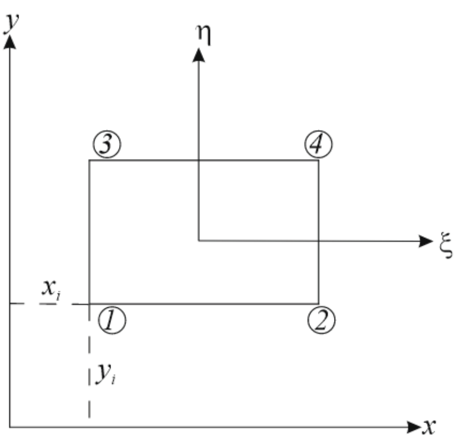


Fig. 3 - Rectangular element

A rectangular master element with four degrees of freedom with $\bar{w}$, $\bar{M}_x$, $\bar{M}_y$, $\bar{M}_{xy}$ as the nodal variables (the total degrees of freedom is 16) is denoted as the VPLTEF16 element.

All variables which must be carried as the primary nodal degrees of freedom, must be approximated by the given interpolation functions and they are inserted into Eq. (12). This functional is extremized with regard to sixteen nodal variables; thereby the sixteen element equations given by Eq. (15) are obtained for the thin VEP-EF interaction problem under wave-type loadings.

$$\begin{bmatrix} k[k_1] & [k_2] & [k_3] & [k_4] \\ [k_2] & -\frac{[k_1]}{\bar{D}(1-\nu^2)} & \frac{\nu[k_1]}{\bar{D}(1-\nu^2)} & 0 \\ [k_3] & \frac{\nu[k_1]}{\bar{D}(1-\nu^2)} & -\frac{[k_1]}{\bar{D}(1-\nu^2)} & 0 \\ [k_4] & 0 & 0 & -\frac{[k_1]}{2\bar{D}(1-\nu)} \end{bmatrix} \begin{bmatrix} \bar{w} \\ \bar{M}_x \\ \bar{M}_y \\ \bar{M}_{xy} \end{bmatrix} = \begin{bmatrix} [k_1]\ \bar{q}_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (15)$$

In Eq. (15), $[k_1]$, $[k_2]$, $[k_3]$ and $[k_4]$ are the submatrices.

3. ILLUSTRATIVE EXAMPLES AND DISCUSSION

A computer program has been written in the FORTRAN to perform the analyses. Numerical examples are considered for simply supported rectangular plate with the following geometrical properties:

- $\frac{a}{2} = \frac{b}{2} = 2\ m$ and thickness of plate h=0.1 m (see Fig. 4)

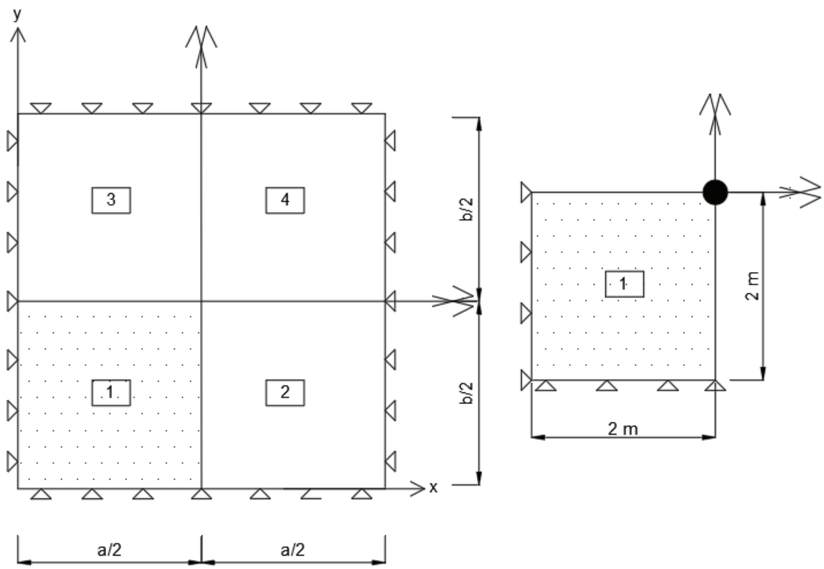


Fig. 4 - Symmetry property of simply supported rectangular plate

Due to the two-way symmetry, we can focus upon a quarter of the plate. The analyses were performed with a mesh size of 4x4. The time histories of wave-type loads considered in the analyses are shown in Fig. 5.

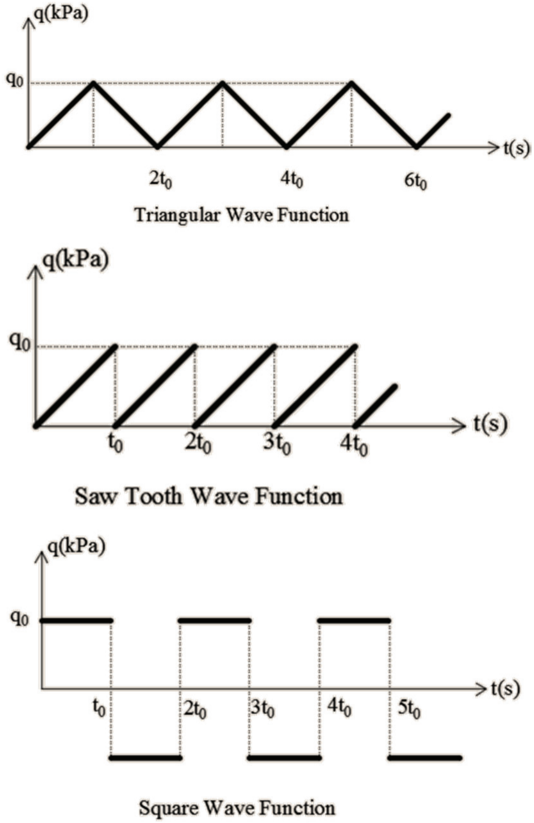


Fig. 5 - Wave-Type load distributions

For describing the viscoelastic behavior of plate material, all numerical examples are employed for the Zener rheological solid model. The Zener model illustrated in Fig. 6 is also known as the standard linear solid.

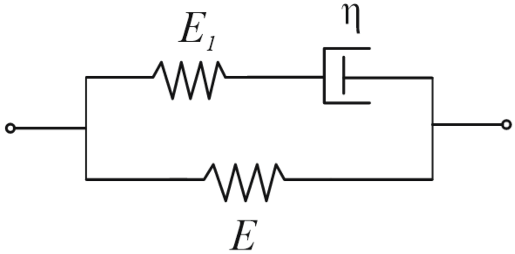


Fig. 6 - Mechanical analog for the Zener solid model

The creep compliance of the Zener solid model is expressed by:

$$J_{(t)} = \frac{1}{E}(1 - e^{-\alpha t}) + \frac{1}{E+E_1}(1 - e^{-\alpha t}) \quad , \qquad \alpha = \frac{E\,E_1}{\eta(E+E_1)} \tag{16}$$

The material properties are $E_1 = E = 98$ MPa, $\eta = 245$ MPa.s and $\nu = 0.3$. The solutions are transformed back to the time domain by using the D&A numerical ILTT technique. For more information about the D&A Laplace inversion process, the interested reader is referred to literature [45].

Example 1:

In this example, quasi-static analysis of a thin VEP subjected to triangular wave-type loading of 1000 N/m^2 considered for t_0 =2 sec and time-varying displacement values at the mid-point of the plate are illustrated in Fig. 7 for different subgrade reactions (k =100 000 kN/m^3, 200 000 kN/m^3 and 300 000 kN/m^3). For numerical inversion, D&A ILTT is employed for aT=5, N=400 and T=20 sec. These results are quite reasonable and indicate that an increase in the value of the subgrade reaction results in the same rate decrease in the central displacement response.

In addition, dynamic and quasi-static response of a thin VEP with foundation interaction with stiffness k=100.000 kN/m^3 subjected to the same wave-type loading for t_0=4 sec is considered. The material density of the plate ρ is assumed to be 2400 kg/m^3. For numerical inversion, D&A ILTT is employed for aT=5, N=600 and T=30 sec. As expected, the VEP starts to vibrate about the quasi-static state but it never approaches the quasi-static state with time when it is subjected to harmonic loading like triangular wave-type loading as depicted in Fig. 8.

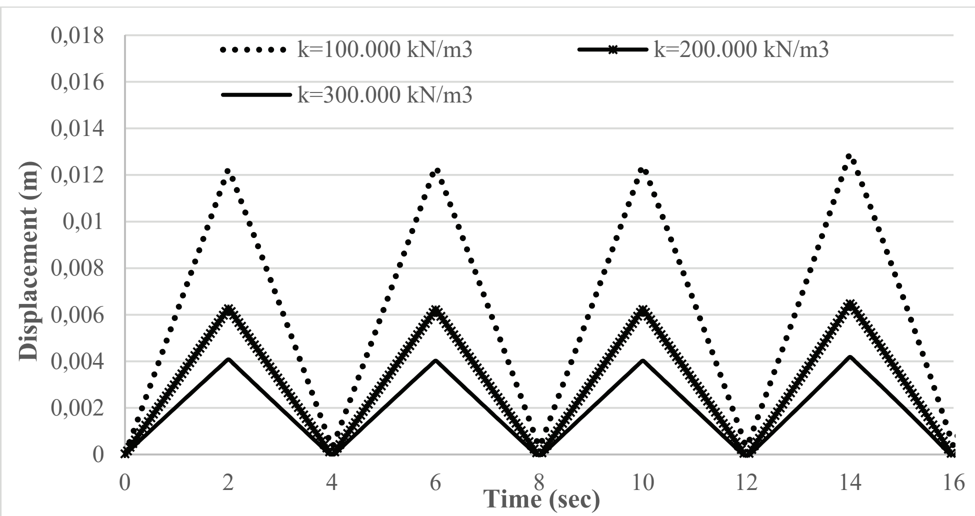


Fig. 7 - Quasi-static central displacement response of viscoelastic thin plate with elastic foundation interaction under triangular wave-type loading

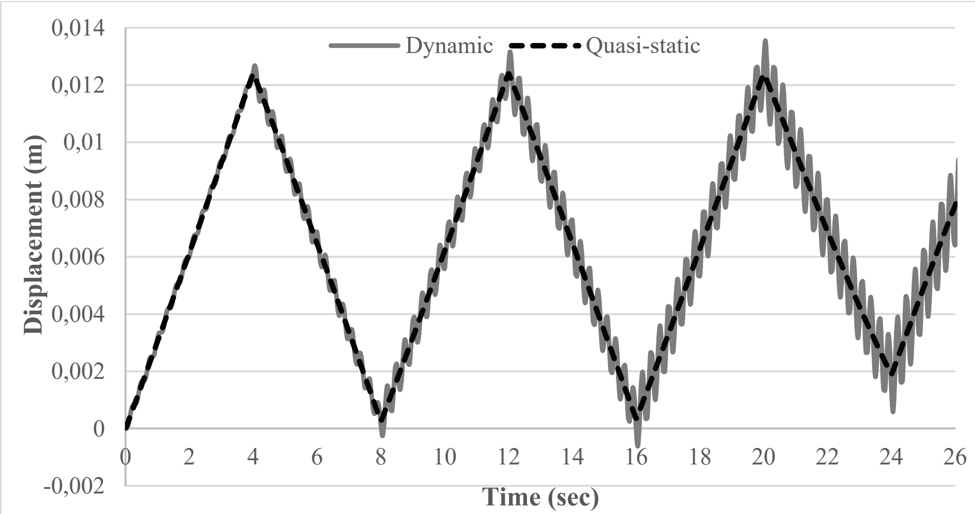


Fig. 8 - Dynamic and Quasi-static central displacement response of viscoelastic thin plate with elastic foundation interaction under triangular wave-type loading

Example 2:

In this example, quasi-static and dynamic analyses of thin VEPs on elastic foundation with stiffness k=100.000 kN/m^3 and subjected to saw tooth wave-type loading of 1000 N/m^2 is considered for t_0=4 sec. The problem is solved by the variation of the central displacement and the results are demonstrated in Fig. 9.

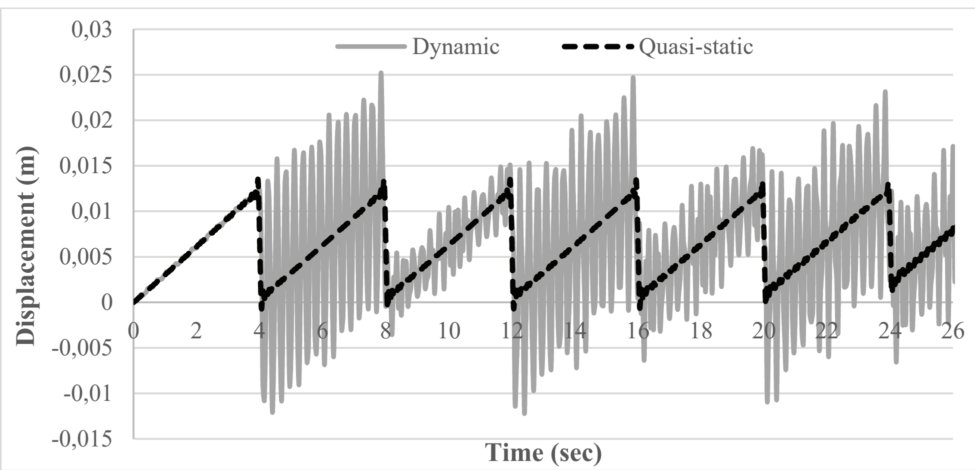


Fig. 9 - Central displacement versus time results under saw tooth wave-type loading

For the numerical inversion, the D&A transform technique is used for ρ=2400 kg/m^3, aT=5, N=600 and T=30 sec. The longer loading time t_0, the springback of the plate is much slower after unloading due to the fact that the dissipation of energy of the plate is more and more, whereas the strain energy stored is less and less with the increment of loading time.

Example 3:

As an example, a thin VEP with no foundation interaction is considered. Two different plate thicknesses are considered. The numerical results are presented in Fig. 10 for central displacement variation with time under the square wave-type loading for t_0= 2 sec. The same material properties as in the previous example are considered. For numerical inversion, the D&A ILTT for aT=5, N=400 and T=20 sec is considered. As expected, the vibration period and displacement of the VEP reduces as the thickness of the plate increases.

Example 4:

In this example, bending moment (M_x) and displacement variation at the mid- point of the thin VEP on the EF with stiffness k=100.000 kN/m^3 subjected to a square wave-type loading (t_0=4 sec) with the amplitude q_0=1000 N/m^2 are investigated. In Fig. 11, quasi-static and dynamic bending moment (M_x) and vertical displacement (w) responses are presented by employing D&A ILTT for aT=5, N=600 and T=30 sec. The same material properties given in the previous example are considered.

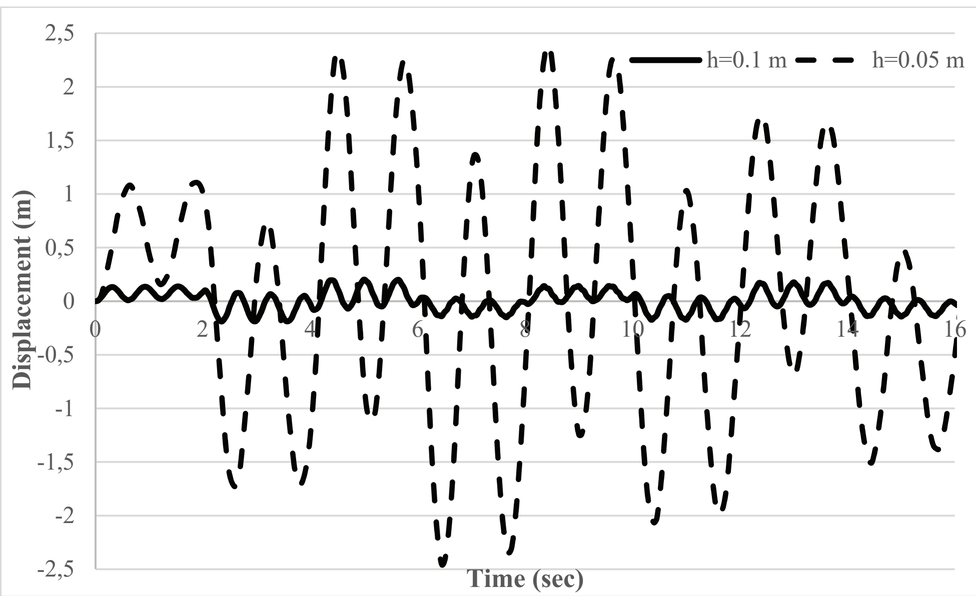
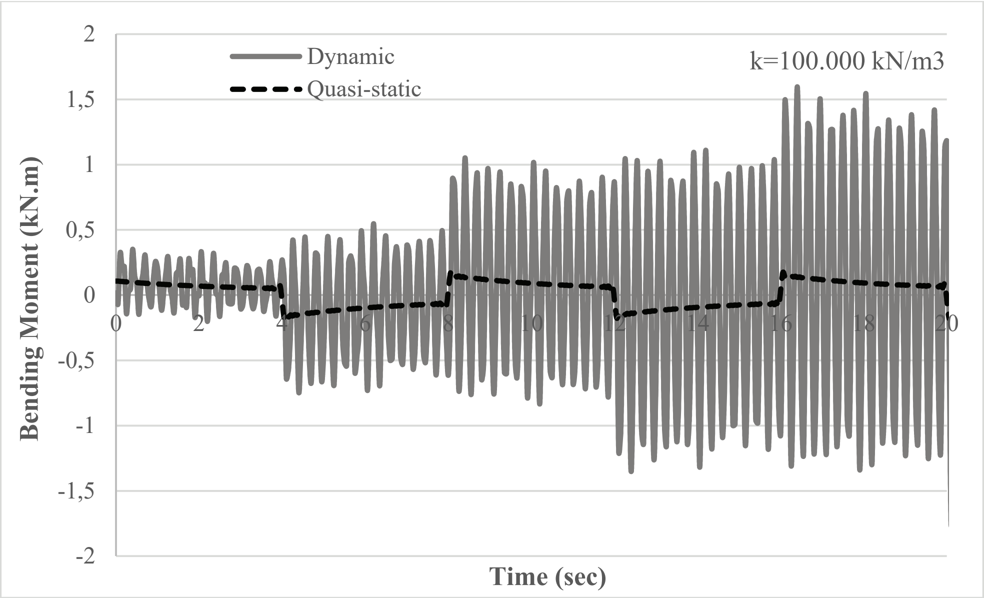


Fig. 10 - Influence of the plate thickness on displacement and vibration period of the viscoelastic thin plate without foundation interaction



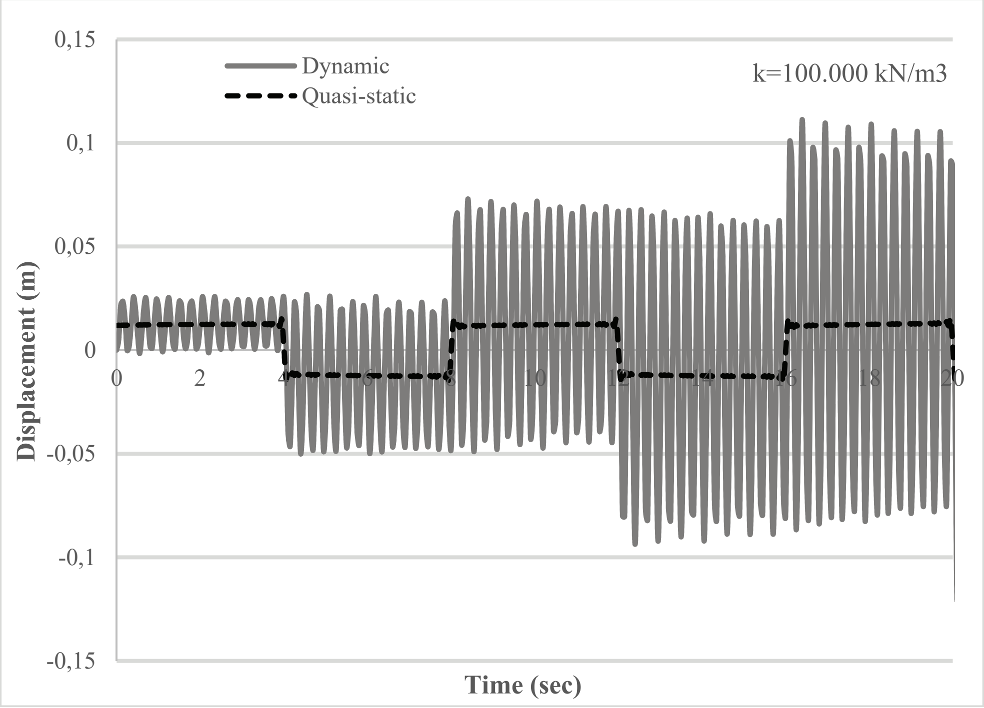


Fig. 11 - Dynamic and Quasi-static central bending moment and displacement responses of viscoelastic thin plate with foundation interaction subjected to square wave-type loading

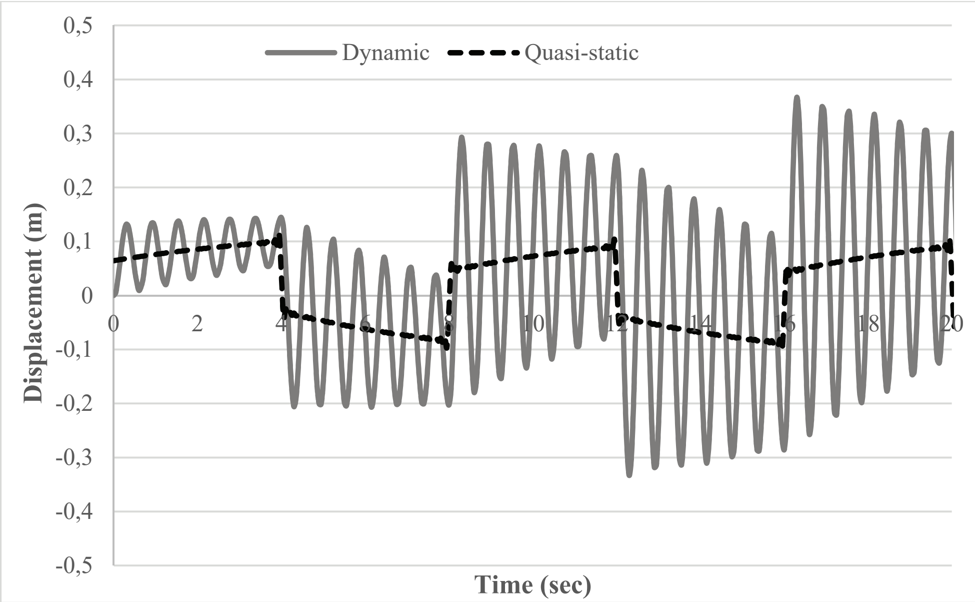


Fig. 12 - Dynamic and Quasi-static central displacement response of viscoelastic thin plate without foundation interaction subjected to square wave-type loading

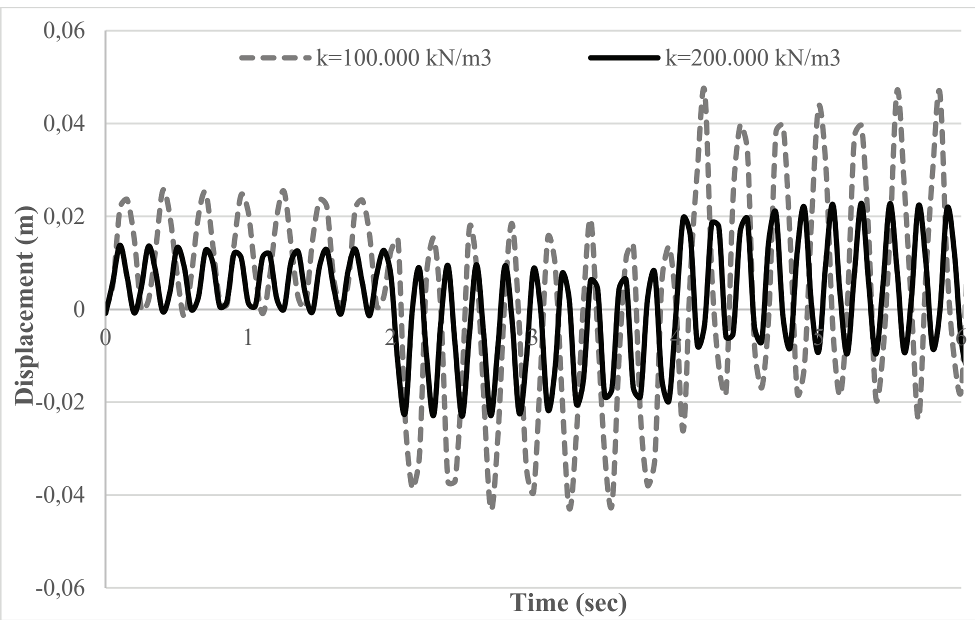


Fig.13 - Dynamic central displacement response of viscoelastic thin plate under square wave-type loading for different Winkler coefficients

Moreover, in Fig. 12, quasi-static and dynamic displacement responses of the plate without foundation interaction are presented in order to make comparison and provide quantitative information about the plate response in the absence and presence of elastic foundation interaction. It can be seen that the frequency of vibration is seriously influenced due to the interaction of the plate with the foundation.

Example 5:

In this problem, the dynamic response of thin VEP on the EF with different subgrade reaction constants (k=100.000 kN/m^3 and 200.000 kN/m^3) is considered. The problem is solved under the square wave-type loading for t_0=2 sec employing D&A ILTT for aT=5 and N=400 and T=20 sec. The same material properties as in the previous example are considered and the variation of displacement at the plate mid-point is given for different values of foundation stiffness in Fig. 13. Again, it is observed that the frequency of vibration is seriously influenced due to the variation of foundation stiffness parameter (k) and it increases when the foundation parameter (k) increases.

4. CONCLUSION

A proposal is presented of a simple numerical analysis method for the dynamic behavior of thin VEP- EF interaction problem under different wave-type loadings by utilizing the functional via a methodical procedure depending on the Gâteaux Differential. The functional and MFE formulation derived for the analysis are in the Laplace-Carson domain and for numerical transformation of the solutions back to the time domain, D&A ILTT is utilized. The analyses are performed with the aid of a FORTRAN code. The performance of the proposed methodology is tested through illustrative examples.

The results are quite reasonable:

i. An increase in the value of the subgrade reaction results in the same rate decrease in the central displacement response and the VEP starts to vibrate about the quasi-static state but it never approaches the quasi-static state with time as expected.

ii. The dynamic behavior of (un)constrained VEP is obtained by the proposed formulation. As expected, the frequency of vibration is seriously influenced due to the interaction of the plate with the foundation and it increases when the foundation parameter (k) increases.

iii. VEPs with different thickness values are analyzed. As expected, the vibration period and displacement of the VEP decreases as the thickness of the plate increases.

iv. Moments can be obtained directly without any mathematical operation by using this new functional, because it can be defined as one of the independent variables in the functional.

For further analysis stages, the behavior of viscoelastic structural members such as laminated composite beams according to different beam theories, shell structures and viscoelastic higher order shear deformation plate structures is planned to be investigated through this methodology.

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