

# THE COMPARISON OF KAM AND ADDITIVE MODEL ON THE PROBLEMS THAT TARGET INTERVALS ARE ALREADY DETERMINED

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## ABSTRACT

*After running Data Envelopment Analysis (DEA) on models, efficiency scores were determined. In addition, DEA models give us target scores too. However; sometimes the target scores are not in the intervals which are determined before, so obtained results will not be realistic. If desired constraints add in DEA normal models, ideal results cannot be obtained. For solving the problem, Kourosh and Arash Method (KAM) has been utilized. For creation of new model, the properties of KAM have been considered. In order to reach the thesis goal, suggested constraints were added to KAM mathematically and behavior of the other input and output changes are examined. For the analysis of large scale problems KAM model was developed in EXCEL Solver software. Results are compared with additive model and shows that KAM model gives better results.*

*Key words: Data Envelopment Analysis, Kourosh and Arash Method, Additive model*

## 1. INTRODUCTION

By arise in positive effects of clean energies and negative effect of fossil fuels on the environment has forced many countries, especially the developed ones, to use renewable energy sources. Nowadays the fastest developing energy source is solar energy. Because solar energy is renewable and environment friendly, systems that convert solar energy to electricity have developed rapidly.

However, there is some study to investigate the impact of solar plants from different cities on Turkey's solar plants in the literature. Therefore, measuring the impact of each city on Turkey's solar plants is necessary to improve Turkey's receipts rank. It can be also a good practice for other countries to improve their receipts from their possible facilities.

In order to measure the mentioned objectives, a recent technique in Data Envelopment Analysis (DEA), called Kourosh and Arash Model (KAM) proposed by Khezrimotlagh et al. (2013) [1] is selected. DEA is a popular non-parametric technique to estimate the performance evaluation of a set of homogenous Decision Making Units (DMUs) with multiple inputs and outputs. It was proposed by Charnes et al. (1978) [2], and has been dramatically improved in many contexts such as Economics, Managements, Business and Industrial Engineering and so on [3]. KAM is also a recent roust model in DEA, which covers many of DEA subjects, and has improved the discrimination power of DEA significantly. It simultaneously ranks and benchmarks both technically efficient and inefficient DMUs, and identifies the most efficient DMUs with a strong logical method.

Some factors were also selected from the Turkish Statistical Institute web page "<http://www.tuik.gov.tr>". The technique of KAM are used and its results are depicted with some appropriate figures as well as clear illustrations and suggestions to improve Turkey's solar plants location. It is illustrated how Turkey should plan for each city to increase its receipts from solar plants.

The rest of this paper is organized into 5 sections. Section 2 is a background on DEA. Section 3 illustrates the used methodology for applying KAM as well as its advantages on measuring the tourism performance. The

results of applying KAM for each city are depicted in Section 4 and the paper is concluded in Section 5.

## 2. PRELIMINARIES

This section tries to review DEA and Additive models, and also KAM method. For interested readers which are looking for deeper results we will refer to several origins.

### 2.1. DATA ENVELOPMENT ANALYSIS

Data envelopment analysis (DEA) is a nonparametric method for evaluating the relative efficiency of decision-making units (DMUs) on the basis of multiple inputs and outputs. In recent years DEA has had important role in application of many fields such as energy (Alp and Sozen, 2011[4], Sozen et al., 2011) [5], banking (Mercan et al., 2003) [6], sport (Alp, 2006 [7]; Anderson and Sharp, 1997 [8]) etc. The first introduction on DEA was practiced by Charnes et al. (1978) [2]. They proposed CCR model which is also called as Constant Return to Scale (CRS). The CCR model evaluates both technical and scale efficiencies via optimal value of the ratio form. The modified version of CCR model is BCC model, which is also called variable returns to

scale, was proposed by Banker et al. (1984). The BCC model is used to estimate the pure technical efficiency of DMUs by reference to the efficiency frontier. The primal form of CCR (CRS) model for the efficiency score of DMU<sub>k</sub> is as follows:

$$\begin{aligned} & \max \sum_{r=1}^s u_r y_{rk} \\ & \text{subject to} \\ & \sum_{i=1}^k u_i x_{ik} \\ & \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} \geq 0; j = 1, 2, \dots, n, \\ & v_i, u_r \geq 0, i = 1, 2, \dots, m, r = 1, 2, \dots, s, \end{aligned}$$

where  $n$  is number of DMUs with  $s$  outputs denoted by  $y_{rk}$ ;  $r = 1, \dots, s$  and  $m$  inputs denoted by  $x_{ik}$ ,  $i = 1, 2, \dots, m$ . And  $U_r, V_i$  are the

weights on output  $r$  and input  $i$ , respectively. The primal form of input-oriented BCC (VRS) model is considered in this paper and it is given as follows,

$$\min \theta_0$$

*subjecto*

$$\theta_0 x_{i0} - \sum_{j=1}^n \lambda_j x_{ij} \geq 0$$

$$\sum_{j=1}^n \lambda_j y_{rj} - y_{r0} \geq 0$$

$$\sum_{j=1}^n \lambda_j = 1$$

$$\lambda_j \geq 0$$

$$i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n, \quad r = 1, 2, \dots, s.$$

where  $\theta_0$  is efficiency score of  $DMU_0$  and  $x_{i0}, y_{r0}$  (all nonnegative) are  $i$ 'th input and  $r$ 'th output of the  $DMU_0$  respectively, and  $\lambda_j$  is intensity of  $DMU_j$ . If the  $\theta_0$  is equal to one, then  $DMU_0$  is called an efficient  $DMU$ .

## 2.2 KOUROSH AND ARASH MODEL (KAM)

This method recently presented by Khezrimotlagh et al. (2013) in the aim of improving foundation of DEA and its first definitions. The method tries to increase DEA's power to distinguish DMUs with multiple inputs and outputs inclusive controllable, non controllable, real and integer data. KAM unlike current DEA models provides a methodology based on an introduced epsilon which is able to measure the efficiency score of DMUs where the weights are available or unknown. Interested readers are referred to Khezrimotlagh et al. (2013).

Let DMUs ( $DMU_i, i = 1, 2, \dots, n$ ) consist of  $m$  non-negative inputs ( $x_{ij}, j = 1, 2, \dots, m$ ) and  $p$  non-negative outputs ( $y_{ik}, k = 1, 2, \dots, p$ ) such that, at least one of the inputs and one of the outputs of each  $DMU_i$  are not zero. Consider an epsilon

Vector  $\varepsilon = (\varepsilon^-, \varepsilon^+)$  in  $\mathbb{R}^{(m+p)^+}$ , where  $\varepsilon^-$  is  $(\varepsilon_1^-, \varepsilon_2^-, \dots, \varepsilon_m^-)$  and  $\varepsilon^+$  is  $(\varepsilon_1^+, \varepsilon_2^+, \dots, \varepsilon_p^+)$ . The linear  $\varepsilon$ -KAM, while  $DMU_l$  ( $l = 1, 2, \dots, n$ ) is under evaluation, is as follows,

$$\begin{aligned} & \max \sum_{j=1}^m w_j^- s_j^- + \sum_{k=1}^p w_k^+ s_k^+ \\ & \text{subject to} \\ & \sum_{i=1}^n \lambda_i x_{ij} + s_j^- = x_{lj} + \varepsilon_j^-, j = 1, 2, \dots, m, \\ & \sum_{i=1}^n \lambda_i y_{ik} - s_k^+ = y_{lk} + \varepsilon_k^+, k = 1, 2, \dots, p, \\ & x_{lj} - s_j^- \geq 0, j = 1, 2, \dots, m, \\ & y_{lk} + s_k^+ - 2\varepsilon_k^+ \geq 0, k = 1, 2, \dots, p, \\ & \lambda_i \geq 0, s_j^- \geq 0, s_k^+ \geq 0 \\ & j = 1, 2, \dots, m, k = 1, 2, \dots, p, i = 1, 2, \dots, n. \end{aligned}$$

$n$ : number of DMUs,

$m$ : number of inputs,

$p$ : number of outputs,

$i$ : index of DMUs,

$j$ : index of inputs,

$k$ : index of outputs,

$l$ : index of evaluated DMU,

$x_{lj}$ : non-negative observed value of input  $j$  of  $DMU_l$ ,

$y_{lk}$ : non-negative observed value of output  $k$  of  $DMU_l$ ,

$s_{lj}^-$ : non-negative slack or potential reduction of input  $j$  of  $DMU_l$ ,

$s_{lk}^-$ : non-negative slack or potential increase of output  $k$  of  $DMU_l$ ,

$\lambda_i$ : multipliers used for computing linear combinations of DMUs' inputs and outputs,

$\varepsilon$ : non-negative real number.

Where  $w_j^-$  and  $w_k^+$  are the user –specified weights obtained through values judgments,  $\lambda_i$  : multipliers used for computing linear combinations of *DMUs* input and outputs.  $s_j^-$  and  $s_k^+$  are non negative slacks, for  $j = 1, 2, \dots, m$ , and  $k = 1, 2, \dots, p$ , note that when epsilon is zero, linear KAM is same as the weighted additive model proposed by charnes et al. (1985).

### 3. APPLICATION

The main aim of this study is to use some of the most important factors that are effective in indicating priority of thirty Turkish cities to plan the location of solar plant. Of course expensiveness of the solar central has important role in selecting these factors. In order to get the best place it is better to consider different conditions and variables like normal pests (flood, earthquake, hail, etc) and analyzing them carefully. Although, some variables like convection, transformers, price of the land are very important factors to determine. In this study we rely on Sozen and Mirzapour (under review) work which were analyzed these parameters and final factors consist of nine inputs and one output as follows,

#### Inputs:

1. Distance to power distribution networks (km).
2. Land cost (Turkish Lira (1\$ 2.5 TL)).
3. The number of earthquakes.
4. The number of flooding rains.
5. The number of severe hails.
6. Snow and blizzard.
7. The number storms and severe hurricanes.
8. The adversity.
9. Human and financial losses.

Output:

Solar monthly average (h): The primary indices for locating solar plants are solar monthly average, which is equal to solar global radiation multiplied by solar duration and divided by month days. Thirty Turkish cities by the mentioned factors are shown in Table 1. In calculations, EXCEL software had been used. Excel DEA solver had important role in calculating these models.

**Table1.** Additive model target scores for input 1

		$x_1^*$	$x_2^*$	$x_3^*$	$x_4^*$	$x_5^*$	$x_6^*$	$x_7^*$	$x_8^*$	$x_9^*$	$y_1^*$
1	BOLU	50.00	5.00	197.00	41.00	23.00	5.00	8.00	6.00	3.00	560.00
2	CANAKKALE	52.38	5.00	43.20	20.01	16.48	4.73	11.17	4.56	3.25	664.00
3	ANKARA	75.79	7.07	40.93	24.25	22.23	4.55	14.15	4.55	3.54	862.00
4	MANISA	62.16	5.80	33.57	19.89	18.23	3.73	11.60	3.73	2.90	707.00
5	AFYON	63.18	6.00	47.98	23.24	19.57	5.26	13.08	5.11	3.70	782.00
6	KAYSERI	195.00	10.00	16.00	95.00	45.00	40.00	69.00	18.00	15.00	1015.00
7	MALATYA	35.00	89.64	105.95	33.19	11.63	13.66	13.79	4.59	4.58	1418.00
8	ICEL	120.00	5.00	51.00	84.00	73.00	30.00	37.00	12.00	16.00	1185.00
9	HATAY	62.34	5.82	33.66	19.95	18.29	3.74	11.64	3.74	2.91	709.00
10	KAYSERI	77.17	83.00	16.00	14.61	21.12	14.85	23.99	1.88	2.18	955.00
11	RIZE	250.00	6.00	10.00	67.00	3.00	13.00	25.00	9.00	13.00	551.00
12	ARTVIN	35.00	16.98	32.72	14.69	10.48	3.94	7.69	2.50	2.10	560.00
13	MUGLA	35.00	3.27	18.90	11.20	10.27	2.10	6.53	2.10	1.63	398.07
14	HAKKARI	30.00	79.00	93.00	29.00	10.00	12.00	12.00	4.00	4.00	1241.00
15	SANLIURFA	150.00	28.00	17.00	48.00	56.00	7.00	11.00	7.00	4.00	862.00
16	USAK	35.00	120.00	371.00	19.00	14.00	5.00	2.00	4.00	5.00	1072.00
17	KASTAMONU	20.00	214.00	72.00	112.00	66.00	17.00	21.00	10.00	17.00	526.00
18	ERZURUM	35.00	35.02	50.90	19.28	10.77	6.35	9.20	3.02	2.72	773.00
19	ELAZIG	76.78	50.00	84.63	35.47	23.20	10.34	17.93	5.84	5.04	1379.00
20	AMASYA	150.00	40.00	64.93	62.15	45.50	8.53	6.00	2.00	11.28	653.00
21	GAZIANTEP	70.00	24.00	48.00	43.00	36.00	3.00	6.00	10.00	7.00	798.00
22	TUNCELI	50.00	6.00	206.00	55.00	28.00	22.00	26.00	20.00	12.00	1388.00
23	NIGDE	75.00	110.00	15.00	10.00	18.00	19.00	30.00	1.00	2.00	1094.00
24	AGRI	35.00	18.76	34.51	15.14	10.51	4.17	7.83	2.55	2.16	581.00
25	KONYA	125.00	5.00	30.00	189.00	117.00	148.00	141.00	78.00	26.00	1016.00
26	KARAMAN	50.00	90.00	12.00	23.00	14.00	30.00	26.00	14.00	6.00	787.00
27	DIYARBAKIR	150.00	14.00	81.00	48.00	44.00	9.00	28.00	9.00	7.00	1706.00
28	SANLIURFA	70.00	61.00	17.00	48.00	56.00	7.00	11.00	7.00	4.00	821.00
29	ANTALYA	35.00	69.49	85.64	28.06	11.31	10.96	12.09	4.01	3.89	1180.00
30	ADANA	35.00	13.82	126.00	34.88	17.66	13.73	16.41	11.61	7.22	959.00



The final decision is relied on results of the Additive and KAM methods. Moreover, technical efficacy of standard Additive and KAM models are provided for these data to compare their results by the other models. Table 1 exhibits Additive model parameters which are estimated target scores by using additive model and Table 2 exhibits also KAM model by  $\varepsilon=0.08$ . Table 1 demonstrates target scores of the mentioned cities by Adding interval which determined before for input 1 in additive model, and Table2 demonstrate KAM model target scores of the mentioned cities by adding interval for input 1. Selected interval for input 1 is between 35 and 50, in other word distance to power distribution networks should be between 35 and 50 kilometer. By running Data Envelopment Analysis (DEA) on models, efficiency scores were determined. In addition, DEA models give us target scores too (Table1.additive model target scores for input 1). suggested constraints were added to KAM mathematically and behavior of the other input and output changes are examined and shown in Table2.

**Table2.** KAM,  $\varepsilon=0.08$  model target scores for input 1

		$x_1^*$	$x_2^*$	$x_3^*$	$x_4^*$	$x_5^*$	$x_6^*$	$x_7^*$	$x_8^*$	$x_9^*$	$y_1^*$
1	BOLU	35.00	1.10	3.30	9.58	1.49	1.85	3.83	1.35	1.80	112.00
2	CANAKKALE	35.00	1.26	4.44	9.70	2.13	1.87	4.03	1.41	1.79	132.80
3	ANKARA	35.00	1.56	6.59	9.92	3.34	1.90	4.40	1.51	1.76	172.40
4	MANISA	35.00	1.33	4.90	9.74	2.39	1.88	4.11	1.43	1.78	141.40
5	AFYON	35.00	1.44	5.72	9.83	2.85	1.89	4.25	1.47	1.77	156.40
6	KAYSERI	35.00	1.79	8.26	10.09	4.28	1.93	4.69	1.59	1.75	203.00
7	MALATYA	35.00	2.40	12.66	10.55	6.75	2.00	5.45	1.80	1.70	283.60
8	ICEL	35.00	2.05	10.12	10.29	5.32	1.96	5.01	1.68	1.73	237.00
9	HATAY	35.00	1.33	4.93	9.75	2.40	1.88	4.11	1.43	1.78	141.80
10	KAYSERI	35.00	14.11	2.87	8.14	2.52	3.67	6.26	1.05	1.59	191.00
11	RIZE	35.00	1.09	3.20	9.57	1.43	1.85	3.81	1.35	1.80	110.20
12	ARTVIN	35.00	1.10	3.30	9.58	1.49	1.85	3.83	1.35	1.80	112.00
13	MUGLA	35.00	3.27	18.90	11.20	10.27	2.10	6.53	2.10	1.63	398.07
14	HAKKARI	35.00	20.77	3.61	7.52	3.57	4.60	7.64	0.95	1.47	248.20
15	SANLIURFA	35.00	5.21	3.37	10.78	10.14	1.68	2.78	1.55	1.14	172.40
16	USAK	35.00	17.97	45.41	14.63	10.90	2.08	1.27	0.72	2.81	214.40
17	KASTAMONU	35.00	6.53	3.97	11.20	13.07	1.63	2.57	1.63	0.93	201.13
18	ERZURUM	35.00	1.43	5.62	9.82	2.80	1.89	4.23	1.46	1.77	154.60
19	ELAZIG	35.00	2.34	12.23	10.51	6.52	1.99	5.38	1.78	1.70	275.80
20	AMASYA	35.00	8.00	13.60	14.20	10.60	1.80	1.20	0.40	2.60	130.60
21	GAZIANTEP	35.00	4.63	3.11	10.59	8.83	1.70	2.88	1.51	1.23	159.60
22	TUNCELI	35.00	2.36	12.33	10.52	6.57	1.99	5.39	1.78	1.70	277.60
23	NIGDE	35.00	17.34	3.23	7.84	3.03	4.12	6.93	1.00	1.53	218.80
24	AGRI	35.00	1.14	3.53	9.60	1.62	1.85	3.87	1.36	1.80	116.20
25	KONYA	35.00	1.79	8.27	10.09	4.29	1.93	4.69	1.59	1.75	203.20
26	KARAMAN	35.00	10.19	2.44	8.51	1.90	3.12	5.44	1.11	1.66	157.40
27	DIYARBAKIR	35.00	2.84	15.80	10.88	8.52	2.05	6.00	1.95	1.67	341.20
28	SANLIURFA	35.00	4.84	3.20	10.66	9.30	1.69	2.84	1.52	1.20	164.20
29	ANTALYA	35.00	2.04	10.06	10.28	5.29	1.96	5.00	1.68	1.73	236.00
30	ADANA	35.00	1.71	7.65	10.03	3.94	1.92	4.58	1.56	1.75	191.80

#### 4. CONCLUSION

In selecting the location of solar plants, we relied on Additive and KAM models results. For these models nine factors as inputs and one factor as output were used. These factors have discussed by Sozen and Mirzapour (under review) as important parameters in selecting the location of solar plants. By Table 1 and Table 2 it is seen that there are different results

between Additive and KAM models by adding interval on our models. Namely, there are high dependence between results of the Additive model and, KAM models.

sometimes the target scores are not in the intervals which are determined before, so obtained results will not be realistic. If desired Constraints add in DEA normal models, ideal results cannot be obtained. For solving the problem, Kourosh and Arash Method (KAM) has been utilized. For creation of new model, the properties of KAM have been considered. In order to reach the thesis goal, suggested constraints were added to KAM mathematically and behavior of the other input and output changes are examined. For the analysis of large scale problems KAM model was developed in EXCEL Solver software. Results are compared with additive model and shows that KAM model gives better results. According to authors in future works it could be adding more constraints and different epsilon quantity and solve models.

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